University of Groningen

Dueling policies

Bosma, Jakob J.

Published in:
European Economic Review

DOI:
[10.1016/j.euroecorev.2016.05.002](https://doi.org/10.1016/j.euroecorev.2016.05.002)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2016

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Bosma, J. J. (2016). Dueling policies: Why systemic risk taxation can fail. *European Economic Review*, 87(3), 132-147. <https://doi.org/10.1016/j.euroecorev.2016.05.002>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Contents lists available at ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/eer

Dueling policies: Why systemic risk taxation can fail[☆]

Jakob J. Bosma

University of Groningen, Faculty of Economics and Business, P.O. Box 800, 9700 AV Groningen, The Netherlands

ARTICLE INFO

Article history:

Received 28 April 2015

Received in revised form

29 April 2016

Accepted 4 May 2016

Available online 7 May 2016

JEL classification:

D83

G01

G18

Keywords:

Systemic risk taxation

Ambiguity

Bailouts

ABSTRACT

Two policy instruments for the banking sector are investigated, namely systemic risk taxation and constructive ambiguity about bailout policy. Bailout expectations can induce moral hazard in the form of excessive risk taking by banks. Systemic risk taxation induces banks to prefer uncorrelated investments, leading to lower systemic risk formation. Constructive ambiguity generates uncertainty about bailout prospects. However, systemic risk taxation also may inform banks about the regulator's concern for financial stability and thereby its bailout policy. This result leads to a trade-off between systemic risk taxation and constructive ambiguity and highlights the need to consider interdependence across policies when evaluating their effectiveness.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Achieving two distinct objectives is possible only if at least two independent policy tools are at a regulator's disposal (Tinbergen, 1952). In a similar vein, the pursuit of one objective with multiple policy tools may render the tools interdependent. In the aftermath of the 2007–2009 global financial crisis, new macroprudential regulation reforms have sought to ensure the stability in the financial sector (Basel Committee of Banking, 2009a,b). Recent studies investigate these proposed regulatory tools independently with the implicit assumption and inference that policy tools are independent. Such an assumption of independence can generate hazardous outcomes if the use of one tool might provide an indication that another tool is likely to be used in tandem. Therefore, this study considers the potential of interdependence between two notable policy instruments recently proposed: systemic risk taxation for banks; and constructive ambiguity in bank bailout support (Acharya et al., 2011; Freixas, 1999; Freixas and Rochet, 2011).¹ By evaluating these two policy tools in a joint framework, the proposed signaling game demonstrates the ineffectiveness of a systemic risk tax if the tax reveals the

[☆] This paper benefited from comments received from participants at the 2014 ESBF meeting at Cass Business School, 2013 EEA Meetings, 2013 FIRS conference, the Dutch Central Bank, and at the University of Groningen. In particular, I thank Itai Agur, Wilko Bolt, Max Bruche, Lammertjan Dam, Falko Fecht, Edward Kane, Michael Koetter, Eric Leeper, Allard van der Made, Joel Sobel, Günter Strobl and two anonymous referees for their valuable input. All errors remain my own responsibility.

E-mail address: j.j.bosma@rug.nl

¹ Acharya et al. (2011, 2012), and Freixas and Rochet (2011) discuss the merits of a systemic risk tax levied on banks as compensation for future explicit or implicit financial assistance by governments in times of financial crises. Adrian and Brunnermeier (2011) suggest an indirect tax for systemically relevant institutions, in the form of risk-weighted capital and liquidity requirements that have been calibrated against a systemic risk measure. A second strategy regulators can adopt to deter banks' risk-shifting behaviour in response to prospective bailout support is to employ constructive ambiguity about bailout policies (Freixas, 1999; Kocherlakota and Shim, 2007; Shim, 2011).

Fig. 1. Game Tree.

regulator's concern for financial stability. That is, taxation limits the regulator's ability to exercise constructive ambiguity about a future bailout policy. If the tax reveals the likelihood of receiving bailout support, banks trade off the costs of the tax for the benefits of a prospective bailout. Introducing a systemic risk tax therefore could even encourage excessive risk taking by banks, rather than achieving its intended goal of curtailing such behaviour.

The methodological approach used to investigate this tradeoff parallels the efforts of [Farhi and Tirole \(2012\)](#) to understand how the time-inconsistency problem of a financial regulator induces banks to coordinate their leverage choices according to the prospect of bailout support. However, the focus of this study is how the tax contributes to interdependence among policy tools, because it may reveal the regulator's concern for financial stability. With respect to endogenous policy decisions, previous studies instead evaluate policy choices in light of speculative exchange rate attacks, anticipated interventions by the IMF, and runs on banks with weak fundamentals ([Angeletos et al., 2006](#); [Chunyang, 2013](#); [Drazen, 2000](#); [Zwart, 2007](#)).

Furthermore, the formation of systemic risk is driven in the model by banks' investment choices, which determine the aggregate correlation structure of banks' returns. This approach therefore builds on models by [Allen and Gale \(2000\)](#) and [Acharya \(2009\)](#) and reflects prior findings about inter-bank return dependence.² As a key extension to Allen and Gale's framework, the proposed model includes a regulator who sets a systemic risk tax for banks and faces a commitment problem in its decision to bail out distressed banks. When a substantial fraction of banks become financially distressed, the societal costs of letting distressed banks fail can force the regulator to initiate bailout support.

In the proposed model, banks have heterogeneous, imperfect prior information about the conditions that force the regulator to initiate bailouts. This uncertainty causes the banks' investment choices to be strategic complements ([Acharya and Yorulmazer, 2007](#); [Farhi and Tirole, 2012](#)). As more banks suffer financial distress the cost of their failure to society increases, which leads banks to believe the regulator is likely to initiate bailout support. These beliefs induce banks to prefer more positively correlated returns across the industry, in that they trade off the cost of failing in a crisis state for the benefits of more likely bailout support. To analyze this coordination problem, I use the global games methodology ([Carlsson and van Damme, 1993](#); [Morris and Shin, 1998, 2001](#); [Morris et al., 2003](#)). The implications of the systemic risk tax are twofold in this setting. First, an increase in the systemic risk tax works as a Pigovian-like tax and induces banks not to prefer jointly correlated investments, such that it lowers systemic risk formation. Second, the tax informs banks about the regulator's concern for financial stability and thereby reveals the likelihood of adopting a bailout policy during financial crises.

This study's results suggest that banks' knowledge of the tax level and their private information about the regulator's inclination to initiate bailouts provide means to infer the likelihood of receiving bailout support. A regulator that appears inclined to initiate bailout support has an incentive to conceal its stance towards bailouts and sets a low tax, in order to assume the identity of a tough regulator that is not inclined to bail out. With a taxation strategy based on imitation, a regulator can generate endogenously the necessary uncertainty, which a weak regulator then can exploit as constructive ambiguity. This strategic uncertainty is driven by the observation of a low tax that does not allow banks to distinguish the two regulator types, such that it may curtail coordination in risk taking on bailout prospects. By taking the bailout policy into account, I argue that a systemic risk tax may not effectively curtail systemic risk formation if the tax's level reveals the likelihood of receiving prospective bailout support to the affected banks.

With respect to policy implications, these results suggest the significant risks associated with drawing inferences about the effectiveness of systemic risk taxation with an assumption of independent macroprudential policy tools. If the signaling aspect of a systemic risk tax is ignored in the policy design, certain levels of taxation may signal the regulator's concern and eliminate ambiguity about bailout policies. A consequence would be that the tax is taken for granted, because a future

² [Maksimovic and Zechner \(1991\)](#) show that the risk characteristics of firms' cash flows are endogenously determined by aggregate investment decisions in the industry. Financially distressed industry peers impose private costs on the leverage of banks and may induce risk-shifting behaviour ([Shleifer and Vishny, 1992](#)). [Rajan \(1994\)](#) connects managerial short-termism and reputation concerns to decreased credit policy leniency when other banks are likely to do the same. [Acharya and Yorulmazer \(2007\)](#) consider the return dependence that results when many banks coordinate on bailout prospects.

bailout is perceived more likely. To credibly install an effective taxation policy, the costs incurred by the regulator to rescue a bank must be sufficiently high. A higher cost associated with a bailout then allows the regulator to generate the necessary commitment not to bail out during a crisis, such that the tax can be set without losing the ability to be ambiguous about its bailout policy. These costs are not necessarily limited to the actual costs of a bank bailout; they also include the political costs that regulators and politicians incur to bail out banks.

2. Model

The model builds on the framework proposed by [Allen and Gale \(2000\)](#) who explain financial bubbles and crises resulting from risk-shifting behaviour of a single investor in a one-period setup. I follow both [Allen and Gale \(2000\)](#) and [Acharya \(2009\)](#) but also extend their framework by including a regulator and many banks, to study the formation of systemic risk based on inter-bank interactions. A financial regulator can implement a systemic risk tax in the first stage of the model and extend bailout support at the end of the first period. The presence of the second stage allows banks to derive value from a bailout. [Fig. 1](#) outlines the stages of the game.

Banks and depositors are active in two periods that are bounded by three dates. Depositors can consume the proceeds of their deposits at the end of the game. Banks are owned and run by the same agents, who maximize their charter value over all periods and readjust their strategies at the start of each period. At the end of each period, the financial regulator evaluates whether it is optimal to bail out financially distressed banks.

2.1. Banks and depositors

A continuum of banks is indexed by i and uniformly distributed over the unit interval, $i \in [0, 1]$. A continuum of depositors chooses to supply funds for deposits at the start of each period, denoted by $D_t > 0$, where $t \in \{0, 1, 2\}$ indexes the dates. The amount D_t is not fixed over the periods. When a bank fails, the deposits are partially destroyed, but following favourable times, any positive returns get reinvested. A market maker is assumed such that all funds available for deposits in a period are equiproportional distributed among the banks, who are perceived as homogeneous by the depositors. Prior to the depositors' choice of savings, banks have not taken any action in the model.

In a given period, banks engage in perfect competition to access the deposit market, and the rate at which banks borrow funds from depositors is denoted by r_t^D . Depositors are paid at the end of a period. The funds obtained from the deposit market constitute the only form of financing for banks. At the start of each of the two periods, banks can decide on their allocation of raised deposits between *risky* assets and a *safe* asset, as in [Allen and Gale's \(2000\)](#) framework. [Appendix A.1](#) provides a detailed exposition of the technology underlying the safe asset.

Payoffs: Banks set strategies at $t=0$; if they survive the first period, they also set strategies at $t=1$. At these instances, banks determine the fraction of received deposits to be invested in the safe asset x_{it}^S and the fraction to be invested in the risky assets x_{it}^R . In addition, banks set the level of idiosyncratic volatility risk associated with the risky assets' return σ_{it} together with their choice of risky asset type class $\rho_{it} \in [\rho^l, \rho^h]$. For $\rho_{it} = \rho^h$, the bank decides to invest in assets to which all other banks have access, and for $\rho_{it} = \rho^l$, the bank invests in an asset class to which only that bank has access. Then, if all banks choose ρ^h , all their returns correlate, whereas if all banks choose ρ^l , there is minimal correlation among their assets' returns.³

A bank fails at the end of a period if the return on the risky asset R_{it+1} and the return on the safe asset r_{t+1}^S are insufficient. If the return on the risky asset is below r_{it+1}^c , the bank will be unable to repay depositors a return r_{t+1}^D . The critical return r_{it+1}^c can be inferred from the budget constraint:

$$r_{it+1}^c x_{it}^R + r_{t+1}^S x_{it}^S = r_{t+1}^D (x_{it}^R + x_{it}^S), \quad t \in \{0, 1\}.$$

Rearranging this condition yields $r_{it+1}^c = r_{t+1}^D + (r_{t+1}^D - r_{t+1}^S) x_{it}^S / x_{it}^R$. If $R_{it+1} \geq r_{it+1}^c$, the pecuniary gain for the bank owners is denoted by

$$(R_{it+1} x_{it}^R + r_{t+1}^S x_{it}^S - r_{t+1}^D (x_{it}^R + x_{it}^S)).$$

In this case, depositors receive $r_{t+1}^D (x_{it}^R + x_{it}^S)$. If the return on risky assets is insufficient, $R_{it+1} < r_{it+1}^c$, bank owners receive nothing, and depositors receive the remaining asset value of the bank, $(R_{it+1} x_{it}^R + r_{t+1}^S x_{it}^S)$.

Furthermore, I follow [Allen and Gale \(2000\)](#) and introduce the costs associated with investing in risky assets. These costs can arise from monitoring efforts, administration, risk management, and they are non-pecuniary. Banks have a parsimonious cost function $c: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ which features: $c(0) = 0$; $c'(0) = 0$; $c'(x) > 0$; and $c''(x) > 0$.

³ This conjecture reflects both empirical and theoretical findings in the literature on inter-bank return dependencies ([Acharya, 2009](#); [Farhi and Tirole, 2012](#); [Maksimovic and Zechner, 1991](#); [Rajan, 1994](#); [Shleifer and Vishny, 1992](#)).

The expected payoff of bank i in a given period can be denoted by $v: [0, \sigma_{\max}] \times \{\mathbb{R}^+\}^4 \rightarrow \mathbb{R}$ and defined by

$$v(\sigma_{it}, x_{it}^R, x_{it}^S, r_{t+1}^D, r_{t+1}^S) = \int_{r_{it+1}^c}^{r_{it+1}^{\max}} (R_{it+1} x_{it}^R + r_{t+1}^S x_{it}^S - r_{t+1}^D (x_{it}^R + x_{it}^S)) dh(\sigma_{it}, \cdot) - C(x_{it}^R), \quad (1)$$

where the return on the risky assets $R_{it+1} \sim h(\sigma_{it}, \rho_i)$, $t \in \{0, 1\}$. The density $h(\sigma_{it}, \rho_i)$ belongs to the joint probability measure $\mathcal{H}(\sigma_{it}, \rho_i, \sigma_{-it}, \rho_{-i})$; indexed by the parameters $\sigma_{it}, \rho_i, \sigma_{-it}, \rho_{-i}$; that feature mean-preserving spreads.⁴ Note that σ_{-it} and ρ_{-i} denote tuples that contain the composite actions of all other banks with respect to their choice of the risk parameters. For ease of exposition, the short-hand notation of $\mathcal{H}$ is used subsequently to denote the probability measure corresponding to the joint distribution of all banks' returns. The choice of risky asset class ρ_{it} does not directly affect the payoff (1), but affects the banks' charter value through the fraction of banks who survive the first period which is discussed in Section 2.2.

At $t=1$, a bank can enter a state of financial distress if the assets generate an insufficient return in the first period, $R_{i1} < r_{i1}^c$. With this financial distress condition, we construct a measure, from among the surviving banks on the set of banks that started in the first period. This measure is denoted by $s \in [0, 1]$, such that it reflects the proportion of banks that survive in the period between $t=0$ and $t=1$. A detailed description of the measure construction is available in Appendix A.2. Note that if a bank is financially distressed it does not automatically fail, because of the possibility of being bailed out in this case still exists. The measure s partly drives the regulator's choice whether to initiate bailout support, as described in the next Section 2.2.

2.2. Bailout policy and systemic risk formation

Banks face two types of risk: idiosyncratic risk and systemic risk. The idiosyncratic risk component, volatility σ_{it} , affects the probability of failure of the bank in a given period. The extent with which a bank's performance is detrimentally affected by other bank failures is denoted as systemic risk. Bank managers do not observe other banks' choices of asset classes. Instead managers infer the likely aggregate action of banks through their belief about whether banks receive bailout support if financially distressed. Equilibrium choices are derived by means of backward induction.

Objective of the regulator: The regulator is concerned with maintaining financial stability. Financial stability reflects in the paper the regulator's concern about the ensuing social costs corresponding to the failure of a substantial number of banks. To minimise these social costs, the regulator can bail out financially distressed banks and levy a systemic risk tax. The bailout decision depends on whether the cost of letting a bank fail outweighs the cost of saving it. The cost of letting bank i fail depends on bank i 's performance and the proportion of banks that have survived s until the bailout decision must be made, as denoted by $C(s)$ and feature $C'(s) < 0$. Therefore, the costs C feature the notion that a single bank's failure might be absorbed by the banking sector, but multiple bank failures may induce the failure of other banks and thus increase the costs associated with their failure (Acharya et al., 2011). The cost that the regulator perceives to be associated with initiating bailout support θ is constant and is not perfectly observed by banks. These costs are not necessarily limited to the actual absolute costs of a bank bailout, but also extend to the relative ease with which regulators and politicians can bail out financial institutions. Steiner (2014) finds empirical evidence that the accumulation of reserves intended to prevent financial crises may relax financing constraints for regulators, such that they might execute bailout policies more readily.

The accuracy with which banks observe θ constitutes the source of the regulator's ability to exercise constructive ambiguity about its bailout decision. This information framework is made explicit in Section 3. The decision to bail out the sector is denoted by $q \in \{0, 1\}$, where $q=1$ means a bailout to the distressed banks and $q=0$ the converse. Therefore, we can regard $\theta - C(s)$ as the net benefit of not bailing out distressed banks. This net benefit expression bears some similarity to the intuition described by Freixas et al. (2003) regarding the cost and benefits of a bailout.

The second tool at the regulator's disposal is the ability to levy a systemic risk tax, $\tau \in [\underline{\tau}, \infty)$, with $\underline{\tau} \geq 0$. The aim of such a tax is to discourage banks from engaging in activities that have the potential to contribute to the formation of systemic risk. In this light, recent literature suggests taxing banks according to a metric that generates a proxy of costs imposed on society by the bank's failure (Acharya et al., 2011; Adrian and Brunnermeier, 2011; Freixas and Rochet, 2011). That is, the tax would be levied according to the probability of bank i facing financial distress, given that at least one other bank is distressed.⁵ The tax therefore aims to lower banks' systemic exposure to one another. However, the tax also generates costs for the regulator measured in terms of the social welfare losses due to taxation.⁶ Because the tax implies an implicit transfer of funds from depositors to the regulator, those funds are no longer invested in productive activities, so they constitute a welfare cost to the regulator that can be denoted by $\delta: [\underline{\tau}, \infty) \rightarrow \mathbb{R}^+$, with the features $\delta'(\tau) > 0$ and $\delta(\underline{\tau}) = 0$.

⁴ The analysis is unaffected in terms of no qualitative differences in results if the expected return is non-decreasing and concave in the risk parameter σ_{it} . The assumption of mean-preserving spreads is made for ease of exposition.

⁵ The tax revenue per bank therefore amounts to $\tau \times \mathbb{P}^i(\{R_{i1} < r_{i1}^c\}, \{s < 1\})$ and is levied after banks have decided on their choice of risky asset class ρ_{it} . Page 11 and 12 contain a detailed discussion on how this tax enters the bank's objective function.

⁶ In a recent issue (April 18, The Economist, 2015), The Economist highlights that the introduction of macroprudential regulation can lead to more illiquid markets, which poses a systemic risk in itself. In this light, banks have cut back on their trading activities since 2008 as market making activities have become more expensive due to higher collateral requirements.

	Bank i fails	Bank i survives
No other bank fails	0	v^{SS}
A fraction s^* banks survive	$\pi_{i1} v^{SS}$	$\hat{\pi}_{i1} v^{SS} + (1 - \hat{\pi}_{i1}) v^{SF}(s^*)$
	0	$r_{i,1}^C$ $R_{i,1}$

Fig. 2. Payoffs at the End of the First Period. Notes: This figure depicts the combinations of bank i 's expected payoff values for four scenarios perceived at the start of the game, $t=0$. That is, v^{SS} denotes the payoff for bank i if all banks survive, or if a bailout is issued for distressed banks; $\hat{\pi}_{i1}$ denotes the probability that a fraction $1-s^*$ of the banks receive a bailout, conditional on bank i not being financially distressed; π_{i1} denotes the probability that bank i receives bailout support, conditional on a fraction $1-s^*$ being financially distressed; and $v^{SF}(s^*)$ denotes the payoff for bank i if it survives and only a fraction s^* of the other banks survive. Finally, τ denotes the systemic risk tax.

Depending on the bailout decision, the benefits associated with the bailout policy and the costs of the tax the following payoff for the regulator can be formulated:

$$(1-q)(\theta - C(s)) - \delta(\tau).$$

The bailout decision of the regulator also follows endogenously from the notion of time inconsistency in bailout policy (Acharya and Yorulmazer, 2007; Brown and Dinc, 2005; O'Hara and Wayne, 1990). If the costs associated with letting a bank fail C exceed the costs to bail out the bank θ , the regulator always bails out the bank. In comments on Panageas (2010), Albuquerque (2010) indicates that the social planner's ability to fund a bailout interacts with the decision to bail out and may give rise to a time-inconsistency problem, similar to the commitment problem faced by the regulator in the current model. Therefore, the bailout decision can be interpreted as an endogenous indicator variable, $q \equiv \mathbf{1}\{\theta < C(s)\}$, such that the objective function can be denoted by⁷

$$W(\theta, \tau, s) = \max\{0, \theta - C(s)\} - \delta(\tau). \quad (2)$$

In turn, banks form expectations about whether a bailout will be available to them if they become financially distressed. In this light, banks predict the probability associated with the event $\theta < C(s)$ as equivalent to the probability of receiving bailout support, because the costs of letting the bank fail exceed the bailout costs in this case. Increases in their portfolio weight ρ_i , assigned to asset classes that contribute to high correlation among banks, result in a higher likelihood of joint failure. Consequently, the probability of receiving bailout support increases as more banks opt to assign more weight to correlated asset classes in their portfolio. This scenario constitutes the nature of the strategic complementarity in banks' choice of asset class ρ_i .

Implications for banks: A bailout event enters each bank's payoff schedule, such that in the event of financial distress, the possibility of receiving a bailout exists, which allows the bank to operate in the second period. Other banks may also receive bailout support when they are financially distressed, which affects the payoff schedule for the surviving banks. For ease of exposition the following four states are considered and depicted in Fig. 2 for $t=1$: In state SS no banks are financially distressed; in state SF bank i is not financially distressed but at least one other bank is; and state FF denotes a state in which bank i is financially distressed along with at least one other bank.

Consider state SF to begin: Bank i is not distressed, yet a fixed fraction $1-s^*$ of banks faces financial distress. The financially distressed banks can be bailed out in this state, and bank i attaches a probability $\hat{\pi}_{i1}$ to this event. The payoffs associated with either action of the regulator are denoted by the charter value v^{SS} in the case of a bailout for other banks, and $v^{SF}(s^*)$ if no bailout is implemented. Based on Lemma 3 (see Appendices A.2 and A.3), it must be that $v^{SS} > v^{SF}(s^*)$ for any fraction of surviving banks $s^* \in [0, 1]$. When a bank fails a fraction of the deposits is destroyed in the bankruptcy process, which lowers the pool of deposits available for investments and thereby all remaining banks' charter value (Acharya and Yorulmazer, 2007; Allen and Gale, 2000). This result implies that a bailout can have a stabilising effect on the entire banking system.

⁷ Note that $q \equiv \mathbf{1}\{\theta < C(s)\}$ equals 1 if $\theta < C(s)$ and 0 otherwise.

Turning to state *FF* which differs from state *SF* in that bank *i* is financially distressed. The probability that bank *i* and other banks receive a bailout is now denoted by π_{i1} . The positive charter values in this state are equal to those in *SS*. If bank *i* were not distressed, the cost of any other bank failing would be strictly lower than in the case in which bank *i* is distressed. This difference can be made explicit by defining banks' ex ante expectations about the costs of failure to society as detailed in Section 3. A graphical depiction of the expected charter values for the various states and their corresponding probabilities is depicted in Fig. 2. Furthermore, if only bank *i* is financially distressed, the regulator will not initiate a bailout policy.

So far, the discussion and presentation of payoffs has focused on the particular case in which a fixed fraction of banks $s = s^*$ survives the first period of the game. By taking all possible states of the banking sector into account at $t=1$, the optimal strategy profile for bank *i* in the first period then can be denoted by the following choices:

$$\begin{aligned} \sigma_{it}^*, x_{it}^{R*}, x_{it}^{S*}, \rho_i^* \in \operatorname{argmax}_{\sigma_{it}, x_{it}^R, x_{it}^S, \rho_i} \{ & v(\sigma_{it}, x_{it}^R, x_{it}^S, r_{it+1}^D, r_{it+1}^S) + \mathbb{E}^{\mathcal{H}}(\{R_{i1} \geq r_{i1}^c\} \cap \{s = 1\})v^{SS} + \int_0^1 \mathbb{E}^{\mathcal{H}}(\{R_{i1} \geq r_{i1}^c\} \cap \{s = s^*\})(\hat{\pi}_{i1}v^{SS} \\ & + (1 - \hat{\pi}_{i1})v^{SF}(s^*))ds^* + \int_0^1 \mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\} \cap \{s = s^*\})(\pi_{i1}v^{SS} - \tau)ds^*, \end{aligned} \quad (3)$$

where $v(\cdot)$ is defined as in (1). For ease of exposition, I omit the arguments σ_{it} and ρ_i , which describe the return density, from the probability and expectation operators $\mathbb{E}^{\mathcal{H}}(\cdot)$ and $\mathbb{E}^{\mathcal{H}}[\cdot]$. Banks incorporate their expected charter value for the second period to determine their strategies. The operand in (3) is bank *i*'s charter value at the start of the first period, which can be written conveniently as⁸:

$$\begin{aligned} V(\sigma_{it}, \rho_i, x_{it}^R, x_{it}^S, r_{it+1}^D, r_{it+1}^S) = & v(\cdot) + \mathbb{E}^{\mathcal{H}}(\{R_{i1} \geq r_{i1}^c\})v^{SS} + \mathbb{E}^{\mathcal{H}}[\hat{\pi}_{i1}v^{SS} + (1 - \hat{\pi}_{i1})v^{SF}(s) - v^{SS} | \{s < 1\}] \\ & + \mathbb{E}^{\mathcal{H}}[\pi_{i1}v^{SS} - (\hat{\pi}_{i1}v^{SS} + (1 - \hat{\pi}_{i1})v^{SF}(s)) - \tau | \{R_{i1} < r_{i1}^c\}, \{s < 1\}]. \end{aligned} \quad (4)$$

The event in which bank *i* and at least one other bank are financially distressed only enters the last term in (4): the expected payoff of being financially distressed, conditional on more banks facing financial distress, $s < 1$. This term is also the only term that depends on bank *i*'s choice of asset class ρ_i , because the expectation is conditioned on the correlation between bank *i*'s returns and the other banks in the sector. Bank *i* will set x_{it}^{R*} , x_{it}^{S*} with an approach similar to the one it used in the first period for the second period. The key difference lies in setting σ_{it}^* , and ρ_i^* , which affect the distribution of returns for all banks and thus $\mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\}, \{s < 1\})$. However, ρ_i only affects the latter term, such that the optimal choice σ_{i1}^* can be expressed in terms of ρ_i^* . The joint probability of the event in which bank *i* fails and at least one other bank fails as well then is increasing in banks' contribution to the overall formation of systemic risk, namely all chosen correlation terms.

The regulator charges the bank an amount $\tau \times \mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\}, \{s < 1\})$ as the systemic risk tax at $t=0$, after the banks set their choice of ρ_i , such that the tax is incorporated in banks' decision about the asset classes. The probability of a crisis event involving bank *i*, $\mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\}, \{s < 1\})$, bears a similarity to the marginal expected shortfall (MES) of Acharya et al. (2011, 2012) and the Delta Conditional Value at Risk (ΔCoVaR) measure of Adrian and Brunnermeier (2011). Both the MES and the ΔCoVaR relate to this joint probability; they are empirical measures of the leftward shift in stock return distribution, driven by a conditioning event such as a market crash or the failure of a number of systemically important financial institutions. In our case, the conditioning event is in which at least one other bank other than bank *i* faces financial distress at $t=1$.

3. Equilibrium analysis

To evaluate the equilibrium decisions of banks at the start of the first period, it is necessary to determine banks' decisions with respect to their choice of asset class. Based on (4), each bank thus solves for:

$$\begin{aligned} \rho^*(\pi_i) \in \operatorname{argmax}_{\rho \in [\rho^l, \rho^h]} \{ & \mathbb{E}^{\mathcal{H}}[\pi_{i1}v^{SS} - \tau | \{R_{i1} < r_{i1}^c\}, \{s < 1\}] - b_i \}, \\ \text{where } b_i := & \mathbb{E}^{\mathcal{H}}[\hat{\pi}_{i1}v^{SS} + (1 - \hat{\pi}_{i1})v^{SF}(s) | \{R_{i1} < r_{i1}^c\}, \{s < 1\}]. \end{aligned}$$

With the assumption of perfect information, each bank has knowledge about the probability of receiving bailout support if it fails along with at least one other bank. The optimal strategy for choice of asset class then can be expressed as:

$$\rho^*(\pi_i) = \rho^h \mathbf{1} \left\{ \pi_i > \frac{b_i + \tau \mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\}, \{s < 1\})}{v^{SS}} \right\} + \rho^l \mathbf{1} \left\{ \pi_i \leq \frac{b_i + \tau \mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\}, \{s < 1\})}{v^{SS}} \right\}. \quad (5)$$

If banks know the regulator's cost of bailout θ , all banks set either $\rho^* = \rho^h$ or all choose $\rho^* = \rho^l$. This result is driven by the complementary nature of the choice of asset class. As more banks opt to assign more weight to the asset class which increases correlation among banks' asset returns, the likelihood of a joint failure of banks increases at the end of the first period. This development reinforces banks' incentives to opt for asset classes that contribute to higher levels of correlation among banks' asset returns. As more banks fail jointly, the regulator is more inclined to initiate bailout support. The strategic

⁸ Further simplification is possible, but retaining the bailout probability in the expectation operator allows us to let π_{i1} depend on banks' choices and the fraction of banks that survive s , in the formation of banks' beliefs about the regulator's type.

complementarity in the choice of asset class therefore results in multiplicity of equilibria, implying either that all banks choose either ρ^h or low ρ^l (Morris et al., 2003).

To avoid multiplicity of equilibria as a result of strategic complementarity, I consider a noisy perturbation in banks' knowledge about the cost for the regulator to initiate bailout support θ . This approach follows the global games methodology and relates closely to the work of Morris et al. (2003), and Angeletos et al. (2006). The imperfect knowledge of banks about θ generates the constructive ambiguity that the regulator can exploit. The perturbation maintains strategic complementarity in banks' asset class choices but also introduces additional uncertainty for banks surrounding the bailout policy. Rather than assuming that constructive ambiguity is present at all times, I explicitly incorporate constructive ambiguity endogenously, which offers the advantage of allowing the investigation of signaling effects on other policy tools for this strategy.⁹ In addition to functioning as an element of cost to banks, the systemic risk tax signals the type of regulator θ banks face.

To evaluate the signaling role of a systemic risk tax, Bayesian–Nash equilibria are considered for two cases. First, in a benchmark pooling equilibrium, the systemic risk tax is fixed and cannot be changed by the regulator. This approach is similar to the manner in which Morris et al. (2003) solve their seminal model on currency crises. In a second case, the regulator is allowed to change the systemic risk tax which results in a signaling game. To this end, let $\tau(\theta)$ be the systemic risk tax set by the regulator with bailout costs $\theta \in \mathbb{R}$. Furthermore, let $\rho^*(\xi_i, \tau(\theta))$ be bank i 's equilibrium choice of asset class based on the received signal $\xi_i \in \mathbb{R}$ about θ and observed tax τ . Last, let $\mu(\theta|\xi_i, \tau)$ denote bank i 's posterior distribution of θ , reflecting bank i 's beliefs about the cost of a bailout being lower than the cost of letting the bank fail. With the posterior distribution, banks infer the probability of receiving bailout support if they are financially distressed or $\pi_i \equiv \mu(\theta < \mathbb{E}^{\mathcal{H}}(C(i, s))|\xi_i, \tau)$, and $\mu(\mathbb{E}^{\mathcal{H}}(C(i, s))|\xi_i, \tau)$ in condensed form.

These expressions allow for an equilibrium definition that characterises the main actions by banks and the regulator in the first period of the model.

Definition 1. Equilibrium characterization: An equilibrium consists of a strategy for the regulator of type $\theta \in \mathbb{R}$, $\tau: [\underline{\tau}, \infty) \rightarrow \mathbb{R}^+$, a cumulative distribution function $\mu: \mathbb{R}^2 \times [\underline{\tau}, \infty) \rightarrow [0, 1]$, and a strategy for banks $\rho: \mathbb{R} \times [\underline{\tau}, \infty) \rightarrow [\rho^l, \rho^h]$, such that

$$\tau^*(\theta) \in \operatorname{argmax}_{\tau \in [0, D_t]} [W(\theta, \tau, s)];$$

$\mu(\theta|\xi_i, \tau)$ is obtained from Bayes' rule, wherever possible, that is for any τ such that $\tau = \tau^*(\theta)$ for some θ ;

$$\rho^*(\xi_i, \tau) \in \operatorname{argmax}_{\rho \in [\rho^l, \rho^h]} \{ \mu(\mathbb{E}^{\mathcal{H}}[C(s)]|\xi_i, \tau) v^{SS} - \tau \mathbb{E}^{\mathcal{H}}(\{R_{i1} < r_{i1}^c\}, \{s < 1\}) - b_i \}.$$

For the cases of both a fixed tax and a strategic tax, these equilibrium expressions are solved subsequently.

With respect to banks' choices for σ_{i0}^* , x_{i0}^{R*} , and x_{i0}^{S*} at the start of the first period, the choices are contingent on bank's choice of ρ_{i0} , as in Acharya (2009), which resembles the findings by Allen and Gale (2000). The choices do not have consequences for the main results of this paper though, so they are omitted.¹⁰

3.1. Fixed non-strategic systemic risk taxation

For this section the systemic risk tax is fixed, $\tau = \tau_0$, with $\tau_0 \in [\underline{\tau}, \infty)$. Therefore, $\tau^*(\theta) = \tau_0$ for all government types $\theta \in \mathbb{R}$. Banks remain uninformed by the systemic risk tax about the cost of bailouts. With imperfect knowledge, banks receive a continuous noisy signal about the regulator's cost of bailouts θ , i.e. $\xi_i = \theta + \varepsilon_i$, where $\varepsilon \sim F$ with density f and $\theta \sim G$ with density g . The signal and the prior knowledge of G allow banks to infer a posterior of θ . According to Bayes' rule, the posterior distribution of bank i about θ can be expressed as:

$$\theta|\xi_i \sim \mu(\theta|\xi_i) := \frac{\int_{-\infty}^{\theta} g(\tilde{\theta}) f(\xi_i - \tilde{\theta}) d\tilde{\theta}}{\int_{-\infty}^{\infty} g(\tilde{\theta}) f(\xi_i - \tilde{\theta}) d\tilde{\theta}} \quad (6)$$

Banks believe that they will receive a bailout if the cost of a bailout is sufficiently low, $\theta \leq \theta^*$. A bank with a signal ξ_i attaches the probability $\mu(\theta^*|\xi_i)$ to the event of receiving a bailout when financially distressed, conditional on at least one other bank being financially distressed. Given this probability, the optimal action of bank i is denoted by $\rho^*(\mu(\theta^*|\xi_i))$. The average action across banks can be denoted, under the true θ by

$$\int_{-\infty}^{\infty} \rho^*(\mu(\theta^*|\xi_i)) f(\xi_i - \theta) d\xi_i.$$

⁹ Assuming that constructive ambiguity holds at all times conditions the analysis on this presumption, such that interactions between policy tools and constructive ambiguity cannot be evaluated.

¹⁰ See proposition 2 in Acharya (2009) and the proof thereof for a detailed exposition of banks' choices.

The critical value θ^* corresponds with the ex ante expectation banks have of the cost of their failure to the regulator $C(s)$. Because the outcome of $C(s)$ is random, banks anticipate that the critical level of bailout costs θ^* equals $\mathbb{E}^{\mathcal{H}}(C(s)|\xi_i)$, which is increasing in banks' choice of ρ_i .¹¹ As banks opt for asset classes that are prone to be correlated, the joint failure probability increases, which in turn increases the expected cost to the regulator of letting a bank fail. For ease of exposition, I assume $\mathbb{E}^{\mathcal{H}}(C(s)|\xi_i)$ to be linearly increasing in the average of banks' asset class choices, such that the threshold condition $\theta^* = \mathbb{E}^{\mathcal{H}}[C(s)|\xi_i]$ can be stated as¹²:

$$\theta^* = \int_{-\infty}^{\infty} \rho^*(\mu(\theta^*|\xi_i))f(\xi_i - \theta^*)d\xi_i. \quad (7)$$

In addition, I assume for simplicity that if all banks choose ρ^l , the average action of banks' asset class choices equals zero, i.e. $\int_{-\infty}^{\infty} \rho^l f(\xi_i - \theta^*)d\xi_i = 0$. Setting this benchmark case does not affect the main qualitative result.

The aggregate action of the banking system is decreasing in θ , so under the critical value θ^* we have

$$\int_{-\infty}^{\infty} \rho^*(\mu(\theta^*|\xi_i))f(\xi_i - \theta)d\xi_i > \theta.$$

The cumulative distribution function of the private signals are increasing in ξ_i , because the cumulative distribution function F is increasing in its argument ε_i . This property implies that the aggregate action of the banking system is continuous and monotonically decreasing in θ . Therefore, (7) is a sufficient condition for the existence of a unique threshold equilibrium.

According to (7) and the monotonicity of μ , there exists a unique signal $\xi^* := \xi(\theta^*|\bar{\pi})$ for which a bank attaches a probability $\bar{\pi}$ to the event of receiving bailout support if it and at least one other bank are distressed. This signal can be inferred from the condition

$$\mu(\theta^*|\xi(\theta^*, \bar{\pi})) = \bar{\pi}.$$

Suppose bank i receives a signal $\xi(\theta^*, \bar{\pi})$ and believes with probability $\bar{\pi}$ that it will receive bailout support when it and at least one other bank are distressed. In addition, suppose bank j receives a signal $\xi_j < \xi(\theta^*, \bar{\pi})$. It then holds that $\mu(\theta^*|\xi_j) > \bar{\pi}$. The fraction of banks with stronger beliefs about receiving bailout support when at least one other bank fails, relative to bank i with belief $\bar{\pi}$, then can be denoted by:

$$\int_{-\infty}^{\xi(\theta^*, \bar{\pi})} f(\xi_i - \theta^*)d\xi_i = F(\xi(\theta^*, \bar{\pi}) - \theta^*).$$

Differentiating this expression with respect to $\bar{\pi}$ results in the density of beliefs among banks, $\gamma(\bar{\pi}|\theta^*)$, under the true θ^* , since μ is monotonically decreasing in ξ_i . Here, γ denotes the density of beliefs about receiving bailout support, distributed across the banking sector for a given type of regulator θ^* . Combined with condition (7), a change of variable ξ_i to $\bar{\pi}$ allows for computing the average action of the banks with respect to their choice of asset class, ρ_i :

$$\begin{aligned} \theta^* &= \int_0^1 \rho^*(\pi) \gamma(\pi|\theta^*) d\pi_i. \\ \text{Let } \hat{\theta} &:= \frac{\theta^* - \rho^l}{\rho^h - \rho^l}. \text{ Stated more conveniently in terms of banks' actions,} \\ \frac{\theta^* - \rho^l}{\rho^h - \rho^l} &= \frac{\int_0^1 \rho^*(\pi_i) \gamma(\pi_i|\theta^*) d\pi_i - \rho^l}{\rho^h - \rho^l} \\ \hat{\theta} &= \int_0^1 1 \left\{ \pi_i > \frac{b_i + \tau_0 \mathbb{E}^{\mathcal{H}}(\{R_{i1} < \Gamma_{i1}^c\}, \{s < 1\})}{vSS} \right\} \gamma(\pi_i|\theta^*) d\pi_i \end{aligned} \quad (8)$$

Based on (5) the parameter $\hat{\theta}$ can be interpreted as an index that indicates the proportion of banks that choose ρ^h , whereas the remaining fraction chooses ρ^l .

At first it seems that (8) implies an increase in the systemic risk tax τ_0 would discourage banks from choosing ρ^h . The inference from this observation is that the regulator is in a position to curtail systemic risk taxation by charging a sufficiently high systemic risk tax. This comparative static result is also the basis of the argument about why systemic risk measures should be used to tax systemically important financial institutions for their contribution to systemic risk (Acharya et al., 2012; Adrian and Brunnermeier, 2011). This result is plausible only if the tax is set independently of future bailout choices. However, if taxation and bailout choices both serve the same purpose of maintaining financial stability, then the tax may reveal future bailout choices, as addressed by Section 3.2.

3.2. Strategic systemic risk taxation

A regulator does not have full freedom in setting a systemic risk tax if it wants to maintain a degree of ambiguity about the future bailout policy. Only a limited set of regulator types exists for which it is optimal to levy a systemic risk tax to induce banks not to coordinate on bailout prospects, that is, not to choose highly correlated assets ρ^h . If the costs associated

¹¹ The expected costs of failure follows the analogy of a system wide shortfall (Acharya et al., 2011; Adrian and Brunnermeier, 2011).

¹² This simplification bears no consequences for the main qualitative results.

with an increased deadweight loss due to a high tax $\tau > \bar{\tau}$ exceed the net costs of letting a bank fail, the regulator finds the low tax $\underline{\tau}$ the optimal choice of systemic risk taxation. Likewise, the costs of letting banks fail can be too low to justify a higher tax, due to the ensuing welfare loss. In this case, banks are unlikely to coordinate, due to their prior belief that they face a tough regulator. These results contrasts with the comparative static derived from (8), which seems to suggest that a high systemic risk tax results in lower systemic risk formation for any type of regulator.

Two equilibria can be identified in a setting in which the regulator can choose $\tau \in [\underline{\tau}, \infty)$. In one, banks are unresponsive to a higher systemic risk tax, so it constitutes the pooling equilibrium; in the other a subset of regulator types benefits from setting a higher systemic risk tax, the separating equilibrium.

Proposition 1. (Equilibria with strategic systemic risk taxation): When the regulator is able to set $\tau^* \in [\underline{\tau}, \infty)$, two equilibria can be identified for the first period of the model, a pooling equilibrium (I) and a semi-separating equilibrium (II):

- I. There exists an equilibrium in which the regulator sets $\tau^*(\theta) = \underline{\tau}$, $\forall \theta \in \mathbb{R}$. Banks' optimal choice, and the regulator's bailout decision, are respectively given by:

$$\rho^*(\xi_i, \tau) = \begin{cases} \rho^h & \text{if } \xi_i < \xi^* \\ \rho^l & \text{otherwise;} \end{cases}$$

$$q(\theta) = \begin{cases} 1 & \text{if } \theta < \theta^* \\ 0 & \text{otherwise.} \end{cases}$$

The private signal $\xi^* = \xi(\theta^*, \cdot)$, the critical cost of initiating no bailout θ^* , and index $\hat{\theta}$ in (8) are the same as in Section 3.1 when $\tau_0 = \underline{\tau}$.

- II. When the regulator sets an optimal tax $\tau^*(\theta) \in [\underline{\tau}, \infty)$ there exists an equilibrium in which

$$\tau^*(\theta) = \begin{cases} \bar{\tau} & \text{if } \theta \in [\underline{\theta}, \bar{\theta}] \subset \mathbb{R} \\ \underline{\tau} & \text{if } \theta \notin [\underline{\theta}, \bar{\theta}]; \end{cases}$$

$$\rho^*(\xi_i, \tau) = \begin{cases} \rho^h & \text{if } \xi_i < \xi^* := \xi(\underline{\theta}, \cdot) \text{ or } \tau \in (\underline{\tau}, \bar{\tau}) \\ \rho^l & \text{otherwise;} \end{cases}$$

$$q(\theta) = \begin{cases} 1 & \text{if } \theta < \underline{\theta} \\ 0 & \text{otherwise.} \end{cases}$$

Here $\underline{\theta} \leq \theta^* \leq \bar{\theta}$, and $\underline{\tau} \leq \bar{\tau}$. Additionally, the equilibrium values $\underline{\theta}$, and $\bar{\theta}$ solve

$$\underline{\theta} = \delta(\tau^*) = \int_{-\infty}^{\infty} \rho^*(\mu(\bar{\theta}|\xi^*))f(\xi^* - \bar{\theta})d\xi_i,$$

where the last term denotes the average action across banks.

Proof. See Appendix B.

The first equilibrium I denotes a pooling equilibrium in which banks are unresponsive to the regulator's choice of τ . The optimal choice of taxation by the regulator in this case is $\underline{\tau}$, because the tax will not affect the bank's choices, and setting it higher only results in a welfare loss. This equilibrium is similar to the one derived in Section 3.1.

In the second equilibrium II, banks are responsive to the regulator's tax, and it is in the interest of the regulator not to raise the tax beyond $\bar{\tau}$. A higher tax only results in additional costs to society, while banks already have been successfully deterred in their choice for the correlated asset class ρ^h by the high tax $\bar{\tau}$. The regulator's choice $\bar{\tau}$ is dominated by the lower tax $\underline{\tau}$ if the net costs to bail out banks are *not* sufficiently high or if banks deem it unlikely for the regulator to initiate a bailout policy. This result is derived from the regulator's objective function (2) and the average action of banks with respect to their choice of ρ . Two conditions prevail that identify the types of regulator that do not prefer $\bar{\tau}$ over $\underline{\tau}$:

$$\int_{-\infty}^{\infty} \rho^*(\mu(\theta|\xi_i))f(\xi_i - \theta)d\xi_i < \delta(\bar{\tau}), \quad \text{and} \tag{9a}$$

$$\theta < \delta(\bar{\tau}). \tag{9b}$$

Condition (9a) yields a $\bar{\theta}$ for which a regulator of type $\theta > \bar{\theta}$ prefers to set $\underline{\tau}$, because the costs of letting banks fail is not sufficiently high. In turn banks are likely to be already deterred due to their private information about $\theta > \bar{\theta}$, which induces a large fraction of banks to believe that a bailout is not a likely outcome during a crisis, in that the costs of a bailout are perceived as too high.

Condition (9b) instead indicates that for $\theta < \bar{\theta} = \delta(\bar{\tau})$, the regulator has no incentive to set $\bar{\tau}$, because the costs of the tax do not outweigh the cost of a bailout. In this case, banks' private information is likely to induce a large fraction of banks to believe that $\theta < \bar{\theta}$, such that a large fraction of them believe they will receive bailout support during a crisis event. However,

their observation of τ for such a regulator renders the banks unsure about whether they face a regulator of a type $\theta < \underline{\theta}$ that is inclined to bail banks out or of a type $\theta > \bar{\theta}$ that is not inclined to do so. This ambiguity is the constructive ambiguity that a weak regulator, $\theta < \underline{\theta}$, can generate by setting a tax τ to imitate the tougher regulator. Setting any other tax between $\underline{\tau}$ and $\bar{\tau}$ would immediately reveal that the regulator is of type $\theta < \underline{\theta}$ and cause the banks to coordinate on bailout prospects, because a regulator of type $\theta \in [\underline{\theta}, \bar{\theta}]$ does not set a tax $\tau < \bar{\tau}$.

4. Discussion

The main results of this study outline the limitation on regulators that want to set an optimal systemic risk tax for banks when the tax reveals the regulator's inclination to initiate bailouts to banks. In Section 3.1, the average action across banks in equilibrium, (8), with respect to their choice of asset class, ρ , suggests that banks' contribution to the formation of systemic risk can be mitigated by setting a higher tax, regardless of the regulator's bailout policy. However, this result hinges on the implicit assumption that the systemic risk taxation policy can be carried out independently from the constructive ambiguity about bailout policy.

This result contrasts with the case in which the tax is strategically set by the regulator, as discussed in Section 3.2. Failing to account for the tax's ability to reflect the regulator's objective of safeguarding financial stability may lead to spurious conclusions about optimal systemic risk taxation. The regulator's objective to safeguard financial stability renders the two policy tools interdependent. If the regulator's inclination to initiate bailout support is high, $\theta < \underline{\theta}$, a higher tax may not be a sufficient deterrent for banks to invest in correlated assets. Accordingly, the banks continue to coordinate, opt for correlated assets, and set ρ^h . A tax higher than $\bar{\tau}$ only results in welfare loss.

The failure of intermediate levels of taxation, $\tau \in (\underline{\tau}, \bar{\tau})$, to deter systemic risk formation is due to the private information that banks have with respect to the regulator's inclination to initiate bailout support. The observation of an intermediate tax induces banks to believe that a sufficiently high tax is apparently sub optimal for the regulator they face, because the regulator is of a type that is likely to initiate bailout support. The regulator with high bailout costs is ruled out, because it is optimal for this regulator not to deviate from the low tax. Therefore, any tax of the level $\tau \in (\underline{\tau}, \bar{\tau})$ that does not reflect the action of a regulator facing relatively high costs associated with the bailout of a bank instead reveals to banks that the regulator is of the type highly inclined to bail out banks $\theta < \underline{\theta}$.

An intermediate tax is thus informative for banks, which impairs the ability of the regulator to be ambiguous about future bailout support to deter systemic risk formation. For regulators with a high inclination to initiate bailout support, it is optimal not to change the systemic risk tax but instead keep banks in the dark about their bailout policy. A regulator with a high inclination to initiate bailout support thus imitates the action of a regulator that is *not* inclined to initiate bailout support. The ensuing ambiguity between both types drives uncertainty for banks, which the regulator can then exploit as constructive ambiguity to lower banks' preferences for correlated assets.

As Condition (9b) shows, the accuracy with which banks perceive the regulator's inclination to initiate bailout support has no influence on the lower threshold $\underline{\theta}$ of the type of regulator that is willing to set $\bar{\tau}$. Accordingly, the results are robust to changes in the specifications of the noisy perturbation. For the set of regulators for which $\theta < \underline{\theta}$, the optimal taxation remains the original, or low tax $\underline{\tau}$, regardless of the distribution of private information across banks. The only requirement is that banks do not have perfect information about the regulator's inclination to initiate bailout support, in order to avoid multiplicity. The upper limit of regulator types for which it is optimal to set $\bar{\tau}$ is increasing in the average action of banks with respect to their choice of asset class.

The derivation of the separating equilibrium implicitly contains an application of the refinements proposed by Cho and Kreps (1987) to determine the optimal levels of taxation for the regulator. The separating equilibrium grants the regulator the freedom to choose the tax. If the regulator sets $\bar{\tau}$ it faces the minimum amount of distortion costs, yet deters banks successfully by inducing banks to choose ρ^l . Other regulators that prefer to set $\underline{\tau}$ do so either because, the costs corresponding to bank failure are small (a tough regulator), or because the distortion cost of a high tax are perceived to be higher than the net bailout costs (weak regulator). Any tax $\tau \in (\underline{\tau}, \bar{\tau})$ could not make any regulator better off. The derived separating equilibrium constitutes therefore also the best separating equilibrium outcome.

In the pooling equilibrium any taxation level $\tau \geq \bar{\tau}$ is equilibrium dominated for the regulator of type $\theta \notin [\underline{\theta}, \bar{\theta}]$. In addition, once the banks' beliefs following this taxation choice are such that they assign probability 1 to the regulator type being $\theta \in [\underline{\theta}, \bar{\theta}]$, this regulator type prefers to deviate to $\bar{\tau}$, such that distortion costs are minimal. While the weak regulator settles on the lowest tax possible $\underline{\tau}$, because any tax $\tau \in (\underline{\tau}, \bar{\tau})$ reveals that $\theta < \underline{\theta}$. Based on the intuitive criterion, the best separating equilibrium therefore constitutes the unique equilibrium.

In Appendix A.4 the case is considered where the systemic risk tax generates revenue y , which is used to finance future bailouts, and thereby lowers the regulator's cost of bailout θ . The introduction of the tax income is an adaptation of the model presented in Section 3.2. For this case we are also in a position to identify a set of regulator types $\theta \in [\underline{\theta}', \bar{\theta}']$ that prefers to set a high tax $\bar{\tau}' > \underline{\tau}$, and a set of regulator types $\theta \notin [\underline{\theta}', \bar{\theta}']$ that prefers to set $\underline{\tau}$. The inclusion of the tax revenue corroborates the earlier findings and demonstrates the robustness of the results.

However, the inclusion of the systemic risk tax's revenue in the regulator's objective function may not necessarily establish an upper bound $\bar{\theta}'$, in which case all regulator types of $\theta \geq \theta'$ prefer to set the high tax $\bar{\tau}'$. In this case even the tough regulator $\theta > \bar{\theta}'$ finds it attractive to set a high tax, because of the income generated by the tax. In this case banks do

not require their private information, because observing tax $\bar{\tau}'$ reveals that the regulator's type is $\theta \geq \theta'$, and a tax $\tau < \bar{\tau}'$ therefore reveals that the regulator is of type $\theta < \theta'$. Since a weak regulator does not gain from setting a tax $\tau \in (\underline{\tau}', \bar{\tau}')$ it prefers to set $\underline{\tau}$, because all banks set ρ^h for when they observe $\tau \in (\underline{\tau}, \bar{\tau}')$. This result again demonstrates the lack of degrees of freedom a regulator faces in setting a taxation policy in light of its future bailout policy that may encourage the hazardous coordination among banks.

5. Conclusion

The interdependence between systemic risk taxation and constructive ambiguity is relevant for their effectiveness, because banks can adjust their risk profile *after* the implementation of a systemic risk tax scheme and *before* a bailout policy is executed. This window of opportunity allows banks to engage in risk-shifting behaviour, driven by the prospect of receiving bailout support. In this setting the level of taxation can be informative about the regulator's future stance on how to maintain financial stability. Banks learn, through the perceived objective of the regulator, the conditions in which bailout support is likely to be initiated. Introducing a systemic risk tax therefore limits the degree of ambiguity the regulator can maintain about its bailout policy for distressed banks. Conversely, if the regulator desires to maintain an ambiguous bailout policy, prospective risk shifting induced by the signalling effect of a systemic risk tax should be incorporated into the decision about the tax level. This finding suggests the existence of a trade-off between the two policy tools, which results from evaluating systemic risk taxation and ambiguous bailout programs in a joint framework.

The implications of this trade-off for macroprudential policy inferences are two-fold. First, to evaluate the effectiveness of regulatory policy tools, those policies must be considered in a joint framework if they serve the single objective of maintaining financial stability. Failing to account for the interdependence between policy tools can lead to spurious outcomes with respect to the policies' effectiveness in handling financial crises. I find that for regulators with a high inclination to initiate bailout support, the introduction of a systemic risk tax can fail to deter systemic risk formation. Second, the model adds the caveat that conditioning a model and its outcomes on the assumption that constructive ambiguity is effective at all times may give rise to spurious results. Maintaining constructive ambiguity imposes restrictions on the regulator's ability to set a systemic risk tax. If constructive ambiguity is assumed to hold at all times, the restrictions on the level of a systemic risk tax are ignored, which can generate risk-shifting behaviour by banks.

To avoid the trade-off, the taxation corresponding to a systemic risk measure is required to be set, in a credible manner, independent from the future bailout policy choice. This condition requires two independent regulatory institutions acting independently from each other: one who sets the systemic risk taxation policy, and another deciding on the bailout policy. In this context, systemic risk taxation may, currently, be most credibly conducted by supranational regulatory authorities. Provided that the two policies can be set independently.

Appendix A. Consequences of bank failure

A.1. Safe assets

The return on the safe asset is denoted by r_t^S , which materializes at the end of a period. Banks channel a proportion of their raised deposits to a sector in the economy that employs a neoclassical risk-free technology. The firms in this sector and banks engage in perfect competition in accessing the market for the safe asset. Therefore, r_t^S equals the marginal rate of return on capital of the risk-free technology. Let $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ denote the production technology of the risk-free asset that features $f'(x) > 0$; $f''(x) < 0$; $\lim_{x \downarrow 0} f'(x) \rightarrow \infty$; and $\lim_{x \uparrow \infty} f'(x) = 0$. In this context, x denotes the total amount invested in the risk-free asset. Perfect competition implies the equilibrium condition $r_t^S = f'(x)$.

A.2. Size of banking sector

To construct a measure of the size of the banking sector at the start of the second period, the set Ω is defined to contain all possible outcomes at $t=1$ with respect to which banks survive and fail. This set can be interpreted as all possible outcomes of tossing a continuum of coins that flip with a fixed, possibly different, probability of landing on heads. Each toss represents a bank, and head and tail events respectively correspond with the complementary events of survival of a bank or a state of financial distress. The event of financial distress for bank i is denoted by $\{R_{it+1} < r_{it+1}^c\}$. A size measure of the banking sector then can be constructed from the field $\mathcal{F}$ that contains the collection of all subsets of Ω . The size of the banking sector in the second period can be denoted by a function $s: \mathcal{F} \rightarrow [0, 1]$, where s is strictly increasing in the proportion of banks that survive the first period.

A.3. Equilibrium strategies and payoff values

Equilibrium at $t = 1$: The state SF is characterized by the survival of bank i and the failure of at least one bank in the first period. The optimal strategy profile of bank i is denoted by:

$$\sigma_{it}^*, x_{it}^{R*}, x_{it}^{S*} \in \operatorname{argmax}_{\sigma_{it}, x_{it}^R, x_{it}^S} v(\sigma_{it}, x_{it}^R, x_{it}^S, r_{t+1}^D, r_{t+1}^S). \quad (10)$$

For this state, v is defined by

$$v^{SF}(\cdot) = \int_{r_{it+1}^c}^{r_{it+1}^{max}} (R_{it+1} x_{it}^R + r_{t+1}^S x_{it}^S - r_{t+1}^D (x_{it}^R + x_{it}^S)) dh(\sigma_{it}, \cdot) - c(x_{it}^R). \quad (11)$$

For state SS, v is expressed as v^{SS} , and can be regarded as a specific case of v^{SF} in which all banks survive. A simplification is derived for v before solving for the arguments in (10). This simplification is summarized in [Lemma 1](#).

Lemma 1. (Return on deposits): In any existing equilibrium, $r_{t+1}^D = r_{t+1}^S = r_{t+1}$, i.e. the return on the safe asset is equal to the return demanded by depositors. This rate of return is equal to the critical return of banks. Hence, $r_{it+1}^c = r_{t+1}^D = r_{t+1}^S = r_{t+1} \forall i$.

Proof. See [Appendix B](#).

[Lemma 1](#) allows for the expression

$$v^{SF}(\cdot) = \int_{r_{it+1}}^{r_{it+1}^{max}} (R_{it+1} - r_{t+1}) x_{it}^R dh(\sigma_{it}, \cdot) - c(x_{it}^R).$$

Bank i 's decision to invest in the risky asset, $x_{it}^{R*}(\sigma_{it}, r_{t+1})$, given σ_{it}, r_{t+1} , is implied by the first order condition¹³:

$$\int_{r_{it+1}}^{r_{it+1}^{max}} (R_{it+1} - r_{t+1}) dh(\sigma_{it}, \cdot) = c'(x_{it}^{R*}).$$

Since density h is from a family of mean-preserving spread densities, the first-order condition can be rearranged as

$$\bar{R}_{it+1} - c'(x_{it}^{R*}) = r_{t+1} + \int_0^{r_{it+1}} (R_{it+1} - r_{t+1}) dh(\sigma_{it}, \cdot), \quad (12)$$

where $\bar{R}_{it+1}$ denotes the mean return value. The right-hand side of (12) is non-negative. A solution for x_{it}^{R*} thus exists, provided that the mean return $\bar{R}_{it+1} \geq c'(D_{it})$.

Lemma 2. (Equilibrium existence at $t = 1$): For state SF, the symmetric equilibrium $(x_t^{R*}, \sigma_t^*, r_{t+1}^*)$ exists at $t = 1$, with $\sigma_t^* = \sigma^{max}$.

Proof. See [Appendix B](#).

[Lemma 2](#) summarizes the equilibrium for state SF at time $t = 1$. For state SS, there exists a symmetric equilibrium as well. The proof is similar to that of [Lemma 2](#), because SS is a special case with $s = 1$. No banks have failed prior to arriving in state SS, yet the equilibrium values may be different for both states for different values of s . Comparative statics associated with these two variables are of interest for inferring the implications of bank failure for surviving banks' charter values.

Lemma 3. (Comparative static for state "SF"): Let the charter value of a bank in state SF be denoted by v^{SF} . Since the equilibrium at $t = 1$ is driven by r_{t+1}^* , v^{SF} is solely expressed in terms of r_{t+1}^* . For similar reasons, r_{t+1}^* is expressed in terms of s . Then the following result holds:

$$\frac{\partial v^{SF}(r_{t+1}^*(s))}{\partial s} > 0, \quad \forall s \in [0, 1].$$

Proof. See [Appendix B](#).

The comparative static in [Lemma 3](#) suggests that if a group of banks fail in the first period, a fraction of the deposit pool is destroyed. This scenario raises the return on the safe asset, so banks substitute safe investments for risky investments. However, no profits are reaped in the market for the safe asset, due to existence of perfect competition. The banks' charter values accordingly decline.

A.4. Financing bailout support with the systemic risk tax

In this section an adaptation of the benchmark cost function of the regulator (2) is considered in which the regulator uses the revenue raised from the systemic risk tax to lower the cost of initiating bailout support θ . The original cost schedule (2)

¹³ Since $c'(x) > 0$, $\forall x$, x_{it}^{R*} denotes a global maximizer of v .

reads:

$$W(\theta, \tau, s) := \max\{0, \theta - C(s)\} - \delta(\tau).$$

The income generated with the tax is derived from the reduction of the bank's charter value (4) due to the levy. Based on the bank's objective function the regulator raises the amount $\tau \times \mathbb{P}^H(\{R_{i1} < r_{i1}^c\}, \{s < 1\})$ in taxes per bank at $t=0$ after the banks set ρ_{it} , which is increasing in the bank's asset class choice ρ_i through the joint failure probability. The amount raised per bank renders the total tax revenue increasing in the level of the tax set by the regulator; and increasing in the probability of a bank failing conditional on other banks being financially distressed.

To specify a general revenue function of the tax, let the revenue function $y: [\underline{\tau}, \infty) \times \mathbb{R}: \rightarrow \mathbb{R}^+$ be defined as $y(\tau, \theta)$. The last argument θ is added to link the tax's income to banks' joint probability of failure, $\mathbb{P}^H(\{R_{i1} < r_{i1}^c\}, \{s < 1\})$, and thereby the aggregate level of correlation which is driven in equilibrium by θ through the private information banks use to decide on the level of ρ_{it} . In the stage where the regulator decides on the tax, banks have only their private information to infer, such that higher values of θ induce banks to not coordinate on bailout support, because the private information reveals for a higher θ also to a larger set of banks a tougher regulator, i.e. a regulator not willing to bail out banks due to high bailout costs. In this light, we have $y'_\tau(\tau, \theta) > 0$ and $y'_\theta(\tau, \theta) < 0$. The choice for letting y directly depend on θ rather than a realization of ρ_i is to take into account the possible change in banks' beliefs after the regulator has set the tax.

The effect of the revenue raised with the tax lowers the benchmark cost θ of a bailout and results in a lower net cost of bailout, or a higher net benefit $\theta - y(\tau, \theta)$, because the tax is used to finance the bailouts.

Taking into account the tax revenue allows for the adaptation of the original cost function of the regulator, resulting in

$$W(\theta, \tau, s) := \max\{0, \theta - y(\tau, \theta) - C(s)\} - \delta(\tau).$$

The original assumptions are that $\delta(\underline{\tau}) = 0$; and if the regulator sets the tax such that no bank has an incentive to set ρ^h we have $\mathbb{E}^H(C(s)|\bar{\tau}') = 0$ as a benchmark. With the addition of the income generated by the tax I assume for simplicity that if the tax is set at its lowest level, then $y(\underline{\tau}, \theta) = 0$.

From the perspective of the banks, a regulator prefers to set $\underline{\tau} < \bar{\tau}'$ if and only if $\mathbb{E}^H[W(\theta, \bar{\tau}', s)] < \mathbb{E}^H[W(\theta, \underline{\tau}, s)]$ which implies that the following two conditions need to hold:

$$\begin{aligned} \theta - \mathbb{E}^H[y(\bar{\tau}', \theta)|\bar{\tau}'] - \delta(\bar{\tau}') &< 0, \\ \theta - \mathbb{E}^H[y(\bar{\tau}', \theta)|\bar{\tau}'] - \delta(\bar{\tau}') &< \theta - \mathbb{E}^H[C(s)|\underline{\tau}]. \end{aligned}$$

These two conditions allow us to identify the types of regulator θ that prefer to set $\underline{\tau}$:

$$\begin{aligned} \theta &< \mathbb{E}^H[y(\bar{\tau}', \theta)|\bar{\tau}'] + \delta(\bar{\tau}'), \quad \text{and} \\ \mathbb{E}^H[C(s)|\underline{\tau}] &< \mathbb{E}^H[y(\bar{\tau}', \theta)|\bar{\tau}'] + \delta(\bar{\tau}'). \end{aligned}$$

Based on the assumption that the tax revenue is low when banks face a tough regulator with high bailout costs, $y'_\theta(\tau, \theta) < 0$, then the first condition provides a lower bound $\underline{\theta}'$. Such that the regulator of type $\theta < \underline{\theta}'$ prefers to set $\underline{\tau}$.

The second condition either provides an upper bound to the type of regulator that prefers to set $\bar{\tau}'$, or is satisfied. First we consider the case where the condition provides an upper bound $\bar{\theta}'$. We have that the expected cost of bank failure, and the expected revenue raised at a given tax level are both decreasing in the bailout cost θ . A higher θ results in a lower likelihood that banks coordinate on bailout prospects by choosing more similar, and therefore more correlated, assets for investments, which in turn lowers both the revenue raised from the tax, and lowers expected future costs of banks failing. The second condition, provides an upper bound if and only if for a particular upper bound value $\bar{\theta}'$ at which the expected cost of banks failing $\mathbb{E}^H(C(s)|\xi_i, \tau')$ are higher than $\mathbb{E}^H(y(\bar{\tau}', \bar{\theta}')|\xi_i, \tau')$ for any $\theta > \bar{\theta}'$. Because for $\theta > \bar{\theta}'$ the condition would not hold.

I find for this case where an upper bound $\bar{\theta}'$ can be established therefore an equilibrium that is of the same type as the equilibrium presented in Proposition 1. I identify again three types of regulators. The weak regulator $\theta < \underline{\theta}'$ who finds it optimal to set a low tax $\underline{\tau}$ in order to imitate the tougher type $\theta > \bar{\theta}'$. The latter tougher type faces such low costs to bail out banks, and despite the added benefit of the tax lowering bailout costs further, finds it still not profitable to set a high tax. Banks have in this case already a prior that they face a tough regulator and a high tax is in this case not sufficient, because it induces only a small fraction of banks to set a lower correlated risk profile ρ^l . The intermediate type $\theta \in [\underline{\theta}', \bar{\theta}']$ finds it again profitable to set $\bar{\tau}'$, because for this type the expected cost of banks failing outweighs the distortionary cost of the tax. If the upper bound can be established the proof of Proposition 1 can be applied and shows that the original qualitative result of Proposition 1 remains unaffected for the equilibrium where the revenue derived from the tax are used to finance future bailout support.

Second, in case the second condition is always satisfied, a new equilibrium can be derived. In this case, an upper limit to the regulator type θ that prefers to set $\bar{\tau}'$ cannot be established; only the lower limit $\underline{\theta}'$ based on the first condition. We are left with the types $\theta < \underline{\theta}'$ to prefer $\underline{\tau}$ and $\theta \geq \underline{\theta}'$ to prefer $\bar{\tau}'$. An interesting feature of this equilibrium is that it renders the private information of banks redundant. The moment banks observe a tax $\bar{\tau}'$, banks know the regulator must be of type $\theta \geq \underline{\theta}'$ and banks are successfully deterred in their choice ρ^h due to the high tax, and all banks set ρ^l . If banks observe $\underline{\tau}$ banks know the regulator is of type $\theta < \underline{\theta}'$ and all banks choose ρ^h , because, unlike the separating equilibrium with an upper bound to θ , there is no possibility banks face a tough regulator with high bailout costs $\theta > \bar{\theta}'$. In this equilibrium there is no constructive ambiguity. Although the case with no upper bound to θ is different from the original separating equilibrium, it

illustrates the signalling aspect of the tax. Ignoring the bailout and the possibility of banks coordinating on bailout prospects can render the tax harmful in both settings.

To illustrate this last point, consider for both cases that it holds that any tax below the tax level $\bar{\tau}'$ set by a weak type $\theta < \theta'$ constitutes an out-of-equilibrium action that induces all banks to coordinate on bailout prospects by setting ρ^h . Both cases therefore demonstrate, as in the original result, that a regulator should be cautious in setting its systemic risk taxation policy. A unique systemic risk tax corresponding to each type of regulator based on a systemic risk measure does not necessarily exist if the tax signals the likelihood of future bailout prospects.

Appendix B. Proofs

Proof. Lemma 1 (Return on deposits): $r_{t+1}^D = r_{t+1}^S = r_{it+1}^c$. Suppose $r_{t+1}^D > r_{t+1}^S \geq 0$. From (11), the critical return is given by

$$r_{it+1}^c = r_{it+1}^D + (r_{it+1}^D - r_{it+1}^S) \frac{x_{it}^S}{x_{it}^R}.$$

In turn, for $r_{t+1}^D > r_{t+1}^S \geq 0$, bank i has no demand for the safe asset. However, this implies that $r_{t+1}^S = \lim_{x \rightarrow 0} f'(x) \rightarrow \infty$, which constitutes a contradiction.

Suppose $0 \leq r_{t+1}^D < r_{t+1}^S$. Then r_{it+1}^c is decreasing in x_{it}^S , and (11) is increasing in x_{it}^S . Bank i has therefore an infinite demand for the safe asset. However, for a limited supply of deposits D_t , the budget constraint or short sale constraint would be violated, which also constitutes a contradiction. Hence, we are left to conclude that $r_{t+1}^D = r_{t+1}^S = r_{it+1}^c \forall i$. $\square$

Proof. Lemma 2 (Equilibrium existence at $t = 1$): For state SF, the symmetric equilibrium $(x_t^{R*}, \sigma_t^*, r_{t+1}^*)$ exists at $t = 1$, where $\sigma_t^* = \sigma_t^{\max}$. The choice of x_t^{R*} is obtained by solving (12), which supports the definition of x_t^{R*} as a function of σ_{it} , r_t , such that $x_t^{R*} \equiv x_t^{R*}(\sigma_{it}, r_t)$. Substitution in (11), and noting Lemma 1 along with first-order condition (12), yields:

$$\frac{\partial v(x_{it}^{R*}(\sigma_{it}, r_t), \sigma_{it}, r_t)}{\partial \sigma_{it}} = x_{it}^{R*}(\sigma_{it}, r_t) \frac{\partial}{\partial \sigma_{it}} \int_0^{r_{it+1}} (r_{t+1} - R_{it+1}) dh(\sigma_{it}, \cdot) > 0.$$

The inequality on the right-hand side holds, because expected losses are increasing in σ_{it} , as set by the bank. Independent of r_{t+1} , it is optimal for banks to set $\sigma_t^* = \sigma_t^{\max}$.

The rate of return on the safe asset is now obtained by an application of Brouwer's fixed point theorem (Mas-Colell et al., 1995, p. 952). The total amount invested in the risk-free asset in state SF amounts to $sD_t - x_t^{R*}(\sigma_t^{\max}, r_{t+1})$. Since f , the technology of the risk-free asset, is C^2 and concave on the domain $\mathbb{R}^+$. The unique equilibrium rate of return on the safe asset can be obtained from the condition:

$$r_{t+1}^* = f'(sD_t - x_t^{R*}(\sigma_t^{\max}, r_{t+1}^*)).$$

Therefore, the equilibrium $(x_t^{R*}, \sigma_t^*, r_{t+1}^*)$ exists at $t = 1$ for state SF. $\square$

Proof. Lemma 3 (Comparative statics for state “SF”): For a bank in state SF at time $t = 1$, the following holds: $\frac{\partial v^{SF}(\cdot)}{\partial s} > 0$. For ease of exposition, the subscript $\sigma_t^{\max}$ is dropped for x_t^{R*} , such that $x_t^{R*} := x_t^{R*}(r_{t+1})$. Since x_t^{R*} is solely determined by (12), the derivative $\frac{dx_t^{R*}(r_{t+1})}{dr_{t+1}}$ is obtained from a total differentiation of (12), which yields:

$$\frac{dx_t^{R*}(r_{t+1})}{dr_{t+1}} = \frac{-1}{c'(x_t^{R*}(r_{t+1}))} \left(1 - \frac{d}{dr_{t+1}} \int_0^{r_{t+1}} (r_{t+1} - R_{it+1}) dh(\sigma_{it}, \cdot) \right) = \frac{-1}{c'(x_t^{R*}(r_{t+1}))} (1 - \mathbb{P}^{\mathcal{H}}(R_{it+1} \leq r_{t+1})) < 0.$$

The second step of the proof is to evaluate the effect of r_{t+1} on the charter value. Combining the results of Lemmas 1 and 2 in (11) yields:

$$\frac{dv(x_{it}^{R*}(r_{t+1}), r_{t+1})}{dr_{it}} = -x_{it}^{R*}(r_{t+1}) \left(1 + \frac{d}{dr_{t+1}} \int_0^{r_{t+1}} (r_{t+1} - R_{it+1}) dh(\sigma_{it}, \cdot) \right) < 0.$$

To prove the first result about the effects of s on the charter value of a bank in state SF, let $r_{t+1}^* = r_{t+1}^*(s)$. Then from the equilibrium condition for the risk-free rate we have

$$r_{t+1}^*(s) - f'(sD_t - x_t^{R*}(r_{t+1}^*(s))) = 0.$$

Applying the implicit function theorem yields:

$$\frac{dr_{t+1}^*(s)}{ds} = \frac{f'(sD_t - x_t^{R*}(r_{t+1}^*(s)))D_t}{1 + f''(sD_t - x_t^{R*}(r_{t+1}^*(s))) \frac{dx_t^{R*}(r_{t+1})}{dr_{t+1}}} < 0,$$

where the denominator is strictly positive. Therefore,

$$\frac{dv(x_{it}^{R*}(r_{t+1}(s)), r_{t+1}(s))}{ds} = \frac{dv(x_{it}^{R*}(r_{t+1}), r_{t+1})}{dr_{it}} \frac{dr_{t+1}^*(s)}{ds} > 0. \square$$

Proposition 1. (Strategic systemic risk tax): *Equilibrium I:* This equilibrium constitutes a pooling equilibrium in which all banks are unresponsive to the regulator's action with respect to τ . Therefore, a regulator of any type $\theta \in \Theta = \mathbb{R}$ finds it optimal to set τ , because the welfare costs associated with the tax δ are increasing in τ . The monotonicity of δ establishes the optimality of τ .

Since the tax τ is set by all types of regulator, the tax is uninformative to banks about the regulator's type θ . Therefore, the beliefs of the banks about the costs of a bailout are pinned down in a way equivalent to 6, such that $\mu(\theta|\xi_i, \tau) \equiv \mu(\theta|\xi_i)$. As in Section 3.1, banks believe, under non-strategic taxation, that a bailout will be initiated if $\theta < \theta^*$. Additionally, as in the case with a fixed systemic risk tax, the bank also finds it optimal to choose ρ^h if $\xi_i < \xi^*$.

If the regulator sets $\tau > \tau$, banks detect a deviation. Let $\bar{\Theta}(\tau)$ denote the set of regulators for which the deviation to $\tau > \tau$ is dominated in equilibrium. Then for any signal ξ_i , we have the following two conditions:

$$\begin{aligned} \mu(\theta \in \Theta|\xi_i, \tau) &= 1, \quad \text{and} \\ \mu(\theta \in \bar{\Theta}|\xi_i, \tau) &= 0, \quad \text{if } \theta \notin \bar{\Theta}(\tau). \end{aligned}$$

The two conditions form the natural restriction that beliefs should assign positive measure to types that could lead to the signal ξ_i , and they require that beliefs assign zero measure to types for which $\tau > \tau$ is dominated in equilibrium.

The next step is to verify that the set of beliefs in equilibrium I is non-empty for all $\tau > \tau$. Banks react to the signal by choosing ρ^l unless they are convinced that the regulator must be of a low type, that is $\xi_i < -\infty$. Borrowing Conditions (9b) and (9a) from Section 3.2, it is possible to pin down the set of θ that set τ such that the strategy is dominated in equilibrium by τ . Then $\bar{\theta}$ solves:

$$\bar{\theta} = \delta(\bar{\tau}) = \int_{-\infty}^{\infty} \rho^*(\mu(\bar{\theta}|\xi_i))f(\xi_i - \bar{\theta})d\xi_i,$$

such that $\bar{\Theta}(\tau) = \mathbb{R}$ for $\tau > \bar{\tau}$, and $\bar{\Theta}(\bar{\tau}) = \mathbb{R} \setminus \{\bar{\theta}\}$. The set of beliefs satisfying these above-mentioned conditions is non-empty for $\tau \geq \bar{\tau}$. If $\tau \in (\tau, \bar{\tau})$, then the set of regulator types for which this strategy is dominated is $\Theta \setminus [\underline{\theta}, \bar{\theta}]$, where

$$\underline{\theta} = \delta(\tau) = \int_{-\infty}^{\infty} \rho^*(\mu(\underline{\theta}|\xi_i))f(\xi_i - \underline{\theta})d\xi_i.$$

Since $\underline{\theta} < \bar{\theta} < \bar{\theta}$, the set of types for which τ is dominated in equilibrium is $\bar{\Theta}(\tau) = \mathbb{R} \setminus [\underline{\theta}, \bar{\theta}]$. *Equilibrium II:* In this equilibrium, banks coordinate on the systemic risk tax. Banks take the average action

$$\int_{-\infty}^{\infty} \rho^*(\mu(\theta|\xi_i))f(\xi_i - \theta)d\xi_i$$

when $\tau < \bar{\tau} \in [\tau, \infty)$, and all banks choose ρ^l as an optimal response when $\tau \geq \bar{\tau}$ because it is not profitable to set ρ^h . It is optimal for the regulator to choose τ if $\tau < \bar{\tau}$, because in this case banks do not respond to the tax, so it is optimal for the regulator to minimize welfare loss δ . Banks are responsive to the tax when $\tau \geq \bar{\tau}$, such that the regulator prefers to set $\bar{\tau} > \tau$ if the regulator wants the banks to respond to the tax. This leaves us with two candidates for the level of taxation, τ and $\bar{\tau}$.

From the perspective of a bank, a regulator prefers to set $\bar{\tau} > \tau$ if and only if the bank infers that $\mathbb{E}^H[W(\theta, \bar{\tau}, s)] > \mathbb{E}^H[W(\theta, \tau, s)]$ which implies that the following two conditions need to hold¹⁴:

$$\begin{aligned} \theta - \delta(\bar{\tau}) &> 0, \quad \text{and} \\ \theta - \delta(\bar{\tau}) &> \theta - \int_{-\infty}^{\infty} \rho^*(\mu(\theta|\xi_i))f(\xi_i - \theta)d\xi_i. \end{aligned}$$

The regulator types that prefer $\bar{\tau}$ over τ then have $\theta \in [\underline{\theta}, \bar{\theta}]$, where $\underline{\theta}$ and $\bar{\theta}$ solve the system:

$$\begin{aligned} \underline{\theta} &= \delta(\bar{\tau}), \quad \text{and} \\ \int_{-\infty}^{\infty} \rho^*(\mu(\bar{\theta}|\xi_i))f(\xi_i - \bar{\theta})d\xi_i &= \delta(\bar{\tau}). \end{aligned}$$

Such that $\delta(\bar{\tau})$ denotes a lower bound. We find an upper bound $\bar{\theta}$, because the expected cost of banks failing $\int_{-\infty}^{\infty} \rho^*(\mu(\theta|\xi_i))f(\xi_i - \theta)d\xi_i$ is decreasing in θ .

¹⁴ Note that the assumptions are made that the average action $\int_{-\infty}^{\infty} \rho^l f(\xi_i - \theta)d\xi_i = 0$ if all banks choose ρ^l , see Section 3.1. These assumptions do not affect the qualitative results derived in this proof.

Banks' beliefs are pinned down by Bayes' rule but differ from equilibrium I, because when the regulator sets $\underline{\tau}$, the corresponding type $\theta \notin [\underline{\theta}, \bar{\theta}]$. Hence, beliefs about a bailout policy being initiated conditional on observing $\underline{\tau}$ are

$$\mu(\theta|\xi_i, \underline{\tau}) \equiv \frac{\mu(\theta|\xi_i)}{1 - \mu(\theta|\xi_i) + \mu(\underline{\theta}|\xi_i)},$$

where $\mu(\theta|\xi_i)$ is defined as (6), and $\mu(\theta|\xi_i, \underline{\tau})$ is decreasing in ξ_i . The last monotonicity result ensures the uniqueness of equilibrium for the case in which banks respond to a strategically set systemic risk tax. Furthermore, the fact that $\Theta = \mathbb{R}$ ensures that the set of regulator types for which $\tau \in (\underline{\tau}, \bar{\tau})$ is dominated in equilibrium by $\underline{\tau}$ is a subset of Θ . $\square$

References

- Acharya, V.V., 2009. A theory of systemic risk and design of prudential bank regulation. *J. Financ. Stab.* 5 (3), 300–394.
- Acharya, V.V., Engle, R., Richardson, M., 2012. Capital shortfall: a new approach to ranking and regulating systemic risks. *Am. Econ. Rev.* 102 (3), 59–64.
- Acharya, V.V., Pedersen, L.H., Philippon, T., Richardson, M.P., 2011. Measuring systemic risk. AFA 2011 Denver Meetings Paper.
- Acharya, V.V., Yorulmazer, T., 2007. Too many to fail—an analysis of time inconsistency in bank closure policies. *J. Financ. Intermed.* 16 (1), 1–31.
- Adrian, T., Brunnermeier, M.K., 2011. CoVaR. FRB of New York, Staff Report No. 348.
- Albuquerque, R., 2010. Comment on: optimal taxation in the presence of bailouts. *J. Monet. Econ.* 57 (3), 117–119.
- Allen, F., Gale, D., 2000. Financial contagion. *J. Polit. Econ.* 108 (1), 1–33.
- Angeletos, G.-M., Hellwig, C., Pavan, A., 2006. Signaling in a global game: coordination and policy traps. *J. Polit. Econ.* 114 (3), 452–484.
- Basel Committee of Banking Supervision, 2009a, December. International Framework for Liquidity Risk Management, Standards and Monitoring. Report.
- Basel Committee of Banking Supervision, 2009b, December. Strengthening the Resilience of the Banking Sector. Report.
- Brown, C.O., Dinç, I.S., 2005. The politics of bank failures: evidence from emerging markets. *Q. J. Econ.* 120 (1), 13–44.
- Carlsson, H., van Damme, E., 1993. Global games and equilibrium selection. *Econometrica* 61 (5), 989–1018.
- Cho, I.-K., Kreps, D.M., 1987. Signaling games and stable equilibria. *Q. J. Econ.* 102 (2), 172–221.
- Chunyang, W., 2013. Bailouts and bank runs: theory and evidence from TARP. *Eur. Econ. Rev.* 64, 169–180.
- Drazen, A., 2000. Interest-rate and borrowing defense against speculative attack. In: *Carnegie-Rochester Conference Series on Public Policy*, vol. 53, pp. 303–348.
- Farhi, E., Tirole, J., 2012. Collective moral hazard, maturity mismatch, and systemic bailouts. *Am. Econ. Rev.* 102 (1), 60–93.
- Freixas, X., 1999. Optimal bail-out policy, conditionality and creative ambiguity. CEPR Discussion Paper 2238, Centre for Economic Policy Research.
- Freixas, X., 2003. Crisis management in Europe. In: Kremers, J.J.M., Schoenmaker, D., Wierds, P. (Eds.), *Financial Supervision in Europe*, Edward Elgar, Cheltenham, pp. 102–119. (Chapter 4).
- Freixas, X., J.C. Rochet, 2011. Taming SIFIS. Working Paper, Zurich University.
- Kocherlakota, N., Shim, I., 2007. Forbearance and prompt corrective action. *J. Money Credit Bank.* 39 (5), 1107–1129.
- Maksimovic, V., Zechner, J., 1991. Debt, agency costs, and industry equilibrium. *J. Finance* 46 (5), 1619–1643.
- Mas-Colell, A., Whinston, M.D., Green, J.R., 1995. *Microeconomic Theory*. Oxford University Press, Inc, New York.
- Morris, S., Shin, H.S., 1998. Unique equilibrium in a model of self-fulfilling currency attacks. *Am. Econ. Rev.* 88 (3), 587–597.
- Morris, S., Shin, H.S., 2001. Rethinking multiple equilibria in macroeconomics. In: *NBER Macroeconomics Annual 2000*, MIT Press, Cambridge, MA.
- Morris, S., Shin, H.S., 2003. Global Games—Theory and Application. In: Dewatripont, M., Hansen, L., Turnovsky, S. (Eds.), *Advances in Economics and Econometrics*. 8th World Congress of the Econometric Society, Cambridge University Press, Cambridge, UK.
- O'Hara, M., Wayne, S., 1990. Deposit insurance and wealth effects: the value of “Too Big to Fail”. *J. Finance* 45 (1), 587–600.
- Panageas, S., 2010. Optimal taxation in the presence of bailouts. *J. Monet. Econ.* 57 (3), 101–116.
- Rajan, R.G., 1994. Why bank credit policies fluctuate: a theory and some evidence. *Q. J. Econ.* 109 (2), 399–441.
- Shim, I., 2011. Dynamic prudential regulation: is prompt corrective action optimal? *J. Money Credit Bank.* 43 (8), 1625–1661.
- Shleifer, A., Vishny, R., 1992. Liquidation values and debt capacity: a market equilibrium approach. *J. Finance* 47 (4), 1343–1366.
- Steiner, A., 2014. Reserve accumulation and financial crises: from individual protection to systemic risk. *Eur. Econ. Rev.* 70, 126–144.
- The Economist, 2015. Frozen—Liquidity in markets. 18 April, 61–62.
- Tinbergen, J., 1952. *On the Theory of Economic Policy*. North-Holland Publishing Company, Amsterdam.
- Zwart, S., 2007. The mixed blessing of IMF intervention: signalling versus liquidity support. *J. Financ. Intermed.* 3 (2), 149–174.