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Distributed Supply Coordination for Power-to-Gas Facilities Embedded in Energy Grids

Desti Alkano and Jacquélien M. A. Scherpen, *Senior Member, IEEE*

Abstract—This paper considers hydrogen and renewable electricity from power-to-gas (PtG) facilities supplied to a gas grid, a mobility sector, and a power grid. The PtG facilities are equipped with hydrogen buffers and fuel cells. The goal is to maximize the expected profit of PtG facilities without exceeding the grid capacities. The decisions on the supply levels are done in a distributed fashion, yet some coordination with the multiple energy grid operators is necessary to obey the grid capacity constraints. The distributed supply coordination is developed using a model predictive control approach based on a dual decomposition combined with the projected gradient method. The approach results in a two-layer optimization problem experienced by each PtG facility and each grid operator. In this way, the grid operators can manage the PtG facilities via dynamic distribution pricing. We study the fairness of the corresponding algorithm and discuss its practical implementation. Simulation results are provided to evaluate the algorithm performance and to investigate the evolution of distribution prices.

Index Terms—Distributed model predictive control, distributed generators, energy storage, intelligent control, modeling.

I. INTRODUCTION

INSECURE energy supply coupled with the drive towards a low-carbon economy has motivated a large-scale implementation of renewable power sources, including solar panels and wind turbines. In general, renewable energy is derived from intermittent sources which are highly affected by weather conditions. With integration of renewable energy into the power grid, a sudden production change could have severe consequences for the grid stability [1]. It is therefore evident that the need for storing excess power production will increase as the penetration of renewable energy increases.

There are several solutions for storing large amounts of electrical energy. Currently, the most cost-effective energy storage options for large-scale power are pumped hydro storage (PHS) and compressed air energy storage (CAES) [2]. However, generally PHS requires reservoirs causing environmental damage by flooding areas up to 10-20 km^2 , whereas CAES requires

an underground cavern which may be difficult to find near renewable energy sources [2], [3].

Power-to-Gas (PtG) is an increasingly popular alternative energy storage option. In this concept, an electrolyzer converts excess power into hydrogen [3]. The produced hydrogen offers an opportunity for accommodation in a gas distribution grid and increases the fuel sustainability in the mobility sector. By including a hydrogen storage device, the PtG facility can be regarded as a means for energy storage since the stored hydrogen can be reconverted into electrical energy at a later time using a fuel cell. In these ways, PtG facilities can create revenue from temporary excesses in the renewable energy supply.

It is desirable that the number of PtG facilities equipped with hydrogen buffers and fuel cells increases to support the integration of renewable energy into the energy grids. However, the capacity of energy grids has limitations hence restricting the aggregated supply levels from the PtG facilities. This restriction is governed by a distribution system operator (DSO) of each energy grid, as the DSO has the responsibility to manage and operate the energy distribution grid. The responsibility makes the DSO act when overloading is detected [5]. It may modify its distribution charge for energy transport and system services utilized by PtG facilities, thereby inducing the PtG facilities to modify their supply levels.

This scheme led us to provide a real-time supply coordination algorithm that automatically interact the PtG facilities and the DSOs through a limited number of message exchanges to search for the optimal energy supply levels for each PtG facility and the optimal distribution charge to be advertised by the operators. In this way, the algorithm helps the PtG facilities to better manage supply with real-time data and helps the DSOs to avoid overloading grids. With this algorithm, we provide some insight on how the DSO of an energy grid reacts to conditions happening in other grids.

Bidirectional communication among energy market operators and supply bidders were examined in [6]–[8]. Additionally in [9], a distributed control strategy was proposed to maximize a network utility with delivery contracts and in [10], the control strategy in a distributed fashion was presented to minimize the imbalance between the consumed and the purchased energy thereby avoiding trading energy at an undesirable price. Two-layer optimization problems are studied in the latter two papers. They solved the problem in a distributed manner using a distributed model predictive control (DMPC) scheme, based on a dual decomposition approach combined

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with a gradient method. Furthermore, recent literature deals with controlling energy producers to match demand in a network, as an implementation of DMPC scheme proposed in [13]. In [11], explicit constraints of energy co-generation facilities, which are micro combined heat and power (μ -CHP) units, are taken into account resulting in a non-convex optimization problem. In [11], a one-layer optimization problem solved locally for each μ -CHP unit using local information is considered. In [14], a distributed coordination approach for embedding biogas from farmers in the gas grid is presented. Here, we use the same optimal control scheme, but applied to PtG facilities embedded in the gas grid, the mobility sector, and the power grid. This requires a new system model and an algorithm to capture mathematically the coordination among PtG facilities and the DSOs of the energy grids.

Our main contributions are twofold. *Firstly*, recent studies such as for instance [6]–[11], deal with a single-grid perspective in developing a coordination framework. In contrast, we deal with *a multiple-grid perspective* at which each PtG facility has more than one type of energy to produce and sell to multiple energy grids. Therefore, it is necessary to take into account all possible current and predicted limitations in the associated energy grids. With this setting, it is natural to formulate the optimization problem in model predictive control framework. The framework allows the PtG facilities to anticipate the future conditions on the energy grids hence maximizing their expected profit for a given time horizon without overloading the energy grids. *Secondly*, solving the maximization problem in a centralized manner may put a huge burden on the central solver requiring the solver to know the up-to-date aggregated supply levels, the grid capacities, the selling prices, the profit functions and all states of all PtG facilities. Furthermore, tracking this information may be impractical because PtG facilities may not be willing to share the information with each other. We therefore solve the optimization problem in a distributed fashion. To do so, we reformulate the problem using a dual decomposition approach combined with a projected gradient method. There are distributed algorithms which are in some sense outperforming the projected gradient method, such as the accelerated gradient method proposed in [12]. However, the gradient method is the simplest way we can implement to perform such bidirectional communication among the PtG facilities and the multiple energy grid operators. In the distributed setting, we provide a pricing mechanism by the DSOs of the energy grids to ensure that the grid capacity constraints are obeyed. The price, the so-called distribution charge, is time-variant and uniform for all PtG facilities. However, our framework offers the possibility so that the PtG facilities may have different characteristics, and thus their profits may be different as well.

Our proposed algorithm is synchronous, assuming that multiple DSOs of the energy grids and the PtG facilities have a common clock to update their supply bids and distribution charges, respectively. Our preliminary results are presented in [15]. In addition to [15], here we provide a full treatment of the model and the method, we study fairness of the proposed algorithm, and we study how the optimal distribution

Fig. 1. An overview of the Power-to-Gas concept, adapted from [4].

charge of an energy grid behaves with respect to both the aggregated supply levels from PtG facilities and the congestion level of other energy grids. Furthermore, in contrast to [15], we explicitly provide the properties of objective functions and their practical interpretations.

The rest of the paper is structured as follows: Section II models dynamics and technical constraints of the PtG facilities, defines the role of the gas grid, the mobility sector, and the power grid connected to the PtG facilities, and introduces objective functions of the PtG facilities. The optimization problem is presented in Section III. Section IV presents an algorithm of the distributed supply coordination, in order to maximize the expected profit of PtG facilities over a finite horizon without exceeding the grid constraints. We present fairness and practical implementation of the algorithm in Section V. Simulation results and a discussion are provided in Sections VI and VII, respectively.

II. MODELING

We first develop a model of a renewable energy generator equipped with a PtG facility. Consider a PtG facility with a hydrogen buffer and a fuel cell. The schematic description of the PtG facility is presented in Fig. 1. The number of technical constraints of the PtG facility are described in Section II-A. We explain a role of the gas grid, the mobility sector, and the power grid connected to PtG facilities in Section II-B. We assume that the strategy of PtG facilities is to maximize their revenue as described in Section II-C and to minimize their cost of hydrogen surplus as formulated in Section II-D, thereby maximizing their (expected) profit.

A. PtG Modeling

We consider regular measurements from a PtG facility $i \in \{1, \dots, n\}$, defined by a time $k \in \{1, \dots, K\}$. Let $v_i(k) > 0$ be the surplus power generation, defined as a difference between the electricity usage of the loads $v_{load}(k) \in \mathbb{R}_+$, the power output from the conventional power generator $v_{conv}(k) \in \mathbb{R}_+$ and the power output from the renewable energy source $v_{renew}(k) \in \mathbb{R}_+$ at a time k , given by $v_i(k) = v_{conv,i}(k) + v_{renew,i}(k) - v_{load,i}(k)$. Strictly speaking, at a time k it is possible that $v_i(k) < 0$ which indicates energy shortage in the power grid and $v_i(k) = 0$ when there is perfect matching between total power demand and

the aggregated supply from both the conventional power and renewable energy generator.

The main task of a PtG facility is to convert the surplus power generation, i.e., if $v_i(k) > 0$, into a gaseous energy carrier, i.e., hydrogen, using an electrolyzer. Here we use a proton exchange membrane (PEM) water electrolyzer because of its high efficiency, i.e., 70% [16].

The produced hydrogen $p_i(k) \in \mathbb{R}_+$ is dependent on the electrolyzer capacity c_i , the electrolyzer efficiency η_i , the maximum heating value h_i , and the minimum fraction of the maximum capacity l_i at which the electrolyzer needs to operate, given by

$$p_i(k) = \begin{cases} \frac{c_i \cdot \eta_i}{h_i} & \text{if } v_i(k) \geq c_i \\ \text{else} \begin{cases} \frac{v_i(k) \cdot \eta_i}{h_i} & \text{if } v_i(k) \geq l_i c_i \\ 0 & \text{if } v_i(k) < l_i c_i. \end{cases} \end{cases}$$

This equation simply defines the produced hydrogen at

$$p_i(k) \in 0 \cup [p_{\min,i}, p_{\max,i}], \quad (1)$$

at time k , where $p_{\min,i} = \frac{l_i c_i \cdot \eta_i}{h_i}$ and $p_{\max,i} = \frac{c_i \cdot \eta_i}{h_i}$. Note that the produced hydrogen $p_i(k)$ is not controllable, as it depends mainly on the intermittent source $v_i(k)$.

As mentioned above, the PtG facility may contribute to increasing the sustainability of a gas grid and to satisfying hydrogen demand of a mobility industrial grid by injecting amounts of hydrogen $g_i(k) \in \mathbb{R}_+$ and $y_i(k) \in \mathbb{R}_+$ to the gas grid and to the mobility industrial grid, respectively. Alternatively, some produced hydrogen can be stored in a hydrogen storage device, denoted by $u_{s,i}(k) \in \mathbb{R}_+$, to be reconverted into electrical energy using a fuel cell, to be sold to the gas grid, and/or to be injected to the mobility industry sector at a later time.

We choose a PEM fuel cell due to its substantially lower cost per kWh of the power output than that of other fuel cells. In addition, the PEM fuel cell has a reasonably high efficiency, 40 – 60% [17]. In practice, the power output from the PEM fuel cell depends on the efficiency of the fuel cell, the amount of electrical energy required per kilogram hydrogen, and the density of hydrogen, which are together characterized by a constant parameter ϵ . Given an amount of stored hydrogen $w_i(k) \in \mathbb{R}_+$, the electrical energy produced by the fuel cell is characterized by $e_i(k) = \frac{1}{\epsilon} w_i(k)$.

Remark 1: It is assumed that there is no excessive demand on the power grid when hydrogen is produced by the electrolyzer of PtG facilities, i.e., $p_i(k) > 0$ implies that there is no excessive demand on the power grid which is denoted by $e_i(k) = 0$.

We assume that there is no leakage of the stored hydrogen over time. Moreover, hydrogen is assumed to be stored at normal temperature and normal pressure, which are respectively 20°C and 1 atm [18]. Given the initial value of stored hydrogen in the storage device $z_i(0)$, we define the dynamics of available hydrogen $z_i(k)$ in the hydrogen buffer of PtG facility i by a linear model, given by

$$z_i(k+1) = z_i(k) - \rho_t u_{t,i}(k) + \rho_s u_{s,i}(k), \quad (2)$$

where $u_{t,i}(k) \in \mathbb{R}_+$ refers to the amounts of hydrogen which are taken from the storage device, whereas ρ_t and ρ_s are respectively the efficiency of discharging and charging the storage device. We set $u_{s,i}(k)$ as a controllable input for the storage device.

Remark 2: It can be argued that there are three reasons why a PtG facility stores hydrogen at a time k . Firstly, it is desirable to store some hydrogen in the storage device when the current energy prices are low compared to the estimates of future energy prices. Secondly, it is necessary to store the produced hydrogen when there is no more demand that needs to be satisfied. However, it can only be done if there is remaining space in the storage device. Thirdly, it is natural to store the remaining produced hydrogen when the corresponding variable production cost to provide energy is higher than the associated revenue.

In fact, the available hydrogen in the storage device has an upper limit determined by the storage capacity of the device S_i of PtG facility i , given by

$$0 \leq z_i(k) \leq S_i. \quad (3)$$

Hence, we have upper limits on $u_{t,i}(k)$ and $u_{s,i}(k)$ given by $u_{t,i}(k) \leq \frac{z_i(k)}{\rho_t}$ and $u_{s,i}(k) \leq \frac{S_i - z_i(k)}{\rho_s}$, respectively.

When the surplus hydrogen that needs to be stored exceeds the remaining space in the storage device, some amount of hydrogen $f_i(k) \in \mathbb{R}_+$ needs to be flared. This amount can be seen as a dump load, enabling the total energy supplied by PtG facility i to match the sum of its production and energy taken from its storage device, given by

$$g_i(k) + y_i(k) + e_i(k) + u_{s,i}(k) + f_i(k) = p_i(k) + u_{t,i}(k). \quad (4)$$

We determine all variables stated in (4) in a volume unit, i.e., Nm^3 meaning that each m^3 is considered at normal condition with the temperature of 20°C and the pressure of 1 atm. The energy injected to the gas grid, the mobility sector, and the power grid per PtG facility i are bounded by

$$\mathcal{X}_i = \{x_i(k) | \underline{x}_i \leq x_i(k) \leq \bar{x}_i\} \quad (5)$$

for all $x = g, y, e$ and all $k \in \{1, \dots, K\}$, where $\underline{x}_i$ and $\bar{x}_i$ are respectively the lower and upper bounds of energy injected in the energy grid $x \in \{g, y, e\}$, where grid g, y, e are the gas grid, the mobility industrial grid, and the power grid, respectively. The constraints (5) represent constraints per PtG i for supply, i.e., valve limitations, etc.

B. Grid Modeling

Let n PtG facilities be embedded in the same gas grid, the same mobility sector, and the same power grid. We refer to such a group of renewable energy producers as a community. The community aims at maximizing its profit from the produced energy without overloading the energy grids.

Overload of the energy grids is defined as a situation in which the total energy supply from PtG facilities exceeds the maximum allowable energy injected in the energy grids. In order to avoid an overload, we have the following constraints:

$$\sum_{i=1}^n x_i(k) \leq c_x(k) \quad \forall x = g, y, e. \quad (6)$$

Remark 3: The constraints (6) are in fact a nonlinear function, dependent on the distribution factor and nodal injection [23], [24]. We however assume these to be external signals, thus allowing us to write the constraint as a time-varying signal.

With the constraints (6), the DSO of an energy grid x ensures that the aggregated supply levels from all PtG facilities will not exceed its grid capacity c_x . More formally on the constraints (6), if $\sum_i^n x_i(k) \leq c_x(k)$, then $x = (x_i, i \in \{1, \dots, n\})$ is feasible.

As stated above, we assume that the strategy of PtG facilities is to maximize their revenue and to minimize their associated costs. We model these two objectives in the following subsections.

C. Revenue Modeling

Let $p_x(k) \in \mathbb{R}_+$ be the selling price signal in an energy grid x at time k . Let $c_{x,i}$ be a time-invariant cost to provide energy to the associated energy grid x . Furthermore, we let $c_{qx,i}$ be time-invariant cost of transmission losses of energy injected to the energy grid x . Each PtG facility may have different responses to the same selling prices $p_x(k)$ for all $x = g, y, e$, as it may not have the same variable production costs $c_{x,i}$ and the same costs of transmission losses $c_{qx,i}$ for each PtG facility i . We can mathematically capture the different responses of diverse PtG facilities to varying selling price signals $p_x(k)$ by using the concept of utility function borrowed from the microeconomics discipline [25]. For each PtG facility $i \in \{1, \dots, n\}$, its corresponding revenue function $U_i(g_i(k), y_i(k), e_i(k))$ depends on the amount of energy injected in the gas grid $g_i(k)$, in the mobility sector $y_i(k)$, and in the power grid $e_i(k)$, characterized by

$$U_i(g_i(k), y_i(k), e_i(k)) = U_{g,i}(k) + U_{y,i}(k) + U_{e,i}(k), \quad (7)$$

where $U_{x,i}(k) = (p_x(k) - c_{x,i})x_i(k) - c_{qx,i}x_i^2(k)$, for all $x = g, y, e$. We are inspired by [26] in choosing quadratic forms associated with the time-invariant costs of transmission losses $c_{qx,i}$. Furthermore, we assume that the revenue functions fulfill the following properties:

- *Increasing revenue function* The PtG facilities are always willing to supply the energy grids, if possible, to maximize their revenue. It is mathematically defined by $\frac{\partial U_i}{\partial x} > 0$, resulting in

$$0 \leq x_i(k) < \frac{p_x(k) - c_{x,i}}{2c_{qx,i}} \quad (8)$$

for all $x = g, y, e$. We can interpret $Q_x = \frac{\partial U_i}{\partial x}$ as marginal revenue.

- *Strictly concave revenue function* That is, the PtG facilities have non-increasing marginal revenue function, given by $\frac{\partial^2 U_i}{\partial x^2} < 0$ for all $x = g, y, e$ at each time k , implying that $c_{qg,i}, c_{qy,i}, c_{qe,i} > 0$.

D. Surplus Hydrogen

From (2), (4), and Remark 2, the control decisions $g_i(k), y_i(k), e_i(k), u_{t,i}(k), u_{s,i}(k)$ influence the amount of

hydrogen available in the storage device $z_i(k)$ and the amount of hydrogen that needs to be flared $f_i(k)$. The surplus hydrogen which are both flared and stored in the storage device causes additional cost for PtG facility i , defined by

$$C_i(k) = c_{f,i}f_i(k) + c_{op,i}z_i(k), \quad (9)$$

where $c_{f,i}$ and $c_{op,i}$ denote the flaring cost and opportunity cost for each Nm^3 hydrogen flared and stored in the storage device, respectively. Notice that the PtG facilities flare their surplus hydrogen if there is no more space remaining in their storage devices. We are inspired by [28]–[31] in choosing a convex function, in particular a linear function, associated with the opportunity cost. We thus simply define the opportunity cost by multiplying the available hydrogen in the storage device $z_i(k)$ by an interest rate $c_{op,i}$, i.e., $c_{op,i} \cdot z_i(k)$. It is calculated repeatedly at each time k . Other ways of determining the opportunity cost, e.g., as proposed in [32], can be adopted for our model. It is based on micro-economic theory where the opportunity cost is not necessarily a linear function of the available inventory in the storage device.

It is desirable for PtG facility i to maximize its revenue function (7) and to minimize its associated costs (9) in order to maximize its profit $V_i(k) = U_i(k) - C_i(k)$ without exceeding the grid capacities. To do so, the PtG facilities need to coordinate their supply bids to avoid overloading grids. We will provide the supply coordination in the following section.

III. PROBLEM DESCRIPTION

Here we define the supply coordination, performing the revenue maximization (7) and the cost minimization (9) while obeying both the grid constraints (6) and local technical constraints (1)–(5) for all $i \in \{1, \dots, n\}$ and $k \in \{1, \dots, K\}$. We model the optimization problem in a model predictive control (MPC) scheme as we aim at taking into account the predictions of all individual constraints of each PtG facility and the predictions of all possible limitations in the associated energy grids. For extensive explanation of the MPC scheme, refer to e.g., [33]–[36].

Let us consider n PtG facilities embedded in the same gas grid, the same mobility sector, and the same power grid. In the MPC scheme, all information about the objective functions, states, and predictions of hydrogen production of all PtG facilities are globally available and governed by a central unit in an effort to solve the optimization problems. We define the optimization problem as

$$\max \sum_{\tau=k}^{k+T} \sum_{i=1}^n \hat{V}_i(\tau), \quad (10)$$

subject to the prediction models of (1)–(6) and given initial conditions denoted by

$$\hat{z}_i(\tau)_{\tau=k} = z_i(k), \quad (11)$$

as well as additional constraints

$$\underline{x}(\tau) \leq \hat{x}_i(\tau) \leq \bar{x}_i(\tau) \quad \forall x = g, y, e \quad (12)$$

where $\tau = k \dots, K + T$ is a new time variable introduced to differentiate between the system time k and the

prediction time τ . The hat-notations are specified to distinguish the prediction parameters from the system model parameters. When injecting energy to all energy grids at time k is not beneficial for PtG facilities, the problem (10) becomes a minimization problem with respect to $z_i(k)$ and $f_i(k)$.

Define $\mathcal{G}_i = \{g_i(\tau) | g_i \leq g_i(\tau) \leq \bar{g}_i(\tau)\}$, $\mathcal{Y}_i = \{y_i(\tau) | y_i \leq y_i(\tau) \leq \bar{y}_i(\tau)\}$, and $\mathcal{E}_i = \{e_i(\tau) | e_i \leq e_i(\tau) \leq \bar{e}_i(\tau)\}$. We have the following assumption on the profit function.

Assumption 1: The profit function $\hat{V}_i(\tau)$ is increasing and strictly concave on the intervals $\mathcal{G}_i, \mathcal{Y}_i, \mathcal{E}_i$.

Notice that all constraints stated in (1)-(5) are compact and convex, whereas the objective function follows Assumption 1. Hence, the problem is a convex optimization problem with one solution at most [9].

When we solve the problem (10) centrally, we assume that there exists a central unit making decisions for all PtG facilities. The central unit only applies the first of the planned actions, i.e., $\hat{g}_1(\tau), \dots, \hat{g}_n(\tau), \hat{y}_1(\tau), \dots, \hat{y}_n(\tau), \hat{e}_1(\tau), \dots, \hat{e}_n(\tau), \hat{u}_{t,1}(\tau), \dots, \hat{u}_{t,n}(\tau)$, and $\hat{u}_{s,1}(\tau), \dots, \hat{u}_{s,n}(\tau)$. It then repeats the optimizations for next time instance.

Due to the competitive nature of energy providers, i.e., PtG facilities, it may happen that they are not willing to share their information to their competitors. We therefore aim at solving the problem (10) in a distributed fashion. In this way, each PtG facility can locally manage their hydrogen allocation based on its own local information, yet it is still necessary to coordinate its supply bids to the DSOs of the energy grids in order to avoid overloading grids. To obtain the distributed solutions, the problem (10) will be reformulated using a dual decomposition approach combined with a projected gradient method. The reformulated problem will contain two-layer optimization problems experienced by each PtG facility and each DSO of the energy grids. With the reformulation, the computations are scalable to implement.

IV. DISTRIBUTED SUPPLY COORDINATION

Recall that the objective function (10) is separable for each supply level $\hat{g}_i(\tau), \hat{y}_i(\tau), \hat{e}_i(\tau)$ and each PtG facility i . However, the supply levels are coupled by the predictions of constraints (6). As we aim at developing a distributed supply coordination, the problem (10) is decoupled using a dual decomposition approach combined with a projected gradient method. Our work is closely related to the work presented in [9] and [10]. However, they only deal with a single-grid perspective in developing a distributed coordination framework. Here we deal with a multiple-grid perspective at which each PtG facility has five controllable inputs with three of them are coupled. Our work thus leads to more complex practical implementations of the algorithm, which will be described in Section V-B. With the algorithm, we provide some insight on how the distributed approach can be realized in the energy markets and how the dual variables associated with three coupling inequality constraints (6) react to each other.

To decouple the problem (10), we first derive a dual function of that problem. Let $\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)$ be the projected dual variables associated with the prediction model of the grid capacities (6) at prediction time τ . It is common in some recent

literature to mention the dual variables as shadow prices, such as in [11] and [19].

Define the Lagrangian function of the problem (10)

$$\begin{aligned} L(\hat{\mathbf{g}}(\tau), \hat{\mathbf{y}}(\tau), \hat{\mathbf{e}}(\tau), \hat{\lambda}_g, \hat{\lambda}_y, \hat{\lambda}_e) \\ = \max_{\tau=k}^{K+T} \hat{V}(\tau) - \hat{\lambda}_g(\tau) \left(\sum_{i=1}^n \hat{g}_i(\tau) - \hat{c}_g(\tau) \right) \\ - \hat{\lambda}_y(\tau) \left(\sum_{i=1}^n \hat{y}_i(\tau) - \hat{c}_y(\tau) \right) - \hat{\lambda}_e(\tau) \left(\sum_{i=1}^n \hat{e}_i(\tau) - \hat{c}_e(\tau) \right) \end{aligned} \quad (13)$$

where $\hat{\mathbf{g}}(\tau) = \{\hat{g}_1(\tau), \dots, \hat{g}_n(\tau)\}$, $\hat{\mathbf{y}}(\tau) = \{\hat{y}_1(\tau), \dots, \hat{y}_n(\tau)\}$, and $\hat{\mathbf{e}}(\tau) = \{\hat{e}_1(\tau), \dots, \hat{e}_n(\tau)\}$. Notice that $\hat{V}(\tau) = \sum_{i=1}^n \hat{V}_i(\tau)$ in (10) is separable in $\hat{g}_i(\tau), \hat{y}_i(\tau), \hat{e}_i(\tau)$, resulting in

$$\begin{aligned} \max_{\tau=k}^{k+T} \sum_{i=1}^n \left(\hat{V}_i - \hat{\lambda}_g(\tau) \hat{g}_i(\tau) - \hat{\lambda}_y(\tau) \hat{y}_i(\tau) - \hat{\lambda}_e(\tau) \hat{e}_i(\tau) \right) \\ - \sum_{x=g,y,e} \hat{\lambda}_x(\tau) \hat{c}_x(\tau) \\ = \sum_{\tau=k}^{k+T} \sum_{i=1}^n \max \left(\hat{V}_i(\tau) - \hat{\lambda}_g(\tau) \hat{g}_i(\tau) \right. \\ \left. - \hat{\lambda}_y(\tau) \hat{y}_i(\tau) - \hat{\lambda}_e(\tau) \hat{e}_i(\tau) \right) \\ - \sum_{x=g,y,e} \hat{\lambda}_x(\tau) \hat{c}_x(\tau). \end{aligned}$$

The dual function of problem (10) is therefore defined by

$$\hat{m}(\hat{\lambda}_g, \hat{\lambda}_y, \hat{\lambda}_e) = \sup L(\hat{\mathbf{g}}(\tau), \hat{\mathbf{y}}(\tau), \hat{\mathbf{e}}(\tau), \hat{\lambda}_g, \hat{\lambda}_y, \hat{\lambda}_e). \quad (14)$$

Hence, we obtain the dual of the problem (10) specified by

$$\begin{aligned} \text{minimize} \quad & \hat{m}(\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)) \\ \text{subject to} \quad & \hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau) \geq 0. \end{aligned} \quad (15)$$

Remark 4: We can interpret the projected dual variables $\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)$ as the distribution charges for energy transport and system services utilized by PtG facilities. The charges experienced by DSOs fluctuate to ensure that the aggregated supply levels from the PtG facilities are not exceeding the grid capacities [37].

The problem stated in (15) is convex. As the projected dual variables $\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau) \geq 0$, any feasible solutions of $\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau) \geq 0$ provide upper bounds of the optimal values of problem (10), given by $\hat{m}(\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)) \geq \sum_{i=1}^n \hat{V}_i(\tau)$, for any feasible $\hat{\mathbf{g}}(\tau), \hat{\mathbf{y}}(\tau), \hat{\mathbf{e}}(\tau)$ and any $\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)$. Additionally, it is confirmed in [38] that if the primal problem is a strictly concave utility function and all constraints are linear, the duality gap does not appear and the dual variables exist. In order to solve problem (15), we use the gradient method for updating $\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)$. At every time τ , the dual variables are initially set at non-negative values,

Fig. 2. Proposed interactions among PtG facilities and multiple energy DSOs at each iteration r_g, r_y, r_e within a time step τ . Given distribution charges $\hat{\lambda}_g^{r_g}(\tau), \hat{\lambda}_y^{r_y}(\tau), \hat{\lambda}_e^{r_e}(\tau) \geq 0$, PtG facilities exchange their supply bids $\hat{g}_i^{r_g}(\tau), \hat{y}_i^{r_y}(\tau), \hat{e}_i^{r_e}(\tau)$ to the energy DSOs. The DSOs then modify their distribution charges based on the aggregated supply levels from all PtG facilities.

i.e., $\hat{\lambda}_g^{r_g=0}(\tau), \hat{\lambda}_y^{r_y=0}(\tau), \hat{\lambda}_e^{r_e=0}(\tau) \geq 0$ and are updated by

$$\hat{\lambda}_x^{r_x+1}(\tau) = \hat{\lambda}_x^{r_x}(\tau) + \alpha_x^{r_x}(\tau) \left(\sum_i^N \hat{x}_i^{r_x}(\tau) - \hat{c}_x(\tau) \right) \quad (16)$$

for all $x = g, y, e$, where r_x is an iteration index within each time step τ , whereas $\alpha_x^{r_x}(\tau)$ is time-variant step size for each iteration r_x . The iterations are terminated within a time step τ when the consecutive updates of $\hat{\lambda}_x^{r_x}(\tau)$ stay within the bounds ξ_x and when the estimates of the grid capacities (6) are met.

Remark 5: The updates (16) are assumed to be done synchronously, i.e., the supply level and distribution charge are iteratively calculated based on updated information at each iteration r_g, r_y, r_e within a time τ . The corresponding algorithm is sketched in Algorithm 1 and illustrated in Fig. 2.

Bidirectional communication among the DSOs and the PtG facilities is captured in the proposed algorithm. The algorithm is distributed and applicable for any non-negative distribution charge initialization. The algorithm will first help the PtG facilities to calculate the optimal supply level for a grid which has the most appealing selling price. The optimal supply levels from the PtG facilities which are injected to a particular grid will then affect the optimal supply levels injected to other energy grids, as the produced hydrogen $p_i(k)$ and the stored energy $z_i(k)$ to supply energy to all energy grids are limited.

Notice that given $\hat{\lambda}_g^{r_g}(\tau), \hat{\lambda}_y^{r_y}(\tau), \hat{\lambda}_e^{r_e}(\tau) \geq 0$, the profit maximization of PtG facility i is locally solved, i.e.,

$$L_i = \max_{\tau=k}^{K+T} \hat{V}_i(\tau) - \hat{\lambda}_g^{r_g} \hat{g}_i^{r_g}(\tau) - \hat{\lambda}_y^{r_y} \hat{y}_i^{r_y}(\tau) - \hat{\lambda}_e^{r_e} \hat{e}_i^{r_e}(\tau) \quad (17)$$

subject to the prediction model of constraints (1)-(5) and (11)-(12).

V. FAIRNESS AND PRACTICAL IMPLEMENTATIONS

Inspired by [21], we study the fairness of the proposed algorithm. We also discuss the practical implementation of Algorithm 1 in this section.

A. Fairness

Recall L_i in (17) representing the realistic profit which is obtained by PtG facility i , given $\hat{\lambda} = [\hat{\lambda}_g^{r_g}; \hat{\lambda}_y^{r_y}; \hat{\lambda}_e^{r_e}]$. Recall the assumption on the profit function $\hat{V}_i(\tau)$ stated in

Algorithm 1: Distributed Supply Coordination

Result: Find $g_i(k), y_i(k), e_i(k), u_{t,i}(k), u_{s,i}(k)$ at each time k of the DMPC scheme.

initialize $\hat{\lambda}_g^{r_g=0}(\tau), \hat{\lambda}_y^{r_y=0}(\tau), \hat{\lambda}_e^{r_e=0}(\tau), \hat{z}_i(\tau) \geq 0$;

for $\tau = k, \dots, K + T$ **do**

while all termination criterion are not met **do**

 Given the updated distribution charges

$\hat{\lambda}_g^{r_g}(\tau), \hat{\lambda}_y^{r_y}(\tau), \hat{\lambda}_e^{r_e}(\tau)$, PtG facility $i = 1, \dots, n$ do their local optimization:

$$\max \hat{V}_i(\tau) - \hat{\lambda}_g^{r_g} \hat{g}_i(\tau) - \hat{\lambda}_y^{r_y} \hat{y}_i(\tau) - \hat{\lambda}_e^{r_e} \hat{e}_i(\tau)$$

s.t. the prediction model of constraints (1)-(5) and (11)-(12).

PtG facilities publish their supply bids

$\hat{g}_i^{r_g}(\tau), \hat{y}_i^{r_y}(\tau), \hat{e}_i^{r_e}(\tau)$ to the associated DSOs.

Given the updated supply bids, DSOs update the distribution charges $\hat{\lambda}_g^{r_g}(\tau), \hat{\lambda}_y^{r_y}(\tau), \hat{\lambda}_e^{r_e}(\tau)$ using according to (16).

DSOs share their distribution charges to PtG facilities.

end

end

implement $g_i, y_i, e_i, u_{t,i}, u_{s,i}$ only for $\tau = k$

calculate $z_i(k), f_i(k)$ and the realistic profit $L_i(k)$

Assumption 1. We assume that its realistic profit $\hat{L}_i$ is also increasing and strictly concave on the interval of the feasible sets to ensure that the PtG facilities supply energy to the grids when it is beneficial.

Let $\hat{h}_i = [\hat{g}_i^*, \hat{y}_i^*, \hat{e}_i^*]$ be the unique maximizer of $\hat{L}_i$. As well known, the Karush-Kush-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality of a constrained convex optimization problem [22]. Based on the KKT conditions, a unique maximizer $\hat{h}_i$ exists where $\frac{\partial \hat{L}_i}{\partial \hat{h}_i} = 0$. More formally, $\hat{h}_i(\hat{\lambda}) = \arg \max \{\hat{L}_i\}$.

Fairness is related to allocating the resource in a way that takes into account the individual capability of each PtG facility to satisfy the demand of the energy grids. Our definition comes from [21]. When we constrain $\hat{V}(\tau)$ stated in (10) to be logarithmic functions, we obtain the so-called proportional fairness which is introduced in [20].

In our work, DSOs provide uniform distribution charge for all PtG facilities connected to their grids. When two PtG facilities a and b have the same parameter values involved in their revenue function (7) and in their cost function (9), they will have equal supply level to be injected to the same grids, i.e., $\hat{h}_a(\hat{\lambda}) = \hat{h}_b(\hat{\lambda})$. Obviously, PtG facilities need to ensure that they have sufficient available hydrogen in their systems, i.e., $\hat{z}_i(\tau)$ and $\hat{p}_i(\tau)$.

The PtG facilities may however react differently to the uniform distribution charge. It is due to the fact that each PtG facility may have different costs associated with producing and transporting some amount of energy to each energy grid and may have the capability to inject different amount of energy than others.

Remark 6: In practice, DSO x may provide a different distribution charge $\hat{\lambda}_{x,i}(\tau)$ to each PtG facility i . For a simple case, let us define $\hat{\lambda}_x(\tau) = \sum_{i=1}^n \hat{\lambda}_{x,i}(\tau)$ for all $x = g, y, e$. Within our problem setting, we still obtain unique solutions for the profit maximization of each PtG facility i as stated in (17), given any value $\hat{\lambda}_{x,i}(\tau) \geq 0$ for all $x = g, y, e$. The solutions jointly solve the global maximization problem (10). The optimality conditions can be obtained by using Karush-Kuhn-Tucker (KKT) conditions [22]. They are summarized in the following statement.

Given the initial values $\hat{\lambda}_{g,i}^{r_g=0}, \hat{\lambda}_{y,i}^{r_y=0}, \hat{\lambda}_{e,i}^{r_e=0} \geq 0$ which may be different for each PtG facility i , the unique solutions for the supply levels $\hat{g}_i^*, \hat{y}_i^*, \hat{e}_i^*$ must satisfy the KKT conditions:

$$\hat{V}'_x \left(\sum_{i=1}^n \hat{x}_i^* \right) = \sum_{i=1}^n \hat{\lambda}_{x,i}^* \quad (18)$$

$$\sum_{i=1}^n \hat{\lambda}_{x,i}^* \left(\sum_{i=1}^n \hat{x}_i^* - c_x \right) = 0, \quad (19)$$

for all $x = g, y, e$.

An arising question is then how DSOs determine such diverse distribution charges such that they are fair for each PtG facility i . The proportional fairness as proposed in [20] may help to do so. It however results in different implementations, since we calculate the optimal solutions $\hat{x}_i^*$ for each PtG facility i in a different way as in [20].

B. Practical Implementations

The dual problem (15) is equivalent to

$$\min_{\hat{\lambda}_g} \min_{\hat{\lambda}_y} \min_{\hat{\lambda}_e} \hat{m}(\hat{\lambda}_g(\tau), \hat{\lambda}_y(\tau), \hat{\lambda}_e(\tau)) \quad (20)$$

Within each time cycle of the DMPC scheme, the sequence of the minimization problems initially depends on the estimated selling price and demand level of the energy grids. If the gas grid has the most appealing expected selling price and demand level among others, $\min_{\hat{\lambda}_g}$ will be first solved. It is due to the fact that each PtG facility aims at maximizing its projected profit.

As a consequence, the total energy supply from all PtG facilities exceeds the capacity of the gas grid. Therefore, the algorithm results in the increase of $\hat{\lambda}_g^{r_g}(\tau)$, resulting in a

Fig. 3. Excess power output of a wind farm for typical five days in the beginning of July 2012. The wind farm was in Groningen, the Netherlands [44].

decrease of $\hat{g}_i^{r_g}(\tau)$. When the increase of $\hat{\lambda}_g^{r_g}(\tau)$ makes PtG facilities unwilling to supply their energy to the corresponding grid, they start to offer their energy to another grid which has the second highest estimated selling price and demand levels, and so on. If the predicted grid constraints (6) are met and the consecutive updates of distribution charges are within bounds ξ_g , the gas grid DSO will stop iterations by publishing the optimal distribution charge $\hat{\lambda}_g^*(\tau)$ to all PtG facilities. The optimal supply levels from the PtG facilities which are injected to the gas grid will affect the optimal supply levels injected to other energy grids as the produced hydrogen $p_i(k)$ and the stored energy $z_i(k)$ to supply energy to all energy grids are limited.

VI. SIMULATION RESULTS

In this section, we first perform numerical testings of our proposed distributed algorithm presented in Section IV. All technical constraints of PtG facilities in Section II-A and the grid capacity constraints in Section II-B are taken into account when we solve the optimization problem stated in Section III in a distributed manner. In addition, the performance of the distributed algorithm is evaluated by applying it for different initial distribution charges and various number of PtG facilities embedded in the energy grids.

We consider three renewable energy generators equipped with the PtG facilities. We assume that they are embedded in the same gas grid, the same mobility sector, and the same power grid. We use realistic electricity load patterns from [39], the capacity of a conventional power generator from [40], and the capacity of installed wind turbines from [41] to provide the excess power pattern. The excess power output pattern is shown in Fig. 3, representing five typical days in the beginning of July 2012. This time horizon is divided into 15-minute samples, resulting in 480 samples.

We implement a prediction horizon of $T = 4$, which means that each PtG facility can take the estimates of the selling price signals and the demand patterns of the next one hour into account. The selling price signals and demand patterns are gathered from [42] and presented in Fig. 4 and Fig. 5.

The parameter values involved in the revenue function (7) and the cost function (9) are given in Table I, as defined in [44]. We set the time-variant step sizes below, inspired by a non-summable diminishing step size proposed in [27], i.e., $\alpha_x^r(k) = \frac{p_x(k) \cdot \alpha}{\sqrt{r_x}}$ for all $x = g, y, e$, with $\alpha = 5e-5$ at iteration numbers r_g, r_y, r_e . We include the selling price

Fig. 4. Selling price patterns of the low-pressure gas grid, the mobility sector, and the power grid [42].

Fig. 5. Demand patterns of the low-pressure gas grid, the mobility sector, and the power grid [43].

TABLE I
THE PARAMETER VALUES IN THE OBJECTIVE FUNCTION [44]

Parameter	Value	Parameter	Value	Parameter	Value
c_g	0.189	c_{qg}	1e-4	c_{op}	2.76e-5
c_y	0.224	c_{qy}	1e-4	α	55e-5
c_e	0.01	c_{qe}	1e-5	c_f	1.0e-4
η_i	1	η_s	1	S	500

signals in these dynamic step sizes to obtain faster convergences when the selling price signals increase. We define the bounds ξ_g, ξ_y, ξ_e at $1e-6$ for each time τ to terminate the iterations. A QP-solver from Gurobi 5.6.3 with YALMIP embedded in MATLAB 2014a is used to find the optimal solutions.

The initial distribution charges are set at zero. Given data shown in Fig. 3, Fig. 4 and Fig. 5, the whole demand on the gas grid and the mobility sector, which is represented as their grid capacities, are fully satisfied. In contrast, there is unsatisfied demand on the power grid, as depicted in Fig. 6. It is natural since the selling price of the power grid presented in Fig. 4 is the lowest one, whereas its demand shown in Fig. 5 is considerably higher than that of other energy grids.

Fig. 6. Unsatisfied demand in the power grid, with given excess power output in Fig. 3, the selling price pattern in Fig. 4, and the demand pattern in Fig. 5.

Fig. 7. Optimal distribution charges.

Fig. 7 shows the optimal distribution charges during $k = 1, \dots, 480$, generated by our proposed algorithm. We see that the distribution charge in the mobility grid $\lambda_y(k)$, green line, has the highest level. It is due the fact that the selling price of the mobility sector is the most appealing price, that makes all PtG facilities aim at satisfying as much demand as possible in the mobility sector. The associated distribution charge is therefore sufficiently increased from zero level to ensure that the corresponding grid constraint is obeyed. It is shown as well in Fig. 7 that the distribution charge in the gas grid $\lambda_g(k)$, black line, varies periodically per day. This phenomenon corresponds to the fact that the selling price on the gas grid also varies periodically each day, as respectively depicted in Fig. 5. It can be concluded from Fig. 7 that the higher selling price and demand level of an energy grid compared to others, the higher distribution charge is needed.

By comparing Fig. 7 next to Fig. 6, we see that when there is unsatisfied demand on the power grid, the distribution charge in the gas grid $\lambda_g(k)$, black line, and the charge in the mobility sector $\lambda_y(k)$, green line, intensely decrease. For instance, on the interval $k = 233, \dots, 287$ the unsatisfied demands on the power grid are up to 447 kNm^3 , resulting in intensely decrease of $\lambda_g(k)$ and $\lambda_y(k)$ up to zero level. It may interpret a reaction of the DSOs of the gas grid and the mobility sector on the current situation of another energy market, i.e., power grid.

The evolutions of the distribution charges and the aggregated supply levels on the gas grid at time $k = 1$ with two different initial values of the distribution charges are shown

Fig. 8. The evolution of distribution charge and aggregated supply bids of 3 PtG facilities when the initial distribution charge is zero.

Fig. 9. The evolution of distribution charge and aggregated supply bids of 3 PtG facilities when the distribution charge is initially set at 0.2 €/Nm³.

in Fig. 8 and Fig. 9. We set the capacity of the gas grid at 28 Nm³. As shown in Fig. 8, the zero initial distribution charge makes all PtG facilities aim at injecting their hydrogen to the gas grid hence the overloading grid is detected by the associated DSO. The DSO then increases its distribution charge thus inducing PtG facilities to reduce their supply level. In contrast, the initial distribution charge set at 0.2 €/Nm³ makes the PtG facilities unwilling to inject their energy to the gas grid. It is shown in Fig. 9. As a result, the corresponding DSO decreases its distribution charge with zero as a lower bound.

A. Performance Evaluation

In this subsection, we address the issues related to the initialization of distribution charges and the scalability of the algorithm. *Firstly*, as shown in Table II, for 3 facilities we simulated two different initial values of distribution charges, i.e., zero initial distribution charges which are denoted by $\lambda^{r=0}(k) = 0$ and the initializations based on the outcome of the previous time step which are defined by $\lambda^{r=0}(k) = \lambda^*(k-1)$. As can be expected, the latter initialization is better than just crudely taking zero as the initial distribution charge. *Secondly*, using the initialization based on the previous time step, we study scalability towards 3, 30, and 50 PtG facilities. In our simulation, the mobility industry has the highest selling price

TABLE II
THE AVERAGE NUMBER OF ITERATIONS (r) WITH DIFFERENT
INITIAL DISTRIBUTION CHARGES $\lambda^{r=0}$ AND DIVERSE
NUMBER OF PtG FACILITIES n

	$\lambda^{r=0}(k), n = 3$		$n, \lambda^{r=0}(k) = \lambda^*(k-1)$	
	0	prev	30	50
Gas DSO	36	17	39	86
Industry DSO	19	8	75	173
Power DSO	2	2	2	2

of hydrogen. More iterations are needed when the number of PtG facilities embedded in the mobility industry is getting larger. Conversely, the mean number of iterations needed in the power grid remains the same and is the lowest number ($r = 2$), as the power grid has the lowest selling price compared to others. The energy demand in the power grid will therefore be fulfilled by PtG facilities after all demand in the gas grid and the industry sector are satisfied. The number of iteration thus provides some insight on how difficult the bidding process between PtG facilities and the energy grid operators is. A larger number of iteration mean that the time it takes to compute the optimal solutions is longer, and hence it affects the size of the time steps that can be taken. For implementation, with many facilities there is therefore a trade-off between the time steps we take and the number of PtG facilities embedded in the energy grids.

VII. DISCUSSION

In this work, we have presented how supply from PtG facilities equipped with hydrogen storage devices and fuel cells can be coordinated, such that their technical constraints and the grid constraints are obeyed when maximizing their expected profit. We have proposed an optimal distribution pricing algorithm in Section IV in order to do the coordination in a distributed manner. Two-layer optimization problems are involved in the algorithm. The low level consists of the individual optimization problem (17) solved to maximize PtG facilities' profit, whereas the high level is due to the requisite in updating the distribution charges to ensure that the grid constraints are met. The distributed algorithm proposed in Section IV caters to the distributed nature of the energy generators in a community.

Our finding that the DSO of an energy grid will immediately react on the current situation of other energy grids is supported by our numerical testings in Section VI-A. Our finding that a good initialization of the distribution charges at every optimization can reduce the computational load is confirmed by our observations presented in Section V-B. In general, the distribution charge intensely decreases, i.e., up to zero level, when large amounts of demand on other energy grids are not satisfied. Conversely, the distribution charge increases when overloading grid is detected.

We terminate the iterations of distribution charges when the predicted grid capacity constraints (6) are met and when the consecutive updates of $\lambda_g^r(\tau)$, $\lambda_y^r(\tau)$, $\lambda_e^r(\tau)$ stay within sufficiently small bounds ξ_g , ξ_y , ξ_e . With these settings, the proposed algorithm has fairly slow convergence rate.

Alternatively, we can set the maximum allowable number of iterations to terminate the iterations while sub-optimality and stability are still guaranteed, as proposed in [10] and [13].

As stated in Remark 5, PtG facilities may not have a common clock to synchronize their updates. Their communication may have time-varying delays. As a consequence, they may optimize their problems based on outdated information. It is therefore of interest for future work to involve this asynchronous scheme in the distributed supply coordination.

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