University of Groningen

Optimal policies for the berth allocation problem under stochastic nature

Ursavas, Evrim; Zhu, Xiang

Published in:
European Journal of Operational Research

DOI:
[10.1016/j.ejor.2016.04.029](https://doi.org/10.1016/j.ejor.2016.04.029)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2016

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Ursavas, E., & Zhu, X. (2016). Optimal policies for the berth allocation problem under stochastic nature. *European Journal of Operational Research*, 255(2), 380-387. <https://doi.org/10.1016/j.ejor.2016.04.029>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Production, Manufacturing and Logistics

Optimal policies for the berth allocation problem under stochastic nature

Evrin Ursavas, Stuart X. Zhu*

Department of Operations, Faculty of Economics and Business, University of Groningen, Groningen, The Netherlands

ARTICLE INFO

Article history:

Received 29 May 2015

Accepted 15 April 2016

Available online 22 April 2016

Keywords:

Pricing

Inventory

Return and expediting

Stochastic dynamic programming

Optimization

ABSTRACT

Key purpose of container terminals is to serve container vessels. Container vessels may be of different types such as large deep-sea vessels or feeders and barges. Container terminal operators have to deploy intelligent strategies for the allocation of their limited resources to the calling vessels of those different types. The presence of uncertainty in the real processing of the operational schedules and arrival of vessels adds to the complexity of the already multifaceted problem. An inefficient decision in the berth allocation phase affects all the other applications connected to this and may increase the service period and costs. In this study, we propose a framework based on stochastic dynamic programming approach to model the berth allocation problem and characterize optimal policies under stochastic arrival and handling times for different types of calling vessels. We find that the optimal control policy is of threshold type depending on the number of vessels in a certain berth group. The derived policies can be used at container terminals for the optimal use of their berthing facilities.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Due to the trend towards sea transportation, efficient port management has become a major issue for the port owners and shipping companies. Typical operations in a port consist of allocation of arriving vessels to berths, allocation of quay cranes to docked vessels at the quayside, routing of internal transportation vehicles, storage space assignment, and gantry crane deployment at the yard side. The allocation of arriving vessels to berths is referred to as the Berth Allocation Problem (BAP). Berth management drives the port management process and the major challenge for this process is to determine the optimal allocation strategy to the calling vessels. An important fact for the terminal operators to consider is the different types of calling vessels. Container terminals not only serve deep-sea vessels but also vessels, such as feeders and barges. Indeed, considering the intention of terminals to stay competitive in the market, these vessels of different types cannot be treated equally. Together with the revenue considerations and navigational reasons, deep sea vessels receive higher attention from the container terminals. Due to limited capacity, it is also possible that some vessel calls may need to be forwarded to other ports/terminals based on collaboration policies among ports. Therefore, the decision maker needs to employ intelligent strategies when allocating their scarce resources to those different

types of vessels. Previous literature has also underlined the importance of employing flexible vessel-berth-order allocation rather than the First-Come-First-Served (FCFS) rule for higher productivity (Imai, Nishimura, & Papadimitriou, 2008).

The fact of serving different types of vessels is not the only aspect that makes the berth allocation problem a complex one. The presence of uncertainty in the real processing of the operational schedules further adds to the complexity of the problem. For instance, in practice, the actual arrival times of vessels are highly uncertain: only half of the vessels arrive on time (Consultants, 2008). Indeed, the solutions of the deterministic berth allocation problem can only be applied in an ideal plant, where cranes are perfectly reliable, container information is accurate, vessel arrival times are precisely known, task handling times are constant and weather is fairly predictable. This of course is far from reality and, thus, the deterministic problem has to be extended to a stochastic version. Mentioned uncertainties will result in the change of the initial plans and both the port owners and the shipping companies will burden extra costs. Therefore it is essential for decision makers to consider the impact of uncertainties to some extent and develop promising methods while making plans.

The berth allocation problem, as being one of the most crucial issues in container terminals, has received a lot of attention from researchers. Approaching the problem considering the practical aspects is important as the decisions given at the berth allocation stage has a direct impact on the following operations within the container terminal. Most literature assumes a deterministic setting

* Corresponding author. Tel: + 31 503638960; fax: + 31 503632032.

E-mail addresses: e.ursavas@rug.nl (E. Ursavas), s.x.zhu@rug.nl (S.X. Zhu).

where information as to the vessel arrivals and handling times are precisely available. Previous work reflecting the stochastic nature has not yet provided analytical solutions and the structure of optimal policies for the berth allocation problem. In this work, we aim to fill this gap in the literature by focusing on the berth allocation problem incorporating the uncertainty in vessel arrival times and handling times. The setting in practice where different types of customers need to be served are together reflected in the novel approach. Our contributions are to propose a framework based on stochastic dynamic programming approach to model the berth allocation problem and characterize optimal policies. For a base model, we find that the optimal control policy is of threshold type depending on the number of vessels in a certain berth group. If the number of vessels in a berth group is below the threshold, then the vessel should be allocated to this berth group; otherwise, it should be allocated to the other berth group. Further, we find that a similar policy can be applied to more general situations, including possibility of forwarding vessels calls, multiple berths at each berth group, and different priorities of vessels. We may conjecture that such a threshold policy is robust.

The remainder of this paper is organized as follows: Next section will put forward the related literature. Following, in [Section 3](#), the model will be proposed. Optimal policy structures will be presented in [Section 4](#). Experiments based on a case at the Port of Izmir will be provided in [Section 5](#). [Section 6](#) will put forward the extensions to the model so as to effectively reflect the different settings in a container terminal. Finally, the last section will be devoted to conclusions and future work.

2. Literature review

Berth allocation problem deals with the allocation of arriving vessels to berths. Without proper berth allocation, it is impossible to activate the subsequent activities competently. Researchers in the literature have brought different approaches to the solution of the problem. In line with its importance, significant amount of research has focused on container terminal operations ([Carlo, Vis, & Roodbergen, 2015](#); [Stahlbock & Voß, 2008](#); [Steenken, Voß, & Stahlbock, 2004](#); [Theofanis, Boile, & Golias, 2009](#); [Vis & De Koster, 2003](#)). Unfortunately, majority of the studies assume a deterministic structure and published works reflecting the stochastic nature are relatively very few. In this review, those studies that capture the stochastic nature of the berth allocation problem will be put forward. Next, studies that reflect the need for differentiating among different vessel types will be summarized.

For the stochastic berth allocation problem, ([Moorthy & Teo, 2006](#)) developed a framework to create robust berth allocations. The problem is modeled as a bi-criteria optimization problem and solved using simulated annealing algorithm. As stated by the authors, the template is relevant only when a substantial number of vessels arrive periodically and within the same period. [Du, Xu, and Chen \(2010\)](#) proposed a feedback procedure for the robust berth allocation problem with stochastic vessel delays similarly using the simulated annealing algorithm. To cope with the uncertainties on vessel delays, some delay scenarios are used, with the assumption that all delay scenarios happen with identical probabilities. [Golias, Boile, and Theofanis \(2007\)](#) proposed a sole conceptual formulation and four solution approaches for the berth allocation problem: Markov chain Monte Carlo based heuristic, stochastic online scheduling based heuristic, deterministic based heuristic and genetic algorithm based approach. [Han, Lu, and Xi \(2010\)](#) implemented a simulation based genetic algorithm approach to solve the integrated berth and quay crane scheduling problem with uncertainty in vessel arrival and operation times. The aim in their study is to minimize the sum of expected value and standard deviation of the service time and the weighted tar-

diness of the vessels. [Hendriks, Laumanns, Lefebvre, and Udding \(2010\)](#) study the robust berth planning problem by considering arrival windows rather than expected arrival times. They propose a mixed integer linear program to find a robust berth plan that minimizes the crane reservation and show that handling time of vessels is highly dependent on the actual arrival time of vessels. [Zhen, Lee, and Chew \(2011\)](#) approached the berth allocation problem under uncertainty by formulating several scenarios and aiming at minimizing the total cost of baseline schedule and expected cost of recourse. [Xu, Chen, and Quan \(2012\)](#) allocated constant buffer times to all vessels to produce robust berth allocation plans. The problem is solved by a scheduling algorithm that integrates simulated annealing and branch-and-bound algorithms. [Zhen and Chang \(2012\)](#) defined robustness as weighted sum of the free slack times in the berthing schedule. A bi-objective model is proposed that minimizes cost and maximizes robustness where weights are determined according to the vessel priorities. [Karafa, Golias, Ivey, Saharidis, and Leonardos \(2013\)](#) formulated the berth allocation problem with stochastic handling times as a bi-objective problem. They used an evolutionary-algorithm based heuristic and a simulation-based Pareto pruning algorithm to solve the problem. However, the arrival times are assumed as deterministic in their model. Regarding studies employing queuing theory we may refer to [Legato and Mazza \(2001\)](#) and [Canonaco, Legato, Mazza, and Musmanno \(2008\)](#). The data used in the studies are based on Gioia Tauro Container Terminal in Italy. First, a queuing network model is proposed and described for the berth allocation problem. Due to the complexity of the problem using an analytical approach to the solution is discouraged and discrete-event simulation model is performed. [Golias, Portal, Konur, Kaisar, and Kolomvos \(2014\)](#) studied a berth scheduling problem with uncertain vessel arrival and handling times. With the objective of minimizing the range of the total service time between the worst and the best performances, the authors formulated a bi-objective optimization problem to obtain a robust schedule and also designed a heuristic to compute the schedule. [Zhen \(2015\)](#) considered the berth allocation problem with uncertain dwell times of ships. The author proposed a stochastic programming formulation with an objective of minimizing the deviations of ship's scheduled berthing time from their expected ones. Further, the author designed some meta-heuristics to solve the models. However, the previous literature has not analytically characterized the structure of optimal policy for the berth allocation problem. Our paper focuses on the development of such policies through optimal controlling of queuing systems. Under the assumption of exponential queueing system, ([Lippman, 1975](#)) first developed a uniformization technique that can formulate the problem of controlling a multiple-server queue into a finite-horizon stochastic dynamic programming. Based on such a technique, [Hordijk and Koole \(1992\)](#) studied the assignment problem to multiple single-server queues in parallel. The authors find out the conditions under which it is cost-efficient to assign customers to a faster server with a shorter queue. [Koole \(1998\)](#) characterized the structure of optimal policies of resource-sharing queueing systems. In the healthcare sector, [Chao, Liu, and Zheng \(2003\)](#) derived an optimal static resource allocation rule for a multiple-single-server queueing system by allowing customer switching. Different from the above papers, we consider the dynamic optimal policies of various scenarios with multiple-server queues in berth groups.

We will continue our literature review with studies that recognize the presence of applying different strategies for different vessel classes within the berth allocation problem. A genetic algorithm based heuristic to solve the formulated nonlinear problem by defining priorities through assigning weights to the vessels was proposed by [Imai, Nishimura, and Papadimitriou \(2003\)](#). In [Guan and Cheung \(2004\)](#) a weight coefficient for each vessel is defined

and they optimized the total weighted flow time using a method that combines tree procedure and a heuristic. Cordeau, Laporte, Legato, and Moccia (2005) considers a multi-depot vehicle routing problem with time windows where they aimed for the minimization of the total weighted service time of all ships. Imai, Zhang, Nishimura, and Papadimitriou (2007) considered a two-objective berth allocation problem: the minimization of total delay time in ship departure and the minimization of total service time. Hansen, Oğuz, and Mladenović (2008) executed ship dependent premiums and penalty costs for handling priority issues and aimed to minimize total costs made up of waiting, handling, earliness and lateness costs. Imai et al. (2008) focused at a terminal where the berth capacities are very limited. They considered the possibility of allocating some ships to another terminal. Golias, Boile, and Theofanis (2009) considered the berth allocation problem with different priority groups of vessels and developed a multi-objective combinatorial optimization formulation and a genetic algorithms based heuristic. Saharidis, Golias, Boile, Theofanis, and Ierapetritou (2010) categorized the vessels into two classes being as preferential or non-preferential. They have considered the maximization of preferential customer satisfaction to be more important than the total throughput of ports. A hierarchical optimization framework that distinguishes between two contradictory objectives port managers come across for the berth allocation problem is developed.

Following the studies in literature, this paper will similarly categorize the vessels into classes based on two distinct types of vessel types namely being deep sea vessels and barge or feeders. Consistent with practice, deep sea vessels are treated differently than barges and feeders which requires a model that is able to differentiate between those two major vessel types. This study aims to fill the gap in the berth allocation literature by considering the existing stochasticity on both arrival times and handling times for different vessel classes. More specifically, previous literature related to the berth allocation problem has not yet provided analytical solutions under uncertainty. The key contribution of our study is to explicitly characterize the structure of the optimal strategies that can be applied to the setting in container terminals. Several settings such as forwarding policy based on collaboration among terminals, vessel differentiation, different berth handling rates that may be present in a container terminal are considered. Optimal policies for the berth allocation problem are proposed for each different setting. The next section will present the associated model formulation.

3. Model description

At a sea port, container terminals having quays or berth groups with different handling rates may exist. This is common especially in large sea port container terminals having enlarged quay spaces. Due to the differences in the handling equipment and proximity to the storage yard the service rates at those terminals may differ. The model proposed in this paper considers the presence of two types of terminals with their own quay spaces. One important example of this setting in practice are ports with indented berth terminals (Beens & Ursavas, 2016). The difference in service rates is eminent in such ports due to the increased ship shore interface.

We model the BAP considering the stochastic nature of the arrival times and the handling times. Handling time of a vessel consists of the required service times for container loading/unloading operations. We assume that the randomness in handling times also reflect the variability in crane assignments. That is, for instance, due to congestion, an idle crane may not be assigned to an arriving vessel immediately.

Based on literature on port operations with real-life case studies, we assume Poisson process for the arrival of vessels and exponential distribution for the handling rates (Imai, Nishimura, &

Papadimitriou, 2001; Karafa et al., 2013; Rodriguez-Molins, Salido, & Barber, 2014). We consider the possible differences in arrival rates for different vessel classes which can be noticed between barges and deep sea vessels. We also recognize the difference in waiting costs for deep sea vessels and barges that is analyzed as an extension in Section 6.3. Upon that, we first model a port, consisting of two types of terminals (berth groups) and use an $M/M/1$ queue to model each type of terminal. Initially, we will assume a single berth at each terminal which later will be extended to a setting with multiple berths at each terminal. The service rate of type- i terminal is denoted as μ_i with an exponential distribution for $i = 1, 2$. Note that the service rates can be different for two types of terminals. Under a multi-user terminal setting, we assume that all the vessels can be handled at each type of terminal. There are two different types of vessels, such as deep sea vessels and barge/feeders. The arrival process of the vessels follows a Poisson process with rate λ_j for type- j vessels for $j = 1, 2$. Each vessel waiting for service in a terminal incurs a waiting cost w per unit per unit time. This in practice is mostly based on the shipping line agreements and terminal's strategies. If one vessel is allocated to type- i berth, there will a handling cost c_i . The objective is to minimize the expected total cost by determining which vessel should be assigned to which terminal.

Denote x_i as the number of vessels allocated to type- i terminal. (x_1, x_2) denotes the state that there are x_1 vessels in terminal 1 and x_2 vessels in terminal 2. Let us define $V_n(x_1, x_2)$ is the minimal total cost function that can be obtained during the last n transitions, starting from state (x_1, x_2) . As stated in Section 11.5.2 of Puterman (1994, p. 563), a discounted continuous-time Markov decision process can be transformed into a discrete-time discounted process. To achieve the transformation, we can define a constant $\Delta = \alpha + \lambda_1 + \lambda_2 + \mu_1 + \mu_2$, where α is the discount factor and $\alpha \geq 0$. The optimality equation can be written as

$$V_{n+1}(x_1, x_2) = \sum_{i=1}^2 \frac{w x_i}{\Delta} + \frac{\lambda_1}{\Delta} \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} + \frac{\lambda_2}{\Delta} \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} + \frac{\mu_1}{\Delta} V_n((x_1 - 1)^+, x_2) + \frac{\mu_2}{\Delta} V_n(x_1, (x_2 - 1)^+). \quad (1)$$

We can assume $\Delta = 1$ by rescaling the values of those parameters. Such an operation will not affect the analysis of the model. Without loss of generality, let $V_0(x_1, x_2) \equiv 0$.

Eq. (1) indicates the relationship between two successive transitions. Based on our model, a transition only occurs when a new vessel arrives or the service of a vessel is completed. The first term represents the total waiting cost of existing vessels. The second and third terms represent the situation when a new vessels arrives, it should be assigned to a certain terminal to achieve a lower cost. The last two terms mean the situation where the service of a vessel is completed. From now on, all the proofs will be put into Appendix.

4. Structural results for the optimal policy

In this section, we start characterizing the structure of the optimal policy for the N -transition problem starting from state (x_1, x_2) in a finite planning horizon for $1 \leq N$. Then, we extend it to the long-run stationary problem in an infinite planning horizon.

Proposition 1. $V_n(x_1, x_2)$ satisfies the following properties:

- (i) $V_n(x_1 + 1, x_2) - V_n(x_1, x_2 + 1)$ is increasing in x_1 and decreasing in x_2 , i.e.,

$$\begin{aligned} V_n(x_1 + 1, x_2) - V_n(x_1, x_2 + 1) \\ \geq V_n(x_1, x_2) - V_n(x_1 - 1, x_2 + 1) \end{aligned} \quad (2)$$

and

$$\begin{aligned} V_n(x_1 + 1, x_2) - V_n(x_1, x_2 + 1) \\ \leq V_n(x_1 + 1, x_2 - 1) - V_n(x_1, x_2); \end{aligned} \quad (3)$$

(ii) $V_n(x_1, x_2)$ is supermodular, i.e.,

$$\begin{aligned} V_n(x_1 + 1, x_2 + 1) - V_n(x_1 + 1, x_2) \\ \geq V_n(x_1, x_2 + 1) - V_n(x_1, x_2); \end{aligned} \quad (4)$$

(iii) $V_n(x_1, x_2)$ is convex in componentwise, i.e.,

$$V_n(x_1 + 1, x_2) - V_n(x_1, x_2) \geq V_n(x_1, x_2) - V_n(x_1 - 1, x_2) \quad (5)$$

and

$$V_n(x_1, x_2 + 1) - V_n(x_1, x_2) \geq V_n(x_1, x_2) - V_n(x_1, x_2 - 1). \quad (6)$$

From Proposition 1, we obtain the following insightful findings:

(i) indicates that when x_1 increases, it is more likely to assign a vessel to the type-2 terminal; when x_2 increases, the statement is reversed; (ii) (the supermodular property) indicates that the cost function $V_n(x_1, x_2)$ has an increasing cost difference with respect to the number of vessels; (iii) (the convex property) indicates that the marginal cost is increasing with the number of vessels.

Next, by Proposition 3.1 of Ross (1983), we have the following proposition, which indicates that when n approaches infinity, $V_n(x_1, x_2)$ converges uniformly to $V(x_1, x_2)$, where $V(x_1, x_2)$ represents the optimal long-run discounted cost with state (x_1, x_2) .

Proposition 2.

$$V_n(x_1, x_2) \rightarrow V(x_1, x_2) \quad \text{as} \quad n \rightarrow \infty$$

for all x_1 and x_2 .

By Proposition 2, $V(x_1, x_2)$ also has the properties in Proposition 1. Following similar argument as in Lemma 1 in the Appendix, we have the following theorem that characterizes the optimal policy of berth allocation.

Theorem 1. Define

$$\hat{y}_1(x_2) = \min\{x_1 \geq 0 : c_1 + V(x_1 + 1, x_2) - c_2 - V(x_1, x_2 + 1) > 0\},$$

and

$$\hat{y}_2(x_1) = \min\{x_2 \geq 0 : c_2 + V(x_1, x_2 + 1) - c_1 - V(x_1 + 1, x_2) > 0\}.$$

Then, $\hat{y}_1(x_2)$ and $\hat{y}_2(x_1)$ are increasing in x_2 and x_1 , respectively. The following control policy for type-1 vessels is optimal: allocate it to terminal 2 if $\hat{y}_1(x_2) \leq x_1$ and accept it at terminal 1 if $x_1 < \hat{y}_1(x_2)$. Similarly, for type-2 vessels, the optimal policy is: allocate it to terminal 1 if $\hat{y}_2(x_1) \leq x_2$ and accept it at terminal 2 if $x_2 < \hat{y}_2(x_1)$.

Theorem 1 shows that the optimal control policy is of threshold-type. For example, for type-1 vessels, when the current number of vessels in terminal 1 is above such a threshold, it is optimal to allocate the vessels to terminal 2; otherwise, it can stay in terminal 1. For a deep sea vessel call with a relatively high waiting cost, the standard preference would be to assign it to a berth with a high service rate, such as an indented berth. Here, the planner first has to control the threshold level before making the decision. Based on the current state of the terminal, the deep sea vessel could also be served at a relatively slower berth.

5. Numerical study

To show the practical application of the model, we conducted a case study based on ship data at the port of Izmir in Turkey. The

Table 1
Sensitivity analysis.

w	$\hat{y}_1(4)$	$\hat{y}_2(3)$	ρ_1	$\hat{y}_1(4)$	$\hat{y}_2(3)$	ρ_2	$\hat{y}_1(4)$	$\hat{y}_2(3)$
1	4.1	3.25	0.5	5.21	1.84	0.5	2.36	5.78
2	4.51	2.95	0.6	4.51	2.95	0.6	3.56	4.98
3	5.02	2.44	0.7	3.42	3.58	0.7	3.98	3.61
4	5.31	2.08	0.8	2.67	3.98	0.8	4.51	2.95
5	5.76	1.99	0.9	1.25	4.46	0.9	5.83	1.53

specifications and the input parameters used for the model were retrieved in consultation with the port staff at Izmir. The port has a container handling capacity of approximately 1 million TEUs and it plays a vital role in the country's economy and import/export trade.

The port operates with weekly expected time of arrivals provided by the lines and constructs a berthing schedule. Two main types of vessel arrivals are noticeable where one class of customer has a special importance due to the volume of business encountered (Hatisaru, 2014). The berth structure is discrete, and the whole quay area, which is 1330 m in length, is partitioned into different sections. There are two berth groups-terminals based on the allocated handling equipment type. These terminals differ in service time rates due to the physical structure and the equipment used. The two main types of quayside handling equipment used are rail mounted quay cranes and rubber tyred quay cranes, where the rubber tyred cranes have a slower handling/service rate than rail mounted quay cranes.

Because of confidentiality, we have to rescale the data provided by the Izmir container terminal as follows: the waiting cost per unit $w = 2$, $\lambda_1 = 6$, $\lambda_2 = 8$, $\mu_1 = 10$, $\mu_2 = 10$, $\alpha = 0.5$.

We first construct an example to illustrate the implementation of the optimal policy presented by Theorem 1. Suppose that there are three vessels in the type-1 terminal and four vessels in type-2 terminal. We have $x_1 = 3$ and $x_2 = 4$. Then, we can compute $\hat{y}_1(4) = 5.51$ and $\hat{y}_2(3) = 2.95$. When a new vessel arrives at the type-1 terminal, since $x_1 = 3$ is below the threshold $\hat{y}_1(4)$, the new arrival at the type-1 terminal will stay there to receive the service. When a new vessel arrives at the type-2 terminal, since $x_2 = 4$ is above the threshold $\hat{y}_2(3)$, the new arrival at the type-2 terminal will be switched to the type-1 terminal. For the existing vessels at a terminal, the order of the service is first-come-first-serve.

Next, to investigate the impact of parameters on the optimal policy $\hat{y}_1(4)$ and $\hat{y}_2(3)$, we perform sensitivity analysis with respect to waiting cost (w) and utilization (ρ) at two terminals. Note that utilization at terminal i is given by the ratio of the arrival rate and the service rate, i.e., $\rho_i = \lambda_i / \mu_i$.

From Table 1, we observe that the threshold ($\hat{y}_1(4)$) for Terminal 1 is increasing in waiting cost and utilization of type-2 terminal, decreasing in utilization of type-1 terminal. The explanation is as follows. On the one hand, when the unit waiting cost increases, since there are less vessels in type-1 terminal than type-2 terminal in the current situation, the operator should allocate incoming vessels to type-1 terminal in order to reduce the total waiting cost. On the other hand, when ρ_1 increases or ρ_2 decreases, type-1 terminal becomes relatively more congested than the current situation. As a result, incoming vessels will be assigned to type-2 terminal more likely. By following similar argument, the threshold ($\hat{y}_2(3)$) for Terminal 2 is decreasing in waiting cost and utilization of type-2 terminal, increasing in utilization of type-1 terminal. Further, we find that utilization of both terminals have a more significant impact on the thresholds than the waiting cost. We have also performed the corresponding experiments for the threshold of other states and observed the same phenomena.

6. Extensions

In this section, we consider the following three extensions of the base model described in Section 3, including possibility of forwarding vessels calls, multiple berths at each terminal (berth group), and different priorities of vessels.

6.1. Possibility of forwarding vessels calls

In practice, due to the limited amount of resources at the container terminal, the terminal controller may choose to forward some vessel calls. This strategy which has recently been in focus with incentives towards collaboration among ports may differ according to management policies of the terminals (See Bichou and Bell, 2007; Hoshino, 2010; Imai et al., 2008; Low, Lam, and Tang, 2009). The decision then is given based on the associated costs for forwarding such calls. Here, we consider such a scenario by allowing the forwarding of vessels. Denote r_i as the forwarding cost per type- i vessel for $i = 1, 2$. Then, we can rewrite (1) as

$$\begin{aligned} V(x_1, x_2) = & \sum_{i=1}^2 wx_i + \lambda_1 \min\{c_1 + V(x_1 + 1, x_2), \\ & c_2 + V(x_1, x_2 + 1), r_1 + V(x_1, x_2)\} \\ & + \lambda_2 \min\{c_1 + V(x_1 + 1, x_2), c_2 + V(x_1, x_2 + 1), \\ & r_2 + V(x_1, x_2)\} \\ & + \mu_1 V((x_1 - 1)^+, x_2) + \mu_2 V(x_1, (x_2 - 1)^+). \end{aligned} \quad (7)$$

By following the similar analysis in Section 4, we obtain the following proposition that describes the optimal control policy.

Proposition 3. Define

$$\bar{y}_1(x_2) = \min\{x_1 \geq 0 : \min\{c_1 + V(x_1 + 1, x_2), c_2 + V(x_1, x_2 + 1)\} - [r_1 + V(x_1, x_2)] \leq 0\}$$

and

$$\bar{y}_1(x_2) = \min\{x_1 \geq 0 : c_1 + V(x_1 + 1, x_2) - c_2 - V(x_1, x_2 + 1) > 0\}.$$

Then $\bar{y}_1(x_2)$ is decreasing in x_2 , and $\bar{y}_1(x_2)$ is increasing in x_2 , and $\bar{y}_1(x_2) \geq \bar{y}_1(x_2)$. The following control policy for type-1 vessel is optimal: forward the vessel if $x_1 > \bar{y}_1(x_2)$; otherwise, allocate it to berth 2 if $\bar{y}_1(x_2) \leq x_1 \leq \bar{y}_1(x_2)$ and allocate it to berth 1 if $x_1 < \bar{y}_1(x_2)$. The same control policy holds for type-2 vessels with $\bar{y}_2(x_1)$ and $\bar{y}_2(x_1)$ given the above similar definitions.

By Proposition 3, we find that the optimal policy is dual-threshold type. Let us take type-1 vessels as an example to illustrate the policy. When the number of vessels in terminal 1 is above the upper threshold $\bar{y}_1(x_2)$, the vessel should be forwarded to reduce the workload; when it is between the upper threshold and the lower threshold $\bar{y}_1(x_2)$, the vessels should be allocated to terminal 2; when it is below the lower threshold, it can stay in terminal 1.

6.2. Multiple berths at each terminal (berth group)

We assume that there exist multiple berths at each terminal. Therefore, we can model each berth group as an $M/M/\hat{N}$ queue, where $\hat{N}$ represents the number of berths at each terminal. Denote k_i as the number of berths at berth group i . Then, we can rewrite (1) as

$$\begin{aligned} V(x_1, x_2) = & \sum_{i=1}^2 wx_i \\ & + \lambda_1 \min\{c_1 + V(x_1 + 1, x_2), c_2 + V(x_1, x_2 + 1)\} \\ & + \lambda_2 \min\{c_1 + V(x_1 + 1, x_2), c_2 + V(x_1, x_2 + 1)\} \end{aligned}$$

$$\begin{aligned} & + \mu_1 (x_1 \wedge k_1) V((x_1 - 1)^+, x_2) \\ & + \mu_1 (k_1 - x_1 \wedge k_1) V(x_1, x_2) \\ & + \mu_2 (x_2 \wedge k_2) V(x_1, (x_2 - 1)^+) \\ & + \mu_2 (k_2 - x_2 \wedge k_2) V(x_1, x_2), \end{aligned} \quad (8)$$

where $x_1 \wedge k_1 = \min\{x_1, k_1\}$ and $x_2 \wedge k_2 = \min\{x_2, k_2\}$.

By following the similar analysis in Section 4, we find that the optimal control policy has the same threshold-type structure as given by Theorem 1. The only difference is that the value of threshold is different due to the assumption of multiple berths at each group.

6.3. Different priorities of vessels

Next, we assume that vessels have different priorities. The priority is measured by the waiting cost. For example, there exists the difference in waiting costs for deep sea vessels and barges. The waiting cost of a vessel with a higher priority is larger than that with a lower priority. We consider two types of vessels with different priorities. Denote w_j as waiting cost per unit for type- j vessels for $j = 1, 2$. Without loss of generality, we assume that $w_1 > w_2$. Therefore, to improve cost-efficiency, type-1 vessels has a higher priority to receive the service than type-2 vessels. We assume that the terminal operator follows a non-preemptive priority scheduling, i.e., type-1 vessels can move ahead of all the type-2 vessels waiting in the same terminal, but the type-2 vessels in service are not interrupted by type-1 vessels. To tackle this priority scheduling problem, we have to define extra state variables to differentiate different types of vessels at each berth group.

Denote x_{ij} as the number of type- j vessels in type- i berth group. Note that x_i means the total number of vessels in type- i berth group. Then, we can rewrite (1) as

$$\begin{aligned} V(x_{11}, x_1, x_{21}, x_2) = & w_1(x_{11} + x_{21}) + w_2(x_1 + x_2 - x_{11} - x_{21}) \\ & + \lambda_1 \min\{c_1 + V(x_{11} + 1, x_1 + 1, x_{21}, x_2), \\ & c_2 + V(x_{11}, x_1, x_{21} + 1, x_2 + 1)\} \\ & + \lambda_2 \min\{c_1 + V(x_{11}, x_1 + 1, x_{21}, x_2), \\ & c_2 + V(x_{11}, x_1, x_{21}, x_2 + 1)\} \\ & + \mu_1 V((x_{11} - 1)^+, (x_1 - 1)^+, x_{21}, x_2) \\ & + \mu_2 V(x_{11}, x_1, (x_{21} - 1)^+, (x_2 - 1)^+). \end{aligned} \quad (9)$$

By following the similar analysis in Section 4, we find that the optimal control policy has the same structure as given by Theorem 1. The difference is the definition of the threshold caused by extra state variables, i.e.,

$$\begin{aligned} \hat{y}_1(x_{11}, x_{21}, x_2) = & \min\{x_1 \geq 0 : c_1 + V(x_{11} + 1, x_1 + 1, x_{21}, x_2) \\ & - c_2 - V(x_{11}, x_1, x_{21} + 1, x_2 + 1) > 0\}, \\ \hat{y}_2(x_{11}, x_{21}, x_1) = & \min\{x_2 \geq 0 : c_2 + V(x_{11}, x_1, x_{21}, x_2 + 1) \\ & - c_1 - V(x_{11}, x_1 + 1, x_{21}, x_2) > 0\}. \end{aligned}$$

7. Conclusions

The tremendous growth in containerization has placed tougher demands on container terminals. To cope with the increased demand and workload, container terminal operators have to ensure the effective usage of their scarce resources. Of the many planning problems present in container terminals, berth allocation problem is the root of all operations and deals with the allocation strategies of the arriving vessels to the available berths. The complexity of the problem arises with the fact that decision makers have to deal with uncertain information as to the vessel arrival times and handling times. Terminal planners have to ensure proper level of service for the different type of vessels they are serving. Development of intelligent strategies considering different vessel types and

most importantly incorporating the presence of uncertainty during the planning stage is of crucial importance.

Motivated from these facts, this study has proposed optimal policies for the berth allocation problem. The study is capable of recognizing the need for differentiating between the two main different types of vessels, namely deep sea vessels and barge/feeders, within the perspective of the container terminal operators. The uncertainty in arrival times of vessels and in handling times is incorporated in the proposed decisions. For the practiced settings we show that the optimal policy is based on a threshold level depending on the number of vessels in a certain berth group. The proposed policy is robust to the extent that it can be applied to the more general settings such as for situations where the vessel calls can be forwarded, different vessel priorities are considered, and diverse berth groups are present.

Due to the model complexity, the berth-allocation-problem literature with uncertainty, such as Golias et al. (2014) and Zhen (2015), focuses on the development of heuristics. Although the heuristic can generate a feasible schedule, it may be difficult for terminal operators to understand how the heuristics works and to learn valuable insights from the solution. Our paper explicitly characterizes the structure of the optimal policy, i.e., a threshold type. We believe such a policy can provides a useful guideline for terminal operators. Further, since the threshold policy mainly depends on the number of existing vessels in the terminals, it is relatively easy to be implemented.

It should be noted that, there exist some limitations for the current study. First, our model considers two types of berth groups. A challenging future direction is to investigate whether the main results still hold for the setting with multiple berth groups. Although it is difficult to prove that the structure of the optimal policy holds for multiple berth groups, our conjecture is that the optimal policy for a berth group will depend on the number of vessels in the rest of berth groups. In other words, the optimal policy could be a multi-dimensional switching curve instead of a threshold for two berth groups. Second, although the service rate is different for different types of berth groups, it is still assumed to be constant in our model. Since the service rate may depend on the type of a vessel, service rate could be treated as a decision variable by terminal operators. To incorporate a variable service rate, we have to handle two types of decision variables: one for berth allocation and the other for the choice of the service rate, which results in further analytical challenges. We believe that the solution technique and results from our model can be used as a base to overcome these challenges. Further, the development of this work will be useful on the understanding and formulation of other problems which can be extended to consider uncertainty on the storage yard allocation problem in a container terminal.

Acknowledgments

The authors are grateful to the referees and the editor for their constructive suggestions that significantly improved this study. The research was partly supported by Netherlands Organisation for Scientific Research under Grant 040.03.021, Royal Netherlands Academy of Arts and Sciences under Grant 530-4CDI18, and National Natural Science Foundation of China under grant 71571125 and 71371158.

Appendix

Before we show the proof of Proposition 1, we need to present four lemmas below.

Denote

$$\hat{x}_1(x_2) = \min\{x_1 \geq 0 : c_1 + V_n(x_1 + 1, x_2) - c_2 - V_n(x_1, x_2 + 1) > 0\},$$

$$\hat{x}_2(x_1) = \min\{x_2 \geq 0 : c_2 + V_n(x_1, x_2 + 1) - c_1 - V_n(x_1 + 1, x_2) > 0\}.$$

Note that $\hat{x}_1(x_2)$ represents the threshold that indicate whether to allocate an incoming vessel to the first terminal based on the number of existing vessels in the second terminal. $\hat{x}_2(x_1)$ represents the threshold that indicate whether to allocate an incoming vessel to the second terminal based on the number of existing vessels in the first terminal.

Lemma 1. To minimize V_{n+1} , the optimal control policy is as follows:

- (1) For the allocation control of terminal 1, there exists $\hat{x}_1(x_2)$ for each x_2 , such that $\hat{x}_1(x_2)$ is increasing in x_2 , and the following policy is optimal: accept it at terminal 1 if $x_1 < \hat{x}_1(x_2)$; allocate it to terminal 2 if $\hat{x}_1(x_2) \geq x_1$.
- (2) For the allocation control of terminal 2, there exists $\hat{x}_2(x_1)$ for each x_1 , such that $\hat{x}_2(x_1)$ is increasing in x_1 , and the following policy is optimal: accept it at terminal 2 if $x_2 < \hat{x}_2(x_1)$; allocate it to terminal 1 if $\hat{x}_2(x_1) \geq x_2$.

Proof. For (i), it is optimal to allocate a vessel to berth group 2 if

$$c_1 + V_n(x_1 + 1, x_2) > c_2 + V_n(x_1, x_2 + 1). \quad (10)$$

From the induction hypothesis of (2) and (3), $c_1 + V_n(x_1 + 1, x_2) - c_2 - V_n(x_1, x_2 + 1)$ is increasing in x_1 and decreasing in x_2 . Hence, $\hat{x}_1(x_2)$ is well defined and increasing in x_2 , and (10) is equivalent to $x_1 \geq \hat{x}_1(x_2)$. Therefore, it is optimal to allocate the vessel to berth group 2 if $\hat{x}_1(x_2) \leq x_1$, and accept it to berth group 1 if $x_1 < \hat{x}_1(x_2)$. By following the similar argument, we can prove that (ii) is true. $\square$

Lemma 2. If $V_n(x_1, x_2)$ satisfies (2), then we have

$$\begin{aligned} & \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ & \quad - \min\{c_1 + V_n(x_1 + 1, x_2 + 1), c_2 + V_n(x_1, x_2 + 2)\} \\ & \geq \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} \\ & \quad - \min\{c_1 + V_n(x_1, x_2 + 1), c_2 + V_n(x_1 - 1, x_2 + 2)\}. \end{aligned} \quad (11)$$

Proof. We prove the lemma in two cases below.

Case 1. Suppose

$$c_1 + V_n(x_1, x_2 + 1) \geq c_2 + V_n(x_1 - 1, x_2 + 2),$$

then from (2) we have

$$c_1 + V_n(x_1 + 1, x_2 + 1) \geq c_2 + V_n(x_1, x_2 + 2).$$

Hence the left hand side (LHS) of (11) can be written as

$$\begin{aligned} LHS = & \min\{c_1 + V_n(x_1 + 2, x_2) - V_n(x_1, x_2 + 2) - c_2, \\ & V_n(x_1 + 1, x_2 + 1) - V_n(x_1, x_2 + 2)\}, \end{aligned} \quad (12)$$

and the right hand side (RHS) can be written as

$$\begin{aligned} RHS = & \min\{c_1 + V_n(x_1 + 1, x_2) - V_n(x_1 - 1, x_2 + 2) - c_2, \\ & V_n(x_1, x_2 + 1) - V_n(x_1 - 1, x_2 + 2)\}. \end{aligned} \quad (13)$$

Notice that from (2) we have

$$\begin{aligned} & c_1 + V_n(x_1 + 2, x_2) - V_n(x_1, x_2 + 2) - c_2 \\ & = [V_n(x_1 + 2, x_2) - V_n(x_1 + 1, x_2 + 1)] \\ & \quad + [V_n(x_1 + 1, x_2 + 1) - V_n(x_1, x_2 + 2)] + c_1 - c_2 \\ & \geq [V_n(x_1 + 1, x_2) - V_n(x_1, x_2 + 1)] \\ & \quad + [V_n(x_1, x_2 + 1) - V_n(x_1 - 1, x_2 + 2)] + c_1 - c_2 \\ & = V_n(x_1 + 1, x_2) - V_n(x_1 - 1, x_2 + 2) + c_1 - c_2, \end{aligned}$$

and

$$\begin{aligned} & V_n(x_1 + 1, x_2 + 1) - V_n(x_1, x_2 + 2) \\ & \geq V_n(x_1, x_2 + 1) - V_n(x_1 - 1, x_2 + 2), \end{aligned}$$

inequality (11) is true in this case.

Case 2. Suppose

$$c_1 + V_n(x_1, x_2 + 1) < c_2 + V_n(x_1 - 1, x_2 + 2).$$

Then, the right hand side of (11) equals

$$RHS = \min\{V_n(x_1, x_2 + 1) - V_n(x_1, x_2 + 1), c_2 - c_1\}.$$

The left hand side satisfies

$$\begin{aligned} LHS &= \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ &\quad - \min\{V_n(x_1 + 1, x_2 + 1), c_1 + V_n(x_1, x_2 + 2)\} \\ &\geq \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ &\quad - c_1 - V_n(x_1 + 1, x_2 + 1) \\ &= \min\{V_n(x_1 + 2, x_2) - V_n(x_1 + 1, x_2 + 1), c_2 - c_1\}. \end{aligned}$$

Note that

$$\begin{aligned} V_n(x_1 + 2, x_2) - V_n(x_1 + 1, x_2 + 1) \\ \geq V_n(x_1 + 1, x_2) - V_n(x_1, x_2 + 1) \end{aligned}$$

from (2). Therefore, (11) is true. $\square$

Lemma 3. If $V_n(x_1, x_2)$ satisfies (2)–(6), then

$$\begin{aligned} &\min\{c_1 + V_n(x_1 + 2, x_2 + 1), c_2 + V_n(x_1 + 1, x_2 + 2)\} \\ &\quad - \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ &\geq \min\{c_1 + V_n(x_1 + 1, x_2 + 1), c_2 + V_n(x_1, x_2 + 2)\} \\ &\quad - \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\}. \end{aligned} \quad (14)$$

Proof. Note that, $\hat{x}_1(x_2)$, which is defined in Lemma 1, is increasing in x_2 , and $x_1 < \hat{x}_1(x_2)$ if and only if

$$c_1 + V_n(x_1 + 1, x_2) \leq c_2 + V_n(x_1, x_2 + 1).$$

We consider the following three cases.

Case 1: $x_1 + 1 < \hat{x}_1(x_2)$. Since $\hat{x}_1(x_2)$ is increasing in x_2 , we have $x_1 + 1 < \hat{x}_1(x_2 + 1)$, $x_1 < \hat{x}_1(x_2)$, and $x_1 < \hat{x}_1(x_2 + 1)$. Hence, (14) is equivalent to

$$\begin{aligned} V_n(x_1 + 2, x_2 + 1) - V_n(x_1 + 2, x_2) \\ \geq V_n(x_1 + 1, x_2 + 1) - V_n(x_1 + 1, x_2), \end{aligned}$$

which is true from (4).

Case 2: $x_1 \geq \hat{x}_1(x_2 + 1)$. This implies $x_1 + 1 \geq \hat{x}_1(x_2 + 1)$, $x_1 \geq \hat{x}_1(x_2)$, and $x_1 + 1 \geq \hat{x}_1(x_2)$, and hence, (14) can be simplified to

$$\begin{aligned} [c_2 + V_n(x_1 + 1, x_2 + 2)] - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq V_n(x_1, x_2 + 2) - V_n(x_1, x_2 + 1), \end{aligned}$$

which is true again due to the supermodularity of $V_n(x_1, x_2)$.

Case 3: $\hat{x}_1(x_2) - 1 \leq x_1 < \hat{x}_1(x_2 + 1)$. This is equivalent to $\hat{x}_1(x_2) \leq x_1 + 1 \leq \hat{x}_1(x_2 + 1)$. In this case, if $\hat{x}_1(x_2) = \hat{x}_1(x_2 + 1) = x_1 + 1$, then $x_1 < \hat{x}_1(x_2)$, and $x_1 < \hat{x}_1(x_2 + 1)$, and hence, (14) is given by

$$\begin{aligned} [c_2 + V_n(x_1 + 1, x_2 + 2)] - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq V_n(x_1 + 1, x_2 + 1) - V_n(x_1 + 1, x_2), \end{aligned}$$

which is true due to the convexity of $V_n(x_1, x_2)$ in x_2 .

If $\hat{x}_1(x_2) < \hat{x}_1(x_2 + 1)$ and $x_1 = \hat{x}_1(x_2)$, then $x_1 + 1 < \hat{x}_1(x_2 + 1)$, $x_1 < \hat{x}_1(x_2)$, and $x_1 < \hat{x}_1(x_2 + 1)$. Hence, (14) becomes

$$\begin{aligned} c_1 + V_n(x_1 + 2, x_2 + 1) - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq V_n(x_1 + 1, x_2 + 1) - V_n(x_1 + 1, x_2). \end{aligned}$$

This is true since

$$\begin{aligned} c_1 + V_n(x_1 + 2, x_2 + 1) - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq V_n(x_1 + 2, x_2 + 1) - V_n(x_1 + 2, x_2) \\ \geq V_n(x_1 + 1, x_2 + 1) - V_n(x_1 + 1, x_2), \end{aligned}$$

where the first inequality is due to the fact that $x_1 + 1 < \hat{x}_1(x_2 + 1)$, and the second inequality is from the supermodularity of $V_n(x_1, x_2)$.

If $\hat{x}_1(x_2) < \hat{x}_1(x_2 + 1)$ and $x_1 = \hat{x}_1(x_2 + 1)$, then $x_1 > \hat{x}_1(x_2)$, $x_1 + 1 > \hat{x}_1(x_2)$, and $x_1 + 1 > \hat{x}_1(x_2 + 1)$. In this case, (14) simplifies to

$$\begin{aligned} [c_2 + V_n(x_1 + 1, x_2 + 2)] - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq V_n(x_1, x_2 + 2) - V_n(x_1, x_2 + 1), \end{aligned}$$

which is true.

Finally, if $\hat{x}_1(x_2) < x_1 < \hat{x}_1(x_2 + 1)$, then $x_1 + 1 > \hat{x}_1(x_2)$. (14) now becomes

$$\begin{aligned} \min\{c_1 + V_n(x_1 + 2, x_2 + 1), c_2 + V_n(x_1 + 1, x_2 + 2)\} \\ - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq c_1 + V_n(x_1 + 1, x_2 + 1) - [c_2 + V_n(x_1, x_2 + 1)]. \end{aligned}$$

Hence, it suffices to have

$$\begin{aligned} c_1 + V_n(x_1 + 2, x_2 + 1) - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq c_1 + V_n(x_1 + 1, x_2 + 1) - [c_2 + V_n(x_1, x_2 + 1)], \end{aligned} \quad (15)$$

and

$$\begin{aligned} [c_2 + V_n(x_1 + 1, x_2 + 2)] - [c_2 + V_n(x_1 + 1, x_2 + 1)] \\ \geq V_n(x_1 + 1, x_2 + 1) - [c_2 + V_n(x_1, x_2 + 1)]. \end{aligned} \quad (16)$$

We find that (15) is true due to the convexity of $V_n(x_1, x_2)$ on x_1 , and (16) is true since

$$\begin{aligned} c_2 + V_n(x_1 + 1, x_2 + 2) - c_1 - V_n(x_1 + 1, x_2 + 1) \\ \geq [c_2 + V_n(x_1 + 1, x_2 + 2)] - [c_2 + V_n(x_1, x_2 + 2)] \\ = V_n(x_1 + 1, x_2 + 2) - V_n(x_1, x_2 + 2) \\ \geq V_n(x_1 + 1, x_2 + 1) - V_n(x_1, x_2 + 1). \end{aligned}$$

Here the first inequality is true since $x_1 < \hat{x}_1(x_2 + 1)$ implies

$$c_1 + V_n(x_1 + 1, x_2 + 1) \leq c_2 + V_n(x_1, x_2 + 2),$$

and the second inequality is from the supermodularity of $V_n(x_1, x_2)$. $\square$

Lemma 4. If $V_n(x_1, x_2)$ satisfies (2)–(6), then

$$\begin{aligned} &\min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ &\quad - \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} \\ &\geq \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} \\ &\quad - \min\{c_1 + V_n(x_1, x_2), c_2 + V_n(x_1 - 1, x_2 + 1)\}. \end{aligned} \quad (17)$$

Proof. From the definition of $\hat{x}_1(x_2)$ in Lemma 1, if $x_1 - 1 \geq \hat{x}_1(x_2)$, then (17) becomes

$$\begin{aligned} [c_2 + V_n(x_1 + 1, x_2 + 1)] - [c_2 + V_n(x_1, x_2 + 1)] \\ \geq V_n(x_1, x_2 + 1) - V_n(x_1 - 1, x_2 + 1), \end{aligned}$$

which is true since $V_n(x_1, x_2)$ is convex in x_1 .

If $x_1 + 1 < \hat{x}_1(x_2)$, then (17) is equivalent to

$$V_n(x_1 + 2, x_2) - V_n(x_1 + 1, x_2) \geq V_n(x_1 + 1, x_2) - V_n(x_1, x_2),$$

which is true again due to the convexity of V_n .

If $x_1 = \hat{x}_1(x_2)$, then $x_1 - 1 < \hat{x}_1(x_2)$ and $x_1 + 1 > \hat{x}_1(x_2)$. In this case, (17) can be written as

$$\begin{aligned} [c_2 + V_n(x_1 + 1, x_2 + 1)] - [c_2 + V_n(x_1, x_2 + 1)] \\ \geq [c_2 + V_n(x_1, x_2 + 1)] - c_1 - V_n(x_1, x_2). \end{aligned} \quad (18)$$

Notice that $c_1 + V_n(x_1 + 1, x_2) \geq c_2 + V_n(x_1, x_2 + 1)$ as $x_1 = \hat{x}_1(x_2)$, (18) is implied by

$$V_n(x_1 + 1, x_2 + 1) - V_n(x_1 + 1, x_2) \geq V_n(x_1, x_2 + 1) - V_n(x_1, x_2).$$

This is true due to the supermodularity of V_n .

Finally, if $x_1 + 1 = \hat{x}_1(x_2)$, then (17) is equivalent to

$$\begin{aligned} c_2 + V_n(x_1 + 1, x_2 + 1) - c_1 - V_n(x_1 + 1, x_2) \\ \geq V_n(x_1 + 1, x_2) - V_n(x_1, x_2). \end{aligned}$$

Since $c_1 + V_n(x_1 + 1, x_2) < c_2 + V_n(x_1, x_2 + 1)$ as $x_1 < \hat{x}_1(x_2)$, it suffices to have

$$\begin{aligned} c_2 + V_n(x_1 + 1, x_2 + 1) - c_1 - V_n(x_1 + 1, x_2) \\ \geq c_2 + V_n(x_1, x_2 + 1) - c_1 - V_n(x_1, x_2), \end{aligned}$$

which is true due to the supermodular of V_n . $\square$

Proof of Proposition 1. We will prove the above proposition by induction. Notice that $V_0(x_1, x_2) \equiv 0$, all properties are satisfied when $n = 0$. Suppose the above relationships are true for n , i.e., $V_n(x_1, x_2)$ satisfies (2)–(6).

For (i), it suffices to show

$$\begin{aligned} V_{n+1}(x_1 + 1, x_2) - V_{n+1}(x_1, x_2 + 1) \\ \geq V_{n+1}(x_1, x_2) - V_{n+1}(x_1 - 1, x_2 + 1), \end{aligned}$$

with V_{n+1} given in (1). Then, we only have to prove

$$\begin{aligned} \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ - \min\{c_1 + V_n(x_1 + 1, x_2 + 1), c_2 + V_n(x_1, x_2 + 2)\} \\ \geq \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} \\ - \min\{c_1 + V_n(x_1, x_2 + 1), c_2 + V_n(x_1 - 1, x_2 + 2)\}, \end{aligned}$$

which is true by Lemma 2.

For (ii), it suffices to show

$$\begin{aligned} \min\{c_1 + V_n(x_1 + 2, x_2 + 1), c_2 + V_n(x_1 + 1, x_2 + 2)\} \\ - \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ \geq \min\{c_1 + V_n(x_1 + 1, x_2 + 1), c_2 + V_n(x_1, x_2 + 2)\} \\ - \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\}, \end{aligned}$$

which is true by Lemma 3.

For (iii), it suffices to show

$$\begin{aligned} \min\{c_1 + V_n(x_1 + 2, x_2), c_2 + V_n(x_1 + 1, x_2 + 1)\} \\ - \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} \\ \geq \min\{c_1 + V_n(x_1 + 1, x_2), c_2 + V_n(x_1, x_2 + 1)\} \\ - \min\{c_1 + V_n(x_1, x_2), c_2 + V_n(x_1 - 1, x_2 + 1)\}, \end{aligned}$$

which is true by Lemma 4.

In summary, we have shown that $V_{n+1}(x_1, x_2)$ satisfies (2)–(6). So, we can conclude that $V_n(x_1, x_2)$ satisfies (2)–(6) for $n \geq 0$. $\square$

References

- Beens, M. A., & Ursavas, E. (2016). Scheduling cranes at an indented berth. *European Journal of Operational Research*. doi:10.1016/j.ejor.2016.02.038.
- Bichou, K., & Bell, M. G. (2007). Internationalisation and consolidation of the container port industry: assessment of channel structure and relationships. *Maritime Economics & Logistics*, 9(1), 35–51.
- Canonaco, P., Legato, P., Mazza, R. M., & Musmanno, R. (2008). A queueing network model for the management of berth crane operations. *Computers & Operations Research*, 35(8), 2432–2446.
- Carlo, H. J., Vis, I. F., & Roodbergen, K. J. (2015). Seaside operations in container terminals: literature overview, trends, and research directions. *Flexible Services and Manufacturing Journal*, 27(2–3), 224–262.
- Chao, X., Liu, L., & Zheng, S. (2003). Resource allocation in multisite service systems with intersite customer flows. *Management Science*, 49, 1739–1752.
- Drewry Shipping Consultants (2008). The drewry container shipper insight - third quarter.
- Cordeau, J. F., Laporte, G., Legato, P., & Moccia, L. (2005). Models and tabu search heuristics for the berth-allocation problem. *Transportation Science*, 39(4), 526–538.
- Du, Y., Xu, Y., & Chen, Q. (2010). A feedback procedure for robust berth allocation with stochastic vessel delays. In *Proceedings of the 2010 8th world congress on intelligent control and automation (WCICA)* (pp. 2210–2215). IEEE.
- Golias, M., Boile, M., & Theofanis, S. (2007). The stochastic berth allocation problem. In *Proceedings of the international conference on transport science and technology (TRANSTEC 2007)* (pp. 52–66).
- Golias, M., Boile, M., & Theofanis, S. (2009). Berth scheduling by customer service differentiation: A multiobjective approach. *Transportation Research Part E*, 45(6), 878–892.
- Golias, M. M., Portal, I., Konur, D., Kaisar, E., & Kolomvos, G. (2014). Robust berth scheduling at marine container terminals via hierarchical optimization. *Computers and Operations Research*, 41, 412–422.
- Guan, Y., & Cheung, R. K. (2004). The berth allocation problem: Models and solution methods. *OR Spectrum*, 26(1), 75–92.
- Han, X., Lu, Z. Q., & Xi, L. F. (2010). A proactive approach for simultaneous berth and quay crane scheduling problem with stochastic arrival and handling time. *European Journal of Operational Research*, 207(3), 1327–1340.
- Hansen, P., Oguz, C., & Mladenović, N. (2008). Variable neighborhood search for minimum cost berth allocation. *European Journal of Operational Research*, 191(3), 636–649.
- Hatisaru, S. (2014). Izmir limani için sahaya inmeye hazır. *Milliyet*, 537–547, 1961186 28.10.2014.
- Hendriks, M., Laumanns, M., Lefebvre, E., & Udding, J. T. (2010). Robust cyclic berth planning of container vessels. *OR Spectrum*, 32(3), 501–517.
- Hordijk, A., & Koole, G. (1992). On the assignment of customers to parallel queues. *Probability in the Engineering and Information Sciences*, 6, 495–511.
- Hoshino, H. (2010). Competition and collaboration among container ports. *The Asian Journal of Shipping and Logistics*, 26(1), 31–47.
- Imai, A., Nishimura, E., & Papadimitriou, S. (2001). The dynamic berth allocation problem for a container port. *Transportation Research Part B: Methodological*, 35(4), 401–417.
- Imai, A., Nishimura, E., & Papadimitriou, S. (2003). Berth allocation with service priority. *Transportation Research Part B*, 37(5), 437–457.
- Imai, A., Nishimura, E., & Papadimitriou, S. (2008). Berthing ships at a multi-user container terminal with a limited quay capacity. *Transportation Research Part E*, 44(1), 136–151.
- Imai, A., Zhang, J., Nishimura, E., & Papadimitriou, S. (2007). The berth allocation problem with service time and delay time objectives. *Maritime Economics & Logistics*, 9, 269–290.
- Karafa, J., Golias, M. M., Ivey, S., Saharidis, G. K., & Leonardos, N. (2013). The berth allocation problem with stochastic vessel handling times. *The International Journal of Advanced Manufacturing Technology*, 65(1–4), 473–484.
- Koole, G. (1998). Structural results for the control of queueing systems using event-based dynamic programming. *Queueing Systems*, 30, 323–339.
- Legato, P., & Mazza, R. M. (2001). Berth planning and resources optimisation at a container terminal via discrete event simulation. *European Journal of Operational Research*, 133(3), 537–547.
- Lippman, S. A. (1975). Applying a new device in the optimization of exponential queueing systems. *Operations Research*, 23(4), 687–710.
- Low, J. M., Lam, S. W., & Tang, L. C. (2009). Assessment of hub status among asian ports from a network perspective. *Transportation Research Part A*, 43(6), 593–606.
- Moorthy, R., & Teo, C. P. (2006). Berth management in container terminal: The template design problem. *OR Spectrum*, 28(4), 495–518.
- Puterman, M. L. (1994). *Markov decision processes*. New York, NY, U.S.A.: John Wiley & Sons, Inc.
- Rodriguez-Molins, M., Salido, M. A., & Barber, F. (2014). A GRASP-based metaheuristic for the berth allocation problem and the quay crane assignment problem by managing vessel cargo holds. *Applied Intelligence*, 40(2), 273–290.
- Ross, S. M. (1983). *Introduction to stochastic dynamic programming*. London: Academic Press.
- Saharidis, G. K. D., Golias, M. M., Boile, M., Theofanis, S., & Ierapetritou, M. G. (2010). The berth scheduling problem with customer differentiation: A new methodological approach based on hierarchical optimization. *The International Journal of Advanced Manufacturing Technology*, 46(1–4), 377–393.
- Stahlbock, R., & Voß, S. (2008). Operations research at container terminals: A literature update. *OR Spectrum*, 30(1), 1–52.
- Steenken, D., Voß, S., & Stahlbock, R. (2004). Container terminal operation and operations research – A classification and literature review. *OR Spectrum*, 26(1), 3–49.
- Theofanis, S., Boile, M., & Golias, M. M. (2009). Container terminal berth planning. *Transportation Research Record: Journal of the Transportation Research Board*, 2100(1), 22–28.
- Vis, I. F., & De Koster, R. (2003). Transshipment of containers at a container terminal: An overview. *European Journal of Operational Research*, 147(1), 1–16.
- Xu, Y., Chen, Q., & Quan, X. (2012). Robust berth scheduling with uncertain vessel delay and handling time. *Annals of Operations Research*, 192(1), 123–140.
- Zhen, L. (2015). Tactical berth allocation under uncertainty. *European Journal of Operational Research*, 247, 928–944.
- Zhen, L., & Chang, D. F. (2012). A bi-objective model for robust berth allocation scheduling. *Computers & Industrial Engineering*, 63(1), 262–273.
- Zhen, L., Lee, L. H., & Chew, E. P. (2011). A decision model for berth allocation under uncertainty. *European Journal of Operational Research*, 212(1), 54–68.