The excitation of the $^{24}\text{Mg}$ $0^+$ and $2^+$ states at 6.43 MeV and 7.35 MeV, respectively, in inelastic hadron scattering has been considered in the collective model where coupling of these states to the states of the ground state band is via a $\beta$-vibration coupled to the static deformation. It is found that coupled channel effects on the $0^+$ state excitation are rather important and moreover the excitation of the $0^+$ state can be satisfactorily explained only if a monopole breathing mode form factor is included in addition to the monopole $\beta$-vibration form factor.

Aside from evidence found [1–3] for a small percentage of fragmented monopole (E0) strength in light nuclei ($A \leq 40$), in only one experiment has a large concentration of E0 strength been reported [4]. Nevertheless, one can consider the existence of the giant monopole resonance (GMR) in light nuclei as yet an open question [5]. A major effort at present is being undertaken to locate this monopole strength in light nuclei either in inelastic hadron scattering at forward angles or from the study of the decay of inelastically excited nuclei in the region where the GMR is expected.

A possible way to investigate the GMR is to study [6] the effective charges of low-lying monopole transitions. By mixing in with low-lying states of the same spin and parity, a giant resonance manifests [6] itself by a renormalization of the transition rates of these low-lying states as calculated from their simple shell model (truncated) configurations. For spherical nuclei where low-lying monopole states could be described as simple shell model configurations, it is possible to obtain state dependent effective charges making certain assumptions on the excitation energy and strength of the monopole resonance. In this way Castel and Satchler [7] were able to explain the enhancement and reduction of the low-lying monopole transitions in $^{206}\text{Pb}$ and $^{90}\text{Zr}$, respectively.

In deformed nuclei, where the configuration of the $\beta$ vibrational $0^+$ state is rather complicated, the renormalization of E0 transition rates due to coupling to the GMR is not easily tractable. However, because the form factor of a monopole (surface) $\beta$-vibration is different from that of the compressional (breathing) mode, there is the hope that by studying inelastic electron or hadron scattering one would be able to find the admixture of the breathing mode into the low-lying monopole transitions by trying to fit the experimental angular distribution with an admixture of the two form factors. Such an attempt has been performed by Morsch and Decowski [8] for $^4\text{He}$, $^{12}\text{C}$ and $^{24}\text{Mg}$ lowest $0^+$ excited states. They claim [8] that while for $^4\text{He}$ and $^{12}\text{C}$ the angular distributions can be fit with either a compressional mode form factor or a $\beta$-vibration type form factor, the contribution of the compressional mode to the excitation of the $0^+$ state at 6.43 MeV in $^{24}\text{Mg}$ is negligible.

In their analysis of the low-lying monopole excitations, Morsch and Decowski [8] used DWBA with form factors obtained by folding the monopole transition densities into an effective projectile–nucleon interaction. While this procedure should be valid for spherical nuclei, its application to strongly deformed nuclei such as $^{24}\text{Mg}$ is rather questionable. In fact, strong channel coupling can affect the differential cross sections for various channels drastically. Such ef-
effects have been observed [9], for example, in single and mutual excitation of $\alpha$-scattering from $^{24}\text{Mg}$ and $^{28}\text{Si}$. Moreover, attempts to fit the experimental 40 MeV [10] and 20 MeV [11] $(p, p')$ data for the $0^+_\beta$ state of $^{24}\text{Mg}$ with DWBA calculations using either macroscopic [10] (vibrating-diffuseness and breathing mode) form factors or microscopic [10,11] form factors failed. This failure could partly be attributed to strong coupled channel (CC) effects, since in the study of inelastic proton scattering from $^{24}\text{Mg}$ at 0.8 GeV, Blanpied et al. [12] concluded from a full CC analysis in the $\beta$-vibrational model that the $0^+$ state at 6.43 MeV can not be described by a pure monopole $\beta$-vibration form factor alone. CC effects on the excitation of the $0^+_\beta$ of $^{24}\text{Mg}$ were studied earlier [13] using a wrong coupling scheme where the same form factor was taken for the $0^+_\text{gs} \rightarrow 2^+$ and $2^+ \rightarrow 0^+_\beta$ transitions but more importantly the $2^+_\beta$ state was omitted from the coupling scheme.

In this letter, we would like to present analysis of the ground state band (gsb) and $\beta$-band, in $^{24}\text{Mg}$ excited by inelastic hadron scattering [1,12,14] in a coupled scheme where proper coupling of the $\beta$-band to the gsb is included. Not only is the $2^+_\beta$ angular distribution well explained (DWBA predictions are out-of-phase with the experimental data) in this scheme, but also the $0^+_\beta$ state can only be described by including in addition to the $\beta$-vibration form factor a breathing mode form factor.

Along similar lines as described in ref. [14], one can show that the coupling of a $\beta$-vibration to a static deformation would lead for the excitation of the $\beta$-band states from states of gsb to a form factor:

$$f_{00\lambda} = (-1)^\lambda \langle \beta_0 | 0 \lambda 0 | \text{gs} 0 \rangle \lambda^{-1} (5/4\pi)^{1/2} \sum_l i_l^{(1)}$$

$$\times \left( |\beta - \beta_2| \langle l \; 0 \; 0 | \lambda 0 \rangle^2 - \xi \langle 0 \; 0 | \lambda \lambda \rangle^2 \right), \quad (1)$$

where $|\beta - \beta_2|$ is the amplitude of the $\beta$-vibration and the second term within the brackets is added to satisfy the condition of conservation of particles. Essentially $\xi$ is determined once $|\beta - \beta_2|$ is determined with the condition

$$\int f_{000}(r) r^2 dr = 0 \quad (2)$$

For the definition of the various symbols see ref. [14]. This form factor was incorporated into CHUCK [15] to perform the full CC calculations. For example, in single and mutual excitation of $\alpha$-scattering from $^{24}\text{Mg}$ and $^{28}\text{Si}$. Moreover, attempts to fit the experimental 40 MeV [10] and 20 MeV [11] $(p, p')$ data for the $0^+_\beta$ state of $^{24}\text{Mg}$ with DWBA calculations using either macroscopic [10] (vibrating-diffuseness and breathing mode) form factors or microscopic [10,11] form factors failed. This failure could partly be attributed to strong coupled channel (CC) effects, since in the study of inelastic proton scattering from $^{24}\text{Mg}$ at 0.8 GeV, Blanpied et al. [12] concluded from a full CC analysis in the $\beta$-vibrational model that the $0^+$ state at 6.43 MeV can not be described by a pure monopole $\beta$-vibration form factor alone. CC effects on the excitation of the $0^+_\beta$ of $^{24}\text{Mg}$ were studied earlier [13] using a wrong coupling scheme where the same form factor was taken for the $0^+_\text{gs} \rightarrow 2^+$ and $2^+ \rightarrow 0^+_\beta$ transitions but more importantly the $2^+_\beta$ state was omitted from the coupling scheme.

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Citations within the same rotational band the usual collective form factor [14] was used.

As it will soon become obvious, breathing mode form factor will be needed to fit the $0^+$ state of the $\beta$-band in addition to the $\beta$-vibration form factor [eq. (1)]. This was taken of the form [16]

$$f_{000} = -3U(r) - r dU/dr, \quad (3)$$

where $U$ is the optical model (OM) potential.

The DWBA and CC calculations were performed using OM parameters of ref. [14]. Coulomb excitation of the $2^+$ states of the $\text{gs}$ and $\beta$-bands was neglected since it had [14] little effect on the angular distributions of the $2^+$ states and its neglect leads only to a slight renormalization of the deformation parameters. In fig. 1, the results of DWBA calculations for the $0^+_\beta$ using the $\beta$-vibration form factor [eq. (1)] and the breathing mode form factor [eq. (3)] are shown as dashed and solid curves, respectively. The two DWBA curves reproduce the first maximum but start to differ from each other and become out of phase with the data in the region of the second and third maxima. For the $2^+_\beta$ state of the $\beta$-band, the DWBA calcula-

*1 The program has been modified to perform the correct couplings to the states of the $\beta$-band.
The question arises as to what happens when the \( \beta \)-vibration and \( \beta \)-band states are coupled together in one scheme where all couplings arising from a \( \beta \)-vibration coupled to a static deformation are included? The results of such a calculation are shown as dashed curves in fig. 2. The only free parameter in this calculation is in principle the coupling parameter \( |\beta - \beta_2| \) [see eq. (1)]. The OM parameters and the deformation parameter of the gs band \( \beta_2 = 0.355 \) are obtained from ref. [14]. The deformation parameter of the \( \beta \)-band \( \beta_2^\beta \) could essentially be obtained [14] by scaling the moments of inertia of the gs and \( \beta \)-bands. However, to anticipate any possible strong effect of \( \beta_2^\beta \) on the angular distribution of the \( 0_1^\beta \) state we have varied both \( |\beta - \beta_2| \) and \( \beta_2^\beta \) to obtain the best \( \chi^2 \)-fit to the \( 0_1^\beta \) differential cross section (dashed curve in fig. 2). It is clear that the reasonable fit at the forward angles obtained to the \( 0_1^\beta \) from DWBA calculations using either the \( \beta \)-vibration or the breathing mode form factors (see fig. 1) is lost. Moreover, the predicted differential cross section for the \( 2_2^\beta \) state was also varied with the parameter \( |\beta - \beta_2| = 0.138 \) and \( \beta_2^\beta = 0.491 \) which give a reasonable estimate to the magnitude of the \( 0_2^\beta \) differential cross section (see dashed curves in fig. 2) overestimates the data of the \( 2_2^\beta \) state by a large factor.

The differential cross sections for elastic scattering of \( \alpha \)-particles from \( ^{24}\text{Mg} \) at \( E_a = 120 \) MeV and inelastic differential cross sections to the \( 2_2^\beta \) state of the gs band obtained from the above CC calculations are not shown here since they are very similar to those shown in fig. 2 of ref. [14].

A good description for the data of the \( 0_1^\beta \) state in the CC scheme described above could only be obtained by including a breathing mode form factor [eq. (3)] in addition to the \( \beta \)-vibration form factor [eq. (1)] connecting the \( 0_1^\beta \) to the gs. A \( \chi^2 \)-search on the two coupling parameters \( |\beta - \beta_2| \) and \( \beta_0 \) (breathing mode) which lead to the best fit to the \( 0_1^\beta \) and \( 2_2^\beta \) differential cross sections resulted in the solid curves in fig. 2. Indeed the results of this CC calculation are quite an improvement even compared to the DWBA curves of fig. 1. The parameters so-obtained are \( |\beta - \beta_2| = 0.0652 \) and \( \beta_0 = -0.0684 \). In this calculation \( \beta_2^\beta \) was also varied. The final value for \( \beta_2^\beta \) obtained from the \( \chi^2 \)-fit is 0.376 which is not much different from the quadrupole deformation of the ground state of 0.355. A \( \chi^2 \)-search on the two parameters \( |\beta - \beta_2| \) and \( \beta_0 \) forcing the latter to be positive resulted in fits which are worse by factors of 2.0 and 3.5 in \( \chi^2 \) for the \( 0_1^\beta \) and \( 2_2^\beta \) data in comparison with those obtained with \( \beta_0 \) negative. Qualitatively the fits were also worse in the sense that they were out of phase with the data for both the \( 0_1^\beta \) and \( 2_2^\beta \) state in the angular region of the 2nd and 3rd maxima. The absolute values of the parameters were, however, similar in both cases.

The good fits to the data of the \( 0_1^\beta \) and \( 2_2^\beta \) states both in magnitude and shape obtained from the same CC calculation (solid curves in fig. 2) attest to the correctness of the suggested coupling scheme. This calculation indicates moreover that the coupling of the breathing mode into the \( 0_1^\beta \) vibrational state is significan-
The isoscalar monopole matrix element obtained using the admixed monopole transition density and following Bernstein procedure [17] is 7.3 fm$^2$ in reasonable agreement with the result of 6.33 ± 0.29 fm$^2$ obtained from electron scattering [18].

We have similarly analyzed the 800 MeV inelastic scattering data of ref. [12] using the reported [12] optical model parameters. Here the value of the deformation parameter $\beta_2$ was fixed to 0.637 by assuming the same scaling to the deformation parameter of the ground state ($\beta_2 = 0.601$; ref. [12]) as was obtained from the analysis of the $^{24}$Mg$(\alpha, \alpha'')^{24}$Mg data. The $\chi^2$-search resulted in a rather good fit to the $0^+_2$ differential cross section (not shown here), with $|\beta - \beta_2| = 0.122$ and $\beta_0 = -0.051$ leading to a monopole matrix element, again using Bernstein procedure [17], of 5.9 fm$^2$.

In summary, the amount of coupling of the monopole breathing mode into the $0^+_2$ $\beta$-vibrational state in $^{24}$Mg turns out to be rather substantial in disagreement with the claim of Morsch and Decowski [8] who found that the angular distribution of the $0^+_2$ state at 6.43 MeV could be fitted with a form factor that corresponds to a monopole surface oscillation ($\beta$-vibration). The appreciable admixture of the breathing mode monopole form factor to the low-lying $\beta$-vibration indicates either that the monopole resonance is not very high in excitation energy or that the coupling matrix element to the $0^+_2$ state is large. A systematic study of these $0^+_2$ $\beta$-vibrational states across the sd-shell nuclei using the above procedure may shed some light on the location and strength of the breathing mode which has escaped detection by direct means of inelastic scattering of electrons and hadrons in these nuclei up to the present.

This work has been performed as part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM) with financial support from the Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek (ZWO).

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