BACKBENDING IN $^{167}$Hf AND $^{168}$Hf AND THE BAND-CROSSING PICTURE

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Using two Compton-suppression spectrometers in an $E_\gamma-E_\gamma$ coincidence experiment, the yrast bands in $^{167,168}$Hf were extended up to $49/2$ and $28\hbar$, respectively. The $i_{13/2}$ positive parity band in $^{167}$Hf experiences backbending at a higher frequency than the first backbending in $^{168}$Hf, and no second backbending is observed in $^{168}$Hf. New information is thereby obtained on the nature and interaction strength of the crossing bands in the vicinity of $N = 96$.

Significant progress, both experimental and theoretical, has recently been achieved in the understanding of nuclear behaviour at high-spin. For example, multiple band structure has been observed in the well-deformed nucleus $^{160}$Yb up to at least $I = 32$ $^1$, and intriguing new phenomena like the existence of a second backbending around $I = 28$ have been reported for the nuclei $^{158}$Er $^2$, $^{160}$Yb $^1,3$ and $^{156}$Er $^4$; these works have included the observation of discrete $\gamma$-rays. From the theoretical side, cranking-model calculations $^5$ allow the interpretation of the irregularities observed in the level sequences in terms of crossings between bands with different degrees of rotation-alignment; in particular, the dominant role of the $i_{13/2}$ neutron orbit for the interpretation of the first backbending in all the even—odd nuclei of the rare-earths region has been emphasized.

In the present paper, experimental data are presented on the discrete $\gamma$-ray spectra in the nuclei $^{168}$Hf and $^{167}$Hf investigated with the $^{159}$Tb($^{14}$N, xn) reactions ($x = 5$ and 6). Using two Compton-suppression spectrometers in coincidence, strong background reduction was achieved, which allowed the identification of yrast states in these two nuclei up to spin values $I = 28$ and $I = 49/2$, respectively; this is 6 to 8 units higher in angular momentum than those observed with conventional techniques. In $^{168}$Hf, after a first strong backbending, the moment of inertia remains almost constant up to the highest spin states observed. In $^{167}$Hf, a backbending is seen in the positive parity band based on the $i_{13/2}$ neutron orbit, at a rotational frequency which is significantly larger than in $^{168}$Hf, and the full alignment gain is determined. These data are well explained by the cranking model, except that the oscillations of the interaction strength between the crossing bands in the odd Hf isotopes should be shifted by two neutron numbers.

A self-supporting metallic terbium target ($\approx 3$ mg/cm$^2$) was bombarded with 95 MeV $^{14}$N ions from the KVI cyclotron of Groningen. A $\gamma-\gamma-\Delta\gamma$ coincidence experiment was performed with two Compton-suppression spectrometers $^6$ facing each other at 90° with
respect to the beam axis. Resulting spectra were corrected for random and background coincidences. Examples are given in fig. 1 along with the deduced yrast sequences. Information on the multipolarity of the γ-rays of interest was deduced from an angular distribution experiment performed with a 110 cm³ Ge(Li) detector located at 15 cm from the target; spectra were recorded for five angles between 90° and 156° with respect to the beam direction. The angular distribution coefficients varied between $A_2/A_0 = 0.20$ and 0.34 and between $A_4/A_0 = -0.01$ and −0.09 for the transitions of interest in the yrast bands of $^{167}\text{Hf}$ and $^{168}\text{Hf}$. This is in agreement with the stretched-E2 character proposed for all these γ-rays. The level schemes and spins proposed in the present work are consistent with excitation functions taken by the authors of ref. [7] using the same reaction with beam energies varying from 70 to 115 MeV.

The main results obtained in the present work are summarized in the decay schemes inserted in fig. 1. The “favoured” band in $^{167}\text{Hf}$, i.e. the one with spin sequence $13/2^+, 17/2^+, ...$, had been established in pre-

Fig. 1 Summary of the relevant data on the nuclei $^{167,168}\text{Hf}$. Summed coincidence spectra gated by selected transitions are presented, together with partial level schemes. In the total projection of all valid coincident events the contribution of the quasi-continuum radiation is clearly seen. Relative γ-ray intensities, corrected for internal conversion and angular distribution, are shown in parentheses.
vious works [7,8] up to \( I^\pi = 37/2^+ \). This result is confirmed by our data, and three more levels could be established. Other \( \gamma \)-rays belonging to levels in \(^{167}\text{Hf}\) have also been observed (not shown in fig. 1).

In particular, the present data confirm the results of refs. [7,8] for the "unfavoured" part of the \( vi_{13/2} \) band, i.e. the one with spin sequence \( 15/2^+, 19/2^+ \ldots \), without adding new levels to it. This is used in the discussion below. Previous studies [7,9] establishing the yrast states in \(^{168}\text{Hf}\) up to \( I^\pi = 20^+ \) are confirmed by the present data, and new \( \gamma \)-rays are proposed which extend the yrast sequence up to \( I^\pi = 26^+ \) (fig. 1). We also propose a 874 keV \( \gamma \)-ray as deexciting the next yrast state. This is based on the presence of a weak line at this energy in the spectra coincident with the 608, 684 and 813 keV transitions (see fig. 1). No firm spin assignment was possible for this transition because its intensity was too weak to analyze the angular distribution.

As discussed in ref. [5], it is possible to deduce the projection \( I_x \) of \( I \) on the rotational axis \( x \) as a function of the rotational frequency \( \omega \) from the experimental energies \( E(I) \) of the levels with spin \( I \) in a rotational band. The difference \( i_x \) between the values of \( I_x \) for members of one band and for those of a reference configuration at the same \( \omega \) is defined as the aligned angular momentum of the excitation. In the present paper, the reference configuration was determined for each rotational band individually according to a prescription proposed by Bengtsson [10]. The expression \( I_x = \delta_0 \omega + \delta_1 \omega^3 + i_x \) was fitted to the experimental data corresponding to frequencies far enough of any backbending or upbending. The parameters \( \delta_0, \delta_1 \) and \( i_x \) were thereby obtained from fits to levels with spin \( \geq 20^+ \) in \(^{168}\text{Hf}\), and from states with spin \( \leq 29/2^+ \) and \( 31/2^+ \) in the "favoured" and "unfavoured" members of the \( vi_{13/2} \) bands, respectively, in \(^{167}\text{Hf}\) and \(^{169}\text{Hf}\) [11]. The resulting values of \( i_x \) for these various bands are shown in fig. 2a as a function of \( \omega \).

The first backbending in \(^{168}\text{Hf}\) occurs at spin \( 14^+ \) (notice that the \( 14^+ \) members of the two interacting bands have both been observed), corresponding to a

![Fig. 2. Plot (a) of the alignment \( i_x \) versus rotational frequency \( \hbar \omega \) for the yrast band in \(^{168}\text{Hf}\) and for the "favoured" and "unfavoured" \( vi_{13/2} \) bands in \(^{167,169}\text{Hf}\). The slightly negative values of \( i_x \) obtained for \(^{168}\text{Hf}\) at low \((\leq 200 \text{ keV}) \hbar \omega \) due are due to the choice of the reference configuration as described in the text. The insert (b) shows the dependence of the derivative \( di_x/d\omega \) on the rotational frequency squared (see discussion in the text) for the yrast band in \(^{168}\text{Hf}\). Data corresponding to the levels with spins 10 to 20 are omitted because they fall outside of the scale.](image-url)
band crossing at $\hbar \omega \approx 260$ keV and resulting in a value of $9h$ for the gain $\Delta i_x$ in aligned angular momentum $i_x$ with respect to the reference. There is no evidence for a possible second backbending in the yrast band of $^{168}$Hf up to spin 28, i.e. for a second band crossing up to $\hbar \omega = 440$ keV, as it has been observed in the lighter nuclei $^{156,158}$Er and $^{160}$Yb [1–4]. On the contrary, the data suggest that if such a phenomenon occurs, it must happen at higher frequencies. This is best seen when plotting the derivative $d\Delta i_x/d\omega$ as a function of $\hbar \omega$. In such a plot, the proximity of a band crossing manifests itself through strong deviation of the data points from a straight line; this is not the case for the highest spin states in $^{168}$Hf as shown in fig. 2b.

A backbending is also present in the “favoured” positive parity band in $^{167}$Hf. For $\hbar \omega \leq 300$ keV, this band exhibits an aligned angular momentum $i_x \approx 5h$, as does its “unfavoured” partner. At $\hbar \omega \approx 330$ keV, a sudden gain $\Delta i_x$ of about $5h$ is observed: to our knowledge, this is the first time that enough experimental data are available on $vi_{13/2}$ bands in odd-$N$ nuclei to accurately deduce the alignment gain. The data on the band of the same nature in $^{169}$Hf display a different behaviour. After an initial aligned angular momentum $i_x \approx 4h$ for the “favoured” and “unfavoured” bands, the curve for the former starts rising at $\hbar \omega \approx 260$ keV, and proceeds rather slowly, without true backbending. Not enough experimental data are available to determine the total alignment gain which is certainly larger than $4h$.

The quasi-particle (q.p) energies of the rotating nucleus have recently been calculated by Bengtsson and Frauendorf [5] using the cranking model; results are available for a large number of nuclei, including the even–even, $N \approx 96$ isotones with $64 \leq Z \leq 76$ [5]. In $^{168}$Hf, a strong backbending is predicted to occur in the yrast band at $\hbar \omega \approx 230$ keV, due to the crossing of the two neutron bands based on the $[643, 5/2^+]$ Nilsson state with their energy conjugate partners, and resulting in an alignment gain of $9.6h$; this is in good agreement with the present experimental data ($\hbar \omega \approx 260$ keV, $\Delta i_x \approx 9h$). The same calculation predicts that the next crossings in the yrast band of $^{168}$Hf should occur at $\hbar \omega \approx 505$ keV for the neutron system, and at $\hbar \omega \approx 445$ keV for the proton system, the latter corresponding to the alignment of a pair of $h_{11/2}$ protons; this would explain why no second backbending has been seen in the discrete spectrum of $^{168}$Hf up to $E_\gamma = 874$ keV, i.e. $\hbar \omega = 437$ keV.

Similar comparisons between theory and experiment are also possible for the odd-$N$ isotopes $^{167,169}$Hf. At low $\hbar \omega$, the favoured and unfavoured bands are predicted to have $i_x$ values of $5.3h$ and $4.3h$, respectively. Their crossings with their energy conjugate partners, which give rise to the first backbending in the yrast band of $^{168}$Hf, is blocked in the odd-$N$ isotopes, since one of the crossing bands is occupied and the other is free [5]. However, at a somewhat larger frequency $\hbar \omega \approx 300$ keV, an unblocked crossing is calculated between the “favoured” $[642, 5/2^+]$ and $[633, 7/2^+]$ neutron bands, with an alignment gain of $\approx 3.5h$. These calculations are in satisfactory agreement with the present experimental data, in particular: the value of $i_x$ at low $\omega$ in $^{167,169}$Hf; the absence of a backbending in $^{167,169}$Hf at the same frequency as in $^{168}$Hf; the occurrence of a backbending in $^{167}$Hf and $^{169}$Hf at $\hbar \omega \approx 330$ keV and of an upbending in $^{169}$Hf above $\hbar \omega = 260$ keV; the alignment gains of $5h$ and $\geq 4h$ in $^{167,169}$Hf, respectively, above these frequencies.

There is however one point where the cranking model calculations disagree with the present experimental data. The different behaviours of the “favoured” bands in $^{167,169}$Hf (with respect to backbending (fig. 2a) suggest that the interaction strength $|V|$ between the interacting bands is notably smaller in $^{167}$Hf than in $^{169}$Hf, since a backbending (upbending) is associated with a small (large) value of $|V|$. The calculations, on the other hand, predict comparable and large strengths for $N = 95$ and 97 (fig. 11 of ref. [5], second paper). This discrepancy would suggest that the calculated characteristic oscillations of $|V|$ as a function of $N$ for this crossing should be shifted by about 2 units of $N$ towards larger $N$, bringing $N = 97$ close to a maximum of $|V|$ in the Hf isotopes.

Finally, the results of the present paper (fig. 2b) suggest that, if a second backbending occurs in the yrast band of $^{168}$Hf, it must be very sharp or must occur at much higher spin values than presently observed ($I > 28h$). Experimental evidence for band-crossings at higher frequencies is presented in the following paper [6]. The interpretation remains, however, difficult since a sharp backbending does not always show up as enhanced intensity in the valley of a $\gamma$–$\gamma$ energy correlation matrix.

In conclusion, the level schemes of $^{167,168}$Hf have been established up to high spins by using two Compton-suppression spectrometers. Valuable information is thereby obtained which allows a detailed com-
parison with cranking model calculations. Further ex-
periments are suggested, to establish the yrast band in
$^{168}$Hf to slightly higher rotational frequencies than
achieved in the present paper, with the hope to detect
the predicted sharp second backbending. From the theo-
retical side, the reason for the shift, in neutron num-
ber, of the maximum of the interaction strength at the
second band crossing in the $i_{13/2}$ neutron bands
should be investigated.

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