Comments on ‘Dynamic Network Actor Models’

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For the analysis of longitudinal network data, important dichotomies are between state and event data; between directed and non-directed relations; and between tie-oriented and actor-oriented models. I congratulate Christoph Stadtfeld, James Hollway, and Per Block on having given us an important and practically useful methodology (Stadtfeld et al., 2017) combining state and event data for actor-oriented modeling of non-directed relations. They propose a fine way for getting maximum likelihood estimates. In my comment I give my view on the position of this new model with respect to the earlier literature, also indicating alternatives for the choice model; I stress the importance of specifying non-constant rate functions; and I give suggestions for how to deal with unknown time orderings.

1 The position in the bestiary of models

For my understanding of the paper, I found it helpful to consider the proximate literature, and assess the position of this model in relation to other publications for statistical dynamic network models.

The Dynamic Network Actor Model (DyNAM) introduced here is a model for a network as a changing state in a continuous-time framework. This is like the Stochastic Actor-oriented Model of Snijders (2001), the Longitudinal

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Exponential Random Graph Model of Koskinen and Snijders (2013), and the Stochastic Actor-oriented Model for non-directed relations of Snijders and Pickup (2016). It can be regarded as a boundary case of the Temporal Exponential Random Graph Model of Hanneke et al. (2010), viz., when this is applied with only one tie difference between consecutive observations.

The DyNAM is a model for network events, like the Relational Event Model of Butts (2008), the model for event streams of Stadtfeld and Geyer-Schulz (2011), the point process model for networks of Perry and Wolfe (2013), and the network event model of Lerner et al. (2013). These network event models focus on the events, with the network giving the structure for the events, and with various different ways of using past network events to determine current event probabilities. The DyNAM has a close combination between the network state and the events in the sense that the events totally determine the state. There is almost a one-to-one correspondence between events and state changes, with the exception that events \( i \rightarrow j \) may occur for ties that already exist, and then the tie will not change. The other mentioned event models are less clear and more general about the link between events and state.

The DyNAM as presented in this paper has the additional property that it is designed for non-directed relations, and ‘yet’ is actor-oriented. The actor-oriented approach comes naturally for directed relations, with actor \( i \) controlling the tie \( i \rightarrow j \), whereas for non-directed relations either a tie-oriented approach is to be followed or something has to be specified for the coordination between both actors involved. The latter approach is followed here, elaborating the game-theoretical idea of Jackson and Wolinsky (1996) that both actors should agree on the creation of the tie. Dissolution of ties could be one-sided, which is mentioned in the paper but not elaborated. It is not much of a problem that tie dissolution is not considered here. The dissolution of ties that require coordination between actors, such as bilateral treaties, will be subject to quite different rules than their creation, so in practice creation and dissolution would have to be modeled distinctly.

2 Modeling coordination

There are potentially a large variety of social situations where actors agree to make a match, and correspondingly a large variety of ways to model this. This
issue is not problematized in this paper, and step 4 in Figure 1 is simply stated: both actors choose their preferred partner out of the \( N - 1 \) alters available, and if they happen to choose each other the match is made. This multinomial-multinomial combination leads to the rates given in equation (4).

The coordination between the two actors may be achieved also in different ways. In the actor-oriented model for network dynamics for symmetric panel data, Snijders and Pickup (2016) presented several different options for this coordination. This includes a multinomial-binomial choice, where actor \( i \) chooses one out of \( N - 1 \), and the chosen \( j \) is faced with an accept-reject choice. An issue with the combination of a multinomial and a binomial choice model, brought to my attention by Christian Steglich, is that the objective functions for binomial and multinomial choice could be different, and it might be advisable to include an offset in the objective function for the binomial choice to represent the difference in the choice situations. A different possibility, still myopic but less strongly so, is a one-step-ahead expected utility rule, suggested to me by Vincent Buskens, where actor \( i \) makes the multinomial choice based on the expected objective function after both choices will have been made, i.e., taking into account the probability that \( j \) will decline the invitation.

Which model is best to represent the coordination between the two actors in creating a tie will depend on the social situation that is defined by the network dynamics, and will influence the parameter interpretation. For the purpose of this paper such a model needs to be phrased as a statistical model; but the rich literature of game-theoretical approaches to this topic (e.g., Jackson and Watts, 2002; Jackson, 2008; Goyal, 2007) may also provide inspiration and give hints with respect to the type of distinctions that could be made. One aspect of the social situation is the exclusiveness of the tie, which may define a vacancy chain. Exclusiveness is quite different for romantic relations (even if not entirely monogamous as in the example for the Add Health data) than for trade partners. Another aspect is the time frame for preparing the tie, where marriages and bilateral treaties between countries could be somewhat similar, and contrast with high school dating.
3 Rate functions

The DyNAM follows the Stochastic Actor-oriented Models for panel data that I started to develop for directed binary networks in Snijders (2001), with the non-directed version presented in Snijders and Pickup (2016), and the event data for directed networks presented by Perry and Wolfe (2013), in building the event rates from the combination of a rate function and an objective function. The rate functions govern the frequency with which actors make choices, while the objective functions govern the probabilities for actors to choose specific other actors for creating or terminating ties. In the DyNAM, the rate is even further removed from the observed frequency of events per actor than for actor-oriented models, because the large majority of choices will not lead to an observed event. This is because there is a large probability that no match is concluded between $i$ and $j$. Mathematically, this is expressed by the fact that

$$\sum_{j:j \neq i} p_{i \rightarrow j}(x, \beta) p_{j \rightarrow i}(x, \beta)$$

in (4) will be much smaller than 1. Note that

$$\sum_{j=1}^{N} p_{i \rightarrow j}(x, \beta) = 1;$$

it is the multiplication of the probabilities from both sides that leads to the difficulties of coordination.

In the development of the Stochastic Actor-oriented Model since 2001, I have followed the line that the specification of the rate function could remain quite simple, because the model was designed for network panel data with –usually– large differences between consecutive observations, and without information about the timing of changes of individual ties, so that little information is available for estimating detailed properties of the rate function. For network event data, however, the case is different. They contain more detailed information about event rates, even though only the successful choices are individually observed. Further, the DyNAM is meant to be applicable for networks of corporate actors which often have important differences of some measure of size, where ‘size’ is a vague term referring to actor characteristics such as importance, status, and visibility. In applications of the Stochastic Actor-oriented Model to such networks I have often found that it was useful to
make rate of change depend on outdegree, more so than on other actor characteristics. I interpret this not as outdegree being the best operationalization of whatever is meant by ‘size’, but as outdegree being a primary expression of the importance of the network for the actors, of the resources they use for establishing a good network for themselves, and hence as the frequency with which they make new choices to try and improve their network.

When the DyNAM is applied to networks of companies, countries, or other corporate actors, I expect that to obtain a well-fitting model it usually will be important to employ a non-constant rate function depending on outdegrees, or on other measures reflecting the number of profitable new ties that still lie waiting.

4 Unknown time orderings

In some cases time orderings are not known exactly, but only categorized in intervals; in other words, time information is censored. In the example of romantic relationships this was because the events were grouped by week. The DyNAM with censored time information can be seen as network panel data with a large number of waves. Thus there is only a gradual difference between network panel data and the DyNAM.

This issue was solved here by analyzing randomized time orderings, with a sensitivity analysis of the results; implicitly assuming that the precise time ordering is not very important. An alternative is to use one of the ways offered by statistical methodology for treating missing data. One approach is a model-based analysis, similar to the maximum likelihood estimation for panel data of Snijders et al. (2010), using the missing data principle of Orchard and Woodbury (1972). Another approach is multiple imputation (Rubin, 1987). For both approaches, if the time categories are narrow so that there are only small groups of events with unknown time ordering, it still would be possible to sample randomly with equal probabilities from the non-observed orderings, and calculate the log-likelihood of each simulated ordering from the DyNAM probabilities. These probabilities then can be used as weights; for the model-based estimation one of the maximum likelihood algorithms presented in McCulloch (1997) could be used, but without the random effects interpretation; for the estimation by multiple imputation, weighted versions of ‘Rubin’s rules’ would be used as in Carpenter et al. (2007, equations 5-6), but without their missing-not-at-random interpretation.
For some applications the exact time ordering will not be important and taking this detailed approach would be superfluous, and might even incur the risk of capitalizing on irrelevant aspects of the model-data combination. For example, bilateral treaties will take a long time to prepare, and the precise date of their inauguration may be quite unimportant, certainly if other actors are informed about the preparations. For some other applications, e.g., for processes that may be regarded as vacancy chains, there may be important differences between probabilities of the unknown time orderings, and an approach along these lines might then be an improvement.

5 Conclusion

Quantitative sociological methodology progresses by the development of models and methods to analyze essential aspects of social processes, and by the construction of good software and application to rich data sets. This paper has given us an important new model, extending the possibilities for longitudinal network analysis for temporally fine-grained event data; with the associated software. I look forward to further applications of this important method.

6 References


