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# Duality symmetry in four-dimensional string actions

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## Abstract

We reduce the dual version of  $D = 10$ ,  $N = 1$  supergravity coupled to  $n$  vector fields to four dimensions, and derive the  $SL(2, R) \times O(6, 6 + n)$  transformations which leave the equations of motion invariant. For  $n = 0$   $SL(2, R)$  is also a symmetry of the action, but for  $n > 0$  only those  $SL(2, R)$  transformations which act linearly on all fields leave the action invariant. The resulting four-dimensional theory is related to the bosonic part of the usual formulation of  $N = 4$  supergravity coupled to matter by a duality transformation.

## 1. Introduction

The symmetries of the low energy effective string action in four dimensions have received much attention recently because of their relation with  $S$ - and  $T$ -duality<sup>1</sup>. The toroidal compactification of the heterotic string can be studied by considering the reduction to four dimensions of  $D = 10$ ,  $N = 1$  supergravity coupled to  $n = 16^2$  abelian vector fields. This revealed an  $O(6, 6 + n)$  symmetry (related to  $T$ -duality) of the action [3], as well as an  $SL(2, R)$  symmetry (related to  $S$ -duality) of the equations of motion [4]. Symmetries of the equations of motion of abelian vector fields involve a linear transformation of the Bianchi identity,  $\partial_\mu i\tilde{F}^{\mu\nu}$  ( $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$ ), and the equation of motion,  $\partial_\mu G^{\mu\nu}$ , where  $G \equiv -2\partial\mathcal{L}/\partial F$ . The linear transformation is of the form:

$$\begin{pmatrix} i\tilde{F} \\ G \end{pmatrix}' = \omega \begin{pmatrix} i\tilde{F} \\ G \end{pmatrix}. \quad (1)$$

The linear transformation must preserve the form of  $G$  as obtained from the action, which restricts the possible  $\omega$ . For a single vector field the group of transformations can be  $Sp(2, R) \simeq SL(2, R)$ , if the vector field couples suitably to scalar fields which are also required to transform under  $SL(2, R)$  [5]. In the absence of scalar fields the group of transformations is  $O(2)$ , which transforms electric and magnetic fields into each other.

The reduction of the *dual* version of  $D = 10$  supergravity has not been studied in as much detail from this point of view, in particular in the presence of additional (non-)abelian vector fields. In the dual version the

<sup>1</sup> For recent reviews, see, e.g. [1,2].

<sup>2</sup> In the following, we will not restrict ourselves to this value of  $n$ .

two-index tensor gauge field is replaced by a six-index antisymmetric tensor gauge field. This is done by a standard duality transformation, which in  $D$  dimensions replaces an  $m$ -form gauge field by a  $D - 2 - m$ -form gauge field. In this paper we will discuss the symmetries of the four-dimensional theory which is obtained by reduction of this dual version to four dimensions.

Our interest in this problem was raised by the work of Schwarz and Sen [6], who established that in the absence of additional vector fields in ten dimensions the resulting four-dimensional theory exhibits an  $SL(2, R)$  symmetry of the action. This result seems surprising from the point of view of  $D = 4$ ,  $N = 4$  supergravity theories, in which  $SL(2, R)$  appears only as a symmetry of the equations of motion<sup>3</sup>. On the other hand, having  $SL(2, R)$  as a symmetry of the action fits well in view of the conjectured string-fivebrane duality in ten dimensions [9,10]. String-fivebrane duality is thought to interchange  $T$ - and  $S$ -duality [6], and therefore the fivebrane effective action is expected to have (in  $D = 4$ ) an  $SL(2, R)$  symmetry. The assumption that the fivebrane effective action is built on the dual version of  $D = 10$  supergravity, then leads to the  $SL(2, R)$  symmetry of the  $D = 4$  action.

To see whether or not the  $SL(2, R)$  symmetry of the action survives the coupling to (abelian) gauge fields, we will work out the reduction to four dimensions in the presence of vector fields, determine the resulting symmetries of the action and equations of motion, and elucidate the relation between this result and  $N = 4$  supergravity theories. Our main result is that for  $n > 0$   $SL(2, R)$  becomes a symmetry of the equations of motion only, together with  $O(6, 6 + n)$ .

The dual version of ten-dimensional supergravity, including its matter coupling and the reduction to four dimensions, was first discussed by Chamseddine [11]. The emphasis in this work was on properties of the resulting scalar potential, in view of possible supersymmetry breaking. The reduction of the bosonic part of the ten-dimensional action *without* additional vector fields was done in [6]. We give the reduction *with* vector fields in Section 2, and obtain the symmetries of the resulting action and equations of motion in Section 3. We draw some conclusions on the relation with  $N = 4$  supergravity in Section 4.

## 2. Reduction from $D = 10$ to $D = 4$

The bosonic part of the ten-dimensional action in the notation and conventions of [12] is

$$\mathcal{L}_{D=10} = \sqrt{|G|} \Phi^{-3} \left\{ -\frac{1}{2} \mathbf{R}(G) + \frac{3}{4} \mathbf{H}_{ABC} \mathbf{H}^{ABC} + \frac{9}{2} (\Phi^{-1} \partial_M \Phi)^2 + \beta \text{tr} \left( -\frac{1}{4} \mathbf{F}^{MN} \mathbf{F}_{MN} \right) \right\} \\ - \frac{1}{4} i \beta \sqrt{2} \varepsilon^{M_1 \dots M_{10}} A_{M_1 \dots M_6} \text{tr} (F_{M_7 M_8} F_{M_9 M_{10}}), \quad (2)$$

where the field strength of the six-form gauge field  $A$  is written in the form

$$\mathbf{H}_{ABC} = \frac{2}{3} i \sqrt{|G|} \Phi^3 \varepsilon^{M_1 \dots M_7} {}_{ABC} \mathbf{R}(A)_{M_1 \dots M_7}, \quad (3)$$

$$\mathbf{R}(A)_{M_1 \dots M_7} \equiv \partial_{[M_1} A_{M_2 \dots M_7]}. \quad (4)$$

The symbol  $\text{tr}$  denotes the trace in the adjoint representation, and  $\beta$  is the inverse squared of the Yang-Mills coupling constant. We use bold face capital letters to denote ten-dimensional fields. Our index conventions are as follows:  $A, B, \dots$  are 10d flat indices,  $M, N, \dots$  are 10d curved indices,  $m, n, \dots$  are 6d curved indices, and  $\mu, \nu, \dots$  are 4d curved indices.

For the reduction of (2) to  $D = 4$  we use the method of [13], which was applied to this case (without vector fields) in [6]. The action has been divided in four parts, the Einstein-Hilbert action, the scalar kinetic part, the vector part and the scalar potential terms:

<sup>3</sup> In  $D = 4$ ,  $N = 4$  supergravity coupled to  $n$  abelian vector multiplets [7]  $O(6, 6 + n)$  is a symmetry of the action, and  $SU(1, 1)$  (isomorphic to  $SL(2, R)$ ) a symmetry of the equations of motion.

Table 1

The four-dimensional fields and their ten-dimensional origin. The third column represents the number of degrees of freedom

	$D = 10$	$D = 4$	# d.o.f	redefined to:
$G$		$g_{\mu\nu}$	2	
		$A_\mu^m$	12	
		$G_{mn}$	21	
$A$		$A^{m_1 \dots m_6}$	1	$\lambda_1$
		$A_\mu^{m_1 \dots m_5}$	12	$B_\mu^m$
		$A_{\mu_1 \dots \mu_4}^{m_1}$	15	$B_{mn}$
		$A_{\mu_1 \dots \mu_3}^{m_1 m_2}$	0	–
		$A_{\mu_1 \dots \mu_4}^{m_1 m_2}$	0	–
$\Phi$		$\lambda_2$	1	
$V$		$V_\mu$	2 dim $G$	
		$V_m$	6 dim $G$	

$$\mathcal{L}_{D=4} = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_P, \quad (5)$$

$$\mathcal{L}_G = -\frac{1}{2} \sqrt{|g|} R(g), \quad (6)$$

$$\begin{aligned} \mathcal{L}_S = & -\sqrt{|g|} G^{mp} G^{nq} \left\{ \frac{1}{8} (\partial_\mu G_{mn}) (\partial^\mu G_{pq}) + \frac{1}{8} (\partial_\mu B_{mn}) (\partial^\mu B_{pq}) \right. \\ & + \frac{\beta}{2} (\partial_\mu B_{mn}) \text{tr} (V_{[p} D^\mu V_{q]}) + \frac{\beta^2}{2} \text{tr} (V_{[m} D_\mu V_{n]}) \text{tr} (V_{[p} D^\mu V_{q]}) \left. \right\} \\ & - \frac{\beta}{2} \sqrt{|g|} G^{mn} \text{tr} (D_\mu V_m D^\mu V_n) - \frac{1}{8} \sqrt{|g|} \text{Tr} [(\partial_\mu M) L (\partial^\mu M) L], \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{L}_V = & -\frac{1}{8} \sqrt{|g|} G_{mn} \mathcal{F}_{\mu\nu}^m T L^T M L \mathcal{F}^{\mu\nu n} - \frac{i}{16} \varepsilon^{\mu\nu\rho\sigma} B_{mn} \mathcal{F}_{\mu\nu}^m T L \mathcal{F}_{\rho\sigma}^n \\ & + \beta \text{tr} \left\{ -\frac{1}{4} \sqrt{|g|} \lambda_2 (F_{\mu\nu}(V) + F_{\mu\nu}^m(A) V_m)^2 - \frac{i}{8} \varepsilon^{\mu\nu\rho\sigma} \lambda_1 F_{\mu\nu}(V) F_{\rho\sigma}(V) \right. \\ & \left. - \frac{i}{8} \varepsilon^{\mu\nu\rho\sigma} (\lambda_1 F_{\mu\nu}^m(A) - F_{\mu\nu}^m(B)) (F_{\rho\sigma}^n(A) V_m V_n + 2F_{\rho\sigma}(V) V_m) \right\}, \end{aligned} \quad (8)$$

$$\mathcal{L}_P = -\beta \sqrt{|g|} \lambda_2^{-1} G^{mp} G^{nq} \left\{ \frac{1}{4} \text{tr} ([V_m, V_n] [V_p, V_q]) + \frac{\beta}{6} G^{rs} \text{tr} ([V_m, V_n] V_r) \text{tr} ([V_p, V_q] V_s) \right\}. \quad (9)$$

The origin of the four-dimensional fields is represented in Table 1. Note that the redefinition of  $A_{\mu_1 \dots \mu_2}^{m_1 \dots m_4}$  to the scalars  $B_{mn}$  involves a duality transformation. Of the other fields that arise from  $A$ , the four-form field is completely absent from the four-dimensional action, while for the three-form field one can solve the equation of motion, which gives rise to the second term in the scalar potential  $\mathcal{L}_P$  [11].

The dilaton degree of freedom  $\lambda_2$  is contained in  $\sqrt{G_{mn}} \Phi^{-3}$ . To obtain the form of the action (5)–(9) a Weyl rescaling of the metric has been performed:  $g_{\mu\nu} \rightarrow \lambda_2^{-1} g_{\mu\nu}$ .  $D_\mu$  is the Yang-Mills covariant derivative.

The kinetic terms of  $A_\mu^m$  and  $B_\mu^m$  in (8) have been written in terms of the doublet

$$\mathcal{F}_{\mu\nu}^m \equiv \begin{pmatrix} F_{\mu\nu}^m(A) \\ F_{\mu\nu}^m(B) \end{pmatrix}. \quad (10)$$

In the absence of Yang-Mills fields ( $\beta = 0$ ), the action is invariant under the  $SL(2, R)$  transformations<sup>4</sup> [6]

$$\mathcal{F} \rightarrow \omega \mathcal{F}, \quad M \rightarrow \omega M \omega^T, \quad \omega^T L \omega = L, \quad (11)$$

<sup>4</sup>Of course, the  $SL(2, R)$  symmetry is also present in the action before the Weyl-rescaling, the redefinition leading to  $B_{mn}$ , and the elimination of  $A_{\mu_1 \dots \mu_3}^{m_1 \dots m_3}$ .

with the  $2 \times 2$  scalar matrix  $M$  and  $SL(2, R)$  invariant metric  $L$  defined by

$$M = \frac{1}{\lambda_2} \begin{pmatrix} 1 & \lambda_1 \\ \lambda_1 & \lambda_1^2 + \lambda_2^2 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (12)$$

Note that the transformation of  $M$  implies that  $\lambda \equiv \lambda_1 + i\lambda_2$  transforms under  $SL(2, R)$  as:

$$\lambda \rightarrow \frac{c + d\lambda}{a + b\lambda}, \quad \omega = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad ad - bc = 1. \quad (13)$$

In  $D = 4$  the pure  $N = 4$  supergravity multiplet has 16, and an abelian vector multiplet 8 bosonic degrees of freedom. The reduction of  $G$ ,  $A$  and  $\Phi$  therefore gives the bosonic degrees of freedom of  $N = 4$  supergravity coupled to six abelian vector multiplets. Each  $V$  in  $D = 10$  gives an additional vector multiplet in  $D = 4$ .

The reduction of the two-index formulation of the  $D = 10$  effective action involves the two-form field  $B$  in place of the field  $A$ . The result in  $D = 4$  involves scalars  $B_{mn}$ , vectors  $B_\mu^m$ , and the four-dimensional two-form field  $B_{\mu\nu}$ . The last gives only one degree of freedom in  $D = 4$ , and can be replaced by a single scalar field by means of a duality transformation. This gives an action  $\mathcal{L}'_{D=4}$ , which has the same bosonic field content as the  $N = 4$  supergravity theories of [7].

By performing these duality transformations in  $D = 4$  we are essentially undoing the  $D = 10$  duality transformation that relates the two-index to the six-index version. Indeed, a last duality transformation on the vectors  $B_\mu^m$  of  $\mathcal{L}'_{D=4}$  gives us precisely the action (5).

For  $\beta = 0$ , i.e. without Yang-Mills fields, the action (5) was obtained by Schwarz and Sen [6] from an action with a doubled number of vector fields, after elimination of an  $SL(2, R)$  invariant half of them.

### 3. Invariances of the equations of motion

If  $\beta \neq 0$ , i.e. if we have extra vector fields  $V_\mu$ , it is not difficult to see that the action (5) is no longer  $SL(2, R)$  invariant. This is true even if all vector fields are abelian.

However, in the case of abelian vector fields, we know that the equations of motion still must be invariant under  $SL(2, R)$ , since this equation of motion symmetry is well-established in the original formulation of  $D = 4$   $N = 4$  supergravity coupled to abelian vector fields [7], and the equations of motion must be the same in both formulations. For non-abelian vector fields this  $SL(2, R)$  invariance of the equations of motion does not hold, and from now on we will assume that all vector fields are abelian.

The part of the action which is not  $SL(2, R)$  invariant is the part of  $\mathcal{L}_V$  containing the  $n$  abelian vector fields  $V_\mu$ . A convenient way of finding the equations of motion invariance is to introduce the doublet [5]

$$\mathcal{D}(V)_{\mu\nu} = \begin{pmatrix} -i\tilde{F}_{\mu\nu}(V) \\ G_{\mu\nu}(V) \end{pmatrix}, \quad \beta\sqrt{|g|}G^{\mu\nu}(V) \equiv 2\frac{\partial\mathcal{L}}{\partial F_{\mu\nu}(V)}, \quad (14)$$

where  $\tilde{F}_{\mu\nu}(V) = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}(V)$ . Then we can write the vector part of the action (where we ignore the invariant kinetic terms for  $A_\mu^m$  and  $B_\mu^m$ ) as

$$\mathcal{L}_V = -\beta\sqrt{|g|}\text{tr}\left\{-\frac{1}{4}G^{\mu\nu}(V)F_{\mu\nu}(V) + \frac{1}{4}\mathcal{D}(V)^{\mu\nu T}L\mathcal{F}_{\mu\nu}^m V_m\right\}. \quad (15)$$

Any linear  $SL(2, R)$  transformation of  $\mathcal{D}$  which leaves the definition of  $G(V)$  invariant is a symmetry of the equations of motion [5]. In this case, we have

$$\begin{aligned} G_{\mu\nu}(V) &= -\lambda_2 F_{\mu\nu}(V) - i\lambda_1 \tilde{F}_{\mu\nu}(V) + C_{\mu\nu}, \\ C_{\mu\nu} &= -\lambda_2 F_{\mu\nu}^m(A)V_m - i\lambda_1 \tilde{F}_{\mu\nu}^m(A)V_m + i\tilde{F}_{\mu\nu}^m(B)V_m. \end{aligned} \quad (16)$$

and there are no restrictions on the  $2 \times 2$   $SL(2, R)$  transformation matrices. Thus,  $\mathcal{D} \rightarrow \omega \mathcal{D}$ , together with the transformations (11), is an  $SL(2, R)$  invariance of the equations of motion. Note that the subgroup of  $SL(2, R)$  transformations

$$\omega = \begin{pmatrix} a & 0 \\ c & \frac{1}{a} \end{pmatrix} \quad (17)$$

is a symmetry of the action (up to a total derivative, in case of the parameter  $c$ ). This subgroup is formed by the transformations which act linearly on the scalar fields. The parameter  $a$  corresponds to a rescaling of the vector fields  $V_\mu$  and  $A_\mu$  by the factor  $a$  and of  $B_\mu$  by  $\frac{1}{a}$ , together with  $\lambda_i \rightarrow \frac{1}{a^2} \lambda_i$ ,  $i = 1, 2$ . The parameter  $c$  shifts  $\lambda_1$  to  $\lambda_1 + c$ , and also  $B_\mu$  to  $cA_\mu + B_\mu$ . The vector  $V_\mu$  is inert under this transformation.

In the same way, it is possible to find additional invariances of the equations of motion. It is well-known that in the original formulation of  $D = 4$ ,  $N = 4$  supergravity coupled to abelian vector fields, the action has an  $O(6, 6 + n)$  invariance, where  $n$  is the number of additional abelian vector fields  $V_\mu^I$ . In our dual formulation, we do not have this symmetry of the action, so it must be an invariance of the equations of motion. The scalar part  $\mathcal{L}_S$  of the action is invariant, which can be seen by introducing the symmetric  $O(6, 6 + n)$  matrix  $N$  of scalars [3],

$$N = \begin{pmatrix} G^{-1} & G^{-1}(B+W) & \sqrt{2\beta}G^{-1}V \\ (-B+W)G^{-1} & (G-B+W)G^{-1}(G+B+W) & \sqrt{2\beta}(G-B+W)G^{-1}V \\ \sqrt{2\beta}V^T G^{-1} & \sqrt{2\beta}V^T G^{-1}(G+B+W) & I_{n \times n} + 2\beta V^T G^{-1}V \end{pmatrix},$$

$$\eta = \begin{pmatrix} 0 & I_{6 \times 6} & 0 \\ I_{6 \times 6} & 0 & 0 \\ 0 & 0 & -I_{n \times n} \end{pmatrix}, \quad (18)$$

where  $\eta$  is the  $O(6, 6 + n)$  invariant metric,  $N^T \eta N = \eta$ . The symmetric  $6 \times 6$  matrix  $(W)_{mn}$  is defined by  $W = \beta V W^T$ , where  $V$  is the  $6 \times n$  matrix of scalars  $(V)_m^I = V_m^I$ . Furthermore,  $B$  is the antisymmetric matrix with components  $(B)_{mn} = B_{mn}$ , and  $G$  is still the internal metric. Then the scalar kinetic part of the action is just

$$\mathcal{L}_S = \frac{1}{16} \sqrt{|g|} \text{Tr} [(\partial_\mu N) \eta (\partial^\mu N) \eta] - \frac{1}{8} \sqrt{|g|} \text{Tr} [(\partial_\mu M) L (\partial^\mu M) L], \quad (19)$$

which is an  $O(6, 6 + n)$  invariant under

$$N \rightarrow \Omega N \Omega^T, \quad M \rightarrow M, \quad \Omega^T \eta \Omega = \eta. \quad (20)$$

The vector part of the action is not invariant. To show that the equations of motion are invariant, we write the complete vector part of the Lagrangian as

$$\mathcal{L}_V = -\frac{1}{4} \sqrt{|g|} G_a^{\mu\nu} F_{\mu\nu}^a, \quad (21)$$

where  $\sqrt{|g|} G_a^{\mu\nu} = -2 \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}^a}$ , and the sum over  $a$  extends over all vector fields  $A_\mu^m, B_\mu^m$  and  $V_\mu^I$  ( $a = 1, \dots, N$  with  $N = 12 + n$ ). From the definition of  $G$  and the Lagrangian (21), we get  $G = F \frac{\partial G}{\partial F}$  which has the general solution

$$G^a = A^a_b F^b + i C^a_b \tilde{F}^b. \quad (22)$$

The  $N \times N$  matrices  $A$  and  $C$  can be read off from the action (8), and they depend on all scalars of the theory. Now the  $O(6, 6 + n)$  equation of motion invariance must be realized on the  $2N$ -dimensional multiplet  $\mathcal{D} = (i\tilde{F}^a, G^a)$  [5]. Indeed, using the known transformation rules (20) for the scalars, we find that

the  $O(6, 6 + n)$  transformations are consistent with the definition of  $G^a$ . As expected, the  $2N$ -dimensional multiplet  $\mathcal{D}$  transforms in a reducible representation which decomposes into two fundamental representations. The resulting  $O(6, 6 + n)$  equations of motion invariance is then given by

$$\begin{pmatrix} i\tilde{F}(A) \\ 2G(B) \\ -i\tilde{F}(V) \end{pmatrix} \rightarrow \Omega \begin{pmatrix} i\tilde{F}(A) \\ 2G(B) \\ -i\tilde{F}(V) \end{pmatrix}, \quad \begin{pmatrix} i\tilde{F}(B) \\ -2G(A) \\ -2G(V) \end{pmatrix} \rightarrow \Omega \begin{pmatrix} i\tilde{F}(B) \\ -2G(A) \\ -2G(V) \end{pmatrix}, \quad (23)$$

together with the transformations (20). Finally we note that the subgroup of  $O(6, 6 + n)$  which acts linearly, or by a constant shift, on the scalars leaves the action invariant (up to a total derivative). This linear subgroup obviously contains  $GL(6) \times O(n)$ .

We have only discussed the equations of motion of the vector fields. The invariance of the other equations of motion under  $SL(2, R) \times O(6, 6 + n)$  is guaranteed by the general arguments given in [5].

#### 4. Conclusions

In the reduction to four dimensions,  $N = 1$  supergravity in  $D = 10$  gives twelve abelian vector fields in  $D = 4$ , of which six belong to the  $N = 4$  supergravity multiplet, and the other six belong to vector multiplets coupled to supergravity. In this paper only the bosonic sector of this theory was considered.

The two versions of  $D = 10$  supergravity (without additional matter) give rise to different results in  $D = 4$ : the two-index version gives  $SL(2, R)$  as a symmetry of the equations of motion,  $O(6, 6)$  as a symmetry of the action. In the six-index version however  $SL(2, R)$  is a symmetry of the action, and  $O(6, 6)$  an equation of motion symmetry. In Section 2 we found that the two actions in  $D = 4$  are related by a duality transformation.

In [8] it was explained how different versions of  $N = 4$  supergravity can be obtained by duality transformations. Basically,  $N = 4$  supergravity coupled to abelian matter contains *two*  $SU(1, 1)$  symmetries: one is an equation of motion symmetry of the vector multiplets, the other is the  $SU(1, 1)$  symmetry of the superconformal  $N = 4$  Weyl multiplet [14]. These two symmetries can be identified in the process of matter coupling, as was done in [7], but it is also possible to allow other isomorphisms between these two groups, determined by an arbitrary, constant  $SU(1, 1)$ -element  $C$ . The equations of motion remain unchanged under this modification, but the symmetries of the action do change. When (part of the) remaining symmetries of the action are gauged, inequivalent theories with local  $N = 4$  supersymmetry are obtained.

The mechanism discussed in [8] can be used to explain the difference between the two reductions from  $D = 10$ . The reduction of the two-index version from  $D = 10$  gives the result of [7] (after a duality transformation of  $B_{\mu\nu}$ ). The reduction of the six-index version corresponds to a particular choice of the  $SU(1, 1)$ -isomorphism in [8]. If the six abelian vector multiplets are coupled to supergravity with the  $SU(1, 1)$ -element (in the notation of [8]):

$$C = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \quad (24)$$

then one indeed finds the action (5). Note that the scalar fields in  $N = 4$  supergravity parametrize the coset  $SU(1, 1)/U(1) \times O(6, 6 + n)/(O(6) \times O(6 + n))$ . In [7,8] this is expressed by imposing suitable constraints on the scalar fields. It has perhaps not been sufficiently realized that dimensional reduction from  $D = 10$  provides a general, explicit solution of these constraints. This solution, and the fact that by a duality transformation a version of  $N = 4$  supergravity can be obtained which has an  $SU(1, 1)$  symmetry of the action is also derived in recent preprints of Zinoviev et al. [15] without reference to  $D = 10$ .

The fact that part of the  $SL(2, R)$  symmetry of the effective action is broken by the coupling to additional vector fields suggests that the dual version of  $D = 10$ ,  $N = 1$  supergravity coupled to matter in the standard way [11] cannot be the low-energy effective action of the fivebrane. In [6], a modification of the ten-dimensional

action including abelian vector fields was suggested which leads to an  $SL(2, R)$  invariant action in  $D = 4$ . It will be interesting to see whether imposing  $SL(2, R)$  symmetry along these or other lines is a useful tool to obtain further insight in the effective action of the fivebrane.

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