University of Groningen

Joint pricing and production decisions with yield uncertainty and downconversion

Lu, Fen; Xu, He; Chen, Pengyu; Zhu, Stuart X.

Published in:
International Journal of Production Economics

DOI:
[10.1016/j.ijpe.2017.12.020](https://doi.org/10.1016/j.ijpe.2017.12.020)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2018

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Lu, F., Xu, H., Chen, P., & Zhu, S. X. (2018). Joint pricing and production decisions with yield uncertainty and downconversion. *International Journal of Production Economics*, 197, 52-62.
<https://doi.org/10.1016/j.ijpe.2017.12.020>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Joint pricing and production decisions with yield uncertainty and downconversion

Fen Lu^a, He Xu^a, Pengyu Chen^{b,*}, Stuart X. Zhu^c

^a School of Management, Huazhong University of Science and Technology, Wuhan, China

^b School of Information Management, Central China Normal University, Wuhan, China

^c Department of Operations, Faculty of Economics and Business, University of Groningen, Nettelbosje 2, 9747 AE, Groningen, The Netherlands

ARTICLE INFO

Keywords:

Yield uncertainty
Downconversion
Pricing

ABSTRACT

We consider a firm who supplies two types of products: high-end and low-end. Because of the uncertainty in the production process, the yield rate of the high-end products is uncertain. The substandard high-end products caused by the yield uncertainty can be transformed into the low-end products with a certain cost. We characterize the optimal pricing and production decisions and develop an algorithm to compute the optimal solution. We also investigate the impact of the yield uncertainty on the firm's performance, and explore how stability of market demand, emergent fulfillment costs, and downconversion cost influence this effect. We find that (i) the profit of the firm deteriorates when the risk of the yield uncertainty is high. In the face of yield uncertainty, the firm prefers to decrease (increase) the production quantity of the high-end (low-end) products; (ii) when the market demands are quite unstable, the emergent fulfillment costs are low, or the downconversion cost is high, the firm has a low incentive to eliminate the yield uncertainty.

1. Introduction

Due to both the technological advances and market differentiation, many manufacturers choose to offer vertically differentiated¹ products to meet demands from different market segments. The production of high-end products often refers to more advanced technology, whereas that of low-end ones only uses relatively mature technology. Hence, the manufacturers often face much higher yield uncertainty when they produce high-end products. In some industries, to reduce the risk and waste caused by yield uncertainty, manufacturers may introduce the practice of “downconversion”, in which the substandard high-end products can be transformed into qualified low-end ones by an acceptable cost. For instance, in the steel industry, WISCO (Wuhan Iron and Steel Corporation), one of the top five largest steel producing companies in the world, provides HiB² and CGO³ steel for high-end and low-end silicon steel markets, respectively. The yield uncertainty of CGO steel can be ignored compared with that of HiB steel, and WISCO can downconvert defective HiB steel into usable CGO steel via some simple processes. In the semiconductor industry, Intel faces the similar situation.

The production of high-end processors (used in servers or high-end PCs) faces yield uncertainty, and high-end processors can be downconverted into low-end processors (used in mobile devices) by an acceptable cost (Kempf, 2004). Meanwhile, manufacturers often have different market powers in vertically differentiated markets. In the example of Chinese steel industry above, WISCO can determine the price in HiB steel market of China. In fact, it was the unique Chinese HiB steel producer before 2009, producing 43.04% of the total domestic produced HiB steel in 2014 (Chinese Metallurgical News, 2014). The demand for HiB steel is increasingly strong with the rapid development of mechanical and electronic industry in China. However, WISCO cannot determine the price in CGO steel market due to fierce competition in the low-end market (Wisco, 2013). Similarly, Intel is a price-maker in the market of high-end processors, while it is a price-taker in low-end processors market (King, 2015).

Motivated by the above phenomena, we intend to investigate the pricing and production strategies faced by a firm supplying different types of products and the impact of the yield uncertainty and downconversion on the firm's performance. To the best of our knowledge, the

* Corresponding author.

E-mail addresses: lufen@hust.edu.cn (F. Lu), xuhe@hust.edu.cn (H. Xu), chenpengyu@mail.ccnu.edu.cn (P. Chen), x.zhu@rug.nl (S.X. Zhu).

¹ Following many product line design works (e.g. Guo and Zhang, 2012; Kuksov and Lin, 2010), in this paper, vertically differentiated products mean a set of products, the quality of which can be uniquely ordered.

² HiB steel is a kind of high-end grain oriented silicon steel used to produce low-loss, low-noise, small-size voltage transformer for ultrahigh voltage environment.

³ CGO steel is a kind of low-end grain oriented silicon steel used to produce common transformer or welding transformer.

above-mentioned research issues have not been studied yet in literature. To close such a gap, we construct a single-period model in which a firm provides both high-end and low-end products targeting vertically differentiated markets. The firm can decide the price of its high-end products, but can only accept the price of the low-end products. The yield rate of its high-end products is uncertain, and a substandard high-end product can be downconverted into a usable low-end product by a certain cost. The demands for high-end and low-end markets are stochastic and independent. We explore the optimal joint pricing, inventory, and downconversion decisions, and analyse the impact of downconversion on the firm's optimal production quantities and profit. We also investigate the impact of the yield uncertainty on the firm's performance, and how stability of market demand, emergent fulfilment costs, and downconversion cost influence this effect.

The contribution of this paper is threefold. First, for a general model, we develop an algorithm to compute the global optimum for the price of the high-end products and the production quantities of both high and low-end products. Second, we explore the optimal product variety strategy, and obtain a sufficient and necessary condition under which the firm will only provide low-end products. In a special case of the all-or-nothing model, we give a sufficient condition under which the firm will always provide low-end products. Third, we find that (i) the profit of the firm deteriorates when the risk of the yield uncertainty is high. In the face of yield uncertainty, the firm prefers to decrease (increase) the production quantity of the high-end (low-end) products; (ii) when the market demands are quite unstable, the emergent fulfilment costs are low, or the downconversion cost is high, the firm has a low incentive to eliminate the yield uncertainty.

The rest of this paper is organized as follows. §2 surveys the existing related literature, followed by §3 on the model preliminaries and §4 on analysis of optimal solution. In §5, we analyse the effects of yield

uncertainty on the profit. §6 performs sensitivity analysis with respect to market variability and key parameters. §7 extends the model to the case of pricing delay. Finally, our concluding remarks are given in §8.

2. Literature review

Our work is closely related to two streams of literature. The first stream studies systems with multiple substitutable products facing stochastic demands. Some researchers consider that customers choose substitute products only when their first-choice items are out of stock. Among them, [Nagarajan and Rajagopalan \(2008\)](#), [Stavrulaki \(2011\)](#) and [Liu et al. \(2013\)](#) explore the strategies of production and substitution in single-period two-product models, whereas [Netessine and Rudi \(2003\)](#), [Tan and Karabati \(2013\)](#) and [Ye \(2014\)](#) explore single-period multi-product systems. In all these works, substitution between products is bi-directional (each product can be used to substitute for the other). There are also some papers that study cases of one-way substitution, in which high-end products can be used to satisfy low-end demands but not vice versa, including [Bassok et al. \(1999\)](#), [Shumsky and Zhang \(2009\)](#), and [Yu et al. \(2015\)](#). Introducing yield uncertainty, [Gerchak et al. \(1996\)](#), [Hsu and Bassok \(1999\)](#), [Han et al. \(2014\)](#), and [Tomlin and Wang \(2008\)](#) study optimal policies in coproduction systems, where multiple products are produced simultaneously in a single production run. [Inderfurth \(2004\)](#) studies decisions in hybrid manufacturing/remanufacturing systems with stochastic returns of used products. Besides them, other scholars consider that substitution may occur even when every alternative product is in stock (e.g., [Dong et al., 2009](#); [Xu et al., 2011, 2016](#); [Zhao et al., 2012](#)). Comparing to above works, there are two significant differences in our research. First, most aforementioned papers do not consider the pricing decision (see [Table 1](#)). Although [Tomlin and Wang \(2008\)](#) also consider the pricing decision, our work is significantly

Table 1
Related literature.

Research papers	Considering factors			Decision Variables	
	Substitution	Yield uncertainty	Stochastic demand	Price	Quantity
Nagarajan and Rajagopalan (2008)	✓		✓		✓
Stavrulaki (2011)	✓		✓		✓
Liu et al. (2013)	✓		✓		✓
Netessine and Rudi (2003)	✓		✓		✓
Tan and Karabati (2013)	✓		✓		✓
Ye (2014)	✓		✓		✓
Bassok et al. (1999)	✓		✓		✓
Shumsky and Zhang (2009)	✓		✓		✓
Yu et al. (2015)	✓		✓		✓
Gerchak et al. (1996)	✓	✓	✓		✓
Hsu and Bassok (1999)	✓	✓	✓		✓
Han et al. (2014)	✓	✓	✓		✓
Tomlin and Wang (2008)	✓	✓	✓	✓	✓
Inderfurth (2004)	✓	✓	✓		✓
Dong et al. (2009)	✓		✓	✓	✓
Xu et al. (2011)	✓		✓		✓
Xu et al. (2016)	✓		✓		✓
Zhao et al. (2012)	✓		✓	✓	✓
Federgruen and Heching (1999)			✓	✓	✓
Chan et al. (2004)			✓	✓	✓
Yano and Gilbert (2004)			✓	✓	✓
Chan et al. (2006)			✓	✓	✓
Gilbert 2000			✓	✓	✓
Zhu and Thonemann (2009)	✓		✓	✓	✓
Cohen (1977)			✓	✓	✓
Rajan et al. (1992)			✓	✓	✓
Zhao and Wang (2002)			✓	✓	✓
Wang (2006)			✓	✓	✓
Li et al. (2015)			✓	✓	✓
Tang et al. (2004)			✓	✓	✓
Tereyaoglu and Veeraraghavan (2012)			✓	✓	✓
Li and Zheng (2006)		✓	✓	✓	✓
Zhu (2012)		✓	✓	✓	✓
Our paper	✓	✓	✓	✓	✓

different from theirs in the following three aspects. First, Tomlin and Wang (2008) use a utility-maximizing demand model to stimulate customer's purchase behavior in the market, while we use a general demand function plus a random forecast error to model the uncertain demand from a more macro standpoint, which is common in the operations management literature (e.g. Li and Zheng, 2006). Second, in our model, the high-end and low-end products are produced by two different processes, while Tomlin and Wang (2008) consider a co-production system, where multiple products are produced simultaneously in a single production run. Third, in our model, “downconversion” refers to the practice of converting a substandard high-end product into a usable low-end one by an acceptable cost, whereas in Tomlin and Wang (2008), downconversion means converting a useable high-end product into a low-end one by some certain cost.

The other relevant stream of research seeks optimal joint pricing and production decisions. A significant amount of research has been done in this area since 1950s, and readers can refer to Petruzzi and Dada (1999) for a comprehensive review. Price determination and restocking in a multi-period setting have been explored by many scholars, including Federgruen and Heching (1999), Chan et al. (2004), Yano and Gilbert (2004), and Chan et al. (2006), etc. Gilbert (2000) and Zhu and Thonemann (2009) extend previous works into a two-product case. Cohen (1977) first analyzes the interaction effect of perishability with optimal pricing and ordering decisions, while Rajan et al. (1992) introduce dynamic pricing to further explore this problem. Zhao and Wang (2002), Wang (2006) and Li et al. (2015) investigate the joint pricing-production decisions in decentralized supply chains. Considering demand updating, Tang et al. (2004) evaluate the benefits of the “advance booking discount” program incorporated with joint pricing and production decisions. Li and Zheng (2006) study the impact of yield uncertainty on the optimal pricing and inventory decision. Tereyagöglu and Veeraraghavan (2012) examine the coordination of operational decisions such as production, salvaging, and sourcing with pricing and scarcity decisions in a market with uncertain demand and conspicuous consumption. Zhu (2012) explores the structure of the optimal pricing and inventory-control policy under the circumstance with return opportunity and expediting delivery. However, most research on integrating pricing strategies with production/inventory control policies has ignored the substitution issue caused by the yield uncertainty that exists in the production process.

In short, Table 1 summarizes aforementioned related papers. We find that studies on product substitutability with random demands always ignore pricing decisions, while few works on joint pricing and quantity decisions consider the effects of yield uncertainty and substitution. Therefore, our new contribution is that we combine elements from these two research streams mentioned above to investigate the optimal joint pricing, production, and downconversion decisions for two vertically differentiated products that target two different market segments with the high-end product facing yield uncertainty.

3. Model description

We study the joint pricing and production decisions faced by a firm who offers two types of products: high-end and low-end. To differentiate these two types of products, we use subscript 1 and 2 for the high-end and low-end products, respectively. For example, q_1 means the planned production quantity of high-end products while q_2 means that of low-end ones. Since an advanced technology is required to manufacture high-end products, there exists a yield uncertainty in the production process of these products. We assume that the yield rate of the high-end products is given by a stochastically proportional yield model, i.e., θq_1 , where θ is a random variable with mean $\bar{\theta}$ and standard deviation σ_θ . We further assume that the unsatisfied demand is backlogged with a unit emergent fulfillment cost e_i and the excess stock can be salvaged with the unit price s_i for product i , where $i = 1, 2$. Denote c_i as unit production cost, where

$c_1 > c_2$. Further, we assume that the number of the defective high-end products caused by the yield uncertainty, i.e., $(1 - \theta)q_1$, is down-converted to the low-end products with a cost of c_T per unit. To avoid the trivial solution, we assume that $e_i > c_i > s_i$ and $c_T < c_2$.

In the market of high-end products, we assume that the firm, as the market leader, can determine the price p_1 . Let d_1 be the mean demand of high-end products at price p_1 . Similar to Li and Zheng (2006), the demand can be written as $D_1 = d_1 + \varepsilon$, where ε is a continuous random variable with zero mean, and the cumulative probability function of ε is $F_1(\cdot)$. This demand function is reasonable for modeling branded products (e.g., WISCO's silicon steel, Intel's processors, etc) that exhibit substantial customer demand. The uncertainty in demand in this situation is mainly due to a random forecast error that is independent of price (Agrawal and Seshadri, 2000). Further, we assume that p_1 is strictly decreasing in d_1 , which is a common assumption in literature. Therefore, there is a one-to-one correspondence between the mean demand of high-end products and the corresponding price, which means that the price is uniquely determined by the mean demand. Thus, we denote the general inverse demand function of high-end products as $p_1 = P(d_1)$. Hence, in the discussion below, without other specification, we will focus on finding the optimal mean demand d_1 . In addition, we assume that $d_1 P(d_1)$ is concave in d_1 . In general, the price should be reasonably chosen in order to guarantee that the demand is nonnegative and the buyer's profit is nonnegative. Hence, we assume that the feasible range of the mean demand is $d_1 \in [\underline{d}, \bar{d}]$, where $\underline{d} \geq 0$ and $\bar{d} < +\infty$.

In the market of low-end products, the firm has to compete with others. Hence, we assume that the price of the low-end products p_2 is given by the external market, and that the random demand D_2 has a cumulative probability function $F_2(\cdot)$. The notations is summarized in Table 2.

The firm's decisions include the price of the high-end products and the planned production quantities for both high-end and low-end products. We consider a single-period model in which the decision sequence is depicted as in Fig. 1. In our research, we assume that the pricing decision is determined before the yield uncertainty is realized, which is common in literature of operations management (e.g., Li and Zheng, 2006; Feng, 2010; Feng and Shi, 2012). The reasons are as follows. First, to avoid price delay, it is necessary for some large firms to determine prices before uncertainty factors are realized, because the decision process may be quite time-consuming and complicated (Blinder et al., 1998). Second, many firms have to determine prices at the early stage because of advertising constraints. For example, Kaya and Özer (2009) point out

Table 2
Summary of Notations.

Symbol	Description
q_i	The planned quantity of type- i products.
c_i	Unit production cost of type- i products.
s_i	Unit salvage value of excess stock of type- i products.
e_i	Unit emergent fulfillment cost of type- i products.
c_T	Unit downconversion cost.
θ	Stochastic yield rate of high-end products.
$\bar{\theta}$	Mean of the stochastic yield rate θ .
σ_θ	Standard deviation of the stochastic yield rate θ .
D_i	Demand of type- i products.
d_1	Mean demand of high-end products.
ε	Random fluctuation of high-end demand.
p_i	Price of type- i products.
$P(\cdot)$	General inverse demand function of high-end products.
$F_1(\cdot)$	Cumulative probability function of ε .
$F_2(\cdot)$	Cumulative probability function of D_2 .
$\Pi(\cdot)$	Expected profit of the firm.
Subscript	
$i \in \{1, 2\}$	Subscript 1 for high-end products, whereas subscript 2 for low-end products.
Superscript	
*	Optimal solution.
c	Solution of the certain yield rate model.

Fig. 1. Decision Sequence.

that telecom executives advocate setting the sales price before their contract manufacturer actually start to produce. Therefore, we focus on the situation where the pricing decision is determined before the yield uncertainty is realized. For completeness of the analysis, we have added an extension in Section 7, which considers the case of pricing delay. We find the properties of the optimal solution still hold even when the firm makes the pricing decision after the yield uncertainty is realized.

The expected profit function of the firm is written as

$$\begin{aligned} \Pi(q_1, p_1, q_2) = & p_1 E(D_1) - c_1 q_1 - E_{\theta, D_1} [e_1 (D_1 - \theta q_1)^+ - s_1 (\theta q_1 - D_1)^+] \\ & + p_2 E(D_2) - c_2 q_2 - E_{\theta, D_2} [e_2 (D_2 - (1 - \theta) q_1 - q_2)^+ \\ & - s_2 ((1 - \theta) q_1 + q_2 - D_2)^+] - c_T (1 - \bar{\theta}) q_1. \end{aligned} \quad (1)$$

4. Analysis of optimal solution

In this section, we first investigate properties of the expected profit function of the firm. Then we develop an algorithm to obtain the global solution for the firm's optimal pricing and production quantities decisions, and explore the firm's optimal product variety strategy.

For simplification, we define $L_i(x) = E[e_i(-x)^+ - s_i(x)^+]$. Consequently, we can rewrite (1) as

$$\begin{aligned} \Pi(q_1, q_2, d_1) = & P(d_1) d_1 - c_1 q_1 - L_1(\theta q_1 - d_1 - \varepsilon) \\ & + p_2 E(D_2) - c_2 q_2 - L_2((1 - \theta) q_1 + q_2 - D_2) - c_T (1 - \bar{\theta}) q_1. \end{aligned} \quad (2)$$

Our analytical results hinge on the following property of profit function given by Lemma 1. In this paper, all proofs are presented in the appendix.

Lemma 1. $\Pi(q_1, q_2, d_1)$ is jointly concave in q_1, q_2, d_1 .

The above lemma confirms that the firm's optimal pricing and production quantities decisions are uniquely given by the first-order conditions. We define

$$G(q_1, d_1) = P(d_1) d_1 - c_1 q_1 - L_1(\theta q_1 - d_1 - \varepsilon) - c_T (1 - \bar{\theta}) q_1, \quad (3)$$

$$H(q_1, q_2) = p_2 E(D_2) - c_2 q_2 - L_2((1 - \theta) q_1 + q_2 - D_2). \quad (4)$$

Then, we obtain

$$\Pi(q_1, q_2, d_1) = G(q_1, d_1) + H(q_1, q_2),$$

where $G(q_1, d_1)$ represents the firm's payoff from the high-end market while $H(q_2, q_1)$ represents the payoff from the low-end market. We denote $d_1(q_1) = \arg\max_{d_1} G(q_1, d_1)$ and $q_2(q_1) = \arg\max_{q_2} H(q_2, q_1)$. Denote $p_1(q_1)$ as the price corresponding to $d_1(q_1)$, i.e., $p_1(q_1) = P[d_1(q_1)]$.

The following proposition characterizes the monotone properties of $d_1(q_1)$, $q_2(q_1)$, and $p_1(q_1)$ with respect to q_1 .

Proposition 1. For any given q_1 , $d_1(q_1)$ is strictly increasing in q_1 ; $q_2(q_1)$ and $p_1(q_1)$ are strictly decreasing in q_1 .

Due to the joint concavity by Lemma 1, we can reshape the firm's expected profit from (2) in terms of a single decision variable, q_1 , as

$\Pi(q_1, d_1(q_1), q_2(q_1))$, which is concave in q_1 . To derive the global optimal solution, we need to check the first derivative with respect to q_1 , i.e.,

$$\begin{aligned} \frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1} = & -c_1 - c_T (1 - \bar{\theta}) + E_{\theta} \theta [e_1 - (e_1 - s_1) F_1(\theta q_1 - d_1(q_1))] \\ & + E_{\theta} (1 - \theta) [e_2 - (e_2 - s_2) F_2((1 - \theta) q_1 + q_2(q_1))]. \end{aligned}$$

Then, we develop the following solution algorithm.

Theorem 1. The optimal pricing and production quantity decisions can be obtained as follows.

- Step 1. If $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1} \Big|_{q_1=0} \leq 0$, then $q_1^* = 0$. Otherwise, $q_1^* > 0$.
- Step 2. Denote $\bar{q}_1$ as the solution such that $E_{\theta} F_2((1 - \theta) \bar{q}_1) = \frac{e_2 - c_2}{e_2 - s_2}$ (that is $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_2} \Big|_{q_2=0} = 0$). We check the sign of $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1} \Big|_{q_1=\bar{q}_1}$. If it is nonnegative, $q_2^* = 0$; otherwise $q_2^* > 0$.
- Step 3. Obtain q_1^* by solving $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1} = 0$. Find d_1^* and q_2^* by substituting q_1^* into $\frac{\partial G(q_1, d_1)}{\partial d_1} = 0$ and $\frac{\partial H(q_1, q_2)}{\partial q_2} = 0$ respectively. Then, we obtain $p_1^* = P(d_1^*)$.

Since $p_1(q_1)$ is decreasing in q_1 and $d_1(q_1)$ is increasing in q_1 , we denote $p_1^{\max}$ as the corresponding price for $q_1 = 0$ and $d_1^{\min}$ as the corresponding mean demand for $q_1 = 0$. The following proposition gives a sufficient and necessary condition under which the firm only produces low-end products.

Proposition 2. $q_1^* = 0$ and $q_2^* > 0$ iff $p_1^{\max} + P'(d_1^{\min}) d_1^{\min} \leq c_1 / \bar{\theta} - (c_2 - c_T) (1 - \bar{\theta}) / \bar{\theta}$.

Note that $c_1 / \bar{\theta} - (c_2 - c_T) (1 - \bar{\theta}) / \bar{\theta}$ represents the expected marginal cost of producing one unit of the high-end product and $p_1^{\max} + P'(d_1^{\min}) d_1^{\min}$ represents the expected marginal revenue of selling one unit of the high-end product. Proposition 2 indicates that when this marginal cost exceeds the marginal revenue, the firm has no incentive to produce the high-end products and only manufacture the low-end products. By Proposition 2, we obtain Corollary 1.

Corollary 1. Whether the firm produces the high-end product or not does not depend on e_2 or s_2 .

Corollary 1 indicates that when the firm does not produce the high-end product, the firm can only rely on the low-end products to satisfy the uncertain low-end demand. Therefore, the firm faces a newsboy problem in the low-end market and an optimal production in the low-end market will balance the underage cost ($e_2 - c_2$) and overage cost ($c_2 - s_2$). With this fact, we observe that the derivative of $\frac{d\Pi}{dq_1} \Big|_{q_1=0}$ is independent of e_2 and s_2 . In other words, e_2 and s_2 mainly affect the production quantity of the low-end products and do not play a role in the marginal revenue and the marginal cost at $q_1 = 0$.

Although the entry condition for the firm to produce the high-end product or not is given by Proposition 2, the condition for low-end products production is unclear with a general stochastic yield model.

To gain more insights, we consider a special case of the random-yield model, i.e., the all-or-nothing model. In the all-or-nothing model, we assume that

$$\theta = \begin{cases} 1, & \text{with probability } \bar{\theta}; \\ 0, & \text{O.W.} \end{cases} \quad (5)$$

Then, we have the following proposition.

Proposition 3. For the all-or-nothing model given by (5), if $e_1 - e_2 < \frac{c_1 - c_2 + c_T(1 - \bar{\theta})}{\bar{\theta}}$, q_2^* is positive.

Proposition 3 indicates that when the unit emergent fulfillment cost of the low-end products is above a certain threshold, it is profitable to produce the low-end products in order to hedge the risk of shortage.

By Propositions 1 and 2, we find that emergent production cost plays a critical role in determining the entry decision. To gain more insights, we further investigate how the firm determines the optimal production quantity in a general random-yield model through numerical experiments.

Similar to Tomlin and Wang (2008), we use a beta distribution to simulate yield uncertainty and normal distributions to represent the randomness of both high and low-end markets in all numerical experiments. For the setting of the experiment, we set $c_1 = 6$, $c_2 = 4$, $c_T = 2$, $s_1 = 2$, $s_2 = 1$, $p_2 = 10$, $d_1 = 30 - p_1$, and θ , ε and D_2 are normally distributed random variables ($\theta \sim \text{Beta}(4, 6)$), $\varepsilon \sim N(0, 2^2)$, $D_2 \sim N(18, 4^2)$. Then, we draw Fig. 2 based on $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1}|_{q_1=0} = 0$ and $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1}|_{q_1=\bar{q}_1} = 0$. From Fig. 2, we can see that the boundary condition for the firm to produce the high-end product or not is a vertical straight line (independent of e_2 , which is consistent with Corollary 1). This vertical line represents $\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1}|_{q_1=0} = 0$. In the left region of this line, e_1 is sufficiently low so that the firm does not produce any high-end products and only satisfies the high-end demand with emergency supply; whereas in the right region, the firm provides high-end products. The sloping curve depicts the threshold ($\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1}|_{q_1=\bar{q}_1} = 0$) for whether the firm provides low-end products or not. Proposition 3 suggests that in the special case of the all-or-nothing model, the sloping curve ($\frac{d\Pi(q_1, d_1(q_1), q_2(q_1))}{dq_1}|_{q_1=\bar{q}_1} = 0$) degenerates into a straight line ($e_2 = e_1 - \frac{c_1 - c_2 + c_T(1 - \bar{\theta})}{\bar{\theta}}$). Under the sloping curve, when e_1 is high and e_2 is low, the firm only produces high-end products and satisfies the low-end demand with the downconverted products or emergency supply. Only if both the unit emergent fulfillment costs are high, i.e., above the sloping curve, the firm has an incentive to

produce both high-end and low-end products.

5. Operational impacts of yield uncertainty

The impact of yield variability is of great interest in business practice. Hence, in this section we study the operational effects of uncertain yield on the profit performance by comparing our model with the certain-yield-rate model. We use the superscript ‘c’ to indicate the solution of the certain yield rate model. We assume that the yield rate is $\bar{\theta}$ (a constant), instead of θ (a stochastic variable). Similar to the model in Section 3, we can write the expected profit function of the certain-yield-rate model as

$$\begin{aligned} \Pi^c(q_1^c, p_1^c, q_2^c) = & p_1^c E(D_1) - c_1 q_1^c - E[e_1(D_1 - \bar{\theta} q_1^c)^+ - s_1(\bar{\theta} q_1^c - D_1)^+] \\ & + p_2 E(D_2) - c_2 q_2^c - E[e_2(D_2 - (1 - \bar{\theta})q_1^c - q_2^c)^+ \\ & - s_2((1 - \bar{\theta})q_1^c + q_2^c - D_2)^+] - c_T(1 - \bar{\theta})q_1^c. \end{aligned} \quad (6)$$

Since we can prove that $\Pi^c(q_1^c, p_1^c, q_2^c)$ is jointly concave in (q_1^c, p_1^c, q_2^c) , the optimum (d_1^*, q_1^*, q_2^*) is uniquely given by solving the following three equations, i.e.,

$$F_1(\bar{\theta} q_1^* - d_1^*) = \frac{\bar{\theta} e_1 + (1 - \bar{\theta})(c_2 - c_T) - c_1}{\bar{\theta}(e_1 - s_1)},$$

$$F_2((1 - \bar{\theta})q_1^* + q_2^*) = \frac{e_2 - c_2}{e_2 - s_2},$$

$$P_1(d_1^*) + P_1'(d_1^*)d_1^* = e_1 - \frac{(1 - \bar{\theta})(c_2 - c_T) - c_1 + \bar{\theta} e_1}{\bar{\theta}}.$$

The following proposition characterizes the relationships between the expected profit with and without yield uncertainty.

Proposition 4. The profit under the certain yield rate is no less than that under yield uncertainty, i.e., $\Pi^c \geq \Pi^*$.

Proposition 4 confirms that the firm obtains a higher payoff without yield uncertainty than he does with yield uncertainty. This result is reasonable because eliminating yield uncertainty helps the firm plan the production quantity of both high-end and low-end products to meet the demands from the markets more accurately, and the firm can also craft the pricing strategy more precisely in the high-end market.

To achieve a clear understanding of the impact of yield uncertainty on the firm's profit, we perform the following experiments. The yield uncertainty is measured by variance(σ^2) of θ . We focus on the impact of variance on the profit gap, where the profit gap is defined as $\frac{\Pi^c - \Pi^*}{\Pi^c} \times 100\%$. In Fig. 3, we set $c_1 = 6$, $c_2 = 4$, $s_1 = 2$, $s_2 = 1$, $c_T = 2$, $e_1 = 15$, $e_2 = 5$, $p_2 = 10$, $d_1 = 30 - p_1$. We use normal distributions for the markets potential ($\varepsilon \sim N(0, 2^2)$, $D_2 \sim N(18, 4^2)$), and a beta distribution for the yield (keeping $\bar{\theta} = 0.4$ and yield variance varying from 0.02 to 0.12). As shown in Fig. 3, the higher the variance of θ , the greater the gap between the firm's profit with and without the yield uncertainty. This is because the variance of θ increases, the profit of the firm with yield uncertainty decreases while that of the firm without yield uncertainty remains the same.

We further investigate how the firm's optimal production quantities react to the degree of yield uncertainty (depicted by $\sigma^2(\theta)$) in Fig. 4, in which the parameter setting is the same as in Fig. 3, explicitly shows that the production quantity of high-end products decreases in yield uncertainty, while that of low-end products increases in yield uncertainty. The intuition is that as yield uncertainty increases, the risk of producing substandard high-end products also increases. To reduce the profit loss caused by the high risk, the firm's optimal strategy is to decrease the production quantity of the high-end products and raise the selling price. As a result, the amount of the downconversion becomes less. The firm has

Fig. 2. Optimal strategy.

Fig. 3. The effect of yield uncertainty.

to increase the production quantity of the low-end products in order to fulfill the demand in the low-end market.

6. Sensitivity analysis

In this section, we numerically study how the stability of market demand, emergent fulfillment costs, and downconversion cost influence the effect of yield uncertainty on profit gap.

First, we investigate the influences of the stability of the high-end and low-end market. In Fig. 5, we set $c_1 = 6$, $c_2 = 4$, $s_1 = 2$, $s_2 = 1$, $c_T = 2$, $e_1 = 15$, $e_2 = 5$, $p_2 = 10$, $d_1 = 30 - p_1$, θ is beta distribution ($\theta \sim \text{Beta}(4, 6)$), ε and D_2 are normally distributed random variables. In Fig. 5(a), we fix $\bar{e} = 0$ and vary $\sigma(\varepsilon)$. We observe that the gap becomes small as the variability of the high-end market increases. In Fig. 5(b), we fix $\bar{D}_2 = 18$ and vary the standard deviation of D_2 . We also observe that the profit gap also narrows in $\sigma(D_2)$. The results show that when $\sigma(\varepsilon)$ or $\sigma(D_2)$ rises, both Π^{c*} and Π^* decrease, but Π^{c*} decreases faster than Π^* . An insight learned from Fig. 5 is that if the high-end or low-end market volatility becomes more fierce, the firm has less incentive to eliminate yield uncertainty.

Next, we examine how emergent fulfillment costs influence the effect of yield uncertainty. For this experiment, we set $c_1 = 6$, $c_2 = 4$, $s_1 = 2$, $s_2 = 1$, $c_T = 2$, $p_2 = 10$, $d_1 = 30 - p_1$, and ε and D_2 are normally distributed random variables ($\varepsilon \sim N(0, 2^2)$, $D_2 \sim N(18, 4^2)$), and θ is beta distribution ($\theta \sim \text{Beta}(4, 6)$). In Fig. 6(a), we fix $e_2 = 5$ and vary e_1 ; whereas in Fig. 6(b), we fix $e_1 = 15$ and vary e_2 . Fig. 6 explicitly shows that the profit gap widens in e_i . We also observe that both Π^{c*} and Π^* decrease with e_i ($i = 1$ or 2), but Π^* decreases faster than Π^{c*} . Therefore, the higher the unit emergent fulfillment costs, the more incentive the firm has to eliminate yield uncertainty.

Finally, we explore how downconversion cost influences the effect of yield uncertainty. In Fig. 7, we set $c_1 = 6$, $c_2 = 4$, $s_1 = 2$, $s_2 = 1$, $e_1 = 15$, $e_2 = 5$, $p_2 = 10$, $d_1 = 30 - p_1$, ε and D_2 are normally distributed random variables ($\varepsilon \sim N(0, 2^2)$, $D_2 \sim N(18, 4^2)$), and θ is beta distribution ($\theta \sim \text{Beta}(4, 6)$). We find that the profit gap narrows in c_T . The reason is that as c_T grows, the downconversion cost significantly counterbalances the profit created by the high-end products. In order to reduce the cost caused by the low yield rate, the firm prefers to use the emergent supply to satisfy the high-end market.

7. Extension: pricing delay

In this section, we consider the impact of the delay of the pricing decision for the high-end products. We assume that the decision sequence is as follows. In the first stage, the firm determines the production quantities for each type of products, the yield uncertainty of the high-end products is revealed, and then the standard high-end products are downconverted into qualified low-end ones. In the second stage, the firm chooses the price of the high-end products, the demand occurs and is fulfilled by the stock available, and then the amount of backlog is satisfied by an emergency supply. We use Π_{pd}^i to denote the profit of pricing delay of stage i .

We solve the firm's problem by backward induction. First, in the second stage, given q_1 , q_2 and the realization of θ denoted as $\hat{\theta}$, the profit of the firm is given by

$$\begin{aligned} \Pi_{pd}^2(d_1 | q_1, q_2, \hat{\theta}) = & P(d_1)d_1 - c_1q_1 - E_\varepsilon \left[e_1(d_1 + \varepsilon - \hat{\theta}q_1)^+ - s_1(\hat{\theta}q_1 - d_1 - \varepsilon)^+ \right] \\ & + p_2E(D_2) - c_2q_2 - E_{D_2} \left[e_2(D_2 - (1 - \hat{\theta})q_1 - q_2)^+ \right. \\ & \left. - s_2((1 - \hat{\theta})q_1 + q_2 - D_2)^+ \right] - c_T(1 - \hat{\theta})q_1. \end{aligned}$$

(a) q_1 decreasing with variance of θ

(b) q_2 increasing with variance of θ

Fig. 4. The effect of yield uncertainty on production quantities.

(a) profit gap decreasing with $\sigma(\epsilon)$ (b) profit gap decreasing with $\sigma(D_2)$ **Fig. 5.** The influence of market volatility. (a) profit gap increasing with e_1 (b) profit gap increasing with e_2 **Fig. 6.** The influence of emergent fulfillment costs.

Then, the following proposition gives the optimal price of the high-end products and characterizes the monotone properties of the price with respect to q_1 , q_2 , and $\hat{\theta}$.

Proposition 5. Under pricing delay, the following statements hold:

- (i) The optimal price of the high-end products is $P^*(\hat{d}_1)$, where $\hat{d}_1$ is the unique solution of

$$P'(d_1)d_1 + P(d_1) + (e_1 - s_1)F_1(\hat{\theta}q_1 - d_1) - e_1 = 0.$$

- (ii) $P^*(\hat{d}_1)$ is strictly decreasing in $\hat{\theta}$ and q_1 , and independent of q_2 .

Proposition 5 (ii) indicates that the price of high-end products is low if the realization of yield uncertainty or the planned quantity is high.

Next, we denote $\Pi_{pd}^{2*}(q_1, q_2)$ as the optimal profit of stage 2 in which q_1 and q_2 are given and θ is realized. In the first stage, the profit of the firm is given by

$$\begin{aligned} \Pi_{pd}^1(q_1, q_2) &= E_{\theta} \Pi_{pd}^{2*}(q_1, q_2) \\ &= E_{\theta} P(\hat{d}_1) \hat{d}_1 - c_1 q_1 - E_{\theta, \epsilon} \left[e_1 (\hat{d}_1 + \epsilon - \theta q_1)^+ - s_1 (\theta q_1 - \hat{d}_1 - \epsilon)^+ \right] \\ &\quad + p_2 E(D_2) - c_2 q_2 - E_{\theta, D_2} \left[e_2 (D_2 - (1 - \theta)q_1 - q_2)^+ \right. \\ &\quad \left. - s_2 ((1 - \theta)q_1 + q_2 - D_2)^+ \right] - c_T (1 - \bar{\theta}) q_1. \end{aligned}$$

The following proposition confirms the existence of the firm's optimal production decision and characterizes the monotone properties of $q_2(q_1)$ with respect to q_1 .

Proposition 6. Under pricing delay, the following statements hold:

- (i) $\Pi_{pd}^1(q_1, q_2)$ is concave in q_1 and q_2 .
(ii) For any given q_1 , $q_2(q_1)$ is strictly decreasing in q_1 .

Fig. 7. The influence of downconversion cost.

8. Concluding remarks

This paper explores the joint pricing and production decisions of a firm providing two types of products: high-end and low-end. When producing high-end products, the firm suffers from yield uncertainty. A substandard high-end product can be downconverted into a qualified low-end one by an acceptable cost. The firm can determine the price of high-end products and only accept the price of low-end products given by the external market. First, we develop an algorithm to compute the optimal solution for the price of high-end products and production quantities of both high and low-end products. We further investigate the operational effects of uncertain yield on the profit performance, and how stability of market demand, emergent fulfillment costs, downconversion

cost influence these effects.

Our key findings are summarized as follows. First, the profit of the firm deteriorates when the risk of the yield uncertainty is high. Moreover, when the degree of yield uncertain increases, the firm decreases the quantity of the high-end products, which is different from the findings by the literature of yield uncertainty. For example, [Henig and Gerchak \(1990\)](#) argue that the firm should increase the production quantity in the face of yield uncertainty. Second, we find that under the conditions that the market demands are quite unstable, or the emergent fulfillment cost is relatively low, or the downconversion cost is relatively high, the firm has a low incentive to eliminate the yield uncertainty, which is novel in comparison of the related literature.

There are several directions for future study. First, we currently use the additive model (a linear function of price plus an uncertain error term) to reflect the effect of the price on the demand. It is possible to allow a more general demand model and investigate the corresponding optimal decisions and the impact of yield uncertainty. Second, it is also interesting to study the multiple-period setting and examine the structure of the optimal pricing and production policies.

Acknowledgments

The authors are grateful for the detailed and insightful comments and suggestions from referees and the constructive suggestions from the editor. These comments and suggestions have helped us to improve this paper significantly. The first and second author were supported in part by National Natural Science Foundation of China under Grant (No. 71511130129). The last author was supported by Netherlands Organisation for Scientific Research under Grant 040.03.021, 040.21.001, and National Natural Science Foundation of China under grant 71571125, 71511130129, 71711530046. The corresponding author was partially supported by China Postdoctoral Science Foundation (No. 2017M622487).

Appendix

Proof of Lemma 1. According to the definition of $L_i(x)$, we know that $L_1(\theta q_1 - d_1 - \varepsilon)$ is jointly convex in (q_1, d_1) and $L_2((1 - \theta)q_1 + q_2 - D_2)$ is jointly convex in (q_1, q_2) . Then, we have the joint concavity of $\Pi(q_1, q_2, d_1)$.

Proof of Proposition 1. Taking the first-order condition of d_1 for $G(q_1, d_1)$, we have

$$\frac{\partial G}{\partial d_1} = P'(d_1)d_1 + P(d_1) + E_\theta[(e_1 - s_1)F_1(\theta q_1 - d_1)] - e_1 = 0. \quad (7)$$

We obtain $d_1(q_1)$ by solving (7).

Taking the first-order condition of q_2 for $H(q_1, q_2)$, we have

$$\frac{\partial H}{\partial q_2} = -c_2 - E_\theta[(e_2 - s_2)F_2((1 - \theta)q_1 + q_2)] + e_2 = 0. \quad (8)$$

Solving (8) gives us $q_2(q_1)$.

Because $G(q_1, d_1)$ is strictly supermodular in (q_1, d_1) and $H(q_1, q_2)$ is strictly submodular in (q_1, q_2) , $d_1(q_1)$ is strictly increasing in q_1 while $q_2(q_1)$ is strictly decreasing in q_1 . Further, by definition, p_1 is strictly decreasing in d_1 . Thus, $p_1(q_1)$ is strictly decreasing in q_1 .

Proof of Proposition 2. Checking the first derivative of $\Pi(q_1, d_1^*(q_1), q_2^*(q_1))$ at $q_1 = 0$, we have

$$\begin{aligned} \frac{d\Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{dq_1} \Big|_{q_1=0} &= -c_1 - c_T(1 - \bar{\theta}) + E_\theta\theta[e_1 - (e_1 - s_1)F_1(-d_1)] \\ &\quad + E_\theta(1 - \theta)[e_2 - (e_2 - s_2)F_2(q_2)]. \end{aligned}$$

For $q_1 = 0$, taking the first-order condition of d_1 and q_2 for $\Pi(q_1, q_2, d_1)$, respectively, we have

$$F_1(-d_1) = \frac{e_1 - (p_1^{max} + P(d_1^{min})d_1^{min})}{e_1 - s_1}, F_2(q_2) = \frac{e_2 - c_2}{e_2 - s_2}.$$

where $p_1^{max} = p_1(0)$ and $d_1^{min} = d_1(0)$.

By substituting them into $\frac{d_1 \Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{d_1 q_1} \Big|_{q_1=0}$, we have $\frac{d_1 \Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{d_1 q_1} \Big|_{q_1=0} \leq 0$ iff $p_1^{max} + P(d^{min})d^{min} \leq c_1/\bar{\theta} - (c_2 - c_T)(1 - \bar{\theta})/\bar{\theta}$. By the concavity of Π , we have $q_1^* = 0$ and $q_2^* = F_2^{-1}\left(\frac{e_2 - c_2}{e_2 - s_2}\right) > 0$.

Proof of Corollary 1. Let $q_1 = 0$, from (7), we obtain

$$F_1[-d_1(q_1)] \Big|_{q_1=0} = \frac{e_1 - (P'(d_1(q_1))d_1(q_1) + P(d_1(q_1)))}{(e_1 - s_1)} \Big|_{q_1=0}.$$

So, $d_1(q_1) \Big|_{q_1=0}$ can be expressed as a function of (e_1, s_1) . We define $d_1(q_1) \Big|_{q_1=0} = u(e_1, s_1)$. Then, we have

$$\begin{aligned} \frac{d\Pi}{dq_1} \Big|_{q_1=0} &= -c_1 - c_T(1 - \bar{\theta}) + \bar{\theta}\{P'[d_1(q_1)]d_1(q_1) + P[d_1(q_1)]\} + (1 - \bar{\theta})c_2 \Big|_{q_1=0} \\ &= -c_1 - c_T(1 - \bar{\theta}) + \bar{\theta}\{P'[u(e_1, s_1)]u(e_1, s_1) + P[u(e_1, s_1)]\} + (1 - \bar{\theta})c_2. \end{aligned}$$

Therefore, $\frac{d\Pi}{dq_1} \Big|_{q_1=0}$ only depends on $e_1, c_1, s_1, c_2, c_T, \bar{\theta}$.

Proof of Proposition 3. Differentiating $\Pi(q_1, d_1^*(q_1), q_2^*(q_1))$ with respect to q_1 , we obtain

$$\begin{aligned} \frac{d\Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{dq_1} &= -c_1 - c_T(1 - \bar{\theta}) + \bar{\theta}[e_1 - (e_1 - s_1)F_1(q_1 - d_1^*(q_1))] \\ &\quad + (1 - \bar{\theta})[e_2 - (e_2 - s_2)F_2(q_1 + q_2^*(q_1))]. \end{aligned}$$

By the first-order conditions $\partial\Pi/\partial d_1 = 0$ and $\partial\Pi/\partial q_2 = 0$, we further have

$$P_1(d_1) + P'_1(d_1)d_1 - [e_1 - (e_1 - s_1)(\bar{\theta}F_1(q_1 - d_1) + (1 - \bar{\theta})F_1(-d_1))] = 0, \quad (9)$$

$$\bar{\theta}F_2(q_2) + (1 - \bar{\theta})F_2(q_1 + q_2) = \frac{e_2 - c_2}{e_2 - s_2}. \quad (10)$$

To find the condition under which $q_2^* = 0$, by (10), we have

$$(1 - \bar{\theta})F_2(\bar{q}_1) = \frac{e_2 - c_2}{e_2 - s_2}.$$

Substituting it into $\frac{d\Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{dq_1} \Big|_{q_1=\bar{q}_1}$, we have

$$\begin{aligned} \frac{d\Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{dq_1} \Big|_{q_1=\bar{q}_1} &= -c_1 - c_T(1 - \bar{\theta}) + \bar{\theta}[e_1 - (e_1 - s_1)F_1(\bar{q}_1 - d_1^*(\bar{q}_1))] \\ &\quad + (1 - \bar{\theta})e_2 - e_2 + c_2 \\ &\leq -c_1 - c_T(1 - \bar{\theta}) + \bar{\theta}(e_1 - e_2) + c_2. \end{aligned}$$

When $e_1 - e_2 < \frac{c_1 - c_2 + c_T(1 - \bar{\theta})}{\bar{\theta}}$, we obtain $\frac{d_1 \Pi(q_1, d_1^*(q_1), q_2^*(q_1))}{d_1 q_1} \Big|_{q_1=\bar{q}_1} < 0$. According to Step 2 of the solution algorithm in [Theorem 1](#), we conclude that $q_2^* > 0$.

Proof of Proposition 4. Because of the convexity of L_1 and L_2 , we have

$$\begin{aligned} \Pi^* &\leq \max_{d_1, q_1, q_2} \left\{ p_1 E(D_1) - c_1 q_1 - L_1(\bar{\theta}q_1 - d_1(p_1) - \varepsilon) \right. \\ &\quad \left. + p_2 E(D_2) - c_2 q_2 - L_2((1 - \bar{\theta})q_1 + q_2 - D_2) - c_T(1 - \bar{\theta})q_1 \right\} \\ &= \Pi^{c*}, \end{aligned}$$

where the first inequality is true due to Jensen's inequality.

Proof of Proposition 5.

(i) Taking the second derivative of Π_{pd}^2 with respect to d_1 , we have

$$\frac{\partial^2 \Pi_{pd}^2(d_1 | q_1, q_2, \bar{\theta})}{\partial d_1^2} = P''(d_1)d_1 + 2P'(d_1) - (e_1 - s_1)f_1(\bar{\theta}q_1 - d_1).$$

Because $d_1 P(d_1)$ is concave in d_1 under the assumption, we have $P''(d_1)d_1 + 2P'(d_1) < 0$, leading to $\frac{\partial^2 \Pi_{pd}^2(d_1 | q_1, q_2, \bar{\theta})}{\partial d_1^2} < 0$. Therefore, $\Pi_{pd}^2(d_1)$ is unimodal in d_1 . The global maximum is given by $\hat{d}_1$, where $\hat{d}_1$ is the unique solution of

$$\frac{\partial \Pi_{pd}^2(d_1 | q_1, q_2, \bar{\theta})}{\partial d_1} = P'(d_1)d_1 + P(d_1) + (e_1 - s_1)F_1(\bar{\theta}q_1 - d_1) - e_1 = 0. \quad (11)$$

- (ii) Because Π_{pd}^2 is strictly supermodular in (d_1, q_1) and $(d_1, \hat{\theta})$, d_1 is strictly increasing in q_1 and $\hat{\theta}$. Further, by definition, p_1 is strictly decreasing in d_1 . Thus, p_1 is strictly decreasing in q_1 and $\hat{\theta}$.

Proof of Proposition 6.

- (i) Taking the first and second derivative of $\Pi_{pd}^1(q_1, q_2)$ with respect to q_1 , we have

$$\begin{aligned}\frac{\partial \Pi_{pd}^1(q_1, q_2)}{\partial q_1} &= \frac{\partial \Pi_{pd}^1}{\partial E_{\theta} \hat{d}_1} \frac{\partial E_{\theta} \hat{d}_1}{\partial q_1} + \frac{\partial \Pi_{pd}^1}{\partial q_1}, \\ \frac{\partial^2 \Pi_{pd}^1(q_1, q_2)}{\partial q_1^2} &= \frac{\partial^2 \Pi_{pd}^1}{\partial (E_{\theta} \hat{d}_1)^2} \left(\frac{\partial E_{\theta} \hat{d}_1}{\partial q_1} \right)^2 + \frac{\partial \Pi_{pd}^1}{\partial E_{\theta} \hat{d}_1} \frac{\partial^2 E_{\theta} \hat{d}_1}{\partial q_1^2} + \frac{\partial^2 \Pi_{pd}^1}{\partial q_1^2}.\end{aligned}\quad (12)$$

We know $\frac{\partial^2 \Pi_{pd}^2}{\partial d_1^2} < 0$ from the proof of Proposition 5, thus $\frac{\partial^2 \Pi_{pd}^1}{\partial (E_{\theta} \hat{d}_1)^2} < 0$. Hence, the first part of (12) is negative.

According to (11) and

$$\frac{\partial \Pi_{pd}^1(q_1, q_2)}{\partial E_{\theta} \hat{d}_1} = E_{\theta} [P'(\hat{d}_1) \hat{d}_1 + P(\hat{d}_1)] + (e_1 - s_1) E_{\theta} F_1(\theta q_1 - \hat{d}_1(q_1)) - e_1,$$

we have $\frac{\partial (\Pi_{pd}^2)}{\partial d_1} = 0$ from the proof of Proposition 5, thus the second part of (12) is equal to zero.

In addition, the third part of (12) is given by

$$\frac{\partial^2 \Pi_{pd}^1}{\partial q_1^2} = -(e_1 - s_1) E_{\theta} [\theta^2 f_1(\theta q_1 - \hat{d}_1)] - (e_2 - s_2) E_{\theta} [(1 - \theta)^2 f_2((1 - \theta)q_1 + q_2)] < 0.$$

Therefore, $\frac{\partial^2 \Pi_{pd}^1(q_1, q_2)}{\partial q_1^2} < 0$. The profit function is concave in q_1 .

Further, we have $\frac{\partial^2 \Pi_{pd}^1(q_1, q_2)}{\partial q_2^2} = -(e_2 - s_2) E_{\theta} [f_2((1 - \theta)q_1 + q_2)] < 0$. So the profit function is concave in q_2 .

- (ii) Since

$$\frac{\partial^2 \Pi_{pd}^1}{\partial q_1 \partial q_2} = -(e_2 - s_2) E_{\theta} [(1 - \theta) f_2((1 - \theta)q_1 + q_2)] < 0,$$

we can show that Π_{pd}^1 is strictly submodular in (q_1, q_2) . Thus, $q_2(q_1)$ is strictly decreasing in q_1 for any given q_1 .

References

- Agrawal, V., Seshadri, S., 2000. Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem. *Manuf. Serv. Oper. Manage.* 2 (4), 410–423.
- Bassok, Y., Anupindi, R., Akella, R., 1999. Single-period multi product inventory models with substitution. *Oper. Res.* 47 (4), 632–642.
- Blinder, A., Canetti, E.R., Lebow, D.E., Rudd, J.B., 1998. Asking About prices: a New Approach to Understanding Price Stickiness. Russell Sage Foundation.
- China Metallurgical News, 2014. 2014. Practicing innovation-driven transformation and upgrading. Dec.29, 2014. http://www.csteelnews.com/xwzx/djbd/201412/t20141229_268517.html.
- Chan, L.M.A., Shen, Z.J.M., Simchi-Levi, D., Swann, J., 2004. Coordination of pricing and inventory decisions: a survey and classification. In: Simchi-Levi, D., Wu, D., Shen, M. (Eds.), *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*. Elsevier Publishers.
- Chan, L.M.A., Simchi-Levi, D., Swann, J., 2006. Pricing, production, and inventory policies for manufacturing with stochastic demand and discretionary sales. *Manuf. Serv. Oper. Manage.* 8 (2), 149–168.
- Cohen, M.A., 1977. Joint pricing and ordering policy for exponentially decaying inventory with known demand. *Nav. Res. Logist. Q.* 24 (2), 257–268.
- Dong, L., Kouvelis, P., Tian, Z., 2009. Dynamic pricing and inventory control of substitute products. *Manuf. Serv. Oper. Manage.* 11 (2), 317–339.
- Federgruen, A., Heching, A., Wang, K., 1996. Combined pricing and inventory control under uncertainty. *Oper. Res.* 47 (3), 454–475.
- Feng, Q., 2010. Integrating dynamic pricing and replenishment decisions under supply capacity uncertainty. *Manage. Sci.* 56 (12), 2154–2172.
- Feng, Q., Shi, R., 2012. Sourcing from multiple suppliers for price-dependent demands. *Prod. Oper. Manage.* 21 (3), 547–563.
- Gerchak, Y., Tripathy, A., Wang, K., 1996. Co-production models with random functionality yields. *IEE Trans.* 28 (5), 391–403.
- Gilbert, S.M., 2000. Coordination of pricing and multiple-period production across multiple constant priced goods. *Manage. Sci.* 46 (12), 1602–1616.
- Guo, L., Zhang, J., 2012. Consumer deliberation and product line design. *Market. Sci.* 31 (6), 995–1007.
- Han, G., Dong, M., Liu, S., 2014. Yield and allocation management in a continuous make-to-stock system with demand upgrade substitution. *Int. J. Prod. Econ.* 156 (5), 124–131.
- Henig, M., Gerchak, Y., 1990. The structure of periodic review policies in the presence of random yield. *Oper. Res.* 38 (4), 634–643.
- Hsu, A., Bassok, Y., 1999. Random yield and random demand in a production system with downward substitution. *Oper. Res.* 47 (2), 277–290.
- Inderfurth, K., 2004. Optimal policies in hybrid manufacturing/remanufacturing systems with product substitution. *Int. J. Prod. Econ.* 90 (3), 325–343.
- Kaya, M., Özer, Ö., 2009. Quality risk in outsourcing: noncontractible product quality and private quality cost Information. *Nav. Res. Logist.* 56 (7), 669C685.
- Kempf, K.G., 2004. Control-oriented approaches to supply chain management in semiconductor manufacturing. In: *Proc. 2004 Amer. Control Conf. IEEE, Piscataway, NJ*, pp. 4563–4576.
- King, I., 2015. Intel forecast shows rising server demand. Available at: <http://www.bloomberg.com/news/articles/20150715/intelforecastshowserverdemandmakesupforpcmarketwoes>.
- Kuksov, D., Lin, Y., 2010. Information provision in a vertically differentiated competitive marketplace. *Market. Sci.* 29 (1), 122–138.
- Li, T., Sethi, S.P., He, X., 2015. Dynamic pricing, production, and channel coordination with stochastic learning. *Prod. Oper. Manage.* 24 (6), 857–882.
- Li, Q., Zheng, S.H., 2006. Joint inventory replenishment and pricing control for systems with uncertain yield and demand. *Oper. Res.* 54 (4), 696–705.
- Liu, W., Song, S., Wu, C., 2013. Impact of loss aversion on the newsvendor game with product substitution. *Int. J. Prod. Econ.* 141 (1), 352–359.
- Nagarajan, M., Rajagopalan, S., 2008. Inventory models for substitutable products: optimal policies and heuristics. *Manage. Sci.* 54 (8), 1453–1466.
- Netessine, S., Rudi, N., 2003. Centralized and competitive inventory models with demand substitution. *Oper. Res.* 51 (2), 329–335.
- Petruzzi, N.C., Dada, M., 1999. Pricing and the newsvendor problem: a review with extensions. *Oper. Res.* 47 (2), 183–194.

- Rajan, A., Rakesh, Steinberg, R., 1992. Dynamic pricing and ordering decisions by a monopolist. *Manage. Sci.* 38 (2), 240–262.
- Shumsky, R.A., Zhang, F.Q., 2009. Dynamic capacity management with substitution. *Oper. Res.* 57 (3), 671–684.
- Stavrulaki, E., 2011. Inventory decisions for substitutable products with stock-dependent demand. *Int. J. Prod. Econ.* 129 (1), 65–78.
- Tan, B., Karabati, S., 2013. Retail inventory management with stock-out based dynamic demand substitution. *Int. J. Prod. Econ.* 145 (1), 78–87.
- Tang, C.S., Rajaram, K., Alptekinoglu, A., Ou, J., 2004. The benefits of advance booking discount programs: model and analysis. *Manage. Sci.* 50 (4), 465–478.
- Tereyaoglu, Lu N., Veeraraghavan, S., 2012. Selling to conspicuous consumers: pricing, production, and sourcing decisions. *Manage. Sci.* 58 (12), 2168–2189.
- Tomlin, B., Wang, Y., 2008. Pricing and operational recourse in coproduction systems. *Manage. Sci.* 54 (3), 522–537.
- Wang, Y., 2006. Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Oper. Res.* 54 (6), 1110–1127.
- Wisco, 2013. The silicon steel production pattern of Wisco has changed for the first time. Nov. 12, 2013. <http://www.wisco.com.cn/mtjj/1582.jhtml>.
- Xu, H., Yao, D.D., Zheng, S.H., 2011. Optimal control of replenishment and substitution in an inventory system with nonstationary batch demand. *Prod. Oper. Manage.* 20 (5), 727–736.
- Xu, H., Yao, D.D., Zheng, S., 2016. Optimal Policies for a Two-Product Inventory System Under a Flexible Substitution Scheme. *Production and Operations Management*. Article in Press.
- Yano, C.A., Gilbert, S.M., 2004. Coordinating pricing and production/procurement decision: a review. In: Chakravarty, A.K., Eliashberg, J. (Eds.), *Managing Business Interfaces*. Kluwer Publishers, Massachusetts.
- Ye, T., 2014. Inventory management with simultaneously horizontal and vertical substitution. *Int. J. Prod. Econ.* 156 (5), 316–324.
- Yu, Y., Chen, X., Zhang, F., 2015. Dynamic capacity management with general upgrading. *Oper. Res.* 63 (6), 1372–1389.
- Zhao, J., Tang, W., Wei, J., 2012. Pricing decision for substitutable products with retail competition in a fuzzy environment. *Int. J. Prod. Econ.* 135 (1), 144–153.
- Zhao, W., Wang, Y., 2002. Coordination of joint pricing-production decisions in a supply chain. *IEE Trans.* 34 (8), 701–715.
- Zhu, K., Thonemann, U.W., 2009. Coordination of pricing and inventory control across products. *Nav. Res. Logist.* 56 (2), 175–190.
- Zhu, S., 2012. Joint pricing and inventory replenishment decisions with return and expediting. *Eur. J. Oper. Res.* 216 (1), 105–112.