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FINAL STATE INTERACTIONS AND y -SCALING IN INCLUSIVE ELECTRON SCATTERING

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The role of final state interactions in inclusive electron scattering off nuclei is investigated in the single-particle Green function formalism with a complex optical potential. In the eikonal approximation the y -scaling properties are discussed. It is suggested that the dominance of the contribution from the imaginary part W of the optical potential to the scaling function at large negative values of the scaling variable y found by Gurvitz and Rina is appreciably reduced if off-shell effects are taken into account. It is shown that if a reasonable off-shell extrapolation of W is used, a satisfactory description of the observed scaling function is obtained for ^4He and nuclear matter.

1. Introduction

During recent years the concept of y -scaling in electron-nucleus scattering has become a topic of increasing interest. The occurrence of y -scaling was first suggested by West [1] and then observed by Sick, Day and McCarthy [2] in ^3He . Originally it was suggested that in the limit of $q \rightarrow \infty$ the scaling function would provide a direct measure of the momentum distribution of nucleons in nuclei. Indeed it has been shown that in the PWIA the longitudinal response function $R_L(\omega, q)$ becomes proportional to a scaling function F^{PWIA} , which in the non-relativistic case is directly related to the nucleon momentum distribution $n(p)$ by $F^{\text{PWIA}}(y) = 2\pi \int_{-\infty}^{\infty} dp p n(p)$, where the scaling variable $y = y(\omega, q)$ can be interpreted as the minimal longitudinal momentum of the hit nucleon.

In practice this simple situation is complicated by several higher order effects, e.g. the presence of non-nucleonic degrees of freedom (especially for $y > 0$) and rescattering of the hit nucleon on the residual $A-1$ nucleus. The analysis of recent data on a heavy nucleus indicates [3] that for $y < 0$ these final state interactions (FSI) cannot be neglected and thus should be treated accurately in order to be able to extract a quantitative nucleon momentum distribution from the data. In this respect the situation is similar to that for thermal neutron scattering from superfluid ^4He [4], where FSI of the recoiling ^4He strongly modifies the PWIA response function.

The role of FSI in inclusive quasi-elastic electron scattering has been investigated by several authors [5-9]. Up to now in studies on the y -scaling properties of the high q data the effect of FSI was mostly neglected or restricted to the inclusion of the real part of the optical potential. However, from a conceptual point of view this is unsatisfactory since at positive energies the single-particle potential is necessarily complex due to the presence of open channels. In fact dispersion relation theory [10] implies that there is a direct relation between the real and imaginary potentials. It has also been pointed out that only in that case the model independent energy sum rules (for $q \rightarrow \infty$) are conserved [5]. For that reason the approach of ref. [9] in which a phenomenological imaginary potential was used is not fully consistent.

A recent investigation of the effects of FSI [11] in the eikonal approximation which includes the imaginary potential led to a surprising result, while scaling was shown to persist, the scaling function at large negative values of y was found to be dominated by the FSI and to grossly overestimate the data.

In this letter we investigate the effect of FSI on the scaling function. We suggest that there are dynamic off-

shell effects that reduce the strength of the imaginary potential, and that inclusion of this effect leads to a satisfactory description of the scaling function

2. Formalism

To show the main features of the inclusion of FSI, we will ignore the radial dependence of the potential (i.e. we use a formalism appropriate for nuclear matter) and compute the longitudinal response in the eikonal approximation. In ref. [12], where the eikonal form of the propagator was studied in detail, it was found that this approximation provides an excellent description of the exact result, for sufficiently high energies of the knocked-out nucleon, $E \gg V$, where V is the strength of the potential

At large values of the energy and momentum transfer ($\omega, \mathbf{q}$) the longitudinal response is dominated by the incoherent part. Furthermore we assume that there is only one bound state. In this case the full spectral function can be replaced by the momentum distribution, and the response function can be written as

$$S_L(\omega, q) = \frac{1}{\pi} \text{Im} \int d\mathbf{p} n(p) [\omega' - T - \Sigma(\omega', |\mathbf{p} + \mathbf{q}|)]^{-1}, \quad (1)$$

where $T = (\mathbf{p} + \mathbf{q})^2/2m$ is the kinetic energy of the knocked-out nucleon (with mass m) and $\omega' = \omega - \varepsilon$, the energy transferred to the nucleon corrected for averaged separation energy ε . The nucleon momentum distribution $n(p)$ is normalized as $\int d\mathbf{p} n(p) = 1$, and $\Sigma = V + iW$ ($W > 0$) is a generalized optical potential. In essence in the eikonal approximation one assumes that the knocked-out nucleon propagates in the direction of $\mathbf{q}$ (which is taken to be the z -axis), this amounts to approximating $\omega' - T = (p_\omega - |\mathbf{p} + \mathbf{q}|)(p_\omega + |\mathbf{p} + \mathbf{q}|)/2m$ in eq. (1) by $(p_\omega - q - p_z)p_\omega/m$, where $p_\omega = \sqrt{2m\omega'}$. In that case the response function $S_L(\omega, q)$ takes on the form of a convolution with a lorentzian [11]

$$S_L(\omega, q) = \frac{1}{\pi} \frac{m}{p_\omega} \int d\mathbf{p} n(p) \frac{\Gamma(\omega', p')}{[\eta(\omega', p') - p_z]^2 + \Gamma^2(\omega', p')}, \quad (2)$$

where the momentum η (the eikonal scaling variable if $W=0$ [12]) is defined by $\eta(\omega', p') = -q + p_\omega - m/p_\omega V(\omega', p')$, and the width Γ by $\Gamma(\omega', p') = m/p_\omega W(\omega', p')$ with $p' = p_z + q$

It is instructive to consider the special case that V and W are independent of the momentum p' of the knocked-out nucleon as in the case of a local optical potential. One can then perform the p_z integral in eq. (2) analytically with the result

$$S_L(\omega, q) = \frac{m}{p_\omega} \left[2\pi \int_{|y|}^{\infty} dp p n(p) + 2 \int_0^{\infty} dp p n(p) \arctan \left(\frac{2p\Gamma}{y^2 - p^2} \right) \right], \quad (3)$$

where the scaling variable is given by $y = -(\eta^2 + \Gamma^2)^{1/2}$. This expression was derived before in ref. [8].

In the spirit of the eikonal method, the natural choice for the optical potential is the “ ip ” approximation. In this case V and W are expressed in terms of the forward nucleon–nucleon scattering amplitudes f_{NN} , $W(\omega') = 2\pi\rho/m \text{Im} f_{NN}(\omega', 0^\circ) = \rho p_\omega/2m \sigma_{NN}(\omega')$ and $V(\omega') = \alpha(\omega') W(\omega')$, where ρ is the nucleon density and α the ratio of the real and imaginary scattering amplitude. If $\sigma_{NN}(\omega')$ and $\alpha(\omega')$ are taken to be energy independent as was assumed in ref. [8], the quantities $m/p_\omega V$ and Γ are independent of the energy ω' and as a result the function $F(y) = p_\omega/m S_L(\omega, q)$ scales in terms of y . It should be noted, however, that for large negative values of $y < -k_F$ (the Fermi momentum) one obtains the rather puzzling results that the second term of eq. (3) due to the FSI, that behaves like $|y|^{-2}$, dominates over the first (PWIA) term, and moreover falls off much slower

than the data. A related problem is that with a constant Γ the energy squared weighted sum rule diverges. This indicates that a more refined treatment of the optical potential is required.

3. Generalized optical potential model

The generalized optical potential for a nucleon with energy E and momentum p' can conveniently be written in the form [10]

$$\Sigma(E, p') = V_{\text{HF}}(p') + \Delta V(E, p') + iW(E, p'), \quad (4)$$

where V_{HF} denotes the Hartree-Fock type potential, which is defined such that the dispersive contribution ΔV vanishes at the Fermi energy E_F .

A convenient parametrization of the empirical V_{HF} is given by [13] $V_{\text{HF}}(p') = V_0 \exp(-\frac{1}{4}c^2 p'^2)$, where c is the range of non-locality. In the following we take $c = 1$ fm and $V_0 = -65$ MeV for nuclear matter, and $V_0 = -30$ MeV for ${}^4\text{He}$.

The energy dependence of the imaginary potential on-the-energy-shell $W_{\text{on}}(E) \equiv W(E, p'(E))$, where $E = [p'(E)]^2/2m + V(E, p'(E))$, is fairly well known from elastic scattering of nucleons off nuclei, especially for not too high energies. For these energies we adopt the parametrization of ref. [13] for the volume integral of W . The high-energy behavior of W is not so well established. For example, in a dilute Fermi-gas model with a hard-core interaction, one expects that for $E \rightarrow \infty$ $W_{\text{on}}(E)$ is proportional to $E^{1/2}$ [14]. On the other hand, experimental data suggest that $W_{\text{on}}(E)$ becomes rather independent of energy for $E \geq 100$ MeV. To investigate the sensitivity of the scaling function to the energy dependence of W we will compare results for two different extrapolations of $W_{\text{on}}(E)$ for $E - E_F > E_m \approx 35$ MeV considered in ref. [13],

$$(i) \quad W_{\text{on}}(E) = \frac{2}{\pi} \frac{P_F^3 r_c^2}{\sqrt{m}} \left((E - E_F)^{1/2} + \frac{E_m^2}{E - E_F} - \frac{3}{2} E_m^{1/2} \right) + W_0,$$

where $P_F = 1.35 \text{ fm}^{-1}$ and $r_c = 0.52 \text{ fm}$ [13] and $W_0 = 11.6$ (10.2) MeV for nuclear matter (${}^4\text{He}$), and

$$(ii) \quad W_{\text{on}}(E) = W_0(\text{constant})$$

We emphasize that in the present situation the use of an on-shell optical potential is not appropriate. In particular for large negative values of y , one deals with an off-shell nucleon with momentum p' larger than the on-shell value $p'(E)$. Intuitively one expects a reduction of the imaginary potential compared to the on-shell value. For example, in a dilute Fermi-gas model, one has from energy-momentum conservation [14]

$$W(E, p') \approx \text{Im} \int d\mathbf{p}_1 \rho(\mathbf{p}_1) \langle \mathbf{p}', \mathbf{p}_1 | t(E) | \mathbf{p}', \mathbf{p}_1 \rangle = 0,$$

for $E \approx E_F$ and $p' > 3k_F$. A few reported calculations [15] using schematic interactions do indeed show that a strong quenching of the off-shell values of $W(E, p')$ for $p' > p'(E)$ as is the case for $y < 0$.

In order to get some idea about the off-shell effect, we use a simple parametrization proposed in ref. [16],

$$W(E, p') = \exp\left(\frac{1}{4}b^2\{p'^2 - [\bar{p}(E)]^2\}\right) W_{\text{on}}(E) \quad \text{for } p' \geq \bar{p}(E) = \sqrt{2mE}, \\ = W_{\text{on}}(E) \quad \text{otherwise,}$$

where the off-shell parameter b will be varied between 0 and 1 fm.

Once W is fixed the real dispersive contribution $\Delta V(E, p')$ can be obtained from a subtracted dispersion relation

$$\Delta V(E, p') = \frac{E - E_F}{\pi} P \int_{-\infty}^{\infty} dE' \frac{W(E', p')}{(E' - E_F)(E - E')} \quad (5)$$

In applying eq (5) one needs also information on the imaginary potential for negative energies. Following ref [13] we have assumed that W_{on} is symmetric around E_F for $|E - E_F| < 35$ MeV and that it falls off exponentially for large negative energy. We emphasize that it is important to treat the real dispersive and imaginary parts of the optical potential consistently. In that case the energy non-weighted sum rule for S_L is satisfied exactly, and there is no need to impose flux conservation by the introduction of a Perey factor (as in refs [3,9]), since the HF potential is taken to be energy-independent.

4. Results and discussion

To illustrate the sensitivity of the scaling function $F(y, q) = p_\omega / m S_L(\omega, q)$ to the treatment of final state interactions (and in particular the effect of W) we compare the theoretical scaling function with experimental ones taken at SLAC for ^4He [17] and nuclear matter [18]. The latter is obtained as an extrapolation from data for $A=4-197$ to $A=\infty$ based upon a decomposition of the reduced cross sections in volume and surface contributions. The data contain longitudinal as well as transverse contributions. However, if the convection current is neglected the reduced longitudinal and transverse structure functions become equal and it is meaningful to define the experimental scaling function as

$$F^{\text{exp}}(y, q) = \frac{p_\omega}{m} \frac{\sigma_{\text{eA}}}{(Z\sigma_{\text{ep}} + N\sigma_{\text{en}})_{\text{off}}} \quad (6)$$

The off-shell electron-nucleon cross section is taken from ref [19], and the nucleon form factors from Hohler et al [20] for the proton and from Blatnik and Zovko [21] for the neutron. In the actual calculations, we use relativistic kinematics, i.e. $\omega' = \omega - \varepsilon = (m^2 + p_\omega^2)^{1/2} - m$ (for nuclear matter), and $\omega' = \omega - \varepsilon = (m^2 + p_\omega^2)^{1/2} + (M_{A-1}^2 + p^2)^{1/2} - m - M_{A-1}$ (for ^4He). As a consequence in the scaling variable η the momentum p_ω is replaced by $(\omega'^2 + 2m\omega')^{1/2}$. The separation energies ε are taken to be 44 MeV for NM and 20 MeV for ^4He .

Figs 1 and 2 show the theoretical calculation compared with experiment for the two high-energy extrapolations of W_{on} , for ^4He and nuclear matter, respectively. For ^4He the nucleon momentum distribution $n(p)$ is parametrized in the form suggested in ref [22] as $n(p) = N[\exp(-\beta_1^2 p^2) + s \exp(-\beta_2^2 p^2)]$. The parameters are taken as $\beta_1 = 1.508$ fm, $\beta_2 = 0.727$ fm and $s = 0.019$, and N is a normalization constant. For nuclear matter we have used the momentum distribution calculated by Jamnion et al [23] for ^{208}Pb #1. The scaling variable is given by $y_e = -[\eta^2(\omega', p_\omega) + \Gamma^2(\omega', p_\omega)]^{1/2}$.

To show the sensitivity on the parameter b that characterizes the off-shellness the results are shown for $b=0, 0.5$ and 1 fm. Since in the present case the optical potential depends on energy and momentum only approximate scaling occurs. As an indication of the size of the scaling deviations we have also indicated (for $b=0$ fm) the spreading in $F(y_e, q)$ for three values of q .

It is seen that for small values of $|y_e| < k_F$ the inclusion of W gives rise to a small reduction of the PWIA result, whereas for larger negative values of y_e there is a strong enhancement of the scaling function which depends sensitively on the high-energy behavior of W . Although it is difficult to draw definite conclusions, the figures suggest that (1) both the data and theoretical calculation favor the choice of $W_{\text{on}} = \text{constant}$ over $W_{\text{on}} \propto (E - E_F)^{1/2}$. In the former case scaling to the PWIA result is recovered for $q \rightarrow \infty$ only (where $\Gamma \rightarrow 0$), and in the

#1 Naturally we first tried to describe the extrapolated "nuclear matter" data of ref [18] using a theoretical momentum distribution for nuclear matter. However, in that case we found that the calculated scaling function exhibits a kink at $y = -k_F$ [coming from the discontinuity of $n_{\text{NM}}(p)$ at $p=k_F$], which does not disappear even if FSI are included, but which is not at all present in the data. Therefore it seems to us that the results of ref [18] describe more likely a heavy nucleus with a large radius than translational invariant nuclear matter. We are investigating this interesting point in more detail.

Fig. 1 The scaling function $F(q, y)$ for nuclear matter calculated at $q = 1.6$ GeV/c, for the PWIA and two high-energy extrapolations of W , and for various values of the off-shell parameter b . To indicate the scaling violation, the results for $b = 0$ fm at $q = 1.2$ and 2.1 GeV/c are also shown. Only experimental data [18] that are more than 100 MeV above the threshold for breakup are shown.

Fig. 2 The same as fig. 1, for ${}^4\text{He}$. The experimental data are taken from ref. [17].

latter case there would be exact scaling if the real potential would vanish but not to the PWIA result, (11) an off-shell modification of the strength of W is required to describe the data.

5. Summary

In this letter we have investigated the effect of final state interactions on the y -scaling function. It is suggested that the large deviations from the IA result coming from the imaginary part of the optical potential (which represents the coupling to two-nucleon knock-out channels) reported by Gurvitz and Rinat [11] are reduced substantially if off-shell effects are taken into account.

We note that a quantitative calculation of the effects of FSI on the scaling function requires a more accurate treatment of the initial state correlations [24,25]. Recently Butler and Koonin [24] using a *G*-matrix approach showed that for large negative values of γ contributions from the 2p2h correlations in the ground state dominate, also leading to two-nucleon knock-out. Therefore a consistent treatment of scaling properties seems to require the inclusion of initial and final state interactions on equal footing.

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