Brief paper

A Hamiltonian control approach for electric microgrids with dynamic power flow solution

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1. Introduction

Microgrids (MGs) constitute a special class of electrical networks that mainly incorporate renewable energy sources in a distributed way. The heterogeneous nature of the sources imposes the necessity of including power electronic converters as an interface to condition the generated energy. Like any electrical network, their purpose is to satisfy the power load demands by achieving (amplitude and frequency) voltage regulation. This two-fold objective must be achieved under severe operation conditions, e.g., inertial less structure (making the network vulnerable to fast changes) and lack of a voltage reference (when islanded operation is considered) (Han et al., 2016).

Due to the features described above, MGs operation must be based on a hierarchical control infrastructure that first guarantees a suitable behavior for voltages and currents to later on steer these variables to specific values that correspond to the fulfillment of the load's power requirements. From the practical point of view, the preferred control structures are based on the use of model-free schemes given by PI algorithms for the converters operation and Droop-based strategies to solve the power-sharing problem. Although the use of this (relatively) simple alternative leads to high dynamical responses, its main drawback lies in the lack of formal stability properties (Rajesh, Dash, Rajagopal, & Sridhar, 2017). From the control theory perspective, most of the contributions reported in the literature do not consider a complete MG's model. A complete model is given by differential–algebraic equations (DAEs) if the dynamical variables associated with the power converters and the transmission lines are considered together with the algebraic variables involved in the power balance equations. With respect to this structure, it is possible to find results, see e.g. Schiffer, Efimov, and Ortega (2019) and Simpson-Porco, Dörfler, and Bullo (2013), where the model is simplified by omitting the converters’ dynamic, other which focuses on finding a solution to the nonlinear algebraic equations (Molzahn & Hiskens, 2019) leaving aside the dynamical variables, and some results where even that the dynamical variables are included (Tucci & Ferrari-Trecate, 2020) then the network is represented after a Kron’s reduction process.

This paper’s main objective is to address the MGs’ voltage regulation and power balance control problems by explicitly including in the controller design both a dynamical model for the network (and the associated converters) and the constraints imposed by the power balance equations. Moreover, a structure-preserving model is used instead of considering a reduced network representation, i.e., sources and loads nodes are separated; although, like in Tucci and Ferrari-Trecate (2020), it is assumed synchronization of the sources clocks. The contribution is a hierarchical control scheme whose stability properties are formally established. The distinctive features of the proposed scheme are: it is developed using energy-based arguments exploiting a port-controlled Hamiltonian (PCH) model for the system; the nonlinear algebraic system that codifies the power balance equations
is dynamically solved by integrating a dynamical system whose solution asymptotically converges to the solution of the former; it is formally proved (using passivity, cascaded systems, and input-to-state stability arguments) that the state trajectories that correspond to a proper MG operation exhibit asymptotic stability properties; from the implementation point of view, the proposed converters controllers only require knowledge of local variables. On the other hand, the main limitation of the result lies in the fact that for establishing the stability properties, it must be assumed that the loads satisfy a passivity condition, leaving the case of constant power loads as a future research topic.

The present contribution improves the results reported in Avila-Becerril, Espinosa-Pérez, and Machado (2017), Avila-Becerril, Espinosa-Pérez, and Cansecro-Rodal (2017), Avila-Becerril, Espinosa-Pérez, and Machado (2019) and Avila-Becerril, Montoya, Espinosa-Pérez, and García (2018) in several directions: for the first time, a complete stability proof of the whole scheme is presented, both a grid-forming (voltage-fed) and grid-following (current-fed) converters are included, and the solution of the set of algebraic equations is based neither on heuristic nor off-line methods.

The rest of the paper is organized as follows. In Section 2, the model of the MG is presented while in Section 3 the control problem is formulated. The main results of the paper are introduced in Section 4. Some numerical results are presented in Section 5 and some concluding remarks in Section 6.

2. Hamiltonian microgrid model

In this section, the model of the considered generic single-phase MG is presented. It is composed of a set of $n_t$ transmission lines interconnected under a defined topology (Avila-Becerril, Espinosa-Pérez, & Fernández, 2016). The network is composed of one grid-forming node, $n_g$ grid-following nodes, and $n_{ld}$ load nodes. In order to consider all the devices involved in the operation of a real MG, in Fig. 1 it is illustrated how a voltage-fed converter is associated with the grid-following node while current-fed converters are associated with the grid-following nodes.

The state of the dynamic system is composed of $x_1, x_2 \in \mathbb{R}$, the inductor current and capacitor voltage of the voltage-fed converter, $x_3 \in \mathbb{R}^{(n_t + 1) \times 1}$, the capacitor voltages at the grid-following and loads nodes, $x_4 \in \mathbb{R}^{(n_{ld})}$, the inductor currents at the transmission lines, and $x_5, x_6 \in \mathbb{R}^{(n_t)}$ the inductor currents and capacitor voltages of the current-fed converters. Furthermore, $x_3$ is divided into $x_{3u} \in \mathbb{R}^{(n_t)}$, which denotes the capacitor voltages at the loads nodes, and $x_{3d} \in \mathbb{R}^{(n_t)}$, which is the vector of capacitor voltages at the grid-following nodes.

Under the conditions described above, the dynamic behavior of the MG is represented by the model

$$D \dot{x} = \left[ J_T(U_2) - R_T \right] x + E_T u_t + I(x_3),$$

where $x = \begin{bmatrix} x_{12}^T \ x_{34}^T \ x_{56}^T \end{bmatrix}^T$, with $x_{12} = \begin{bmatrix} x_1 \ x_2 \end{bmatrix}^T$, $x_{34} = \begin{bmatrix} x_{3u} \ x_{3d} \ x_5 \ x_6 \end{bmatrix}^T$, and $x_{56} = \begin{bmatrix} x_5 \ x_6 \end{bmatrix}^T$, while $u_t \in \mathbb{R}$ is the control input of the voltage-fed converter, $U_2 = \text{diag}(u_{x_2})$ is composed of the $n_t$ control inputs of the current-fed converters, $D = \text{diag}\{ I_1, C_1, C_3, L_4, L_5, C_6 \}$, and

$R_T = \text{diag}\{ R_1, R_2^{-1}, R_3^{-1}, R_4, R_5, R_6^{-1} \}$.

The internal interconnection structure of the system is given by

$$J_T(U_2) = \begin{bmatrix} J_{12} & J_{13} & 0 \\ -J_{11} & J_{34} & J_{35} \\ 0 & J_{32} & J_{56}(U_2) \end{bmatrix} = -J_T(U_2),$$

with

$$J_{12} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, J_{34} = \begin{bmatrix} 0 & -H_2 \\ H_2 & 0 \end{bmatrix},$$

$$J_{56} = \begin{bmatrix} 0 & U_2 \\ -U_2^T & 0 \end{bmatrix},$$

$$J_{11} = \begin{bmatrix} 0 & 0 \\ 0 & -H_1 \end{bmatrix}, J_{32} = \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

where $H_1 \in \mathbb{R}^{1 \times n_t}$ and $H_2 \in \mathbb{R}^{(n_t + n_{ld}) \times n_t}$ express how transmission lines are connected to the grid-forming node and to the grid-following and loads nodes, respectively (Avila-Becerril et al., 2016), while $I$ is a generic identity matrix. Finally, the voltage source $V_n \in \mathbb{R}$ is included in the vector $E_T = [V_n, \ 0, \ 0, \ 0, \ 0, \ 0]$ and the current sources $I_6 \in \mathbb{R}^{n_t}$ and the constitutive relationships of the loads compose the vectors $I(x_3) = \begin{bmatrix} 0 \\ -I_{34}(x_3) \\ I_{56}(x_3) \end{bmatrix}$, $I_{34}(x_3) = \begin{bmatrix} I_3(x_3) \ 0 \end{bmatrix}$, $I_{56} = \begin{bmatrix} 0 \\ I_6 \end{bmatrix}$, where $I_3(x_3) = [I_{3u}^T(x_{3u}), 0]^T$ with $i_{3u}(x_{3u}) \in \mathbb{R}^{n_t}$ the loads currents vector.

Remark 1. Note that model (1) corresponds to a classical widely accepted representation for both the power converters (Shuai et al., 2016) and the electrical network (Kundur, Balu, & Lauby, 1994), which is presented in a PCH form (Vander Schaft & Maschke, 2013).

Remark 2. It must be pointed out that the existence of the grid-forming node in the considered model comes from the assumption that the network operates in islanded mode. The model can be refurbished in an easy way to consider connected operation mode by converting the grid-forming node into an additional grid-following node.

Remark 3. In the presented model, only power converter interfaced energy sources are considered for the sake of presentation clarity. The inclusion of conventional sources, like synchronous
Consider model (1) constrained by the power flow Eqs. (5). For a given \( P^*(t) \) and \( x^* \) that simultaneously satisfy (1)–(5), design a control law

\[
u_1 = u_1(x, x^*) \quad ; \quad i = 1, \ldots, n_i
\]

such that \( x^* \) is an asymptotically stable solution of the closed-loop system guaranteeing internal stability.

**Remark 5.** The control problem formulation is stated under the usual and necessary assumption that a proper operating condition of the network (small signal stability) can be achieved in the sense that the demanded power is less than the combined power capacity of the sources.

4. Main results

To state the proposed solution to the formulated control problem, it is first necessary to identify the so-called admissible trajectories of system (1) given by \( x^* \) and defined as a dynamic behavior that complies with the constraints imposed by the model structure. From model (1), the corresponding admissible states for \( x_{12} \) and \( x_{56} \) must be solutions of

\[
D_{12}x^*_{12} = \begin{bmatrix} I_2 & -R_1 \end{bmatrix}x^*_{12} + E_1u^*_1 + J_1x_{34}. \quad (6)
\]

\[
D_{56}x^*_{56} = -R_{56}x^*_{56} + E_2(x^*_{56})u^*_2 - J_2x_{34} + I_{56}. \quad (7)
\]

where \( D_0 = \text{diag} \{ L_i, C_i \}, R_{ij} = \text{diag} \{ R_i, R_i^{-1} \}, \) and \( E_1 = [V_0, 0]^\top \). Note that the right-hand side of (7) was obtained exploiting the property presented in Remark 4.

After straightforward computations, assuming that \( V_0 \) is strictly positive and \( x^*_1, x^*_2 \) are given prescibed values, it can be shown, in accordance with (2), that

\[
x^*_1 = C_2x^*_2 + R_2x^*_3 + H_1x_4, \quad (8)
\]

and the solution of

\[
C_2x^*_2 = -R_2x^*_3 - x^*_2 \cap u^*_2 + I_6, \quad (9)
\]

constitute an admissible solution of (6)–(7) if and only if

\[
u_1^* = V_0^{-1} \begin{bmatrix} L_1x^*_1 + R_1^{-1}x^*_1 + x^*_1 \end{bmatrix}, \quad (10)
\]

\[
u_2^* = \begin{bmatrix} E_2(x^*_{56})E_2(x^*_{56}) \end{bmatrix}^{-1} \begin{bmatrix} E_2(x^*_{56}) \end{bmatrix} \times \begin{bmatrix} D_{56}x^*_{56} + R_{56}x^*_{56} + J_2x_{34} - I_{56}. \end{bmatrix}
\]

The behavior presented above serves to design a globally stabilizing control for the trajectories \( x^*_{12}, x^*_{56} \), which is presented in the next proposition.

**Proposition 1.** Consider the system (1) and the admissible trajectories (8)–(9) with their corresponding control inputs (10). Assume that

A.1 The voltage \( V_0 > 0 \) and the currents \( I_6 > 0 \) are known.

A.2 The states \( x_1, x_{56}, x_{34} \) and the port-current \( H_1x_4 \) are available for measurement.

A.3 The functions \( x^*_2 \) and \( x^*_3 \) are known bounded with bounded first and second derivatives.

A.4 The load ports variables fulfill the passivity condition \( x_{12}^2H_1(x_{12}) \geq 0 \).

Define the error variables \( \tilde{x}_{12} = x_{12} - x^*_1, \; \tilde{x}_{56} = x_{56} - x^*_2 \) and the feedback control

\[
u_1 = -K_1y_1 + u^*_1; \; K_1 > 0; \quad (11)
\]

\[
u_2 = -K_2y_2 + u^*_2; \; K_2 > 0,
\]

where \( y_1 = V_0x_1 \) and \( y_2 = E_1^\top(x^*_{56})x_{56} \). Under these conditions

\[
limit_{t \to -\infty} \tilde{x}_{12} = 0; \; \lim_{t \to -\infty} \tilde{x}_{56} = 0 \text{ guaranteeing internal stability.}
\]
Proof. Assumptions A.1 and A.2 are necessary to make implementable the proposed controller.

Under the definition $\tilde{u} = u - u^*$, the error dynamic takes the form

$$D_{12}\frac{dx_{12}}{dt} = [J_{12} - R_{12}]x_{12} + E_1\tilde{u}_1,$$  
(12)

$$D_{56}\frac{dx_{56}}{dt} = [J_{56}(U_2) - R_{56}]x_{56} + E_2(x_{56})\tilde{u}_2.$$  
(13)

This system appears in cascade form with the subsystem

$$D_{34}x_{34} = [J_{34} - R_{34}]x_{34} - I_4(x_4) + \Gamma'(\tilde{x}, x^*),$$  
(14)

where

$$\Gamma'(\tilde{x}, x^*) = \begin{bmatrix} H\tilde{x}_5 + Hx_5^* \\ -H_1\tilde{x}_2 - H_1'x_2^* \end{bmatrix}$$  
(15)

is a not vanishing bounded perturbation if A.3 holds.

Concerning the system (12)-(13), considering the globally positive definite function

$$W_1(x) = \frac{1}{2}x_{12}^T D_{12} x_{12} + \frac{1}{2}x_{56}^T D_{56} x_{56},$$

it is obtained that

$$\dot{W}_1(x) = -x_{12}^T R_{12} x_{12} - \tilde{x}_{56}^T R_{56} x_{56} + y_1 \tilde{u}_1 + y_2 \tilde{u}_2.$$  

Hence, taking into account the control law (11) and invoking LaSalle’s invariance principle, it can be shown that the trajectories of (12)-(13) asymptotically converge to the largest invariant set contained in $y_1 = 0, y_2 = 0$, which corresponds to the equilibrium point $x_{12} = x_{56} = 0$.

The internal stability properties of the closed-loop systems are related with system (14). Notice that under $\Gamma'(\tilde{x}, x^*) = 0$, and considering the time-derivative of the globally positive definite function

$$W_2(x_{34}) = \frac{1}{2}x_{34}^T D_{34} x_{34}$$

evaluated along (14), if A.4 holds then it is possible to conclude that the equilibrium point $x_{34} = 0$ is Globally Asymptotically Stable (GAS). Thus, under the presence of the perturbation, when $x_{12} = x_{56} = 0$, the system exhibits Input-to-State stability properties from the bounded input $\Gamma'(0, x^*)$ to the state $x_{34}$. The proof is finished applying well-known results from cascaded systems theory (see for example Proposition 4.1 of Sepulchre, Jankovic, & Kokotovic, 2012) to conclude that $x_{12}$ and $x_{56}$ tend to zero while $x_{34}$ remains bounded. □

Remark 6. From an implementation viewpoint, it is important to point out that the voltages $x_{34}$, which correspond to the upper entry of the vector $J_{34}x_{34}$, and the currents $H_{1}x_{3}$ involved in the controller’s structure are output variables of each converter, i.e., variables composed of linear combinations of the voltages and currents of the transmission lines $x_{34}$. Therefore, they are local variables with respect to each converter. This fact states the local nature of the proposed control scheme if $K_2$ is a diagonal matrix.

Remark 7. One topic that deserves attention concerns the assumed synchronization of the power converters’ clocks. In this sense, further research is necessary to take into account the clock-drift problem reported in Schiffer, Hans, Kral, Ortega, and Raisch (2016).

Remark 8. This version of the proposed control law assumes knowledge of converters’ parameters and measurement of local variables. However, the control scheme is very suitable to be reformulated to obtain adaptive and observer-based versions. This claim is based on the fact that adaptive control of bilinear systems is a well-known topic, while some recent results on observer design for nonlinear PCH systems have been obtained in Rojas, Granados-Salazar, and Espinosa-Pérez and Rojas, Rueda-Escobedo, Espinosa-Pérez, and Schiffer (2020).

The next step of the design is related to the integration of a dynamic algorithm that, for a prescribed power balance, can compute the corresponding value of $x^*_5$. To this end, the scheme presented in Mylvaganam, Ortega, Machado, and Astolfi (2018) states a viable alternative. The main contribution of this result is the proof that if a nonlinear algebraic equation $h(z) = 0$ can be written in the form $Q - \mathcal{P}(z)z = 0$ with $Q$ a known term and $\mathcal{P}(z)$ a matrix of proper dimension, then it is possible to build a dynamical system of the form

$$\dot{\xi} = Q - \mathcal{P}(z)z,$$  
(16a)

$$\dot{z} = \hat{U}(\xi, z, \theta),$$  
(16b)

$$\hat{\theta} = \hat{W}(\xi, z, \theta),$$  
(16c)

for which all the equilibrium points $(\xi^*, z^*, \theta^*)$ are locally asymptotically stable. Hence, for any $(\xi(0), z(0), \theta(0))$ starting sufficiently close to one equilibrium point, it is possible to compute $z^*$ solution of $Q - \mathcal{P}(z^*)z^* = 0$ as $\lim_{z \to \infty} z = z^*$.

To show that the power balance equations exhibit the considered structure in Mylvaganam et al. (2018), note that if the desired MG steady-state behavior corresponds to a sinusoidal operation regime at a frequency $\omega_0$, then the Fourier transforms of (3) and (4) satisfy

$$X_3(j\omega_0) = Y^{-1}(j\omega_0) \left\{ -I_L(j\omega_0)X_5(j\omega_0) \right\} + Y_2(j\omega_0)X_3(j\omega_0)$$

with $Y_1(j\omega_0)$ and $Y_2(j\omega_0)$ suitable transfer functions and $X_3$, $X_5$, $I_L$ are generalized Fourier transforms of their corresponding time depending variables evaluated at a frequency $\omega_0$. Thus, by recalling that in the frequency domain power is given by the product of voltage $X_3(j\omega_0)$ times the complex conjugate of currents $I_L^*(j\omega_0)$, $X_5^*(j\omega_0)$, it is obtained that

$$\begin{bmatrix} P_2 + jQ_2 \\ P_3 + jQ_3 \end{bmatrix} = G(j\omega_0) \left\{ \begin{bmatrix} I_L(j\omega_0) \\ X_5(j\omega_0) \end{bmatrix} \right\},$$  
(17)

where $P_2$, $P_3$, $Q_2$, $Q_3$ are the active and reactive powers at the loads and the grid-following nodes, respectively, while

$$G(j\omega_0) = Y^{-1}(j\omega_0) \left\{ -I_L(j\omega_0)X_5(j\omega_0) \right\} + Y_2(j\omega_0)X_3(j\omega_0)$$

is a function of $I_L$, $X_3$ and $X_5$.

Since (17) exhibits the form $Q - \mathcal{P}(z)z = 0$, by defining $Q = \left\{ P_2, P_3, Q_2, Q_3 \right\}$ and $z = \left\{ z_1, z_2, z_3, z_4 \right\}$ with $z_1 = \text{Re}(I_L)$, $z_2 = \text{Re}(X_5)$, $z_3 = \text{Im}(I_L)$ and $z_4 = \text{Im}(X_5)$, then, for a given $Q$ and a given $x^*$, the corresponding $x^*_5$ can be computed as $\lim_{z \to \infty} x_5 = x^*_5$ where $x_5 = A\sin(\omega_0t + \phi)$ with $A = \sqrt{z_1^2 + z_4^2}$ and $\phi = \arctan(z_4/z_2)$.

Remark 9. It is interesting to mention that the system with structure given by (16) clearly exhibits the hierarchical structure of the proposed control scheme. This system operates in an upper level with respect to the power converters’ controllers in the sense that from the knowledge of the voltage reference $x^*_5$, the demanded power $P_2$, $Q_2$ and the available power at the grid-following nodes $P_3$, $Q_3$ determines the corresponding value of $x^*_5$ which is fed for the local power converters’ operation.

In the next proposition, the complete control scheme proposed in this paper is presented. It is composed of the basic stabilization mechanism introduced in Proposition 1, operating with the information provided by a system of the form (16). In this case, instead of considering A.2, it is assumed that only $x^*_5$ is prescribed.
**Proposition 2.** Consider the system (1) and let \( x^* \) and \( u^* \) be defined as in (8)–(10). Assume A.1, A.3–A.4 hold and that

\( \text{A.6} \) The function \( \hat{x}_5 \) is obtained from (16a)–(16c) and \( \hat{x}_6 \) is solution of

\[
\begin{align*}
C_0 \hat{x}_6 &= -R_0 x_0 \hat{x}_3 \hat{u}_2 + I_0; \quad \hat{x}_5 = \text{diag} \{ \hat{x}_{5i} \}.
\end{align*}
\]

Consider the feedback control

\[
\begin{align*}
u_1 &= -K_1 y_1 + u^*_1; \quad K_1 > 0, \tag{18} \\
u_2 &= -K_2 \hat{y}_2 + \hat{u}_2; \quad K_2 = K_2^* > 0,
\end{align*}
\]

where \( y_1 = V_x \hat{x}_1, \hat{y}_2 = E^T \hat{x}_{56} (x_{56} - \hat{x}_{56}) \) and \( u^*_i \) is obtained from (10) substituting \( x^*_i \) by \( \hat{x}_{56} \).

Under these conditions \( \lim_{t \to \infty} \hat{x}_{12} = 0; \lim_{t \to \infty} \hat{x}_{56} = 0 \) guaranteeing internal stability.

**Proof.** After some manipulations it is obtained that the expression of the output \( \hat{y}_2 \) can be written as

\[
E^T \hat{x}_{56} (x_{56} - \hat{x}_{56}) = E^T (x^*_{56}) \hat{x}_{56} + \Gamma_1 (x, x^*, \hat{x}),
\]

where

\[
\Gamma_1 (x, x^*, \hat{x}) = E^T (x^*_{56}) x^*_{56} - E^T \hat{x}_{56} \hat{x}_{56} - E^T (x^*_{56}) (x^*_{56} - \hat{x}_{56})
\]

On the other hand, \( -K_2 E^T (x^*_{56}) \hat{x}_{56} = u_{2a} - u^*_2 \) with \( u_{2a} \) the control law presented in (11). Thus, the controller proposed in (18) takes the form

\[
u_2 = u_{2a} + \hat{\Gamma} (x, x^*, \hat{x}), \tag{19}
\]

where \( \hat{\Gamma} (x, x^*, \hat{x}) = K_2 \Gamma_1 (x, x^*, \hat{x}) + \hat{u}_2 - u^*_2 \) is such that it vanishes when \( x_{56} \to x^*_{56} \), i.e., it is a vanishing perturbation.

Taking into account expression (19) the whole closed-loop control system is defined by the dynamic behavior of the error state \( \hat{x} = (\xi - \xi^*, z - z^*, \theta - \theta^*) \) which, see (16), can be conceptualized as

\[
\hat{x} = \mathcal{F} (\hat{x}),
\]

with \( \hat{x} = 0 \) locally asymptotically stable, the system

\[
\begin{align*}
D_{12} \hat{x}_{12} &= [J_{12} - R_{12}] \hat{x}_{12} - E_1 K_1 y_1, \\
D_{56} \hat{x}_{56} &= [J_{56} (U_2) - R_{56}] \hat{x}_{56} - E_2 (x^*_{56}) K_2 \hat{y}_2 + E_2 (x^*_{56}) \hat{\Gamma} (x, x^*, \hat{x}),
\end{align*}
\]

with \( \hat{x}_{12} = \hat{x}_{56} = 0 \) globally asymptotically stable and subject to a vanishing perturbation, and

\[
D_{34} \hat{x}_{34} = [J_{34} - R_{34}] x_{34} - I_3 (x_1) + \Gamma (\hat{x}, x^*),
\]

for which \( x_{34} = 0 \) is globally asymptotically stable, for the unperturbed system, and that exhibits input-to-state stability properties from the input \( \Gamma (0, x^*) \) to the state \( x_{34} \).

The proof is concluded by applying cascaded systems arguments (see for example Proposition 4.1 of Sepulchre et al. (2012)) showing that \( (\hat{x}, \hat{x}_{12}, \hat{x}_{34}) = 0 \) is locally asymptotically stable while \( x_{34} \) remains bounded. \( \square \)

5. Simulation results

The usefulness of the proposed controller is numerically illustrated in this section. The illustrative network consists of one grid-forming node (voltage-fed converter) and one grid-following node (current-fed converter) both connected to a load. The parameter values were taken from Avila-Becerril et al. (2019) where \( |x^*| = 180 \text{ V} \) and \( f_0 = 60 \text{ Hz} \). The prescribed power profiles for the load and the grid-following nodes, that correspond to \( \mathcal{Q} \) in (17), were as in Figs. 3 and 4, respectively. To show the potentiality of the contribution, instead of considering a passive load (as considered in theory), a CPL was included in the system.

It is important to mention that the initial condition for the voltage associated with the grid-forming node was set to zero during the experiment. This choice was aimed to resemble the case of sudden islanding operation since the transient response of this variable illustrates the capacity of the controller to achieve resynchronization under this condition.

Concerning the voltage regulation objective, at the top of Fig. 2 it is shown how \( \hat{x}_2 \) tends to zero while \( x_{3a} \) and \( x_{3b} \) remain bounded. In the middle of this figure, the phase achieved by each node is included, while at the bottom the achievement of the synchronization objective is illustrated. Regarding the power tracking goal, in Fig. 3 and Fig. 4, the behavior of the active and reactive powers for the grid-following and the load nodes, respectively, are presented. It is worth noting the remarkable achieved convergence despite the sudden changes introduced in the desired behavior. Finally, the convergence to zero of the error states is shown in Fig. 5.

6. Conclusion

In this paper, a controller that guarantees voltage regulation and power balance for an islanded MG was presented. The considered model is given by a set of DAES without disregarding any element of the network. The control scheme exhibits a hierarchical structure including local control laws for the power converters to steer their associated voltages and currents to some prescribed
values obtained from a dynamic algorithm whose solutions satisfy a prescribed power network balance. The (local) asymptotic stability properties of complete closed-loop system are formally stated. The usefulness of the contribution was illustrated in a numerical context.

References


