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CHIRAL SYMMETRY BREAKING BEYOND THE LANDAU GAUGE

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By using novel Ansätze for the longitudinal and transverse parts of the fermion-gauge boson vertex that respect the Ward identity and multiplicative renormalizability, we demonstrate that chiral symmetry breaking occurs in vector-like gauge theories in such a way that the critical coupling constant and dynamical mass function are gauge-independent, at least in leading order.

The dynamics of chiral symmetry breaking in vector gauge theories can be studied by analyzing the Dyson-Schwinger equation for the fermion propagator. Miransky and co-workers showed that, for both asymptotically free and non-free theories with massless bare fermions, chiral symmetry breaking occurs above a certain critical coupling [1]. They also found that, with the fixed or running coupling just above the critical level, the dynamical fermion mass function has the ultra-violet asymptotic $M(p^2) \sim (p^2)^{-1/2}$; to within logarithms. These results were obtained in the Landau gauge, and we shall show here that they are gauge-independent in a general covariant gauge. Our results support to certain recent technicolor models, in which this mass function is a crucial ingredient in the suppression of flavour-changing neutral currents [2]. Here we will treat a fixed coupling constant. The more involved analysis with a running coupling constant (i.e., as in QCD), which also leads to gauge-invariant results, will be published elsewhere [3].

The Dyson-Schwinger equation for the fermion propagator $S(p)$, expressed at euclidean momenta, is

$$S^{-1}(p) = \not{p} + \Sigma(p), \quad (1)$$

where the fermion self-energy $\Sigma(p)$ is given in terms of the full fermion-fermion-boson vertex $\Gamma^\nu(p, p')$ and full boson propagator $D^{\mu\nu}(k)$ as

$$\begin{aligned} \Sigma(p) &= \frac{g^2 C_2(R)}{(2\pi)^4} \int d^4 p' \gamma^\mu S(p') \Gamma^\nu(p, p') D^{\mu\nu}(p' - p). \end{aligned} \quad (2)$$

The fermion inverse propagator can be written

$$S^{-1}(p) = \alpha(p^2) - \not{p} \beta(p^2). \quad (3)$$

If there is a solution of (1) in which α is not identically zero, chiral symmetry is broken.

Little attention has been given to the problem of maintaining gauge covariance in the study of non-perturbative chiral symmetry breaking. We recently found [4] that, with a bare vertex function, gauge-dependence was so severe that, in all but the Landau gauge, the theory degenerated into the trivial free case ($\alpha=0; \beta=1$) as the ultra-violet cut-off was removed. An attempt was made to respect the ultra-violet Ward identity by replacing Γ^ν by $\gamma^\nu \beta$, but this basic difficulty in gauges other than that of Landau remained unchanged [5]. The new feature here is to separate the vertex function into a longitudinal part, which contributes only to the Ward identity; and a transverse part, which contributes only to the transverse (Landau gauge) part of the Dyson-Schwinger equation. The longitudinal part is given by the Ward identity, whereas the Ansatz of ref. [5] is used for the transverse part (although the justification for the latter is now that of renormalizability).

Let us truncate (1) by replacing $D^{\mu\nu}$ by a free massless propagator in a general covariant gauge:

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$$D^{\mu\nu}(k) \rightarrow (1/k^2) \times [(\delta^{\mu\nu} - k^\mu k^\nu/k^2) + a k^\mu k^\nu/k^2], \quad (4)$$

where a is the gauge-fixing parameter. We have split the propagator into transverse and longitudinal parts; for Landau gauge ($a=0$) only the transverse $D_T^{\mu\nu}(k)$ is present. Let us also separate the vertex function into transverse and longitudinal parts; $\Gamma^\nu = \Gamma_T^\nu + \Gamma_L^\nu$;

$$\Gamma_T^\nu(p, p') = (\delta^{\nu\rho} - k^\nu k^\rho/k^2) \Gamma^\rho(p, p'), \quad (5)$$

$$\Gamma_L^\nu(p, p') = (k^\nu k^\rho/k^2) \Gamma^\rho(p, p'). \quad (6)$$

The Ward identity determines the longitudinal part:

$$\Gamma_L^\nu(p, p') = (k^\nu/k^2) [S^{-1}(p') - S^{-1}(p)]. \quad (7)$$

On the other hand, the transverse part must reduce on mass-shell to

$$\bar{u}(p) \Gamma^\rho(p, p) u(p) \rightarrow Z_1 \bar{u}(p) \gamma^\rho u(p), \quad (8)$$

and since $Z_1 = Z_2 = \beta(m^2)$, this motivates the Ansatz

$$\Gamma_T^\nu(p, p') = (\delta^{\nu\rho} - k^\nu k^\rho/k^2) \gamma^\rho \beta(p_{\max}^2), \quad (9)$$

with $p_{\max}^2 = \max(p^2, p'^2)$.

Let us decompose the self-energy into transverse and longitudinal parts: $\Sigma = \Sigma_T + \Sigma_L$;

$$\Sigma_T = \frac{g^2 C_2(R)}{(2\pi)^4} \int d^4 p' \gamma^\mu S(p') \gamma^\nu \times \beta(p_{\max}^2) D_T^{\mu\nu}(p' - p), \quad (10)$$

$$\Sigma_L = \frac{ag^2 C_2(R)}{(2\pi)^4} \int d^4 p' \frac{\not{p} - \not{p}'}{(p' - p)^4} S(p') S^{-1}(p), \quad (11)$$

where in the last term we have made the replacement

$$\int d^4 k \frac{\not{k}}{k^4} = 0. \quad (12)$$

Integrating over angles, and using notations $x = p^2$ and $\lambda = g^2 C_2(R)/16\pi^2$, we obtain

$$\Sigma_T(x) = 3\lambda \int_0^\infty dy \frac{\beta(x_{\max})}{x_{\max}} \frac{y\alpha(y)}{\alpha^2(y) + y\beta^2(y)}, \quad (13)$$

and

$$\Sigma_L = a\lambda [f(x) + \not{p} g(x)] S^{-1}(p), \quad (14)$$

where

$$f(x) = \int_x^\infty dy \frac{\beta(y)}{\alpha^2(y) + y\beta^2(y)}, \quad (15)$$

and

$$g(x) = -\frac{1}{x^2} \int_0^x dy \frac{y\alpha(y)}{\alpha^2(y) + y\beta^2(y)}. \quad (16)$$

The Dyson-Schwinger equation is equivalent to this coupled nonlinear system of equations for $\alpha(x)$ and $\beta(x)$:

$$\alpha(x) [1 - a\lambda f(x)] + a\lambda x g(x) \beta(x) = \Sigma_T(x),$$

$$\beta(x) [1 - a\lambda f(x)] - a\lambda g(x) \alpha(x) = 1. \quad (17)$$

To look for chiral symmetry breaking, we study the bifurcation equations for a small solution $\delta\alpha$ obtained by linearizing the system; viz,

$$\delta\alpha(x) h(x) - \frac{a\lambda\beta(x)}{x} \int_0^x dy \frac{\delta\alpha(y)}{\beta^2(y)} = 3\lambda \int_0^\infty dy \frac{\beta(x_{\max})}{x_{\max}} \frac{\delta\alpha(y)}{\beta^2(y)}, \quad (18)$$

and

$$1/\beta(x) = h(x) = 1 - a\lambda \int_x^\infty dy \frac{1}{y\beta(y)}. \quad (19)$$

Let us make sharp infra-red and ultra-violet cut-offs by defining the functions $\alpha(x)$ only for $\mu \leq x \leq A$. In such a case (19) is equivalent to the differential equation

$$(1/\beta)' = a\lambda/x\beta, \quad (20)$$

along with the boundary condition $\beta(A) = 1$, so that

$$\beta(x) = (x/A)^{-a\lambda}. \quad (21)$$

Let us then express (18) as a linear integral equation for the fermion mass function $M(x) = \delta\alpha(x)/\beta(x)$, namely

$$M(x) = \frac{\lambda(3+a)}{x} \int_\mu^x dy (y/x)^{a\lambda} M(y) + 3\lambda \int_x^A \frac{dy}{y} M(y). \quad (22)$$

Eq. (22) is equivalent to the differential equation

$$x^2 M''(x) + 2xM'(x) + 3\lambda(1+a\lambda)M(x) = 0, \quad (23)$$

Along with these boundary conditions:

$$[x^{-a\lambda}M(x)]|_{\mu} = 0, \quad [xM(x)]|_{\Lambda} = 0. \quad (24)$$

The general solution of (23) can be expressed in terms of the parameter

$$\tau = 3\lambda(1+a\lambda), \quad (25)$$

which is gauge-independent to leading order in λ . One would anticipate other contributions of order λ^2 in a calculation consistent to that order. This requires a more refined Ansatz for Γ_T .

We find the following solutions of (23) in three regimes of τ :

$$\begin{aligned} M(x) &= A x^{-1/2+\sigma} + B x^{-1/2-\sigma} \quad (0 < \tau < \tfrac{1}{4}), \\ &= x^{-1/2} (C + D \log x) \quad (\tau = \tfrac{1}{4}), \\ &= E x^{-1/2+i\rho} + F x^{-1/2-i\rho} \quad (\tau > \tfrac{1}{4}), \end{aligned} \quad (25)$$

where

$$\sigma = (\tfrac{1}{4} - \tau)^{1/2}, \quad \rho = (\tau - \tfrac{1}{4})^{1/2}. \quad (27)$$

For $\tau < \frac{1}{4}$, the boundary conditions require

$$\frac{(\frac{1}{2}-\sigma)(\frac{1}{2}-\sigma+a\lambda)}{(\frac{1}{2}+\sigma)(\frac{1}{2}+\sigma+a\lambda)} = \left(\frac{\Lambda}{\mu}\right)^{2\sigma}. \quad (28)$$

These conditions cannot be met, so that nontrivial solutions of (22) do not occur for $\tau < \frac{1}{4}$. For $\tau = \frac{1}{4}$, one must have

$$4 \frac{1+a\lambda}{1+2a\lambda} + \log \frac{\Lambda}{\mu} = 0, \quad (29)$$

which is also impossible. For $\tau > \frac{1}{4}$, the boundary conditions require

$$\frac{(\frac{1}{2}-i\rho)(\frac{1}{2}-i\rho+a\lambda)}{(\frac{1}{2}+i\rho)(\frac{1}{2}+i\rho+a\lambda)} = \left(\frac{\Lambda}{\mu}\right)^{2i\rho}. \quad (30)$$

The transcendental equation (30) for fixed Λ, μ has an infinite set of solutions λ_n which become dense over the whole domain $\tau > \frac{1}{4}$ as the ratio of cut-offs Λ/μ becomes large. Therefore, the critical value is $\tau_c = \frac{1}{4}$. Thus, to leading order, the critical coupling

$$(g^2 C_2(R)/4\pi)_{\text{crit}} = \tfrac{1}{3}\pi \quad (31)$$

is independent of a .

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