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Dijkstra, Jacob; Mulders, Jaap Oude

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EFFICACY, BELIEFS, AND INVESTMENT IN STEP-LEVEL PUBLIC GOODS

Jacob Dijkstra

*Department of Sociology and Interuniversity Center for Social Science
Theory and Methodology (ICS), University of Groningen, Groningen,
The Netherlands*

Jaap Oude Mulders

*Department of Sociology, University of Groningen, Groningen,
The Netherlands*

A central concept for understanding social dilemma behavior is the efficacy of an actor's cooperative behavior in terms of increasing group well-being. We report a decision and game theoretical analysis of efficacy in step-level public goods (SPGs). Previous research shows a positive relation between efficacy and contributions to SPGs and explains this relation by a purely motivational account. We show, however, that from a decision and game theory perspective an increasing relationship is not general, but only follows from very specific assumptions about players' information and beliefs. We offer 3 examples of how the predicted efficacy–contribution relation depends on players' information and beliefs. We discuss the implications of our results for the social psychology of efficacy in social dilemmas.

Keywords: Bayesian equilibrium, beliefs, efficacy, information, social dilemmas, step-level public goods

1. INTRODUCTION

Social dilemmas are situations in which individually rational and selfish choices lead to lower collective well-being than individually nonrational or unselfish choices (Dawes, 1980; Messick & Brewer, 1983). Since social dilemmas occur frequently in important arenas in social life (see the classic treatments of Hardin, 1968; Olson, 1965; Ostrom, 1990) and because they present many theoretical challenges (e.g., explaining why individuals do not choose the actions that maximize their individual well-being, explaining why factors such as cheap talk, that theoretically should not affect behavior, in fact do affect it), studying individuals' choices in these dilemmas is one of the main topics in diverse branches of the social sciences (e.g., Fehr & Gächter, 2002; Kollock, 1998; Simpson, 2004; Willer, 2009).

Address correspondence to Jacob Dijkstra, Assistant Professor of Sociology, Department of Sociology/ICS, University of Groningen, Grote Rozenstraat 31, 9712 TG Groningen, The Netherlands. E-mail: j.dijkstra@rug.nl

A central concept for understanding individual behavior in a social dilemma is *efficacy*, which is defined in the context of a social dilemma by Kerr (1996) as “a judgment of the degree to which one’s cooperative behavior will increase the chances of the group achieving some valued collective outcome” (pp. 211–212). Efficacy in a social dilemma thus pertains to the perceived necessity of one’s cooperative behavior for increasing the well-being of the entire group, and is treated throughout the sociological and social psychological literature as a *motivational* concept.

Efficacy affects individual behavior through a direct causal link (i.e., individuals who *think* their contribution matters more in fact contribute more often; see Kerr & Kaufman-Gilliland, 1994; Kerr, 1996), but is also an important mediating and moderating factor. For example, individuals feel less efficacious in larger groups, even though this may not objectively be the case (Kerr, 1989). Feelings of efficacy can be affected by certain types of leadership communication (Vasi & Macy, 2003). Also, the effects of pregame communication depend crucially on the perceived efficacy that the communication messages imply (Kerr, 1996; Van de Kragt, Orbell, & Dawes, 1983).

Efficacy in social dilemmas has most systematically been studied in step-level public goods dilemmas (SPG; Van de Kragt et al., 1983). In an SPG, every individual in a group of N individuals decides whether or not to invest in the public good, which is only provided if the sum of contributions exceeds some threshold, commonly called the provision point (Marwell & Ames, 1980). Incentives are such that if the public good is produced, all individuals are better off, including those who invested. If the investments are insufficient, however, and the public good is not produced, those who have invested incur a net loss. Thus, investing in the SPG is potentially beneficial for all members of the group, but implies a risk for the investor.¹

Efficacy in an SPG is thus an individuals’ judgment that her contribution is necessary to produce the public good. The dichotomy in both the individual contribution and collective outcomes in the SPG provides a clear framework to study the effects of efficacy: any contribution can make the difference between the public good being provided or not, and individual judgments regarding the necessity to contribute are vitally important. Most research suggests a positive linear relation between efficacy and contributions in an SPG.

From a *decision and game theoretical* perspective, however, such a monotonically increasing relationship between efficacy and investment is not at all self-evident. Thus, in their investigation of the “mobilizer’s dilemma,” Vasi and Macy (2003) note the possibility that an increased sense of efficacy (through the reception of so-called “empowerment messages”) might induce *free-riding*. In fact, taking the SPG studied in Kerr and Kaufman-Gilliland (1994) as the basis, we will show below that there are potentially many rational decisions and game theoretical equilibria, some of which indeed have a monotonically *increasing* relationship between efficacy and investment, whereas others have a monotonically *decreasing* relationship between efficacy and investment or a *nonmonotonous* relationship between efficacy and investment.

¹Note that an SPG is strictly speaking not always a social dilemma, since individually rational choices can be collectively efficient if an individual investment pushes the group over the threshold of the production of the SPG. However, since there always also exists an inefficient equilibrium in which no one invests, there is a tension between individual and collective interests, as in pure social dilemmas.

We then provide three concrete examples for which we prove that *all* symmetric game theory equilibria have the same monotonically decreasing, nonmonotonic, or monotonically increasing efficacy–investment relation. The key social psychological element distinguishing these different equilibria is individuals’ *beliefs* about the (distribution of the) efficacy of others, an element that is often ignored or assumed to have little impact in the existing literature on the effects of efficacy in social dilemmas.

There are important exceptions to the neglect of beliefs in theorizing about the SPG. For instance, van de Kragt et al. (1983) and Bornstein and Rapoport (1988) do explicitly argue that (perceived) efficacy (or “criticalness”) is affected by beliefs about what others do. In these papers the idea is that *preplay communication* affects beliefs, which affect perceptions of efficacy, which in turn affect behavior. In these studies, communication affects beliefs about the *behavior* of others (not their efficacy) in the SPG, basically because the agreement on the “minimal contributing set” (a minimal coalition of players that can produce the SPG; see van de Kragt et al., 1983) is both *self-committing* (i.e., part of a Nash equilibrium) and *self-signaling* (i.e., players want the others to play their best responses to the agreement if and only if they themselves intend to play according to the agreement). Thus, using the cheap-talk theory of Farrel and Rabin (1996), the concept of “reasonableness” that van de Kragt et al. invoke can be interpreted as the belief that self-committing and self-signaling messages are regarded as credible by “reasonable” players.

Rapoport (1985) builds a decision theoretic expected utility model of investment in the SPG, and in doing so also explicitly models players’ beliefs about the investment decisions of others. Rapoport considers three different assumptions that might govern players’ beliefs about the decisions of others: (a) the belief that all other players have the same probability p to invest (homogeneity assumption); (b) the belief that each other player j has an individual probability p_j to invest (heterogeneity assumption); and (c) the belief that the set of all other players is partitioned in a set of disjoint subsets k , with players in the same subset having the same probability p_k to invest (partial homogeneity assumption). Rapoport and Bornstein (1987) analyze this model in the context of intergroup competition involving an SPG, the provision point of which depends on the investments of the other group.² Although Rapoport’s is not a game theoretical equilibrium analysis, we will see that the partial homogeneity assumption maps onto the notion of *symmetric equilibrium* used in this article.

Whereas the studies mentioned above do explicitly consider beliefs when modeling the individual decision to invest or not in the SPG, we go one step further. In this article we will set up the decision theoretic model in the most general way, allowing for *any* belief structure, and then show that in *equilibrium* the relationship

²In the same intergroup competition context, Rapoport, Bornstein, and Erev (1989) report results of a first experiment that show a *decreasing* relationship between efficacy and investment probability. However, in this experiment higher efficacy was associated with higher costs of investment, so that for players with higher efficacy the monetary payoff to the investment was smaller. In a second experiment, in which higher efficacy is associated with proportionally larger costs *and* benefits of investment such that the ratio of benefits to costs is equal for all players, these authors report *no* relation between efficacy and investment.

between efficacy and investment can take many different forms and, in particular, need not be a monotonically increasing one.

In addition to sometimes accounting for beliefs in a nonstrategic manner, the social psychological hypothesis concerning the effects of (perceived) efficacy on investment in social dilemmas or other group tasks is sometimes differentiated according to characteristics of the group task or game (e.g., Kerr & Bruun, 1983; Kerr, 1989). However, this differentiation does typically not refer to beliefs in the game theoretical sense, and hypotheses are posited without reference to such beliefs.

In the next sections we present a decision and game theoretical analysis of the SPG such as it is studied in for instance Kerr (1992), Kerr and Kaufman-Gilliland (1994), and Kerr, Garts, Lewandowski, and Harris (1997). We explicitly adopt the payoff parameters from the latter two studies. The theoretical analysis yields two main results: (a) the monotonically increasing relationship between efficacy and investment in the SPG found in empirical studies like the one of Kerr and Kaufman-Gilliland (1994) and Kerr et al. (1997) can from a decision and game theoretical perspective only be rigorously derived if more assumptions are made concerning the beliefs individuals have regarding the distribution of efficacy across all individuals, and (b) more information about those distributions on the part of individuals making the decisions can lead to predicted behavior that shows monotonically decreasing or nonmonotonic relationships between efficacy and investment. This second result suggests that the empirically observed monotonically increasing relationship between efficacy and investment in the SPG is caused by a *lack* of information about the efficacy of others, on the part of participants. This lack of information should theoretically be regarded as a scope condition under which the monotonically increasing relationship between investment and efficacy is predicted.

The purpose of this article is emphatically not to contend that the motivational account of the efficacy hypothesis is mistaken. Scientifically, it might well be the most potent explanation, and data show that efficacy is an important factor in determining individual behavior in social dilemmas. The purpose of this article is simply to show that (a) from a decision and game theoretical perspective, the efficacy hypothesis is less straightforward than it might at first seem to be from a purely motivational account; (b) in many instances there might exist an alternative, or at least richer, explanation for the observed relation between efficacy and investment that deserves scrutiny; and (c) we can in future experiments try to manipulate participants' beliefs in a precise and controlled manner to further assess the merit of the decision and game theoretical model. In the end, this might well further corroborate the social psychological motivational account.

The rest of the article is organized as follows. In the next section we define the basic SPG around which the discussion in this article revolves, and in the section after that we set up the model. The two subsequent sections contain detailed analyses of the decision theoretic and game theoretic problems, respectively. In the penultimate section we present three concrete numerical examples of belief structures that induce different forms of the efficacy–investment relationship. For these games we prove that all symmetric equilibria have the same form of efficacy–investment relation (i.e., monotonically decreasing, nonmonotonic, or monotonically increasing). The article concludes with a general discussion of the implications of the results for the investigation of efficacy in social dilemmas and specifically in SPGs.

2. THE BASIC SPG

The SPG in Kerr and Kaufman-Gilliland (1994; hereafter KKG) has the following form. There are five individuals (hereafter called players) whom each have an endowment of \$10. Each player separately and in ignorance of what others are choosing, decides whether to invest the \$10 or keep them to themselves. Investment means the investor loses her \$10. If the SPG is produced, all players earn \$15, regardless of whether or not they invested, leaving investors with \$15 and noninvestors with \$25.

Production of the SPG is a function of the investments in the following way. Each player is assigned a number (called share) between 1 and 50 (inclusive) that determines the weight that individual's investment carries in the production of the SPG. If the sum of all the weights of the investors is 51 or higher, the SPG is produced. Thus, an individual's share determines her objective efficacy.

Importantly, it is common knowledge that the shares sum to 100, but each player knows only her own share and that no one share is larger than 50. Thus, concerning the shares of the other players, a given player knows only that they sum to 100 minus her own share. The empirical finding of the monotonically increasing relationship between efficacy and investment found in KKG thus comes down to a significantly positive statistical relation between share and the probability of investment.

Two further design aspects of the KKG experiment should be mentioned. First, although participants play the game repeatedly, they get no feedback about the results between rounds. Thus, game theoretically we are dealing with a one-shot game, and we will treat it as such below. Second, players are not provided with any information concerning the distribution of others' shares. However, to reach a decision on whether or not to invest, players will likely form beliefs about this. While these beliefs were not elicited in the study of KKG, we will show that from a decision and game theoretical perspective they are of paramount importance for the predicted relationship between efficacy and investment.

3. SETTING UP THE MODEL

To spell out the decision and game theoretical models of the SPG, we first introduce some notation. Let θ_i denote player i 's share, and let $\bar{\theta}$ denote the vector of shares, one for each of the five players. Prior to being told her share (or prior to "Nature" determining her share, in the parlance of game theory), player i has beliefs concerning all vectors $\bar{\theta}$ that have $\sum_j \theta_j = 100$, and $\theta_j \leq 50$, for all j . We can summarize these beliefs in a (personal) probability distribution over all such vectors. Thus, player i 's beliefs are denoted as cumulative distribution $P_i(\bar{\theta})$, with $p_i(\bar{\theta})$ denoting the associated probability density function.

Each player must make a choice whether to invest or not, and for generality we allow the possibility that players "mix" their actions, that is, that they invest with a certain *probability*. A player's strategy in this game can then be described as a map from her shares to the set of all probability distributions over the actions "invest" and "not invest." Thus, a player's strategy is a map that takes as input

one of the possible shares she can have and has as output the probability that the player invests.³ Therefore, in terms of the decision and game theoretical models of the SPG, the monotonically increasing relationship between efficacy and investment hypothesized in social psychological theory and found in experiments means that the probability of investment is increasing (or at least nondecreasing) in θ_i . However, depending on the beliefs $P_i(\bar{\theta})$, virtually any form of relationship between these two variables can be sustained as a rational individual choice or in equilibrium.

In the next two subsections, we will first show that from an individual decision-making perspective (i.e., decision making as a “control problem”) there is nothing particularly special, necessary, or inevitable about the monotonically increasing relation between efficacy and investment probability. Then, we proceed to show that from a game theoretical perspective (i.e., decision making as a “strategic problem”) the same is true.

4. INDIVIDUAL DECISIONS IN THE SPG: THE CONTROL PROBLEM

Since players know their own share, and only their own share, we must consider their conditional (or posterior) beliefs $P_i(\theta_{-i}|\theta_i)$ and $p_i(\theta_{-i}|\theta_i)$, when analyzing their investment decisions, where θ_{-i} denotes the vector of shares of the players other than i . Let $s_i(\cdot)$ denote the strategy of player i ; that is, $s_i(\theta_i)$ denotes the probability that player i invests when her share is θ_i . Similarly, let $s_{-i}(\cdot)$ denote the vector of strategies of all players other than i , and let $s(\cdot)$ denote the vector of strategies of all players. For a given share vector θ_{-i} , let $X(s_{-i}(\theta_{-i})|\theta_{-i})$ denote the random variable that equals the sum of the shares of the investors among the players other than i . As is expressed in the notation, the distribution of this random variable is induced by the strategies of the players.

Then from the perspective of player i with share θ_i , the probability that the SPG will be produced when i *does not* invest is $\sum_{\theta_{-i}} p_i(\theta_{-i}|\theta_i) \Pr[X(s(\theta_{-i})|\theta_{-i}) \geq 51]$, whereas the probability that the SPG is produced when i *does* invest is $\sum_{\theta_{-i}} p_i(\theta_{-i}|\theta_i) \Pr[X(s(\theta_{-i})|\theta_{-i}) + \theta_i \geq 51]$. In forming these two equations we have implicitly assumed that the $p_i(\theta_{-i}|\theta_i)$ are discrete probability densities. By replacing the summation sign by an integral sign (and integrating over the four-dimensional space covered by the θ_{-i}), one can deal with continuous probability densities. Since this does not affect the results of the analyses, and since the summation over discrete probabilities is more intuitive, we have chosen to use summation signs. The two equations above simply express the probabilities that the investments of all players sum to at least 51, given player i 's action and her beliefs about the conditional distribution of shares. Note how the probability that the SPG is produced *given* a certain vector of shares $\bar{\theta}$, in general depends on the strategies of all the players. For instance, if two players

³One could also argue that a player's strategy is a map from her share *and her beliefs* to the set of probability distributions over her actions. However, since $P(\cdot)$ is given and fixed, a player's posterior beliefs (i.e., beliefs after she learns her own share) are entirely determined by her share. Modeling strategies as depending only on players' own shares is therefore equivalent to modeling strategies as depending on players' own shares and her beliefs about the shares of others.

who together have shares of at least 51 each invest with probability .5, while all others do not invest, the probability that the SPG is produced is .25.⁴

Thus, a player's (subjective) probability that the SPG will be produced depends on (a) her share or efficacy, (b) her beliefs concerning the distribution of efficacies of the others, (c) her action (i.e., investment probability) given her share, and (d) the strategies of the other players. With this probability in hand we can write player i 's expected utility as a function of her strategy, her beliefs, and the strategies of the others, given her share θ_i as

$$u_i(s_i(\theta_i), P_i(\cdot), s_{-i}(\cdot)) = s_i(\theta_i) \left\{ 15 \sum_{\theta_{-i}} p_i(\theta_{-i} | \theta_i) \Pr[X(s(\theta_{-i}) | \theta_{-i}) + \theta_i \geq 51] \right\} + \\ (1 - s_i(\theta_i)) \left\{ 15 \sum_{\theta_{-i}} p_i(\theta_{-i} | \theta_i) \Pr[X(s(\theta_{-i}) | \theta_{-i}) \geq 51] + 10 \right\}$$

Her decision task now comes down to choosing a strategy $s_i(\cdot)$ that maximizes $u_i(s_i(\theta_i), P_i(\cdot), s_{-i}(\cdot))$, for each θ_i that has strictly positive probability under $P_i(\cdot)$.

The formalization of the individual decision problem above makes clear that to actually reach a decision, player i must form expectations concerning the strategies of the others, in addition to holding beliefs regarding their shares, and these two are linked. Thus, a player has beliefs concerning the probabilities of different vectors of shares across the other players. Since these players' strategies depend on their shares, the focal player also must have expectations or beliefs about this share-strategy link. These two sets of beliefs together determine the subjective probability mixture in terms of the other players' behaviors against which the focal player has to maximize her utility.

But from this formulation it is immediately clear that, depending on her beliefs and expectations, player i might very well rationally choose a strategy $s_i(\cdot)$ that is *not* monotonically increasing in her share θ_i . Deferring numerical examples until the next section, let us give the intuition here.

If a player expects others to use strategies that are monotonically *decreasing* in their shares, and if under her beliefs $P_i(\cdot)$ low shares are sufficiently probable (for instance if she believes share distributions are very likely to be right-skewed, with one player having a high share and the others having a low share), she herself rationally chooses a strategy that is decreasing in her share. Thus, if "Nature" assigns her a low share, she invests her \$10, and if "Nature" gives her a high share she lets the others pick up the tab. Given her beliefs about the distribution of shares and her expectations concerning the behavior of others, this is then the strategy that maximizes her expected utility over all possible shares she might have.

This intuitively shows that, in the formalization of the individual decision problem there is nothing per se that justifies the hypothesis that a player's probability to invest will be increasing in her share. In other words, the hypothesis of a positive relationship between efficacy and investment does not follow from a formal analysis of the individual decision problem *unless* we make more assumptions concerning players' beliefs and expectations. Typically, in social psychology such assumptions are not explicitly made, at least not in the detail sufficient to allow rigorous derivation

⁴Note that we assume *independent* strategies, although players' *shares* are of course generally dependent, since they sum to 100.

of the hypothesis. The previously mentioned paper of Rapoport (1985) constitutes an exception in that it considers three different possible belief structures. However, in the model of Rapoport the players' objective efficacies (i.e., shares) are all the same and the beliefs directly concern players' expectations regarding the actual *investment decisions* of the others.

Up to this point we have analyzed the individual decision problem confronted by the player, and concluded that without further assumptions we cannot derive the positive relation between efficacy and investment. Next, we show the same is true for equilibria in the SPG.

5. EQUILIBRIA IN THE SPG: THE STRATEGIC PROBLEM

Given the fact that the θ_i are private information, the appropriate equilibrium concept to consider is the Bayes-Nash equilibrium (hereafter BNE; see for instance Fudenberg & Tirole, 1991). Compared to the individual decision problem from the previous subsection, BNE imposes two restrictions on the beliefs and strategies of the players. The first one is that the beliefs must be "correct" or "consistent." The standard way to impose this restriction is to assume that the probability distribution over the vectors of shares is *common knowledge*. Thus, for each player i , $P_i(\cdot) = P(\cdot)$, and $P(\cdot)$ is common knowledge. The second restriction is that strategies are in equilibrium, that is, that for each player i and share θ_i that has strictly positive probability under $P(\cdot)$, $u_i(s_i^*(\theta_i), s_{-i}^*(\cdot)) \geq u_i(s'_i(\theta_i), s_{-i}^*(\cdot))$, for any strategy s'_i , where $s_i^*(\cdot)$ and $s_{-i}^*(\cdot)$ denote the equilibrium strategies of player i and all other players, respectively.

Again, and as in the individual decision-making problem, nothing in the equilibrium concept of BNE implies that equilibrium strategies are necessarily monotonically increasing in the shares. Thus, as before, without making further specific assumptions about players' beliefs, it is not possible to rigorously derive the hypothesis that there is a positive relation between efficacy and investment probability. In fact, it is easy to construct "beliefs" (i.e., common knowledge probability distributions over the share vectors) such that equilibrium strategies are (a) monotonically decreasing in the shares, (b) nonmonotonic in the shares, or (c) monotonically increasing in the shares.

We will now present a very simple example of each case, involving very limited beliefs, for expositional clarity. However, lest the simplicity of these examples convey the impression that only such limited beliefs can give rise to the different relations between efficacy and investment, we stress here that the point is completely general: without making further assumptions on players' beliefs, the efficacy hypothesis cannot be derived from a decision and game theoretical analysis.

6. THREE EXAMPLES

To simplify the computations of the equilibria in the examples below, we impose two restrictions. Most importantly, we focus on *symmetric* BNE. In these equilibria the map from share to probability of investment is identical for all players. Thus, even though different players will have different shares in any given play of the game, their maps from shares to investment probabilities are the same. Concretely

this means that players with the same share have the same equilibrium investment probability. Note that this in no way affects our main point: depending on beliefs there do exist symmetric BNE that are *not* monotonically increasing in shares.

In addition, there might also be a more substantive reason for choosing symmetric BNE. Theoretically, players are distinguished from each other based only on their shares, or efficacies. Therefore, it seems reasonable to look for symmetric BNE in which players who are equally efficacious have the same investment probability. The focus on symmetric BNE also has a more direct payoff: in the appendix we show that in the three games below *all* symmetric BNE in each example, have the same form of relationship between efficacy and investment probability. Thus, if one is persuaded by the symmetry argument (and is looking for equilibrium in the first place), one is lead to predict monotonically decreasing and nonmonotonic investment behavior in the first and second example, respectively. Moreover, the criterion of symmetry in equilibrium selection nicely maps onto the partial homogeneity assumption made by Rapoport (1985). Under this assumption, the set of (other) players is partitioned in a set of disjoint subsets. Players in the same subset are then believed to have the same investment probability. Although the model of Rapoport considers a setting of complete information in which all players are equally efficacious, partial homogeneity is readily applicable to our model simply by letting the shares of the players determine the partition. Thus, two players are in the same subset if and only if they have the same share, and players in the same subset have the same equilibrium investment probability.

The second restriction to simplify the equilibrium computations is that we choose beliefs that, given the symmetry of the BNE, are “degenerate”: Given a player’s share, she knows with certainty what type of vector of shares she is in. For instance, in the first example, if a player knows she has a share of 15, she knows that three of the other players also have a share of 15, and one player has a share of 40. Since the BNE is symmetric, she then knows which equilibrium strategies the others will play, regardless of the actual *identity* of the player with the share of 40. In other words, after a player learns her share, the game becomes one of complete information. Once again, these restrictions do not affect the generality of our point, as the equilibria that we find are precisely that: equilibria.

In all the examples below, “no one investing” is always an equilibrium. Since this equilibrium is trivially monotonic, one could claim that there always exists a monotonic equilibrium. However, the efficacy hypothesis is clearly that the efficacy–investment relation is nondecreasing *and* yields strictly positive investment probabilities for at least some (perhaps relatively high) levels of efficacy. Thus, in the remainder we ignore the no-investment equilibrium and focus on equilibria that have a strictly positive probability of investment by at least some players.

6.1. Example 1: A Symmetric BNE with Monotonically Decreasing Efficacy–Investment Relation

Suppose that it is common knowledge that $P(\cdot)$ is as follows: the probability that all have a share of 20 is .5, and the probability that one player has a share of 40, while the others have a share of 15, is .1. Note that with five players there are five ways in which one can have a share of 40 while the others have a share of 15.

Thus, the probabilities of the possible share vectors sum to 1. Then $s_i(15) = 1$ and $s_i(20) = s_i(40) = 0$ is the unique pure strategy symmetric BNE.

It is easy to see this constitutes an equilibrium. First of all, assume a player finds she has a share of 20. Then she knows from her prior beliefs that the share vector is the one in which each player has a share of 20. Then she knows that in equilibrium, none of the other players invests. Her best reply to this is not to invest, either. If she finds she has a share of 15, she knows from her prior beliefs that three others also have a share of 15, and one other player has a share of 40. Then she knows that in equilibrium, the other players with a share of 15 invest, whereas the player with the share of 40 does not. Her best reply to this is to invest. Finally, if she finds she has a share of 40, she knows from her prior beliefs that the other players all have a share of 15. Then she knows that in equilibrium, the other four all invest, and her best reply to this is not to invest herself. In addition, there also exists a symmetric BNE in mixed strategies. The proof that this equilibrium has the same monotonically decreasing relationship between efficacy and investment probability is provided in the Appendix.

Thus, if the efficacies are such that each has a share of 20, it is common knowledge that the players are in this vector, and the only symmetric BNE is that no one invest. This is a fact that KKG do not mention: in the game they (and we) investigate the only equilibria if the shares are divided equally are asymmetric in a complete information setting. Rapoport (1985), however, effectively shows the same thing (for the complete information case), by demonstrating that under the homogeneity assumption (i.e., all other players are believed to have the same investment probability), no expected utility maximizing player should invest.

In any one of the other vectors, in which one player has share 40 and the others have share 15, the latter invest with positive probability and the former does not. Thus, if these were the players' beliefs and we focused on symmetric equilibrium, we would predict a *negative* relation between efficacy and investment probability.

6.2. Example 2: A Symmetric BNE with Nonmonotonic Efficacy–Investment Relation

Suppose that it is common knowledge that $P(\cdot)$ is as follows: The probability that all have a share of 20 is .5, and the probability that one player has a share of 2, one player has a share of 50, while the three others have a share of 16, is .025, for each of 20 vectors that can thus be formed. Then $s_i(2) = 1$ and $s_i(16) = s_i(20) = 0$ and $s_i(50) = 1$ is the unique symmetric BNE in pure strategies.

It is easy to see this is an equilibrium. Since the case in which a player learns she has a share of 20 has already been covered in Example 1, we leave it aside here. If a player learns she has share 2, she knows from her prior beliefs that three others have a share of 16, and one other player has a share of 50. Then she knows that in equilibrium, the other players with a share of 16 do not invest, whereas the player with the share of 50 does. Her best reply to this is to invest. If she finds she has a share of 50, she knows from her prior beliefs that one other player has a share of 2, and three other players have a share of 16. Then she knows that in equilibrium only the player with the share of 2 invests, and her best reply to this is to invest herself. Finally, if she learns she has a share of 16, she knows from her prior beliefs that one player has a share of 2, another has a

share of 50, and two others also have a share of 16. She then knows that in equilibrium only the former two invest, and her best reply to this is to not invest herself.

The proof that any mixed strategy equilibrium must also be nonmonotonic is easily derived from the fact that in any equilibrium with strictly positive investment probability, the player with a share of 50 must have a strictly positive probability to invest (see Appendix).

Thus, if the players are in a vector where one player has a share of 2, one player has a share of 50, while the three others have a share of 16, this is common knowledge, and the only symmetric equilibrium in pure strategies is that the *least* efficacious player and the *most* efficacious player together shoulder the burden and produce the SPG. Thus, if these are the players' beliefs and we focus on symmetric equilibrium, we would predict a nonmonotonic, U-shaped relation between efficacy and investment probability.

6.3. Example 3: A Symmetric BNE with Monotonically Increasing Efficacy-Investment Relation

Suppose that it is common knowledge that $P(\cdot)$ is as follows: the probability that all have a share of 20 is .5, and the probability that two players have a share of 35, while the three others have a share of 10, is .05, for each of the 10 vectors that can thus be formed. Then $s_i(10) = s_i(20) = 0$ and $s_i(35) = 1$ is the unique symmetric BNE in pure strategies.

It is again easy to see this is an equilibrium. We again omit the case already covered, where a player learns she has a share of 20. If a player learns she has share 10, she knows from her prior beliefs that two others also have a share of 10, and two other players have a share of 35. Then she knows that in equilibrium, the other players with a share of 10 do not invest, whereas the players with the share of 35 do. Her best reply to this is not to invest. If she finds she has a share of 35, she knows from her prior beliefs that one other player also has a share of 35 and three other players have a share of 10. Then she knows that in equilibrium, only the other player with the share of 35 invests, and her best reply to this is to invest herself. There also exists a symmetric BNE in mixed strategies. The proof that this equilibrium has the same monotonically increasing relationship between efficacy and investment probability is in the Appendix.

Thus, if the players are in a vector where two players have a share of 35 while the three others have a share of 10, this is common knowledge, and all symmetric equilibria have the players with a share of 35 invest with positive probability and produce the SPG. If these are the beliefs and if one focuses on the symmetric equilibrium, one would in this case predict a positive relation between efficacy and investment probability, as has been empirically found in many experiments.

The three examples above used the same SPG game with the same payoffs. The only element that differed between them was the belief structure. Using the symmetric pure strategy equilibria derived above we can lump the efficacy levels from the three examples and relate them to the investment probabilities. We then get

$$(s_i(2), s_i(10), s_i(15), s_i(16), s_i(20), s_i(35), s_i(40), s_i(50)) = (1, 0, 1, 0, 0, 1, 0, 1),$$

which clearly displays a nonmonotonic relationship between efficacy and investment behavior.

7. SO WHAT? IMPLICATIONS FOR THE SOCIAL PSYCHOLOGY OF EFFICACY IN SOCIAL DILEMMAS

Efficacy is a key concept in understanding cooperative behavior in social dilemmas. Many empirical studies have established a positive relation between efficacy and contribution rates in social dilemmas, especially the SPG. The relation between efficacy and contribution rates is implicitly assumed to be monotonically increasing, which is substantiated by experimental evidence of SPG dilemmas.

However, the exposition above has made clear that *from a decision and game theoretic perspective* the hypothesis that there be a positive relationship between a player's efficacy and her investment probability is not warranted, without making further, explicit assumptions on the players' beliefs. In our view, this has at least two implications for the investigation of the efficacy–investment relation.

First, the empirical finding that this relation is positive needs further explanation in terms of players' beliefs about the distribution of efficacy. Thus, in future research, players' beliefs about the shares that others have, and their expectations about the strategies of others (i.e., expectations about how others adjust their investment probabilities to their share) should be elicited. Only then can it be clarified whether players' investment decisions are best responses given their beliefs and expectations or whether players' make these decisions based on other rules. It could for instance be argued that players in a new experimental situation are uncertain about how to behave and use their shares as “anchors” for some heuristic, such as “the higher my share, the more I should invest.” Or perhaps players follow a norm of the form “those with higher shares should invest more,” or “the strongest shoulders should carry the heaviest burden.” Or perhaps players think others *expect* those with higher shares to invest more, without themselves thinking this necessarily should be the case (for instance, players could have read this article and think others have not). We would most certainly not want to suggest that any of these explanations of the positive efficacy–investment relation is more unlikely than the game theoretical best response explanation. All we are saying is that we cannot know unless we know players' beliefs and expectations.

A particularly appealing possible explanation for the empirically found positive efficacy–investment relationship seems to be that participants are “naïve cost-benefit analyzers.” In the SPG from KKG the direct costs of investment are \$10 and the expected benefit is \$15 times the impact the investment has on the probability of production of the public good. An increase in one's share or efficacy does not affect the direct investment costs, but seems to increase the expected benefit (although, as we have argued above, this of course depends on how the decisions of others depend on their shares and on how one believes the shares are distributed, but this is exactly why we call this reasoning naïve cost-benefit analysis). Since the apparent cost-benefit balance of the act of investment becomes more positive, investing becomes more attractive, and the individual's propensity to invest increases. This explanation of the efficacy hypothesis seems to parallel the motivational account from social psychology. In fact, we think the two are identical. If veridical, and if efficacy-driven individual motivation would be the only factor determining behavior in the SPG, this would imply that players ignore the strategic nature of the situation and approach it as a pure control problem.

Second, our theoretical result is that by making the probability distribution over the vectors of shares explicit and common knowledge in future experiments, we can directly manipulate players' beliefs in a controlled way. Thus, if we were to conduct experiments with the share distributions from the three examples being common knowledge, we would expect to see different efficacy–investment relations across distributions. In those experiments, the game theoretical model would then constitute a clear benchmark against which actual decisions of experimental subjects could be contrasted.

Finally, the modeling of the current article sheds new light on a central part of the experimental results reported in KKG and Kerr et al. (1997). As mentioned before, these studies found a strong positive association between efficacy and investment probability. Moreover, whereas preplay communication was shown to increase investments, the efficacy–investment relation was the same across communication conditions. However, in these studies communication took place *before* the actual assignment of shares or efficacies. Thus, participants in these experiments could only discuss general strategies to play the game, and a strategy such as “invest whenever your share is X or more” is a simple and readily available decision rule. If followed by a sufficient number of participants, this rule indeed leads to a monotonically increasing relationship between efficacy and investment probability. However, if a given share distribution would have been known at the time of discussion, say the (2, 16, 16, 16, 50) distribution from Example 2 above, one could easily imagine a very different outcome of the communication process. Indeed, one can envision a conflict arising between the three players with a share of 16 and the player with a share of 2, in which the latter would advocate a “higher efficacy means higher investment” kind of rule, whereas the former would argue along the lines of our symmetric pure strategy equilibrium. After all, given that the player with a share of 50 must always invest for the SPG to be produced, the group needs only one additional investor. Designating the single player with a share of 2 to be this investor seems at least as convincing as designating one of the players with a share of 16 (which would entail the additional issue of *which* of the three players with a share of 16 should make the investment). Of course, the player with the share of 2 will argue that exactly because of the fact that *any* additional investor will do, the players with the share of 16 and 2 are structurally equivalent and their shares effectively mean nothing, and so on. It is not clear why the single player with a share of 2 would always gain the upper hand in this conflict. It might not even be true that preplay communication in this case is conducive to the production of the SPG. Thus, one additional interesting hypothesis from the current article is that the effects of communication on investment crucially depend on the amount of information concerning efficacy available at the time of discussion.

With this article we hope to have convincingly shown how the analysis of a simple formal model of the decision situation and strategic interactions involved in the production of SPGs can elicit scope conditions of previously formulated, more intuitive hypotheses, and how this analysis can in addition yield many interesting hypotheses for future research. Even if in this future research the game theoretically derived hypotheses were to be completely rejected, this would in our opinion yield improved social psychology and better informed behavioral game theory.

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APPENDIX: THE EFFICACY-INVESTMENT RELATION IN THE THREE EXAMPLES

In each of the examples, the prior beliefs are such that a player learning her own share amounts to her learning the type of vector the game is in: the equal share vector, or the unequal share vector. She then knows with certainty what the equilibrium investment probabilities of the others are, and the game is effectively one of complete information. Thus, to show that all symmetric BNE have the same efficacy-investment relationship for each of the three examples, all we have to show is that the complete information games associated with each of the four vectors used in those three examples each only have symmetric equilibria corresponding to the BNE.

Throughout, let y_k denote the equilibrium probability that a player with share k invests. Also, let $u_i(1)$ and $u_i(0)$ be player i 's expected utility of investment and noninvestment, respectively. We begin with the vector in which each player has a share of 20, which is part of each example. We have $u_i(1) = 15[1 - (1 - y_{20})^4 - 4y_{20}(1 - y_{20})^3]$ and $u_i(0) = 15[1 - (1 - y_{20})^4 - 4y_{20}(1 - y_{20})^3 - 6y_{20}^2(1 - y_{20})^2] + 10$. This yields $u_i(0) - u_i(1) = 10 - 90y_{20}^2(1 - y_{20})^2 > 0$. That the latter inequality is true can be seen from the fact that $y_{20}^2(1 - y_{20})^2$ has three stationary points at 0, $\frac{1}{2}$, and 1, the first and last being minima and the middle being the maximum. Thus, the minimizer of $u_i(0) - u_i(1)$ is $y_{20} = \frac{1}{2}$, and yields $u_i(0) - u_i(1) = 4\frac{3}{8} > 0$. Thus, $u_i(0) > u_i(1)$ for any $y_{20} \in [0, 1]$ and the best reply of player i is to never invest. Hence, the only symmetric equilibrium is $y_{20} = 0$.

Example 1

Now consider the unequal vector from Example 1, in which one player has a share of 40 and the others each have a share of 15. First note that in pure strategies, the only symmetric equilibrium is that the players with a share of 15 invest, and the player with the share of 40 does not, which is the symmetric pure strategy equilibrium from the main text. Thus, we have to show that any existing mixed strategy equilibrium has a monotonically decreasing relationship between efficacy and investment probability.

Consider player i with $\theta_i = 40$. For this player we have $u_i(1) = 15[1 - (1 - y_{15})^4]$ and $u_i(0) = 15y_{15}^4 + 10$. Player i is willing to randomize over investing and not investing if and only if these two actions have the same expected utility, yielding the requirement $5 - 15(1 - y_{15})^4 - 15y_{15}^4 = 0$. Numerical search using the method of Newton-Raphson yields two solutions, namely $y_{15} \cong 0.24, 0.76$. Now consider player j with $\theta_j = 15$. First suppose that $y_{15} \cong 0.24$. Then we have $u_j(1) = 15[y_{40} + 0.014(1 - y_{40})] = 15[0.014 + 0.986y_{40}]$, and $u_j(0) = 15[y_{40}(1 - (1 - y_{15})^3)] + 10 = 15[0.56y_{40}] + 10$, which are both linear in y_{40} , and for which $u_j(0) > u_j(1)$, for any $y_{40} \in [0, 1]$. Thus, $y_{15} \cong 0.24$ does not constitute an equilibrium because the players with a share of 15 are never willing to play this mixture.

Now suppose that $y_{15} \cong 0.76$. Then we have $u_j(1) = 15[0.44 + 0.66y_{40}]$ and $u_j(0) = 15[0.99y_{40}] + 10$, for which it is again true that $u_j(0) > u_j(1)$, for any $y_{40} \in [0, 1]$. Therefore, $y_{15} \cong 0.76$ does also not constitute an equilibrium, because the players with a share of 15 are never willing to play this mixture. Thus, the player with a share of 40 cannot be made to mix, since the players with a share of 15 are not willing to

play any of the mixtures necessary to make the player with a share of 40 indifferent between investing and not investing. Therefore, in equilibrium the player with the share of 40 will play a pure strategy.

First, suppose $y_{40} = 1$, and again consider player j with $\theta_j = 15$. We know that there is no symmetric pure strategy equilibrium with $y_{40} = 1$, and we are thus led to look for a possible mixed strategy equilibrium. We have $u_j(1) = 15$ and $u_j(0) = 15[1 - (1 - y_{15})^3] + 10$, which are equal if and only if $y_{15} \cong 0.13$ (solution found using the method of Newton-Raphson). For $y_{15} \cong 0.13$, however, $u_i(1) = 6.41 < 11.95 = u_i(0)$ for player i with $\theta_i = 40$. Thus, $y_{40} = 1$ is not a best reply, and cannot be part of a symmetric equilibrium.

Second, suppose $y_{40} = 0$, and once more consider player j with $\theta_j = 15$. Then $u_j(1) = 15y_{15}^3$ and $u_j(0) = 10$, which are equal if and only if $y_{15} = \sqrt[3]{\frac{2}{3}} \cong 0.87$. Then, $u_i(1) \cong 15 < 18.59 = u_i(0)$ for player i with $\theta_i = 40$, and $y_{40} = 0$ is a best reply.

Thus, in addition to the symmetric pure strategy BNE with $s_i(15) = 1$ and $s_i(20) = s_i(40) = 0$, there exists a symmetric mixed strategy BNE with $s_i(15) = \sqrt[3]{\frac{2}{3}}$ and $s_i(20) = s_i(40) = 0$. In both equilibria, the relationship between efficacy and investment probability is monotonically decreasing, which is what we set out to prove.

Example 2

Next consider the unequal share vector of Example 2, in which one player has a share of 2, one other player has a share of 50, and the three remaining players have a share of 16. The only symmetric pure strategy BNE is the one mentioned in the main text, where only the players with the shares of 2 and 50 invest, and the others do not. We now inspect possible mixed strategy equilibria.

First observe that in any equilibrium with positive investment probability, we must have $y_{50} > 0$. Since $y_{20} = 0$, the only possibility to have a monotonic relationship between efficacy and investment is an increasing one. But this would imply that $y_2 = y_{16} = 0$, which would mean that $y_{50} = 0$ is a best reply for the player with share 50. Thus, any equilibrium with strictly positive investment probability is nonmonotonic, which is what we wanted to prove.

Example 3

Finally, consider the unequal share vector of Example 3, in which two players have a share of 35 and three players have a share of 10. The unique symmetric BNE in pure strategies has only the players with a share of 35 investing, establishing a monotonically increasing relationship between efficacy and investment. We now consider mixed strategy equilibria.

First consider player i with $\theta_i = 10$. We then have $u_i(1) = 15[2y_{35}(1 - y_{35})(2y_{10}(1 - y_{10}) + y_{35}^2)]$ and $u_i(0) = 15[2y_{35}(1 - y_{35})y_{10}^2 + y_{35}^2] + 10$. Let $f(y_{10}, y_{35}) = u_i(1) - u_i(0) = 30y_{35}(1 - y_{35})(2y_{10}(1 - y_{10}) - y_{10}^2) - 10$. Player i is willing to play a mixed strategy only if $f(y_{10}, y_{35}) = 0$, but we will show that $f(y_{10}, y_{35}) < 0$. To do this, find the first order derivative of $f(y_{10}, y_{35})$, and set it to zero. We get

$Df(y_{10}, y_{35}) = \begin{bmatrix} 30y_{35}(1 - y_{35})(2 - 6y_{10}) \\ 30(1 - 2y_{35})(2 - 3y_{10})y_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. The set of stationary points of $f(y_{10}, y_{35})$ that satisfies this equation is $\{(y_{10}, y_{35}) : (0, 0), (0, 1), (\frac{2}{3}, 0), (\frac{2}{3}, 1), (\frac{1}{3}, \frac{1}{2})\}$. Then compute the Hessian of $f(y_{10}, y_{35})$, to get $D^2f(y_{10}, y_{35}) = \begin{bmatrix} -180y_{35}(1 - y_{35}) & 30(1 - 2y_{35})(2 - 6y_{10}) \\ 30(1 - 2y_{35})(2 - 6y_{10}) & -60(2 - 3y_{10})y_{10} \end{bmatrix}$. Evaluation of the determinant of the Hessian reveals that the first 4 stationary points are all saddle points with an image of $f(0, 0) = f(0, 1) = f(\frac{2}{3}, 0) = f(\frac{2}{3}, 1) = -10$. The fifth stationary point is a local *maximum* (which can also be seen by just inspecting $f(y_{10}, y_{35})$) with $f(\frac{1}{3}, \frac{1}{2}) = -7\frac{1}{2}$. Inspection of $f(y_{10}, y_{35})$ along the boundaries of its domain, $y_{10} = 0$, $y_{10} = 1$, $y_{35} = 0$, and $y_{35} = 1$ reveals that here we always have $f(\cdot, \cdot) \leq -10$. Thus, $f(y_{10}, y_{35}) < 0$ for any $\{(y_{10}, y_{35}) : 0 \leq y_{10} \leq 1, 0 \leq y_{35} \leq 1\}$, player i never plays a mixed strategy in equilibrium, and $y_{10} = 0$.

Given that $y_{10} = 0$, it is straight-forward to see that there exists a mixed strategy with $y_{35} = \frac{2}{3}$. Thus, in addition to the symmetric pure strategy equilibrium from the main text, there exists a single symmetric mixed strategy equilibrium with $s_i(10) = s_i(20) = 0$ and $s_i(35) = \frac{2}{3}$. This equilibrium also has the monotonically increasing relationship between efficacy and investment probability, which is what we set out to prove. *QED*.