EVIDENCE FOR AN ENERGY DEPENDENT FORM FACTOR
OF THE \( \alpha \)-PARTICLE OPTICAL POTENTIAL

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Detailed angular distributions for elastic \( \alpha \)-particle scattering from \( ^{90}\text{Zr} \) over a wide angular range have been obtained at 40, 59, 80, 100 and 118 MeV. A distinct “break” in the energy dependence of the shape parameters of the optical is observed.

Due to a paucity of \( \alpha \)-particle scattering data over a wide energy range and also because of ambiguities in the determination of optical model parameters comparatively little was known about the energy dependence of the \( \alpha \)-nucleus optical potential until very recently [1–4]. It has been shown that the discrete ambiguity in the real well depth can be resolved by taking data at high energies out to sufficiently large scattering angles [1, 2, 5–7]. However, it is still an unanswered question whether or not \( \alpha \)-particle scattering can be consistently described over wide angular and energy ranges by the optical model with fixed geometrical parameters (form factors) and a smooth variation of the potential strengths with energy as one would expect for instance on the basis of the folding model as it is usually applied to \( \alpha \)-particle scattering [4, 8, 9].

It is to this question that we address ourselves in this letter in which the first results of a continuing study of the energy and \( A \)-dependence of \( \alpha \)-particle scattering are presented.

Using non-analysed \( \alpha \)-particle beams from the variable energy cyclotron of the University of Groningen differential cross sections for scattering of \( \alpha \)-particles from \( ^{90}\text{Zr} \) have been obtained over a wide angular range at laboratory energies of 40.0, 59.1, 79.5, 99.5 and 118 MeV. The incident energies are known to an accuracy of 1% or better. Isotopically enriched and self-supporting targets of thicknesses ranging from 1 to 5 mg/cm\(^2\) were used. The scattering angles were known within 0.01\(^\circ\) and the angular resolution varied between 0.50\(^\circ\) for angular regions with rapidly varying cross sections and 1\(^\circ\) at larger angles. The total energy resolution was always less than 300 keV. The errors assigned to the differential cross sections were the statistical errors or 5\%, whichever was larger, added to the error due to the inaccuracy in the scattering angle.

The data for elastic scattering are shown in fig. 1, together with the optical model fits to be discussed below. At some of the large scattering angles an estimated upper limit for the value of the differential cross section is indicated by the horizontal line of a downward arrow. The angular distributions clearly exhibit an oscillatory structure and, except at \( E_\alpha = 40 \) MeV, a smooth fall-off at large angles.

While a detailed account of the analysis of these data will be published elsewhere (see also ref. [2]) this letter deals with one aspect of the results from an analysis in terms of a six-parameter optical model in which both real and imaginary potential are of Woods–Saxon shape.

Discrete real well depth families were determined by fixing \( V \) to a number of values between 0 and 300 MeV and varying the remaining five parameters to obtain a best fit to the data. The number of discrete ambiguities in the real well depth decreases with increasing \( \alpha \)-particle energy and only one real potential family was found to fit the 118 MeV data over the whole angular range in agreement with the findings for other nuclei (see refs. [1] and [5–7]). For the lower energies the corresponding family was located by requiring a smooth variation of the volume integral \( J_R \) per pair of interacting particles [10] with energy. The best fit parameters within the unique potential family are given in table 1, the corresponding fits are shown as full in fig. 1.

It is seen from the table that the 79.5, 99.5 and 118 MeV data can be described with almost identical radial and diffuseness parameters \( r \) and \( a \) and a linear energy dependence in \( V \) and \( W \) \( ^\dagger \). At lower energies, "Footnote see next page."
Fig. 1. Differential cross sections for elastic scattering of $\alpha$-particles from $^{90}$Zr. The downward arrows at large scattering angles correspond with estimated upper limits for the cross section. Solid curves are best optical model fits with the parameters given in table 1. Dashed curves are fits with fixed-geometry potentials (see text).

However, the best fit “geometrical parameters” $r$ and $a$ have significantly different values, and in particular for $r_R$ and $a_R$ a systematic variation with energy is found. The difference in the quality of fit for a “fixed geometry” with $r_R$, $a_R$, $r_I$ and $a_I$ being the same as for the highest energies and a search on $V$ and $W$ (dashed curves) and for calculations in which the geometrical parameters were allowed to vary (solid curves) is shown in fig. 1. Though the positions of the maxima and minima in the angular distributions are reproduced rather well, even for the 26.2 MeV data of Mailandt et al. [11] which were also analysed (but which are not included in fig. 1), the fits with the fixed geometry fail to give the correct magnitudes at low energies. The discrepancies between the calculated curves and the data increase with decreasing energy. Extensive searches with different geometrical parameters showed that the failure to fit the low-energy data with the fixed-geometry potential is mainly due to the different shape of the real potential.

Thus the arresting feature from the analysis of the present data is that while the three highest energies can be fit with essentially the same potential (with a smooth variation in $V$ and $W$) this potential fails to account for the low-energy data ($E_\alpha < 60$ MeV). Instead a definite change in the shape of the real potential is required to fit the low-energy data.

Because of the success of fitting data at 80, 100 and 118 MeV with the same potential it is tempting to regard this as the “true” $\alpha$-nucleus potential which at low energies is distorted by exchange effects or by the term in Feshbach’s generalized optical potential which represents virtual excitations [12]. Both these effects are expected to be most important at low energies. It is also conceivable that the Woods–Saxon shape is not adequate to describe the $\alpha$-scattering. Since at different energies different radial parts of the potential may be dominant in determining the scattering process, such an inadequacy may result in energy-dependent Wood–Saxon parameters. Preliminary attempts with different but fixed potential shapes and especially with those obtained from the folding model, however, did not give better results. Similarly calculations with an $\ell$-dependent imaginary potential and a fixed-geometry have not been successful, which is not surprising as cross sections at forward angles in general are not affected by introducing an $\ell$-dependent absorption [13].

For the fixed geometry which is defined in the text and for which the parameters are given in the last line of table 1 these energy dependencies are given by $V(E_\alpha) = 160 - 0.26E_\alpha$ (MeV), $W(E_\alpha) = 17.0 + 0.025E_\alpha$ (MeV).
Table 1
Best-fit optical model parameters for the unique potential family at $E_\alpha = 118$ MeV and for corresponding families at lower energies, and comparison of $\chi^2/n$ values for best-fit potentials and potentials with fixed geometry.

<table>
<thead>
<tr>
<th>$E_\alpha$ (MeV)</th>
<th>$V$ (MeV)</th>
<th>$r_R$ (fm)</th>
<th>$a_R$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_I$ (fm)</th>
<th>$a_I$ (fm)</th>
<th>$J_R$ (MeV.fm$^3$)</th>
<th>$\chi^2/n$ for fixed geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.0</td>
<td>110.0</td>
<td>1.491</td>
<td>0.509</td>
<td>14.99</td>
<td>1.566</td>
<td>0.310</td>
<td>404</td>
<td>6.0</td>
</tr>
<tr>
<td>59.1</td>
<td>120.0</td>
<td>1.354</td>
<td>0.671</td>
<td>20.80</td>
<td>1.511</td>
<td>0.579</td>
<td>350</td>
<td>14.0</td>
</tr>
<tr>
<td>79.5</td>
<td>140.0</td>
<td>1.228</td>
<td>0.815</td>
<td>18.70</td>
<td>1.568</td>
<td>0.575</td>
<td>330</td>
<td>9.1</td>
</tr>
<tr>
<td>99.5</td>
<td>135.0</td>
<td>1.227</td>
<td>0.814</td>
<td>19.44</td>
<td>1.571</td>
<td>0.563</td>
<td>318</td>
<td>5.1</td>
</tr>
<tr>
<td>118</td>
<td>130.0</td>
<td>1.231</td>
<td>0.821</td>
<td>20.03</td>
<td>1.572</td>
<td>0.568</td>
<td>309</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Also compound nucleus and/or coupled-channel effects can be excluded with some certainty. Estimates of the compound nucleus contributions (for instance from the back-angle cross sections) show that they are negligible for $E_\alpha \geq 40$ MeV. Because of the small coupling parameters for $^{90}$Zr [14] the effect of coupling of the elastic channel to inelastic channels is expected to be small, and even if coupled-channel effects were included in the present analysis they would most likely reflect themselves in the imaginary potential and not in the shape of the real well [15].

We are thus lead to believe that the deviations in the shape of the real potential towards low energies are real. Furthermore similar discrepancies seem to be present in the $\alpha$-scattering from $^{40}$Ca [16], $^{58}$Ni, and from $^{60}$Ni [17], and it must therefore be concluded that the variation in the shape of the real potential below a certain $\alpha$-particle energy is a basic property of the $\alpha$-nucleus potential which still awaits a satisfactory explanation.

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