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Journal of Computer Assisted Learning

Procedural versus content-related hints for word problem solving: an exploratory study

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Abstract

For primary school students, mathematical word problems are often more difficult to solve than straightforward number problems. Word problems require reading and analysis skills, and in order to explain their situational contexts, the proper mathematical knowledge and number operations have to be selected. To improve students' ability in solving word problems, the problem solving process could be supported by procedural and content specific guidance or with only procedural support. This paper evaluates the effect of two types of hints, procedural only and content-procedural, provided by a computer programme presented in two versions. Students of grade 6 were randomly assigned to these two versions, which offered five lesson units consisting of eight word problems each. The results indicate that on average the students in the procedural-content hints group ($n=54$) finished about just as many problems in the programme as their counterparts in the procedural-only condition ($n=51$). However, the participants in the first group solved more problems correctly and improved their problem-solving skills more as indicated by the scores on the problem-solving post-test. Apart from presenting our analysis of the findings of this study, also its limitations and its possible implications for future research are discussed in this paper.

Keywords

learning analytics, MOOC, network analysis, small group, social sensors, virtual learning environments.

Introduction

Word problem solving plays an important role in mathematics education (Martin & Mullis, 2013; Timmermans, Van Lieshout, & Verhoeven, 2007; Verschaffel, Greer, & De Corte, 2007). Word problems are mathematical assignments embedded in a textual problem which has to be solved using mathematical knowledge and procedures (Foshay & Kirkley, 2003; Verschaffel, Greer, & De Corte, 2000). The complexity of word problems depends on the proportion of relevant and irrelevant textual and

numerical information, the way in which it is presented, and the type of mathematical operations involved. These factors influence the degree of difficulty as experienced by students in providing a correct representation of the problem situation (Boonen, Van Wesel, Jolles, & Van der Schoot, 2014; Verschaffel et al., 2000). In primary education, word problems are widely used in mathematical textbooks and (inter)national mathematics tests (Martin & Mullis, 2013; Van der Schoot, 2008). Many students have difficulty in solving word problems successfully (e.g., Schoenfeld, 2002; Swanson, Orosco, & Lussier, 2014).

Based on research with successful problem solvers, Schoenfeld (1992) presented the following episodes in the problem solving process: reading the word problem (Read), analysing what the problem is about (Analyse), exploring the mathematical knowledge required to solve

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the problem (Explore), planning how to use this mathematical knowledge (Plan), implementing a solution plan (Implement) and finally verifying the correctness of the answer (Verify). With complex word problems, which require a combination of number operations, unsuccessful problem solvers spend most of their time on implementing a plan (which involves doing calculations based on the numbers and operators suggested in the text). What is first required however, is properly analysing and exploring the problem situation described (Dreyfus & Eisenberg, 1996; Goos, 2002; Schoenfeld, 1992). Those who have difficulty in solving word problems often do not know how to use their reading skills and mathematical knowledge to analyse and explore the problem situation adequately. As a result, they do not succeed in formulating a correct solution plan (Cooper & Harries, 2005; Verschaffel et al., 1999).

To overcome the difficulties in word problem solving, computer programmes have been deployed which offer instructional support and feedback tailored to the needs of the individual student (Aleven & Koedinger, 2000; Li & Ma, 2010). Examples of computer programmes based on the episodes of Schoenfeld (1992) can be found in various studies (e.g., Kramarski & Friedman, 2014; Teong, 2003). In these studies, the episodes form a framework which students can use to solve the problem items systematically. In combination with the episodes, the computer programmes provide cues and hints to help the students in developing a structured approach. The aim of the episode Read/Analyse is to help students visualize the problem situation (Boonen et al., 2014; Swanson et al., 2014), while the episodes Explore and Plan are focussed on assistance in finding a solution that fits the problem situation (Fuchs et al., 2010; Hegarty & Kozhevnikov, 1999; Mayer, 1992).

Computer programmes based on Schoenfeld's episodes vary in the type of hints they provide. Some programmes offer hints containing general, procedural advice about how to use the episodes. Here the aim is to teach the student how to develop a systematic approach. There are also hints which provide problem-specific help with each episode. These hints give precise directions on how to read, analyse and explore the item and how to plan its final solution. In this study, the first type of hints is called *procedural-only hints* and the second type *procedural-content hints*.

Procedural-only hints

In explaining his discovery of the problem-solving episodes when studying novice and expert problem solvers at school, Schoenfeld (1992) concludes that the first group spends relatively little time on the first episodes (Read/Analyse, Explore and Plan). Novices read a problem only superficially and mainly focus on the obvious numbers and operators in the problem as the basis for their calculations (they implement the first plan that comes to their mind). While experts spend relatively much time on the first episodes, they also pay ample attention to the problem situation and then carefully construct a solution plan that fits the problem context. Schoenfeld argues that novice problem solvers often do not lack the mathematical knowledge to solve a problem, but that they do not have the ability to approach the problem situation systematically. They already fail at the first step: reading/analysing and exploring the problem. Schoenfeld taught students how to improve their problem-solving skills by asking them questions about their use of the episodes. This approach stimulates students' metacognitive skills by making them think about their actions in solving the problem in a systematic way (Jacobse, 2012; Schoenfeld, 2013). Several computer programmes offer procedural hints to teach students how to follow the episodes and think systematically during the word problem solving process. These procedural hints point students to the various strategies which they can use in the episodes (e.g., in the episode Read/Analyse: underline the key elements in the problem text or make a schema of the problem situation; Hill & Hannafin, 2001). A better knowledge of the use of the episodes and strategies is expected to improve students' problem-solving abilities.

Examples of two studies in which procedural support is offered to teach students the use of the episodes are Kramarski and Friedman (2014) and Teong (2003). Kramarski and Friedman (2014) used the IMPROVE approach, in which hints are offered containing procedural support aimed at fostering the students' self-questioning skills. IMPROVE is an acronym for Introducing new concepts, Metacognitive questioning, Practising, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment (Mevarech & Kramarski, 1997). It provides a step by step method. These steps have been inspired by Schoenfeld's episodes and teaching approach.

The hints contain questions which provoke students to use adequate strategies in each of the IMPROVE steps.

Also Teong (2003) used the different episodes of the problem solving process as proposed by Schoenfeld (1992) in creating a systematic approach for students. Teong designed the CRIME method. The acronym stands for a problem solving framework consisting of the following stages: Careful reading, Recalling possible strategies, Implementing possible strategies, Monitoring, and Evaluation. The stages defined by this method include the availability of hints. These hints are aimed at teaching students to think critically throughout the problem-solving process about how to proceed, and ultimately how to solve the problem item correctly.

The results of both studies indicate that students who used the programme containing procedural hints better improved their problem-solving skills than students who used the programme without hints. However, offering procedural hints is not always a guarantee for successfully teaching students how to solve word problems. Students may also need content-related help (e.g., if they fail to select the relevant numbers and operations to solve the problem) (e.g., Bulu & Pedersen, 2010; Lee, Lim, & Grabowski, 2010).

Procedural-content hints

Solving word problems with the aid of episodes also involves cognitive tasks. To analyse the problem situation, students have to connect it to their own knowledge of the world. Furthermore, to find a fitting solution plan, they have to relate the problem's numerical information to their mathematical knowledge (e.g., Jacobse & Harskamp, 2012). The cognitive activities involved here are content specific and different for each problem. If an item is complex or if the problem situation described differs from that presented in other word problems, content-related hints may be helpful. Content-related hints aim to bridge the gap between what is known by the student and the knowledge required to solve the problem (Liu, Bera, Corliss, Svinicki, & Beth, 2004). Content-related (or domain) knowledge is an important factor in successful problem solving (Murphy & Alexander, 2002). According to Mayer (1992), content-related support can be effective in assisting students in both problem representation and problem solution. For example, a content-related hint which presents a schema containing numbers and key words has been considered to increase

students' understanding of the episode Read/Analyse. In a similar vein, a content-related hint which offers suggestions as to which number operations to execute helps in carrying out the episodes Explore and Plan.

One may conclude that in order to enhance students' problem-solving skills to a higher level, a combination of procedural and content-related hints is advisable. Content-related hints provide domain-specific information on applying the episodes and strategies. This information bridges the gap between the student's current level of understanding and the insight required for solving the problem (Crippen & Earl, 2007).

Chang, Sung, and Lin (2006) observed students' use of content-related hints in the first four episodes of problem solving. The students in their study were first taught to make a schematic representation of the problem (Read/Analyse). Then, additional support was given. The students could fill in the operands and operator(s) of a problem in a second schema while hints were provided on how to plan the solution to the problem (Explore and Plan). Finally, the students calculated the answer, which was checked by the computer programme (Implement and Verify). Afterwards, the students could review their solution by watching a demonstration of the problem-solving process. In this study, the students' problem-solving skills improved considerably compared to those of a control group who worked with a similar programme without hints.

In a research literature review, Powell (2011) acknowledged that teaching students how to use schematic diagrams can help improve their understanding of word problems. In the approach described in the review, students first had to analyse the problem's key elements and then incorporate them into a schematic diagram. Powell (2011) concludes that if students can transform a word problem into a correct visual representation (diagram), they can generally also take a step further by constructing a solution plan and finding the correct answer (see also Boonen et al., 2014). Powell suggested that especially for students with learning difficulties it might be profitable to learn how to recognize types of problems as defined by their number operations (for instance: addition, subtraction, multiplication, division or combination problems). If students learn to draft the different word problem types and incorporate their key numbers and operators in a schema, they are already half way in formulating a correct solution plan. As one of the founders of this view, Jitendra et al. (2013) conducted considerable

research on this topic. For instance, Jitendra et al. (2013) taught grade three students with mathematical difficulties to read and analyse addition and subtraction types of word problems and transform them into visual representations. They then compared the students who followed the experimental programme to those who worked with a programme without instruction in word problem types. They found that in two of the three tests, the experimental group improved its problem-solving skills more than the control group.

Jacobse (2012) and De Kock and Harskamp (2014) took a different view on the use of visual representations in word problem solving. They developed a computer programme based on Schoenfeld's episodes, containing procedural hints in combination with content-related hints. The procedural part mostly included questions or suggestions about which strategy to use while the content-related element offered advice on how to apply the desirable strategy to the problem item. The content-related hints had the form of a visual representation. The episodes and hints were presented to the students in the following framework of statements: 'I read carefully', 'I know which numbers and operations to use', 'I make a plan' and 'I check my answer'. With each episode there was a hint which could be addressed. The procedural component included a suggestion or question. The content-related information consisted of a visual schema of the problem situation (further worked out in each of the episodes) and a brief textual explanation on how to read, analyse, explore or plan the problem situation depicted. Usage of hints was free for the students, and the studies' results show that students used hints in 50% of the problems. The students working with the hints programme outperformed the students who did not use a programme or used the programme without hints.

Another example of a combination of procedural and content-related hints in a computer programme is presented by Meijer and Riemersma (2002). When solving a problem, students were first given a procedural hint which instructed them on how to read and analyse the item, explore the strategies to formulate a plan and come up with a solution. If students were in need of more help they could consult a content-related hint which gave them suggestions plus a half-finished plan to solve the problem. The results show that only the high ability students made good use of the procedural and content-related hints and profited from working with the

computer programme. The other students only used the hints sparingly and were more focussed on finishing the items in time than on solving them correctly. If hints can be used freely, it is often necessary to stimulate students in using them whenever they do not succeed on their own, as well as to provide them with feedback on their results (Aleven & Koedinger, 2000; Baker et al., 2008).

Next to the distinction in type of hints, there are also differences in the ways in which word-problem solving can be taught. One approach is direct instruction (e.g., Fuchs et al., 2009; Jitendra et al., 2013; Teong, 2003). Here, students are instructed to follow the different problem-solving steps in a strict order. Teong (2003) successfully applied this instructional approach in training students how to use the episodes of Schoenfeld. Fuchs et al. (2009) and Jitendra et al. (2013) used direct instruction to train students to first read and analyse the key elements in a problem, and then to identify the type of problem. The students learned, for instance, to distinguish between addition and subtraction or division and multiplication. They were also taught how to draw visual representations of the different problem types, which served as the basis for the solution process. The direct instruction approach is particularly focussed on solving word problems using a step-by-step model. What the above-mentioned studies have shown is that after direct instruction, students' problem solving skills specifically improved for the types of problems trained. There is, however, little evidence of transfer to other types of problems (Mevarech & Kramarski, 2003; Renkl, 2011).

Furthermore, according to Schoenfeld (1992) the direct instruction approach is not designed to be used in the process of problem solving. The episodes should not be used as a step-by-step model which is followed in a fixed order. Instead, students should be invoked to think about their use of the episodes. In a guided instruction approach, therefore, students learn to develop their own ways of solving word problems. Instruction is delivered by merely offering prompts and hints, and presented in a systematic framework of episodes. Students can use this framework as a guideline to estimate what they have accomplished so far. It is argued that the instruction should be particularly directed at the episode in which help is really needed. In this manner, students are stimulated to determine for themselves whether they need assistance or whether they can proceed on their own (Schoenfeld, 2002).

Jacobse and Harskamp (2009); De Kock and Harskamp (2014) and Meijer and Riemersma (2002) used guided instruction to teach higher grades of primary education and lower grades of secondary education the use of Schoenfeld's episodes in combination with procedural-content hints. The students were taught how to use a framework of episodes combined with hints. It was discussed with the students when to address the hints as provided in the computer programme. In addition, feedback on successful hints usage was offered. In this way, the students learned how to work effectively with this method, which improved their problem-solving skills. The guided instruction approach clearly seems to be more suitable for the transfer of trained problem-solving skills to different problem types (Mevarech & Kramarski, 2003). In this study we chose the guided instruction approach.

Research question

Although effective ways of enhancing students' word problem solving proficiency have extensively been researched, it is still unclear which type of hints in computer programmes contributes to the effectiveness (Devolder, Van Braak, & Tondeur, 2012): procedural hints, posing procedural questions (Kramarski & Friedman, 2014) or a combination of procedural and content-related information containing visual schemes and explanations (De Kock & Harskamp, 2014). Both approaches seem promising in improving students' problem solving abilities. The main research question of this study is therefore:

'Is there a difference in effect between the use of procedural-content hints versus the use of procedural-only hints on students' problem-solving skills during a word problem solving programme and afterwards in a post-test?'

Both types of hints have shown to be effective in furthering students' problem-solving skills. But, we expect procedural-content-related hints to have more impact. As, apart from advice on which strategies to use in the episodes, this type of hint also provides problem-specific suggestions on how to apply the strategies.

Hypothesis 1:

Students in the procedural-content condition will complete more problems in the programme correctly than students in the procedural-only condition.

We further suggest that clearer directions during the programme about how to solve the problems and how the episodes contribute to the process enhance students' understanding of in word problem solving and their skills (Jitendra et al., 2015).

Hypothesis 2:

Students in the procedural-content condition will perform better in a word problem solving post-test than their counterparts in the procedural-only condition.

Method

To answer the research question, a quasi-experimental research design was used based on two conditions. In each condition the students were asked to work with a different version of a computer programme. One group of students used a programme containing hints with procedural and content-related information and the other group worked with a programme containing hints with only procedural information. In both conditions the students were free to either seek help or not. This free choice is an important characteristic of guided instruction (De Jong, 2005). Given the non-obligatory nature of the hints usage in both versions of the programme, the results particularly depended on the way in which the students used the hints.

The programme offered 40 word problems divided equally over five lessons. In each problem item, the students had access to hints. Prior to the intervention, they received a pre-flight training in Schoenfeld's episodes and in the use of the hints. Examples were given of cases where students could not proceed without help, and it was explained how the hints could support the solution process. Hereafter, all students participated in a paper and pencil pre-test in which 20 word problems had to be solved in 30 min. After the pre-test, the students in each class were randomly assigned to one of the two conditions. The experiment included five lessons of 30 min in which eight word problems were offered. During the experiment, the students worked individually on the computers outside the classroom under supervision of the researcher. They were escorted in small groups to and from the experimental setting (a room or quiet corner) in the schools. After the five lessons, the post-test was administered in which all students participated.

Participants

The participants in this study were students from four sixth grade classes in the Netherlands, in total 105 students of whom 56 were assigned to the procedural-content condition and 51 to the procedural-only condition. The average ages of the groups were 11.62 years ($SD = .51$) for the procedural-content condition and 11.57 years ($SD = .49$) for the procedural-only condition. In both groups the students had the same age, while the procedural-content group comprised 28 girls and 26 boys and the procedural-only group 27 girls and 24 boys.

Materials

Problem solving tests

Both before and after the experiment, all students had to take a problem solving test. The pre-test included 20 items (see Figure 1). The post-test contained the same 20 items with slight adaptations; for example, the setting of the problem situation was changed whereas the numbers and operations involved stayed the same. The items were similar to those in the Dutch standardized tests for primary school grade 6 (Cito, 2002). CITO is the Dutch institute for educational testing and assessment and the acronym stands for Central Institute for Test Development. For each item solved correctly a score of 1 was awarded. In total the students could obtain 20 points for the pre-test and 20 for the post-test. The reliability scores of the pre- and the post-test as indicated by Cronbach's α

were both .88. Moreover, the tests correlated highly ($r = .83$, $p < .001$), which indicates that they both measured the same characteristic: skill in word-problem solving.

Computer programme

An adapted version of a web-based computer programme (De Kock & Harskamp, 2014) was used. The computer programme contained 40 word problems, which were presented to the students in five lessons (eight problems each lesson). The level of the eight items went from easy to difficult. The difficulty level depended on the amount of irrelevant numerical and textual information included, and the type and number of mathematics operations involved. In each lesson the students started with easy items and finished with the more difficult ones. In this way, they could try to solve as many problems as possible. The need for hints was expected to grow as the problem items became more difficult.

The items included fractions, percentages, the metric system, money and decimals, requiring addition, subtraction, multiplication, division or a combination of these number operations.

The students could log into the computer programme with their username and password. After a successful log-in, they could start with the first item. The duration of each lesson was precisely 20 min, which was indicated by a timer. It started to count down as soon as the first item was started.

<p>Teacher Marit wants to make her classroom nicer.</p> <p>For this reason, she sets out to buy new indoor plants.</p> <p>The plants are € 2,50 a piece at the garden centre.</p> <p>At the cash register the teacher gets a € 6,00 discount</p> <p>After the discount, she pays € 21,50.</p> <p>How many plants has she bought?</p>
<p>During the summer holidays the Fransen family goes to France.</p> <p>Mrs Fransen drives $\frac{1}{3}$ of the distance.</p> <p>Mr Fransen drives the remaining 320 km.</p> <p>How many kilometres was the total distance?</p>

Figure 1 A Sample of Two Word Problems of the Post-Test

Figure 2 Interface of the Computer Programme in Both Conditions

At the left of the computer interface in Figure 2, the word problem is displayed which had to be solved by the students. In the middle of the screen, the framework of episodes and hints is presented. Each hint could be requested once during each word problem. When a solution was found, the student could press the key 'I check my answer', after which the answer could be typed in (screen below at the right). This was followed by feedback, depending on the correctness of the answer. The feedback was presented as a pop-up. In total, students had two attempts to answer a word problem item. After a correct answer or two incorrect answers the solutions provided could be compared to a model answer. This

answer was presented in the same screen as where the hints were displayed.

Two versions had been developed for the programme. In both of them, the word problems and the computer programme interface were similar (see Figure 2), except for the information provided by the hints (see Figure 3). In both conditions, hints were offered to support the problem solving episodes (Schoenfeld, 1992). 'Read and analyse' was supported by the hint 'I read carefully'. In the procedural-content condition, however, this hint consisted of a visual scheme to clarify the problem situation, while procedural support was given in the form of a question (e.g., Formulate in your own words what the

Figure 3 Hints Provided in the Episodes of Problem Solving in the Two Experimental Conditions

question is you have to answer, or: Which numbers and words do you need to understand the problem?). Next, if it was not clear to the student which mathematical knowledge had to be used to solve the problem, the hint 'I know the type of sum' could be addressed. This hint refers to the Explore episode. In the procedural-content condition, the hint consisted of a scheme depicting the mathematical relationships in the problem situation. The procedural suggestion was to make a sketch of the situation and note down the number operations required to solve the problem. Finally, there was the hint 'I make a plan'. Here the students in the procedural-content condition were provided with a half-finished plan (a table with steps showing how to calculate the solution). In the procedural condition, a procedural hint was offered in the form of a question: Which numbers have to be used, and how do you go about solving the problem?

Hints usage was not compulsory, while the students were stimulated to think critically about which hints they really needed to facilitate a successful problem solving process.

Procedure

The experiment started with a problem solving pre-test. After randomly assigning the students to the two conditions (procedural and procedural-content), they were granted 30 min to complete this paper and pencil examination.

The whole intervention entailed five lessons during which the students worked with the computer programmes. Under supervision of the researcher, the students were divided into small groups outside the classroom. For all students a computer was reserved so that they could work individually. In each lesson eight word problems were offered. The students were asked to complete as many problems as they could in 20 min. Before each lesson, 5 min were scheduled for the necessary organizational aspects of the intervention. Before the first lesson, a pre-flight training was given of 40 min. During this training, the use of hints was practised and the

students were asked to predict whether or not they would need them. After that, the students were trained in the systematic use of the hints. After the training, the students worked with the programme during 3 weeks. All students participated in the problemsolving post-test. By comparing the results of both tests the effect of the training could be determined.

Data analysis

From the log files, descriptive statistics were gathered about student usage of the computer programme. Means and standard deviations were calculated to clarify how many word problems were finished by students, the result of the finished word problems, and if they used help in solving these problems. Small differences of the pre-test problem solving were taken into account when comparing the students' success in hints use and their post-test results between the two conditions.

Results

Comparability of the conditions before the experiment

Within class, the students were randomly assigned to the two conditions. As a check for randomization, a word problem solving pre-test was administered at the beginning of the experiment (see Table 1).

Table 1 presents the mean scores of the students in both conditions. It seems that the students in the procedural-only condition performed slightly better in the pretest, although an independent *t*-test indicated that the differences between the two groups were not significant ($t(103) = .685$, $p = .410$, $d = .16$).

Implementation of the research conditions

The intervention consisted of five lessons, in which all students were able to participate, except one pupil in the procedural-content condition who missed two sessions. The average time spent in each lesson on working

Table 1. Students' Pre-Test Scores: Means and Standard Deviations

	Procedural-content condition ($n = 54$)		Procedural-only condition ($n = 51$)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problem solving pre-test (0–20)	9.61	5.15	10.49	5.73

Table 2. Number of Word Problems Finished and Solved: Means and Standard Deviations

	Procedural-content condition (<i>n</i> = 54)	Procedural-only condition (<i>n</i> = 51)
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)
Word problems finished (0–40)	38.22 (3.27)	38.52 (3.94)
Correctly solved (0–40)	27.72 (7.71)	25.82 (8.58)
Correctly solved (0–40)*	28.18 (5.31)	25.33 (5.31)

*= corrected for the scores of the problem solving pre-test.

in the computer programme was 14.48 (*SD* = 1.98) min in the procedural-content condition and 14.02 (*SD* = 2.11) min in the procedural-only condition. Table 2 shows an overview of the use of the computer programme for both conditions.

Table 2 demonstrates that the mean number of word problems finished was almost equal for both conditions. These means are close to the maximum number (40) of word problems offered. The students tried to solve almost all of the problems but did not always succeed. In both conditions they finished on average around 38 of the 40 problems in the programme. But was there a difference between the two conditions in terms of solving the problems correctly? On average, the procedural-content group seemed to solve more problem items correctly than the procedural-only group. An ANCOVA with ‘pre-test’ as covariate, ‘condition’ as independent variable, and ‘number of items solved’ correctly as dependent variable pointed out that the procedural-content group had managed to solve more problems correctly than the procedural-only group ($F(1,102) = 7.521$, $p = .007$).

Effectivity of the hints

While working on the problems, the students could request hints from the programme, which were provided in the following problem-solving episodes: Read/analyse (I read carefully), Explore (I know the type of sum) and Plan (I make a plan). The hint information differed

between the two conditions. This difference is indicated in Figure 3. But was there also a difference in hints use between the two groups? In the procedural-content condition, the hint ‘I read carefully’ was requested in on average 9.00 of the problems (*SD* = 9.68). It was consulted in on average 7.49 of the items (*SD* = 10.34) in the procedural-only condition. The second hint, ‘I know the type of sum’, was requested on average 8.65 (*SD* = 9.59) times in the procedural-content condition and 6.94 (*SD* = 10.17) times in the procedural-only condition. Finally, the third hint, ‘I make a plan’, was used by the procedural-content group 8.48 (*SD* = 9.57) times and by the procedural-only group 7.12 (*SD* = 10.16) times. There was no significant difference between the two conditions in average number of times the different hints were used. Moreover, the use of the three hints was strongly interrelated, with correlations ranging between $r = .91$ and $r = .99$ in both conditions. The results show that the students in both conditions frequently employed all three hints when they decided to use help.

As the students in the procedural-content condition did finish more problems correctly, it would be interesting to establish if this achievement was related to their use of hints. Table 3 shows how successfully the hints were used by the students in both conditions.

Table 3 indicates that the students used (one or more) hints with some of the problem items. In the procedural-only condition, they used hints with on average 8.06 of the problems and in the procedural-content condition with 10.43 of the problems. The difference is not

Table 3. Usage of Hints and Problems Solved: Means and Standard Deviations

	Procedural-content condition (<i>n</i> = 54)	Procedural-only condition (<i>n</i> = 51)
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)
Hints used (with 0–40 problems)	10.43 (9.61)	8.06 (10.24)
Correctly solved with hints (0–40)	6.24 (6.60)	4.04 (6.08)
Correctly solved with hints (0–40)*	6.19 (6.36)	4.09 (7.01)

*= corrected for the scores of the pre-test problem solving.

significant. In order to find out if students in the procedural-content condition were more successful in solving the problems with the help of hints, an ANCOVA was performed with the 'problem-solving pre-test' as covariate, 'condition' as independent variable and 'number of word problems solved correctly using hints' as dependent variable. The analysis revealed no significant interaction effect for 'condition' and 'pre-test' on the number of problems solved with the aid of hints ($F(1,101) = .208, p = .650$). However, contrary to our expectations, there was no main effect of 'condition' ($F(1,102) = 2.840, p = .095$), although an implicit tendency towards a positive effect of the usage of hints in the procedural-content condition may have been established (effect size $d = 0.32$). The students in both conditions used hints just as frequently, but the procedural-content group tended to be more successful in their hints use.

Difference between the two research conditions in the students' posttest performance

It was expected that the procedural-content group would perform better in the post-test than the procedural-only students (see Table 4).

In sum, the mean score of the procedural-content group seems higher than that of the procedural-only group. In the analysis of the post-test results, the small difference between the two conditions in the pre-test was taken into account. We conducted an ANCOVA with 'pre-test scores' as covariate, 'condition' as independent variable and 'post-test scores' as dependent variable. Table 4 includes the corrected means of the post-test scores for both conditions.

The ANCOVA reveals a significant main effect for the procedural-content condition ($F(1,102) = 7.135; p = .009$). The effect is of an intermediate size ($d = .53$). There is no interaction effect of 'pre-test' and 'condition' on the post-test results ($F(1,101) = .029; p = .865$). The results indicate that the procedural-content condition led to a

higher transfer of the problem-solving skills than the procedural-only condition.

Conclusion

For improving students' skills in solving mathematical word problems, experts have advised the use of the episodes Read/Analyse, Explore, Plan, Implement and Verify as proposed by Schoenfeld (1992). One form of support in this approach is the provision of hints in each of these episodes. The hints can be provided by a computer programme which also offers individual help and feedback. There are two types of hints. The first includes procedural advice on how to proceed from one episode to another. The second type contains both procedural information and content-related directions on how to proceed in order to solve the specific problem item successfully (procedural-content hints.).

Different studies have been conducted to examine the effectiveness of each type of hints during the problem-solving process. The results show that both types may improve students' skills in solving mathematical word problems (Ritter, Anderson, Koedinger, & Corbett, 2007; Rittle-Johnson & Koedinger, 2005). However, there is as yet only little research answering the question which type (procedural-only or procedural-content hints) is more beneficial for students. And also little is known about how hints influence students' behaviour during problem solving. This study has been aimed at investigating whether there is any difference in effect between procedural-content hints and procedural-only hints in students' problem solving achievements. To this end, two versions of a computer programme were designed containing five lessons of eight problems each. In one version procedural hints were provided and in the other procedural-content hints. It was observed that the students of both conditions used hints in approximately 25% of the word problems. This percentage

Table 4. Students' Post-Test Scores: Means and Standard Deviations

	Procedural-content condition ($n = 54$)		Procedural-only condition ($n = 51$)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Problem-solving post-test (0–20)	13.57	4.92	12.80	5.41
Problem-solving post-test (0–20) *	13.92	2.81	12.44	2.82

* = corrected for the problem-solving pre-test scores.

could be expected, because also easy items were included. If the items are all complex, a hints use of about 50% of the problems can be expected. Students may not always address hints when they need them (Aleven & Koedinger, 2000; Baker, Corbett, & Koedinger, 2004). In our study, the hints use may not have been optimal because it is usually perceived as decreasing the speed of the problem-solving process. The students may have only addressed them when they had no idea how to go about solving the problem. In our study, if hints were addressed they were mostly used in all three episodes. Other studies also show that students do not always use hints as frequently as they should, but once they decide to do so, they use all of them (Baker et al., 2008).

We studied students' success with hints and observed a trend in favour of the procedural-content condition. Here the students solved more problems correctly after the use of hints than the students in the procedural-only condition. These results were in line with our expectations: procedural-content hints provide more specific information on how to solve a problem item than procedural hints. Procedural-content hints offer visual schemas of the problem situation during the Read/Analyse episode while they clarify the problem's mathematical relationships in the Explore episode. For the Plan episode they provide a half-completed solution plan which the student has to finish. The procedural-only hints, on the other hand, merely provide general suggestions and directions as regards the use of episodes, but offer no content-related help specifically tailored to the problem item at hand. Both conditions however, provide feedback on the correctness of the answers.

Owing to the specific nature of the hint information, the procedural-content condition may be considered as more effective in helping students improve their problem-solving skills. Nevertheless, in the computer programme students with the procedural-content hints were hardly more successful than the students in the procedural only hints (effect size $d=0.32$). Perhaps, the amount of practice with the hints (five lessons of eight problems with an average of 25% of hints use) was too small to show the extra effect of procedural-content hints. However, the students in the procedural-content group seemed to have learned more about problem solving. As expected, they managed to solve more word problems correctly in the post-test.

Discussion and recommendations

A limitation of this study is that we found a positive effect, but that we have not been able to uncover the precise factors which improved the students' problem-solving skills after usage of the procedural-content hints. In this research, the effectiveness of hints was determined by the number of correctly solved word problem items in the computer programme, and the results of the post-test. In order for our research to be given a more solid empirical basis, we argue that it has to be replicated in other studies (Makel & Plucker, 2014). Furthermore, we are interested in finding more information on *how* procedural-content hints can be used more effectively than procedural-only hints and improve students' problem-solving skills more. One way of increasing our understanding could be by means of think-aloud procedures.

Next, the added value of the different types of hints could be investigated for students with different levels of prior knowledge. In this study, we found no indication for such an interaction-effect, but it may still be found with the aid of the think-aloud approach. Perhaps, for instance, procedural-content hints are particularly profitable for low ability problem solvers. Moreover, more information could be gathered by asking students to make visualizations of the problem situations, without and with the use of content-related hints. In this way, it could be determined how the hints are beneficial for students' abilities in the construction of schemas, which is believed to be a crucial part of the problem-solving process (e.g., Powell, 2011).

With respect to hints use improvement, another interesting topic of research is the reason why students decide to address hints. Gaining more insight into students' motivations for addressing hints could be obtained by adopting the think-aloud method. For example, after reading a word problem, students can be asked to indicate if they believe they can solve it on their own or if they need help. If they think they need help, the students can be asked which kind of help they would prefer: concrete suggestions, procedural help, examples of other students with a similar problem, etc. Knowledge of why students would choose help would be valuable for further improving the hints-based computer programmes, so that students could be supported even better in enhancing their skills in solving mathematical word problems.

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