University of Groningen

Heuristic policies for the stochastic economic lot sizing problem with remanufacturing under service level constraints

Kilic, Onur A.; Tunc, Huseyin; Tarim, S. Armagan

Published in:
European Journal of Operational Research

DOI:
[10.1016/j.ejor.2017.12.041](https://doi.org/10.1016/j.ejor.2017.12.041)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2018

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Kilic, O. A., Tunc, H., & Tarim, S. A. (2018). Heuristic policies for the stochastic economic lot sizing problem with remanufacturing under service level constraints. *European Journal of Operational Research*, 267(3), 1102-1109. <https://doi.org/10.1016/j.ejor.2017.12.041>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Decision Support

Heuristic policies for the stochastic economic lot sizing problem with remanufacturing under service level constraints[☆] Onur A. Kilic^{a,b,*}, Huseyin Tunc^{b,c}, S. Armagan Tarim^d^a Department of Operations, University of Groningen, Groningen, The Netherlands^b Department of Policy and Strategy Studies, Hacettepe University, Ankara, Turkey^c École Polytechnique Fédérale de Lausanne, CH-1015, Lausanne, Switzerland^d Department of Management, Cankaya University, Ankara, Turkey

ARTICLE INFO

Article history:

Received 14 December 2015

Accepted 28 December 2017

Available online 3 January 2018

Keywords:

Inventory

Stochastic lot-sizing

Remanufacturing

Heuristic

Mixed integer programming

ABSTRACT

In this paper, we address the stochastic economic lot sizing problem with remanufacturing under service level constraints. The problem emerges in hybrid production systems where demand can be met via two alternative sources: manufacturing new products and remanufacturing returned products. The deterministic counterpart of this problem has been considered in the literature and it is shown to be NP-Hard. We focus on the case where period demands and returns are stochastic. The optimal solution to this problem is not a deterministic production schedule but a control policy, yet its structure has not yet been characterized. We propose two heuristic policies for the problem that make use of simple decision rules to control manufacturing and remanufacturing operations and present mathematical models thereof.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction

The environmental consciousness has been rapidly improving over the last decades. As a result of this, there has been a growing interest in environment-friendly manufacturing practices. Hybrid manufacturing and remanufacturing is a common example of such practices that appears in many different industries. Here, besides manufacturing new products, returned products are remanufactured into as good as new products. This practice is often found in industries producing high-value products such as automotive parts, personal computers, cameras, medical instruments, engines, tires, aviation and military equipment, and furniture (see e.g. Ferguson & Toktay, 2006; Golany, Yang, & Gang, 2001; Kelle & Silver, 1989; Toktay, Wein, & Zenios, 2000; Van Der Laan, Salomon, Dekker, & Van Wassenhove, 1999).

It is more difficult to manage inventories in hybrid manufacturing and remanufacturing systems as compared to traditional manufacturing systems. That is due to the need of synchronizing manufacturing and remanufacturing operations to match sup-

ply and demand. Also, it is often not possible to make use of traditional inventory control methods in hybrid systems. Hence, effective production planning and inventory control in such systems has become an important direction for academic research.

A challenging problem in this context is the Economic Lot Sizing with Remanufacturing (ELSR) which is a direct extension of the well-known economic lot sizing problem (see e.g. Teunter, Bayindir, & Van Den Heuvel, 2006). ELSR is a discrete-time inventory control problem with dynamic demands where the aim is to find a replenishment plan so as to minimize the sum of production costs of manufacturing and remanufacturing operations and holding costs of serviceable and return inventories. Lot sizing problems are considered to be crucial since they are often embedded into material requirement planning systems. This is also the case for ELSR. There has been an increasing academic interest in ELSR due to its practical relevance (see e.g. Baki, Chaouch, & Abdul-Kader, 2014; Li, Baki, Tian, & Chaouch, 2014; Pan, Tang, & Liu, 2009; Pappasopoulos, Konstantaras, & Skouri, 2015; Pineyro & Viera, 2009; Retel Helmrich, Jans, van den Heuvel, & Wagelmans, 2014; Schulz, 2011; Sifaleras, Konstantaras, & Mladenović, 2015; Teunter et al., 2006; Yang, Golany, & Yu, 2005). It is also shown that the problem is NP-hard (Retel Helmrich et al., 2014).

An important limitation of ELSR is the assumption that both demand and return quantities are deterministic, i.e. known with certainty at the beginning of the planning horizon. It is clear that

[☆] This paper is an extended version of the work presented at IFAC MIM 2013 Conference (Kilic (2013)).

* Corresponding author at: Department of Operations, University of Groningen, Groningen, The Netherlands.

E-mail address: o.a.kilic@rug.nl (O.A. Kilic).

this assumption is unrealistic in most practical situations. Therefore, integrating uncertainty into ELSR is of practical concern. There is a stream of research on hybrid manufacturing and remanufacturing systems where stochastic demands and returns are taken into consideration (see e.g. Ahiska & King, 2010a; Benedito & Corominas, 2013; Kiesmuller, 2003; Kiesmuller & Scherer, 2003; Li, Liu, Cao, & Wang, 2009; Mahadevan, Pyke, & Fleischmann, 2003; Panagiotidou, Nenes, Zikopoulos, & Tagaras, 2017; Toktay et al., 2000; Van Der Laan et al., 1999; Van Der Laan & Teunter, 2006; Vlachos & Dekker, 2003). However, the contributions that fall into this research domain either concentrate on systems where demands and returns – although being stochastic – are stationary over time, or neglect the lot-sizing component of the problem by ignoring fixed manufacturing and remanufacturing costs. The assumption on stationarity can be reasonable when demand and return rates change rather slowly. However, it does not hold for most products as product life cycles are getting shorter due to rapidly changing customer preferences and technological progress. The fixed production costs, on the other hand, are essential for systems where costly setup operations are present.

The literature on hybrid manufacturing and remanufacturing systems with fixed manufacturing and remanufacturing costs and non-stationary stochastic demands and returns is very limited. Ahiska and King (2010b) address a case where demand and return distributions change as the product move from one stage of the product life cycle to the next. They propose an approach where the overall problem is approximated by several stationary problems each of which captures a stage of the product life cycle. A heuristic policy is adopted for each stage of the life cycle whose parameters are computed by using stationary analysis. Although being practical, this approach is reasonable if the stages of the product life cycle span relatively large intervals to justify stationary analysis. Hilger, Sahling, and Tempelmeier (2016) consider a multi-item manufacturing and remanufacturing system where demands and returns are stochastic and non-stationary. They consider the lot-sizing problem with a service level constraint which makes sure that expected stockouts cannot exceed a specified proportion of expected demands. They model and solve the problem by using a static-uncertainty strategy (see e.g. Bookbinder & Tan, 1988) where all replenishment decisions, i.e. the timing and the size of all manufacturing and remanufacturing lots, are made at the beginning of the planning horizon. As such, their approach is not responsive to realizations of demands and returns over time.

The current paper addresses the production and inventory control problem in hybrid manufacturing and remanufacturing systems with fixed production costs facing non-stationary stochastic demands and returns – herein referred to as the stochastic economic lot sizing with remanufacturing (SELSR). We contribute to the literature by proposing two novel heuristic policies for this challenging problem as well as presenting mathematical models of these policies. The heuristic policies follow the so-called static-dynamic uncertainty strategy (see e.g. Bookbinder & Tan, 1988) where production periods are determined in advance, whereas production lot sizes are determined on production periods based on simple decision rules. This strategy is not necessarily optimal, yet it is shown to be rather cost effective (see e.g. Tunc, Kilic, Tarim, & Eksioglu, 2013). Also, it is widely implemented in supply chains where a strong coordination and cooperation between players is of concern and it is either required or preferred that replenishment schedules are known in advance. These involve a variety of industrial settings such as managing joint replenishments (Silver, Pyke, & Peterson, 1998), shipment consolidation in logistics (Mutlu, Cetinkaya, & Bookbinder, 2010), and buying raw materials on fluctuating price markets (Kingsman, 1985). The proposed policies are both characterized by fixed production schedules, but they differ on how the size of manufacturing and remanufacturing lots

are determined in manufacturing and remanufacturing periods and offer different degrees of flexibility in terms of production quantities. They both account for the quality of service by means of an α -service level approach whereby it is guaranteed that the probability of experiencing a stockout is below α for any period over the planning horizon (Silver et al., 1998). This is a practical approach towards modeling the quality of service, because it does not require one to determine the financial cost of losing customer goodwill. We develop certainty equivalent mixed integer programming (MIP) models for both policies by exploiting their specific characteristics. These models can be used approximate the cost-optimal parameters of the proposed policies while considering α -service level constraints.

The remainder of the paper is organized as follows. In Section 2, we provide the problem description and set the notation. In Section 3, we introduce the heuristic control policies and carry out some preliminary analysis. In Section 4, we present the mathematical models of the proposed policies. In Section 5, we conduct a numerical study discuss our findings. Finally, in Section 6 we conclude.

2. Problem definition

SELSR can be defined as follows. A hybrid manufacturing and remanufacturing system faces random demands and returns over a finite planning horizon of N time periods. The demands and returns over these periods are independent but not necessarily identically distributed random variables. The (non-negative) demand and return in period n are denoted as D_n and R_n , respectively. It is possible to meet demand either by manufacturing new products or remanufacturing returned products. It is assumed that remanufactured products are as-good-as-new. Therefore, products procured from both sources are serviceable. The manufacturing and remanufacturing lead times are assumed to be negligible. Both manufacturing and remanufacturing incurs fixed and variable costs; the respective fixed costs K^m and K^r are charged for each period where manufacturing and remanufacturing operations are initiated, and the respective variable costs c^m and c^r are charged for each unit of manufactured and remanufactured products. The production system keeps inventories for serviceables and returns. The respective holding costs h^s and h^r are incurred for each unit of excess serviceable and return inventories carried from one period to the next. It is assumed that demands not satisfied on time are backordered and a sufficient level of service quality is imposed by means of an α -service level. That is, the non-stockout probability in any period should be higher than a pre-specified threshold α . The problem is to decide in each period whether to manufacture and/or remanufacture, and if so what should be the production quantities so as to minimize the expected total cost over the planning horizon while making sure that a sufficient non-stockout probability is achieved.

In the reminder of the paper, we use the following notational convention. We let $[u, v]$ denote the set $\{u, u+1, \dots, v\}$ for $u \leq v$, and $\emptyset$, otherwise. For any sequence $x_1, x_2, \dots$, we let $x_{[u,v]} := \sum_{n \in [u,v]} x_n$ where the empty sum is zero. For any real number x , we let $x^+ = \max\{x, 0\}$. We use E for the expectation operator and $\mathbb{1}\{\cdot\}$ for the indicator function.

The sequence of events over an arbitrary period n is as follows. First, serviceable and return inventories X_{n-1}^s and X_{n-1}^r carried from the previous period are observed, and the random period return R_n is realized. Then, following the production control policy in use, manufacturing and remanufacturing quantities Q_n^m and Q_n^r ($Q_n^r \leq X_{n-1}^r + R_n$) are determined; and production operations are commenced. Next, the random period demand D_n is realized. Finally, depending on production quantities as well as realized demands and returns, serviceable inventory $X_{n-1}^s + Q_n^m + Q_n^r - D_n$ and return inventory $X_{n-1}^r - Q_n^r + R_n$ are carried over to the next period.

The dynamics of the serviceable and return inventories can be captured by the following inventory conservation equations:

$$\begin{aligned} X_n^s &= X_{n-1}^s + Q_n^m + Q_n^r - D_n \\ X_n^r &= X_{n-1}^r - Q_n^r + R_n \end{aligned} \quad (1)$$

The α -service level constraint for period n can be expressed as a lower bound on the non-stockout probability by

$$\Pr\{X_n^s \geq 0\} \geq \alpha. \quad (2)$$

The total cost charged in period n is comprised of fixed and variable production costs and holding costs, and it can be written as

$$K^m \mathbb{1}\{Q_n^m > 0\} + K^r \mathbb{1}\{Q_n^r > 0\} + c^m Q_n^m + c^r Q_n^r + h^s (X_n^s)^+ + h^r (X_n^r)^+. \quad (3)$$

It is important to observe in (3) that the holding costs are charged on on-hand inventories $(X_n^s)^+$ and $(X_n^r)^+$. In what follows, we will have a closer look at the on-hand serviceables and returns. We begin by serviceable inventories. Let us recall that the service quality is imposed by means of an α -service level. This service measure is often used in systems where the service quality is very high; common values are 0.95 and 0.99 (Axsäter, 2007; Silver et al., 1998). Because of this, any feasible production policy that satisfies the α -service level constraint (2) should guarantee that $X_n^s \geq 0$ holds almost always. Hence, it is reasonable to assume that cases with $X_n^s < 0$ are rare events and ignore them in cost computations. Then, we can approximate the on-hand serviceable inventory with the serviceable inventory as $(X_n^s)^+ \approx X_n^s$. This is a common approach in the stochastic lot sizing literature (see e.g. Bookbinder & Tan, 1988; Chen & Krass, 2001; Koca, Yaman, & Ak-türk, 2015; Tarim & Kingsman, 2004; Tempelmeier, 2007) and it is justifiable when the service level α is close to one – as is usually the case, and period demand distributions are light-tailed. We now turn our attention to the return inventories. For any feasible production policy, the remanufacturing quantity should be bounded by the available return inventory. That is $Q_n^r \leq X_{n-1}^r + R_n$. This suggests the return inventory $X_n^r = X_{n-1}^r - Q_n^r + R_n$ is always non-negative. Therefore, the return inventory is equivalent to the on-hand return and we have $(X_n^r)^+ = X_n^r$.

In light of the discussion provided above, we will make use of the approximate total cost expression

$$K^m \mathbb{1}\{Q_n^m > 0\} + K^r \mathbb{1}\{Q_n^r > 0\} + c^m Q_n^m + c^r Q_n^r + h^s X_n^s + h^r X_n^r \quad (4)$$

rather than (3) in the analysis to follow.

3. Heuristic policies

In this section, we introduce two heuristic policies: All-or-Nothing Policy (AoN) and Threshold Policy (THR). Both of these policies are characterized by simple decision rules. They follow a static-dynamic uncertainty strategy where manufacturing and remanufacturing periods are set at the beginning of the planning horizon. Nevertheless, they offer different levels of flexibility in production quantities and differ on how manufacturing and remanufacturing quantities are determined. In what follows, we provide formal definitions of these policies and present some preliminary analysis to set the grounds for the mathematical models to be introduced.

3.1. AoN

In AoN, the size of manufacturing lots are determined at the beginning of the planning horizon, whereas the size of remanufacturing lots are dynamically determined in an all-or-nothing fashion. That is, in each manufacturing period a predetermined amount

of new products are manufactured regardless of the actual inventory levels, and in each remanufacturing period all returns are remanufactured so that the post-production return inventory is reduced to zero. AoN is apt for production systems where a rigid manufacturing schedule is preferred. This is the case for instance in material requirements planning environments. Nevertheless, because AoN disregard actual inventory levels when determining manufacturing lot sizes, it is plausible to expect that it should not be as competitive as more dynamic policies in terms of cost performance.

The parameters of AoN include the following: (i) the set of manufacturing periods $\Pi^m \subseteq [1, N]$ and manufacturing quantities for these periods $q = \{q_n : n \in \Pi^m\}$, and (ii) the set of remanufacturing periods $\Pi^r \subseteq [1, N]$.

Based on the policy parameters, the decision rules for manufacturing and remanufacturing in period n can be expressed as

$$Q_n^m = \begin{cases} q_n & \text{if } n \in \Pi^m \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad Q_n^r = \begin{cases} X_{n-1}^r + R_n & \text{if } n \in \Pi^r \\ 0 & \text{otherwise} \end{cases}. \quad (5)$$

It is easy to observe that the static decision rule for manufacturing leads to deterministic manufacturing quantities. The decision rule for remanufacturing, on the other hand, is dynamic and result in stochastic remanufacturing quantities whose values depend on the actual return inventories.

We now consider the expected values of serviceable and return inventories and manufacturing and remanufacturing quantities. Because the inventory conservation equations (1) as well as the decision rules for manufacturing and remanufacturing (5) are all linear (provided the sets of manufacturing and remanufacturing periods), expected serviceable and return inventories can be written as

$$EX_n^s = EX_{n-1}^s + EQ_n^m + EQ_n^r - ED_n$$

$$EX_n^r = EX_{n-1}^r - EQ_n^r + ER_n$$

where expected manufacturing and remanufacturing quantities are

$$EQ_n^m = \begin{cases} q_n & \text{if } n \in \Pi^m \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad EQ_n^r = \begin{cases} EX_{n-1}^r + ER_n & \text{if } n \in \Pi^r \\ 0 & \text{otherwise} \end{cases}. \quad (6)$$

The analysis presented so far characterizes the expected values of serviceable and return inventories for given policy parameters. However, in order to express α -service constraints, we need to have the complete distributional information of the serviceable inventory level in each period of the planning horizon. To that end, we provide an alternative expression for the serviceable inventory X_n^s based on accumulated manufacturing quantities, demands, and returns.

Let $q_{[1,n]}$ be the accumulated manufacturing quantity over time; i.e. $q_{[1,n]} = \sum_{u \in \Pi^m: u \leq n} q_u$. This value can immediately be derived for the policy parameters. Then, if we sum the serviceable inventory expression in (1) over all periods up to and including period n and substitute (5), we obtain

$$X_n^s = q_{[1,n]} - (D_{[1,n]} - R_{[1,k]}) \quad (7)$$

where $k \in [0, n]$ is the most recent remanufacturing period prior to period n ; i.e. $k := \max\{u \in \Pi^r \cup \{0\} : u \leq n\}$. The case with no remanufacturing period up-to period n is captured by $k = 0$. It should be clear that k depends on n as in $k = (k)_n$, but we shall make the dependency implicit for the sake of notational simplicity.

Fig. 1 graphically illustrates the derivation of (7) by using a network flow representation of an example instance. Here, each period is represented by a serviceable node (below) and a return node (above). The serviceable and return inventories carried from one period to the next are depicted by arcs directed from

Fig. 1. A network illustration of AoN.

left to right (arcs with zero flows are not included). The set of manufacturing periods is $\Pi^m = \{1, 4\}$ and the set of remanufacturing periods is $\Pi^r = \{3, 5\}$. The remanufacturing quantities (in remanufacturing periods) are depicted by an arcs directed from return nodes to serviceable nodes. The manufacturing quantities (in manufacturing periods) are denoted by arcs labeled as q . Finally, demand and return in each period are respectively denoted by arcs labeled as D, R . Let us consider the serviceable inventory at the end of the 8th period X_8^s and recall that the last remanufacturing period prior to the 8th period is the 5th period. That is, $n = 8$ and $k = 5$. The network flow representation makes it easy to see that the flow conservation dictate $X_8^s = (q_1 + q_4) + (R_1 + \dots + R_5) - (D_1 + \dots + D_8)$ which is equivalent to (7).

We now show that (7) can be used to derive a compact expression of α -service level constraints. Let us observe that the α -service level constraint (2) for period n can be written as $\Pr\{q_{[1,n]} \geq D_{[1,n]} - R_{[1,k]}\} \geq \alpha$. We can re-write this as

$$q_{[1,n]} \geq F_{nk}^{-1}(\alpha) \quad (8)$$

where $F_{nk}^{-1}(\cdot)$ is the inverse cumulative distribution function (percentile function) of the random variable $D_{[1,n]} - R_{[1,k]}$. Because $F_{nk}^{-1}(\alpha)$ is a constant, we can say that (8) is an interpretation of the α -service level constraint of period n as an lower bound on the accumulated manufacturing quantity up-to period n .

We now derive an equivalent representation of this constraint that will be useful for the purposes of this study. Because $X_n^s = q_{[1,n]} - (D_{[1,n]} - R_{[1,k]})$, we also have that $EX_n^s = q_{[1,n]} - (ED_{[1,n]} - ER_{[1,k]})$. If we substitute this into (8), we obtain an alternative representation of the service level constraint as

$$EX_n^s \geq F_{nk}^{-1}(\alpha) - (ED_{[1,n]} - ER_{[1,k]}). \quad (9)$$

This inequality expresses the α -service level constraint for period n as a lower bound on the expected serviceable inventory level at the end of period n , and it will be functional in the mathematical model we present in the following section.

3.2. THR

In THR, the size of both manufacturing and remanufacturing lots are dynamically determined following critical threshold values which we refer to as “manufacture-up-to” and “remanufacture-down-to” levels. THR works as follows. In a manufacturing period, if the serviceable inventory is below the manufacture-up-to level, then the serviceable inventory level is raised to the target inventory level by manufacturing. In a remanufacturing period, if the return inventory is above the remanufacture-down-to level, then the return inventory level is decreased to the target level by remanufacturing. If both manufacturing and remanufacturing is scheduled in the same period, then the remanufacturing rule takes precedence over the manufacturing rule. THR’s decision rules are significantly different as compared to AoN’s. First, THR’s decision rule

for manufacturing decision rule is dynamic. Second, THR’s decision rule for remanufacturing is more general as compared to AoN’s. That is, the all-or-nothing rule is nothing but a special case of remanufacture-down-to rule where the target level is zero. Therefore, THR, in general, is more responsive to realizations of stochastic demands and returns.

The parameters of THR include the following: (i) the set of manufacturing periods $\Pi^m \subseteq [1, N]$ and manufacture-up-to levels for these periods $w_n^m = \{w_n^m : n \in \Pi^m\}$, and (ii) the set of remanufacturing periods $\Pi^r \subseteq [1, N]$ and remanufacture-down-to levels for these periods $w_n^r = \{w_n^r : n \in \Pi^r\}$.

Based on the policy parameters, we can express the decision rules for manufacturing and remanufacturing in period n as

$$Q_n^m = \begin{cases} (w_n^m - Q_n^r - X_{n-1}^s)^+ & \text{if } n \in \Pi^m \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad Q_n^r = \begin{cases} (X_{n-1}^r + R_n - w_n^r)^+ & \text{if } n \in \Pi^r \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

We observe that the dynamic manufacturing and remanufacturing decision rules lead to stochastic production quantities whose realizations depend on the actual serviceable and return inventories. Furthermore, in periods where both manufacturing and remanufacturing take place, the manufacturing quantity also depends on the remanufacturing quantity.

It is important to see that the decision rules (10) are not linear, as the non-negativity operator resets possible negative production quantities to zero. This significantly complicates the analysis of expected production quantities. To alleviate this issue, we approach expected production quantities under somewhat restrictive assumptions. We shall assume that the initial serviceable inventory is always lower than the manufacture-up-to level, i.e. $Q_0^r + X_{-1}^s \leq w_0^m$, and the initial return inventory level is always higher than the remanufacture-down-to level, i.e. $X_{-1}^r + R_0 \geq w_0^r$. Despite being restrictive, these assumptions are tenable as it does not make sense to “plan” negative manufacturing and remanufacturing quantities in presence of fixed manufacturing and remanufacturing costs. Hence, one would expect them not to hold very rarely if optimal policy parameters are employed.

The assumptions explained above ensure that production quantities are indeed non-negative, and thereby allow us to drop the non-negativity operators in (10). Then, provided the sets of manufacturing and remanufacturing periods, expected serviceable and return inventories can be written as

$$EX_n^s = EX_{n-1}^s + EQ_n^m + EQ_n^r - ED_n \\ EX_n^r = EX_{n-1}^r - EQ_n^r + ER_n$$

where expected manufacturing and remanufacturing quantities are

$$EQ_n^m = \begin{cases} w_n^m - EQ_n^r - EX_{n-1}^s & \text{if } n \in \Pi^m \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

Fig. 2. A network illustration of THR.

$$EQ_n^r = \begin{cases} EX_{n-1}^r + ER_n - w_n^r & \text{if } n \in \Pi^r \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

We now provide an alternative expression for the serviceable inventory X_n^s at period n that will be functional in establishing the α -service constraint. We begin by defining the following three critical periods: $j \in [0, n]$ is the most recent manufacturing period prior to period n , i.e. $j := \max\{u \in \Pi^m \cup \{0\} : u \leq n\}$; $i \in [0, j]$ is the most recent remanufacturing period prior to period j , i.e. $i := \max\{u \in \Pi^r \cup \{0\} : u \leq j\}$; and $k \in i \cup [j+1, n]$ is the most recent remanufacturing period prior to period n , i.e. $k := \max\{u \in \Pi^r \cup \{0\} : u \leq n\}$. Notice that i, j and k all depend on n as in $(i, j, k)_n$, but for notational simplicity we shall make the dependency implicit.

Let us consider the interval $[j, n]$. Because there is no remanufacturing period over the intervals $[i+1, j]$ and $[k+1, n]$, we know that the total remanufacturing quantity over the interval $[j, n]$ is equal to that of the interval $[i+1, k]$. Due to the fact that the respective post-production return inventory levels at the beginning and at the end of this interval are w_i^r and w_k^r , we have that the total remanufacturing quantity over the interval $[i+1, k]$ is equal to $w_i^r + R_{[i+1, k]} - w_k^r$. Hence, we can say that the total remanufacturing quantity over the interval $[j, n]$ equals $w_i^r + R_{[i+1, k]} - w_k^r$. As for the manufacturing, we know that there is only one manufacturing period over the interval $[j, n]$ and it is the first period of this interval. The post-production serviceable inventory in this manufacturing period is w_j^m , and the total demand over the interval $[j, n]$ is $D_{[j, n]}$. We can thus write the serviceable inventory level at period n as

$$X_n^s = w_j^m + w_i^r - w_k^r - (D_{[j, n]} - R_{[i+1, k]}). \quad (12)$$

Fig. 2 graphically illustrates the derivation of (12) by using a network flow representation of an example instance. This network is essentially the same as the one illustrated in Fig. 1, except here we do not need the manufacturing flows. We are interested in the serviceable inventory at the end of the 8th period X_8^s , while knowing that the post-production return inventories at the 3rd and the 5th periods are w_3^r and w_5^r , respectively; and the post-production serviceable inventory at the 4th period is w_4^s . That is, $n = 8$, $i = 3$, $j = 4$, and $k = 5$. We observe that the flow conservation dictates $X_8^s = w_4^s + w_3^r - w_5^r + (R_4 + R_5) - (D_4 + \dots + D_8)$. This is indeed equivalent to the expression provided in (12).

We now use (12) to derive an expression of α -service level constraints. We can express the α -service level constraint for period n as $\Pr\{w_j^m + w_i^r - w_k^r - (D_{[j, n]} - R_{[i+1, k]}) \geq \alpha\}$. If we let $F_{ijkn}^{-1}(\cdot)$ be the inverse cumulative distribution function of the random variable $D_{[j, n]} - R_{[i+1, k]}$, then we obtain the following α -service level constraint:

$$w_j^m + w_i^r - w_k^r \geq F_{ijkn}^{-1}(\alpha) \quad (13)$$

It is also possible to derive an alternative representation of the α -service level constraint where it is expressed as a lower bound on the expected serviceable inventory level. Because $X_n^s = w_j^m + w_i^r - w_k^r - (D_{[j, n]} - R_{[i+1, k]})$, we have that $EX_n^s = w_j^m + w_i^r -$

$w_k^r - (ED_{[j, n]} - ER_{[i+1, k]})$. If we substitute this into (13), we obtain the following service level constraint:

$$EX_n^s \geq F_{ijkn}^{-1}(\alpha) - (ED_{[j, n]} - ER_{[i+1, k]}) \quad (14)$$

This represents the α -service level constraint for period n as a lower bound on the expected serviceable inventory level at the end of period n . It will be of use in the mathematical model we present in the following section.

4. Models

In this section, we present certainty equivalent MIP models to compute the parameters of AoN and THR. Because the mathematical models of both policies have many common characteristics, we introduce them together and point out their differences as necessary.

We begin by presenting the decision variables we employ. For ease of exposition, we use the same decision variables for the mathematical models of AoN and THR. These are listed as follows:

- y_n^m / y_n^r binary variable that equals 1 if period n is a manufacturing/remanufacturing period, and 0, otherwise
- EQ_n^m / EQ_n^r expected manufacturing/remanufacturing quantity at period n
- EX_n^s / EX_n^r expected serviceable/return inventory level at period n

The decision variables express the timing of manufacturing and remanufacturing periods and expected values of production quantities and inventory levels. Thus, they do not explicitly represent policy parameters. This rather unconventional approach enables us to build compact models for AoN and THR in a unified fashion as we introduce in the following.

We first explain how decision variables are used to characterize the parameters of AoN and THR. Recall that both AoN and THR are defined by fixed sets of manufacturing and remanufacturing periods Π^m and Π^r . These periods are specified by indicator variables y_n^m and y_n^r for each period n . That is $\Pi^m = \{n \in [1, N] : y_n^m = 1\}$ and $\Pi^r = \{n \in [1, N] : y_n^r = 1\}$. AoN is characterized by static manufacturing quantities q_n for manufacturing periods $n \in \Pi^m$. For all manufacturing periods n , the manufacturing quantity amount to its expectation and expressed by $q_n^m = EQ_n^m$. It is easy to observe that this strictly follows (6). THR, on the other hand, is characterized by manufacturing-up-to and remanufacture-down-to levels w_n^m and w_n^r for manufacturing and remanufacturing periods $n \in \Pi^m$ and $n \in \Pi^r$, respectively. For all manufacturing periods n , the manufacturing-up-to level equals the expected post-production serviceable inventory level as $w_n^m = EX_{n-1}^s + EQ_n^m + EQ_n^r$. For all remanufacturing periods n , the remanufacture-down-to level equals the expected post-production return inventory level as $w_n^r = EX_{n-1}^r + ER_n - EQ_n^r$ which also implies $w_n^r = EX_n^r$. Notice that these are in line with (11).

We now introduce the objective function and constraints of AoN and THR models. The objective function is the same in AoN and THR models and it minimizes the total expected cost over the planning horizon which is comprised of fixed and variable production costs and inventory holding costs. We write the objective function building on the single-period cost expression provided in (4). We replace indicator functions by indicator variables for manufacturing and remanufacturing periods. Then, we take the expectation of the single-period cost expression and accumulate it over the planning horizon. This results in the following objective function:

$$\min \sum_{n=1}^N [K^m y_n^m + K^r y_n^r + c^m EQ_n^m + c^r EQ_n^r + h^s EX_n^s + h^r EX_n^r] \quad (15)$$

The conventional inventory conservation constraints for expected serviceable and return inventory levels can be written as follows:

$$EX_n^s = EX_{n-1}^s + EQ_n^m + EQ_n^r - ED_n \quad \forall n \in [1, N] \quad (16)$$

$$EX_n^r = EX_{n-1}^r + ER_n - EQ_n^r \quad \forall n \in [1, N] \quad (17)$$

These constraints characterize the dynamics of expected inventories, and ensure that the expected manufacturing and remanufacturing quantities are in line with the expected serviceable and return inventory levels.

The expected manufacturing and remanufacturing quantities can be non-negative only in manufacturing and remanufacturing periods. We express these conditions as

$$EQ_n^m \leq My_n^m \quad \forall n \in [1, N] \quad (18)$$

$$EQ_n^r \leq My_n^r \quad \forall n \in [1, N] \quad (19)$$

where M is a sufficiently large number.

The constraints presented so far apply to both AoN and THR. We now look at the policy-specific constraints. AoN is characterized by the all-or-nothing property such that if remanufacturing takes place in a given period, then all the return inventory must be remanufactured. This can be guaranteed by means of the following constraint:

$$EX_n^r \leq M(1 - y_n^r) \quad \forall n \in [1, N] \quad (20)$$

We demonstrated in the previous section that under specific conditions the service level constraint can be expressed as a lower bound on the expected serviceable inventory level as in (9) and (14). In the following, we integrate these lower bound expressions of service level constraints into our formulation – while making sure that the associated conditions hold.

Let us begin with AoN. We know that (9) is a valid expression of the service level constraint for period n , if the only remanufacturing period in the interval $[k, n]$ is its first period. Then, conditioning on whether this holds, we can express the service level constraint as

$$EX_n^s \geq \left(F_{kn}^{-1}(\alpha) + ER_{[1,k]} - ED_{[1,n]} \right) \left(y_k^r - \sum_{t=k+1}^n y_t^r \right) \quad \forall k \in [1, n], \forall n \in [1, N]. \quad (21)$$

It is easy to see that (21) is equivalent to (9) except its right-hand-side is multiplied with the term $y_k^r - \sum_{t=k+1}^n y_t^r$. This term takes the value of 1 if the above condition holds, and some value smaller than or equal to 0, otherwise. Because α is, by definition, very close to 1, we can safely assume that $F_{kn}^{-1}(\alpha) + ER_{[1,k]} - ED_{[1,n]}$ is always non-negative. Therefore, the constraint is binding only if the expression equals 1, and if so, it sets an appropriate lower bound on the expected serviceable inventory level.

Next, we turn our attention to THR. We know that (14) is a valid expression of the service level constraint for period n , if the only remanufacturing periods in intervals $[i, j]$ and $[k, n]$ are their first periods, and the only manufacturing period in the interval $[j, n]$ is its first period. Then, conditioning on whether these hold, we can write the service level constraint as

$$EX_n^s \geq \left(F_{ijn}^{-1}(\alpha) + ER_{[i+1,k]} - ED_{[j,n]} \right) \times \left(y_j^m + y_i^r + y_k^r - \sum_{t=j+1}^n y_t^m - \sum_{t=i+1}^j y_t^r - \sum_{t=k+1}^n y_t^r - 2 \right) \quad \forall i \in [1, j], \forall k \in i \cup [j+1, n], \forall j \in [1, n], \forall n \in [1, N]. \quad (22)$$

Note that (21) is equivalent to (9) except its right-hand-side is multiplied with the expression in the parenthesis on the right. Here, the terms $y_i^r - \sum_{t=i+1}^j y_t^r$ and $y_k^r - \sum_{t=k+1}^n y_t^r$ both take the value of 1 if the conditions on remanufacturing periods hold, and the term $y_j^m - \sum_{t=j+1}^n y_t^m$ takes the value of 1 if the condition on manufacturing period holds. Therefore, the expression in the parenthesis equals 1 if all conditions hold, and some arbitrary value smaller than or equal to 0, otherwise. As we know that $F_{ijn}^{-1}(\alpha) + ER_{[i+1,k]} - ED_{[j,n]}$ is always non-negative, the constraint sets an appropriate lower bound on the expected serviceable inventory level if all conditions are hold, and otherwise it becomes redundant.

These last constraints finalize MIP models. For convenience, complete models for AoN and THR are provided below.

min (15)

subject to (16)–(21) (constraints for AoN)

(16)–(19), (22) (constraints for THR)

$$EX_0^s = 0, EX_0^r = 0, y_0^m = 1, y_0^r = 1$$

$$EX_n^s, EX_n^r, EQ_n^m, EQ_n^r \geq 0 \quad \forall n \in [1, N]$$

$$y_n^m, y_n^r \in \{0, 1\} \quad \forall n \in [1, N]$$

In order to deploy the proposed models, α -quantile values for cumulative demand and return distributions should be readily available. There is variety of exact and approximate methods that can be used for the numerical computation of quantiles, yet the applicability of these methods strongly depend on the underlying distribution. We stress that the models presented here are not designed based on any distributional assumption. Therefore, we do not provide a specific method to compute quantiles, and rather refer the reader to the statistical inference literature which provides a general and complete discussion on numerical the computation of quantiles (e.g. Avramidis & Wilson, 1998; David & Nagaraja, 1970).

5. Numerical study

The aim of our numerical study is to investigate the cost performance of AoN and THR policies as well as the computational performance of the associated mathematical models. In what follows, we first provide the experiment design, and then present and discuss our findings.

The design of our numerical study is partly adopted from Schulz (2011), and extended to capture stochastic demands and returns. This is a full factorial design where a variety of factors are taken into account that may have an impact on the performance of the proposed approaches. We generate the test bed as follows. We consider fixed production costs $K^m = \{500, 2000\}$ and $K^r = \{500, 2000\}$; and assume zero variable production costs $c^m = 0$ and $c^r = 0$. We use serviceable holding cost $h^s = 1$ and return holding costs $h^r = \{0.2, 0.5\}$. We assume that period demands and returns are normally distributed with a fixed coefficient of variation over the planning horizon. This enables us to fully characterize demand and return distributions given mean and coefficient of variation of demand and returns. We consider planning horizon lengths $N = \{6, 9, 12, 15, 18\}$ and coefficient of variation values $\rho \in \{0.1, 0.2, 0.3\}$. We draw mean demands from a uniform distribution on the interval $[0, 200]$. We consider two cases for mean returns to reflect relatively low and high return rates where mean returns are drawn from uniform distributions on intervals $[0, 50]$ and $[0, 100]$, respectively. We generate 10 random problem instances for each combination of planning horizon length and return rates. Finally, we consider service levels $\alpha = \{0.95, 0.99\}$. The numerical design described thus far leads to 4800 test instances in total.

For each test instance, we obtain the parameters of AoN and THR by solving the mathematical models presented in the pre-

Table 1
Results of the numerical study.

		AoN					THR					DIF
		MIP			Simulation		MIP			Simulation		
		OBJ	TIME	GAP	SIM	ERR	OBJ	TIME	GAP	SIM	ERR	
N	6	3396.97	0.01	0.00	3396.94 (± 1.46)	0.00	3342.05	0.03	0.00	3342.47 (± 1.15)	0.02	2.16
	9	4957.30	0.02	0.00	4957.24 (± 2.34)	0.00	4813.09	0.20	0.00	4813.93 (± 1.54)	0.02	3.61
	12	7129.29	0.08	0.00	7129.21 (± 3.66)	0.00	6774.28	2.15	0.00	6775.62 (± 1.84)	0.03	6.04
	15	8557.58	0.29	0.00	8557.35 (± 4.83)	0.00	8057.79	14.61	0.00	8061.2 (± 2.35)	0.06	6.96
	18	10986.83	1.50	0.00	10986.71 (± 7.04)	0.00	10055.47	188.74	0.21	10060.43 (± 2.67)	0.06	10.29
K^m	500	5308.18	0.52	0.00	5308.12 (± 3.85)	0.00	4785.13	60.25	0.08	4789.44 (± 1.37)	0.07	9.14
	2000	8703.00	0.24	0.00	8702.86 (± 3.88)	0.00	8431.94	22.05	0.00	8432.01 (± 2.44)	0.00	2.49
K^r	500	6546.14	0.65	0.00	6546.05 (± 3.92)	0.00	6152.68	76.14	0.08	6155.80 (± 2.03)	0.06	6.02
	2000	7465.05	0.11	0.00	7464.92 (± 3.81)	0.00	7064.39	6.16	0.00	7065.66 (± 1.79)	0.02	5.60
h^r	0.2	6709.37	0.38	0.00	6709.31 (± 3.82)	0.00	6304.93	38.34	0.04	6306.65 (± 1.84)	0.03	6.26
	0.5	7301.81	0.38	0.00	7301.67 (± 3.91)	0.00	6912.14	43.96	0.04	6914.81 (± 1.97)	0.04	5.36
α	0.95	6732.49	0.32	0.00	6732.40 (± 3.87)	0.00	6408.40	24.61	0.00	6409.93 (± 1.96)	0.03	4.94
	0.99	7278.69	0.44	0.00	7278.58 (± 3.86)	0.00	6808.67	57.68	0.08	6811.53 (± 1.86)	0.05	6.69
ρ	0.1	6209.20	0.19	0.00	6209.21 (± 1.94)	0.00	6013.08	6.31	0.00	6013.11 (± 1.05)	0.00	3.26
	0.2	7006.12	0.38	0.00	7006.04 (± 3.87)	0.00	6617.60	30.48	0.00	6618.53 (± 1.94)	0.02	5.85
	0.3	7801.46	0.57	0.00	7801.22 (± 5.79)	0.00	7194.92	86.65	0.13	7200.54 (± 2.74)	0.09	8.33
ER	$U(0, 25)$	6876.43	0.36	0.00	6876.25 (± 3.77)	0.00	6480.11	35.11	0.03	6480.91 (± 1.74)	0.02	6.00
	$U(0, 50)$	7134.76	0.40	0.00	7134.72 (± 3.96)	0.00	6736.96	47.18	0.06	6740.55 (± 2.08)	0.06	5.63
	Average	7005.59	0.38	0.00	7005.49 (± 3.86)	0.00	6608.53	41.15	0.04	6610.73 (± 1.91)	0.04	5.81

vious section. As discussed earlier, AoN and THR models assume that the effect of stockouts on expected inventory levels is negligible, and the THR model further assumes that post-production serviceable and return inventory levels in manufacturing and remanufacturing periods are always equal to the respective manufacturing-up-to and remanufacturing-down-to levels. Due to these assumptions, these models are not exact. As such, their objective values do not necessarily match with the true expected costs of the policies they provide. We assess the error due to the aforementioned assumptions by simulating the policies provided by AoN and THR models. We conduct 10^6 simulation runs for both policies. We run all numerical experiments on a 3.10 gigahertz Intel Xeon CPU with 8 gigabytes RAM. We use Gurobi v7.01 as a MIP solver and set a computational time limit of half an hour per instance.

Table 1 presents the results on the mathematical models and the simulations for AoN and THR. The statistics reported on mathematical model are as follows; OBJ is the objective value of the best solution obtained by the model, TIME the solution time in seconds, and GAP is the relative optimality gap at termination. The statistics reported on the simulation are the following; SIM is the simulated cost (of the best policy obtained by the model) and its 95% confidence interval, and ERR is the relative difference between the objective value of the model and the simulated cost. Finally, DIF is the relative difference between the simulated costs of AoN and THR policies. As we have 4800 test instances in total, it is not viable to present the aforementioned statistics for each test instance. The statistics provided in Table 1 are therefore average values over all instances identified by the same pivot parameter.

The results we obtained from the numerical study are summarized in the following. We first concentrate on the reliability of AoN and THR models. We observe that the error due to the restrictive assumptions of the mathematical models is very small. ERR is less than 0.005% for AoN and 0.04% for THR. For AoN, we see that OBJ is always within the SIM confidence interval. This shows that the effect of neglecting expected stockouts is so small that it cannot even be observed in the simulation study. For THR, on the other hand, OBJ may not fall into the SIM confidence interval. As such, the simulation study validates the error due to the assumption that post-production serviceable and return inventories

always match manufacturing-up-to and remanufacturing-down-to levels. Nevertheless, it also shows that the effect of this assumption is rather limited as ERR is very small. For THR, we notice that ERR is particularly larger for problem instances characterized by longer planning horizons, lower fixed production costs, higher levels of demand uncertainty, and higher return rates.

Next, we now turn our attention to the computational performance of AoN and THR models. Here, we observe that the computational performance of the AoN model is better than the THR model. TIME is significantly lower for the AoN model as compared to the THR model. For instance, if we consider problem instances with planning horizons of 18 time periods, TIME is 1.50 seconds for the AoN model whereas it is 188.74 seconds for the THR model. The AoN model solves all instances to optimality within the time limit. Hence, its GAP is zero. The THR model, on the other hand, fails to solve 22 instances out of 4800. Nevertheless, it averages a GAP of only 0.04%. As expected, the main determinant of TIME is length of the planning horizon. But, for both AoN and THR models, we also see that TIME is higher for problem instances with lower fixed production costs and higher demand and return uncertainty.

Finally, we discuss the cost performance of AoN and THR. The results clearly demonstrate that AoN is significantly more expensive than THR. The average DIF is 5.81% over all instances. This is not unexpected since AoN is less responsive to the realizations of random demands and returns as compared to THR. Nevertheless, it is possible to observe that DIF is not uniform over different parameter settings. We particularly observe that DIF is increasing on the length of the planning horizon. Also, DIF is higher for problem instances characterized by lower fixed production costs and higher demand and return uncertainty.

6. Conclusion

In this paper, we addressed SELSR – a prominent inventory control problem that emerges in hybrid production systems. Because the problem is beyond being tractable, we proposed two heuristic policies. These policies are characterized by simple decision rules to control manufacturing and remanufacturing operations. They both follow a static-dynamic uncertainty strategy where manufac-

turing and remanufacturing periods are set at the beginning of the planning horizon. Yet, they offer different levels of flexibility in production quantities and differ on how manufacturing and remanufacturing lot sizes are determined. We present deterministic equivalent MIP models of the proposed policies which can be solved by off-the-shelf commercial solvers.

The work presented in this study can be regarded as an initial attempt towards SELSR. There are many avenues for further research. The problem can be addressed by considering more sophisticated control policies. Because the problem is NP-hard even when demands and returns are deterministic, a tractable optimal solution approach appears to be out of reach. Nevertheless, it could be possible to design dynamic policies that make more effective use of realized demand and return information, thereby offering a better cost performance. It is also important to consider production lead times which were neglected in this work. Our models and methods can easily be extended to systems with positive but identical manufacturing and remanufacturing lead times by replacing the inventory level with the inventory position. In this context, lead times affect policy performance as does the demand uncertainty, and it turned out that the relative performance of a policy can be significantly correlated with the extent of demand uncertainty. Therefore, it appears to be a challenging yet relevant research objective to address systems with arbitrary manufacturing and remanufacturing lead times. Another research direction is to consider variants of the problem characterized by different service level measures and/or stockout penalty costs. Also, the problem can be investigated under the assumption that stockouts are lost rather than backordered.

Acknowledgments

We thank the editor and three anonymous reviewers for their constructive feedback on earlier drafts of the manuscript. Onur A. Kilic and Huseyin Tunc are supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under Grant No. MAG-114M389.

References

- Ahiska, S. S., & King, R. E. (2010a). Inventory optimization in a one product recoverable manufacturing system. *International Journal of Production Economics*, 124(1), 11–19.
- Ahiska, S. S., & King, R. E. (2010b). Life cycle inventory policy characterizations for a single-product recoverable system. *International Journal of Production Economics*, 124(1), 51–61.
- Avramidis, A. N., & Wilson, J. R. (1998). Correlation-induction techniques for estimating quantiles in simulation experiments. *Operations Research*, 46(4), 574–591.
- Axsäter, S. (2007). *Inventory control*: 90. Springer Science & Business Media.
- Baki, M. F., Chaouch, B. A., & Abdul-Kader, W. (2014). A heuristic solution procedure for the dynamic lot sizing problem with remanufacturing and product recovery. *Computers & Operations Research*, 43, 225–236.
- Benedito, E., & Corominas, A. (2013). Optimal manufacturing policy in a reverse logistic system with dependent stochastic returns and limited capacities. *International Journal of Production Research*, 51(1), 189–201.
- Bookbinder, J. H., & Tan, J. Y. (1988). Strategies for the probabilistic lot-sizing problem with service-level constraints. *Management Science*, 34(9), 1096–1108.
- Chen, F. Y., & Krass, D. (2001). Inventory models with minimal service level constraints. *European Journal of Operational Research*, 134(1), 120–140.
- David, H. A., & Nagaraja, H. N. (1970). *Order statistics*. Wiley Online Library.
- Ferguson, M. E., & Toktay, L. B. (2006). The effect of competition on recovery strategies. *Production and operations management*, 15(3), 351.
- Golany, B., Yang, J., & Gang, Y. (2001). Economic lot-sizing with remanufacturing options. *IIE Transactions*, 33(11), 995–1003.
- Hilger, T., Sahling, F., & Tempelmeier, H. (2016). Capacitated dynamic production and remanufacturing planning under demand and return uncertainty. *OR Spectrum*, 38(4), 849–876.
- Kelle, P., & Silver, E. A. (1989). Purchasing policy of new containers considering the random returns of previously issued containers. *IIE Transactions*, 21(4), 349–354.
- Kiesmuller, G. P. (2003). A new approach for controlling a hybrid stochastic manufacturing/remanufacturing system with inventories and different leadtimes. *European Journal of Operational Research*, 147(1), 62–71.
- Kiesmuller, G. P., & Scherer, C. W. (2003). Computational issues in a stochastic finite horizon one product recovery inventory model. *European Journal of Operational Research*, 146(3), 553–579.
- Kilic, O. A. (2013). A MIP-based heuristic for the stochastic economic lot sizing problem with remanufacturing. In *IFAC Proceedings Volumes*: 46(9), pp. 742–747.
- Kingsman, B. G. (1985). *Raw materials purchasing: An operational research approach*: 4 (1st). Pergamon Press.
- Koca, E., Yaman, H., & Aktürk, M. S. (2015). Stochastic lot sizing problem with controllable processing times. *Omega*, 53, 1–10.
- Li, C., Liu, F., Cao, H., & Wang, Q. (2009). A stochastic dynamic programming based model for uncertain production planning of re-manufacturing system. *International Journal of Production Research*, 47(13), 3657–3668.
- Li, X., Baki, F., Tian, P., & Chaouch, B. A. (2014). A robust block-chain based tabu search algorithm for the dynamic lot sizing problem with product returns and remanufacturing. *Omega*, 42(1), 75–87.
- Mahadevan, B., Pyke, D. F., & Fleischmann, M. (2003). Periodic review, push inventory policies for remanufacturing. *European Journal of Operational Research*, 151(3), 536–551.
- Mutlu, F., Cetinkaya, S., & Bookbinder, J. H. (2010). An analytical model for computing the optimal time-and-quantity-based policy for consolidated shipments. *IIE Transactions*, 42, 367–377.
- Pan, Z., Tang, J., & Liu, O. (2009). Capacitated dynamic lot sizing problems in closed-loop supply chain. *European Journal of Operational Research*, 198(3), 810–821.
- Panagiotidou, S., Nenesis, G., Zikopoulos, C., & Tagaras, G. (2017). Joint optimization of manufacturing/remanufacturing lot sizes under imperfect information on returns quality. *European Journal of Operational Research*, 258(2), 537–551.
- Parsopoulos, K. E., Konstantaras, I., & Skouri, K. (2015). Metaheuristic optimization for the single-item dynamic lot sizing problem with returns and remanufacturing. *Computers & Industrial Engineering*, 83, 307–315.
- Pineyro, P., & Viera, O. (2009). Inventory policies for the economic lot-sizing problem with remanufacturing and final disposal options. *Journal of Industrial and Management Optimization*, 5, 217–238.
- Retel Helmrich, M. J., Jans, R., van den Heuvel, W., & Wagelmans, A. P. (2014). Economic lot-sizing with remanufacturing: complexity and efficient formulations. *IIE Transactions*, 46(1), 67–86.
- Schulz, T. (2011). A new silver-meal based heuristic for the single-item dynamic lot sizing problem with returns and remanufacturing. *International Journal of Production Research*, 49(9), 2519–2533.
- Sifaleras, A., Konstantaras, I., & Mladenović, N. (2015). Variable neighborhood search for the economic lot sizing problem with product returns and recovery. *International Journal of Production Economics*, 160, 133–143.
- Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling* (3rd). Wiley.
- Tarim, S. A., & Kingsman, B. G. (2004). The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. *International Journal of Production Economics*, 88(1), 105–119.
- Tempelmeier, H. (2007). On the stochastic uncapacitated dynamic single-item lot-sizing problem with service level constraints. *European Journal of Operational Research*, 181(1), 184–194.
- Teunter, R. H., Bayindir, Z. P., & Van Den Heuvel, W. (2006). Dynamic lot sizing with product returns and remanufacturing. *International Journal of Production Research*, 44(20), 4377–4400.
- Toktay, L. B., Wein, L. M., & Zenios, S. A. (2000). Inventory management of remanufacturable products. *Management Science*, 46(11), 1412–1426.
- Tunc, H., Kilic, O. A., Tarim, S. A., & Eksioğlu, B. (2013). A simple approach for assessing the cost of system nervousness. *International Journal of Production Economics*, 141(2), 619–625.
- Van Der Laan, E., Salomon, M., Dekker, R., & Van Wassenhove, L. (1999). Inventory control in hybrid systems with remanufacturing. *Management Science*, 45(5), 733–747.
- Van Der Laan, E. A., & Teunter, R. H. (2006). Simple heuristics for push and pull remanufacturing policies. *European Journal of Operational Research*, 175(2), 1084–1102.
- Vlachos, D., & Dekker, R. (2003). Return handling options and order quantities for single period products. *European Journal of Operational Research*, 151(1), 38–52.
- Yang, J., Golany, B., & Yu, G. (2005). A concave-cost production planning problem with remanufacturing options. *Naval Research Logistics*, 52(5), 443–458.