A PHENOMENOLOGICAL APPROACH TO $\alpha$-CLUSTERING IN HEAVY NUCLEI

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We suggest that $\alpha$-clustering may play an important role in the structure of heavy nuclei and propose a phenomenological model for treating it. We discuss the structure of spectra in two simple limits of this model.

Nuclei with proton number $>82$ and neutron number $>126$ display somewhat unusual properties. (i) Low-lying excited $0^+$ states do not behave as expected on the basis of the collective model ($\beta$-bands) [1]. (ii) Low-lying ($\approx 300$ keV) negative parity bands with small ($\approx 5-10$) alpha-hindrance factors occur in these spectra. Moreover, the absence of double excitations of these bands makes an interpretation in terms of harmonic octupole vibrations difficult [2]. (iii) Recent ($d, 6\text{Li}$) reaction studies show unusually large population of groups of excited states [3]. These phenomena, together with the observed large $\alpha$-decay probabilities [4] suggest that $\alpha$-clustering effects may play a role in heavy nuclei as important as that played in light nuclei. Contrary to light nuclei, however, the study of $\alpha$-clustering effects in heavy nuclei suffers from the absence of a simple yet detailed framework for its discussion. With the exception of a few nuclei in the vicinity of $^{208}_{82}\text{Pb}_{126}$ [5], microscopic calculations are difficult to perform because of the large configuration space involved. The purpose of the present note is to propose a simple phenomenological scheme within which $\alpha$-clustering phenomena in nuclei can be studied.

The study of $\alpha$-clustering effects in heavy nuclei is made difficult by the fact that they involve a mixture of collective aspects (as evidenced by the rotational nature of the spectra) and particle aspects (as evidenced by the $\alpha$-decay probabilities). For this reason, we base our discussion on the interacting boson model [6] where both aspects are, to a certain extent, present. In this model, normal collective features are generated by considering a set of $N_T$ bosons able to occupy two levels with $\mathbf{J}^P = 0^+, 2^+$. The bosons represent correlated pairs of nucleons and, in first approximation, the total number of bosons, $N_T$, is taken as the sum of the number of proton, $N_p$, and neutron, $N_n$, pairs in the valence shell [7]. This model has group structure $U(6) \otimes U(2)$. In the shell model, $\alpha$-clustering states are constructed by raising four particles from the lowest configurations to some excited ones. Similarly, one can imagine that $\alpha$-clustering states can be constructed in the interacting boson model by raising two pairs from the lowest to some excited configuration. If pairs, either protons or neutrons, are excited individually, the excitation leads to pair-vibration modes [8]. These have been discussed extensively by Broglia et al. [9] within the framework of the Bohr-Mottelson model and by Duval and Barrett [10] within the framework of the interacting boson model. Here we shall be concerned with the simultaneous excitation of proton and neutron pairs.
At first sight, one might expect the resulting states to occur at an energy which is the sum of the proton and neutron pair-vibrational excitation energies and, thus, to be unimportant for the structure of low-lying states. However, because of the strongly attractive proton–neutron interaction, pairs of bosons in the excited configurations can bind to form $\alpha$-clusters at an energy much less than that needed to excite a single pair vibration.

An important question is, then, what is the structure of the spectrum when $\alpha$-clusters are present. This depends on the angular momenta of the elementary modes of excitation from which the $\alpha$-cluster states are built. In general, one expects modes with $J^P = 0^+$, $1^-$, $2^+$, $3^-$, ... . In the normal configurations, the $J^P = 1^-$ mode is absent since it corresponds to a displacement of the center of mass. Thus, one usually truncates the angular momentum of the normal modes (bosons) to $J^P = 0^+$, $2^+$. For the excited configurations, however, the $J^P = 1^-$ mode is not spurious since it corresponds to oscillations of the cluster relative to the core. Indeed, this may be the dominant mode. Thus, we suggest that, in first approximation, only modes with $J^P = 0^+$, $1^-$ be retained in the excited configurations as shown in fig. 1. For the same reason for which the group structure of the normal configurations is $U(6) \otimes U(2)$, that of the excited configurations is $U(4) \otimes U(2)$. The four-dimensional space spanned by the $1 + 3$ components of the $s^*, p^*$ bosons. Detailed calculations of the properties of nuclei can then be performed by allowing the $N_p$ ($N_n$) pairs to occupy the levels shown in fig. 1. These calculations are similar to a shell-model calculation with configuration mixing except that they are done for bosons rather than fermions. A computer program to perform such calculations is being written.

The salient features of this model can be most easily considered if we neglect the difference between proton and neutron bosons. This approximation, usually called IBA-1, is a good approximation to the exact diagonalization whenever the proton–neutron boson hamiltonian is invariant under proton–neutron boson transformations ($F$-spin invariance) [7]. For $\alpha$-clusters this is a potentially dangerous approximation since it places the excitation of spurious four-neutron (or four-proton) clusters on the same footing as genuine two-neutron–two-proton excitations which would represent $\alpha$-clustering. However, for simplicity, we neglect in this note the differences between proton and neutron bosons since this does not affect the simple phenomenological considerations which follow. In this case, the group structure of the normal configurations is simply $U(6)$ while the group structure of the $\alpha$-cluster configurations is $U(4)$. We consider the limiting case in which the normal $\alpha$-cluster configurations do not interact so that the spectrum contains two distinct kinds of states. First, states in which all bosons occupy the $(s, d)$ levels. For the axially symmetric deformed nuclei in the actinide region, the appropriate group chain is

$$U(6) \supset SU(3) \supset O(3) \supset O(2).$$

The spectrum of these states is given by eq. (2.5) of ref. [11] and is shown in fig. 2a. Second, states in which all bosons occupy the $(s^*, p^*)$ levels. These states would represent a kind of $\alpha$-particle condensate. The structure of spectra with $U(4)$ symmetry has been discussed in ref. [12] in connection with molecular rotation–vibration spectra. The group chain appropriate for this problem is

$$U(4) \supset O(4) \supset O(3) \supset O(2).$$

The spectrum is given by eq. (9) of ref. [12] and is shown in fig. 2b. It remains to determine the distribu-
tion of bosons between normal and $\alpha$-cluster states. Two neutron separation energies in the actinide region reveal an interesting discontinuity in slope when the number of valence neutrons is equal to the number of valence protons. This suggests a possible competition between normal and maximally $\alpha$-condensed configurations for the ground state. The major difference between the normal and $\alpha$-condensed spectra of fig. 2 is the occurrence of low-lying negative parity states in the latter. In the present model, this would appear to be the spectral signature of $\alpha$-clustering.

The cleanest indication of $\alpha$-clustering should be low hindrance factors for $\alpha$-decay. However, this requires the construction of a suitable operator. In the present model, we wish to permit $\alpha$-decay only from the $(s^*, p^*)$ configurations in which $\alpha$-clusters pre-exist. Alpha decay from the normal $(s, d)$ configurations should lead to $\alpha$-hindrance factors on the order of $10^3$ and can be ignored. Both normal and $\alpha$-cluster configurations could contribute to $\alpha$-transfer reactions where it is merely necessary to transfer four nucleons with the quantum numbers of the $\alpha$-particle and not necessarily a real $\alpha$-particle. Thus, the present model may also be useful in explaining apparent discrepancies between $\alpha$-decay rates and $\alpha$-transfer cross-sections.

The assumptions leading to fig. 2 are extreme even within the context of this simple model. It is nonetheless interesting to note that the observed spectra of light actinide nuclei are suggestive of $\alpha$-clustering. The example of $^{222}$Ra is given in fig. 3 and shows states with small $\alpha$-hindrance factors and a spectrum similar to that of fig. 2b. A weak interaction between the spectra of figs. 2a and 2b could account for the observed level spacing.

Before making detailed comparisons with observed spectra, one should solve the model hamiltonian without arbitrary restrictions on the number of bosons in normal and $\alpha$-cluster configurations retaining those new terms in the hamiltonian which can change the distribution of bosons. It seems reasonable to assume that the underlying fermion states leading to the $(s, d)$ and $(s^*, p^*)$ bosons are quite different. Thus, the underlying two-particle interaction can, at most, change a single boson, $s$, to a single boson, $s^*$. The new terms required in the present hamiltonian arise from a two-step process (e.g. $s^2 \to ss^* \to s^*2$) and are determined by a single parameter, 

$$\delta = \frac{(s|H|s^*)^2}{\Delta E},$$  \hspace{1cm} (3)
where $\Delta E$ represents the excitation energy of a single $s^\ast$-boson. This picture is quite similar to the “nuclear coexistence” picture of $^{16}\text{O}$ and $^{40}\text{Ca}$.

Finally, the normal configurations may also have a mode with $J^P = 3^-$ (an octupole or $f$-boson) [6]. This mode can couple and interfere with the negative parity levels arising from $\alpha$-clustering. Indeed, it has been suggested [13] that the occurrence of low-lying $J^P = 0^-$ bands in the light actinides is related to octupole vibrations, although the results of ref. [2] seem to contradict this suggestion. It would be interesting to investigate the mixing of octupole and $\alpha$-clustering modes. In light nuclei, both modes have been observed [14].

In conclusion, we have suggested that $\alpha$-clustering may play an important role in the structure of heavy nuclei and have proposed a simple model to study the observed properties of these nuclei. The key feature of this model is the occurrence of low-lying negative parity bands and small $\alpha$-hindrance factors. Although we have concentrated on the actinides, where $\alpha$-clustering may already occur in the ground state, it is quite possible that similar effects may be present in other nuclei at higher excitation energies. We have in mind particularly the $^{38}\text{Sr}$, $^{40}\text{Zr}$, $^{42}\text{Mo}$ isotopes with neutron number >50 and the $^{62}\text{Sm}$, $^{64}\text{Gd}$, and $^{66}\text{Dy}$ isotopes with neutron number >82.

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