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Condition-based maintenance for systems with economic dependence and load sharing

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ABSTRACT

Redundancy is often essential for achieving high system availability. An additional benefit of installing redundant components is that the total system load can be shared among components, thus preventing fast deterioration. On the one hand, this provides an incentive to replace failed components as soon as possible, as a component failure increases the load on the remaining components. On the other hand, however, redundancy gives rise to maintenance clustering and postponement opportunities, to reduce the maintenance frequency and thereby lower downtime and maintenance set-up costs. To date, this trade-off has not been investigated under a condition-based maintenance regime. To address this, we consider a parallel system that is subject to both failure dependence through load sharing and economic dependence through maintenance set-up costs. We formulate our system as a Markov Decision Process, and obtain the optimal replacement decisions that minimize the long-run average cost per time unit. Through a numerical investigation and a sensitivity analysis, in which we vary the degree of load sharing, and the maintenance set-up cost, and the degradation process, we obtain key insights into the optimal policy structure. Standard threshold policies, that replace a component as soon as its deterioration exceeds a certain threshold, can be far from optimal, while ignoring or misinterpreting the load sharing effects between components can also lead to a significantly more expensive maintenance policy.

1. Introduction

Maintenance costs can constitute a large part of the total production costs. For example, Gräber (2004) states that up to 30 percent of the production costs for power plants is spent on maintenance, while Bevilacqua and Braglia (2000) state that the maintenance cost can represent between 15 and 70 percent of the total production costs. For manufacturing firms, Mobley (2002) confirms that maintenance costs form up to 60 percent of the production costs, and that up to a third of these costs may be due to unnecessary or poorly executed maintenance.

For this reason, much research has been performed in the field of maintenance strategies over the past decades, of which reviews have been written by McCall (1965), Wang (2002), Van Horenbeek et al. (2013), and Olde Keizer et al. (2017a). Whereas a corrective maintenance strategy (where maintenance is performed upon failure) results in high failure costs, and preventive maintenance strategies (upon reaching

a prespecified age or with a certain periodicity) result in high maintenance costs, Condition-Based Maintenance (CBM) aims to balance and simultaneously minimize these costs (Camci, 2009). In addition, CBM increases the productivity, availability, and system safety by reducing the number of failures (Rao, 1996; Camci, 2009). When applying CBM, maintenance actions are based on condition measurements, such as vibration or temperature readings (Wang et al., 2007).

For a single-component system, CBM is often successfully implemented through a deterioration threshold; upon reaching this threshold, a maintenance action is initiated. Such a strategy may however not result in a close-to-optimal solution for multi-component systems, because several dependencies can exist between these components. According to Thomas (1986), three types of dependencies can be distinguished: economic dependence, structural dependence, and failure dependence. Whereas economic dependence is often present in the form of a fixed maintenance set-up cost (which means that combining, or clustering,

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maintenance actions can be profitable), structural dependence means that some (working) components need to be dismantled to maintain a failed component. In addition, failure dependence can arise when the system load is shared among the components, and the resulting component load affects the deterioration speed. The failure of a component then increases the load on the remaining components, and thus leads to faster deterioration. A system can also incorporate redundancy, by installing more components than strictly necessary. In such cases, the performance of the system depends on the states of all components (Zhou et al., 2013).

This research is inspired by the following real-life example that we encountered at a gas company that pumps up and distributes gas. Storage options for gas are very limited, so the company must continuously produce gas to meet demand from both companies and households. To ensure a high availability of the pumps, redundant components are included for the most critical and failure-prone components, which are placed in a parallel setting. An additional benefit of installing redundant components is that the load (the amount of gas to be distributed) can be shared, thus reducing the load on each individual component, thereby implying a lower failure rate. On the one hand, this provides an incentive to replace failed components as soon as possible. On the other hand, high maintenance set-up costs are often involved when replacing a component. Maintenance actions including (set-up) safety precautions usually take a few days and happen several times per year. The economic dependence on a parallel system indicates that postponing and clustering corrective maintenance on a failed component can be profitable, rather than performing immediate corrective maintenance. To date, this trade-off has not been investigated when it is possible to monitor component conditions (i.e., CBM). Our research is the first to combine the system elements of redundancy, load sharing and economic dependence for CBM systems. Next, we will discuss the state-of-the-art for these separate elements, and then point out our exact contribution.

CBM has been considered by others for systems with redundancy. Most authors consider systems where one or more components are placed in parallel. Extra components can either serve as cold or warm standbys. Wang et al. (2009) and Tan et al. (2011) consider cold standby systems, where a component on standby does not deteriorate and there is no economic dependence with the operating component. Koochaki et al. (2012) and Olde Keizer and Teunter (2015) consider warm standby systems where there is economic dependence, and they therefore propose and analyze opportunistic maintenance policies. The cold or warm standby systems in these studies can be interpreted as 1-out-of- N systems, where the system operates as long as at least one of the parallel components is functional. Lu and Jiang (2007), Huynh et al. (2013, 2015), and Olde Keizer et al. (2016) study the more general k -out-of- N system. However, none of the contributions on CBM systems with redundancy consider load sharing.

Load sharing has been considered for CBM systems by quite a few authors, but without (a clear focus on) redundancy. Some consider series systems, which by nature do not have redundancy built in as all components need to function in order for the asset to function. Do et al. (2015), for instance, determine optimal inspection intervals for a two-component series system where degradation of one component affects the wear rate of the other. Other authors have considered systems with parallel elements (besides series elements), more closely related to redundancy. Marseguer et al. (2002) consider complex multi-component system with both series and parallel elements. However, maintenance decisions for a component are based on just its own degradation level and not those of other components, despite the presence of load sharing. Liang and Parlikad (2015) also consider complex systems, where some critical items (such as wires in a bundle) can be considered as being in parallel, although more than one may be needed for the system to function. Correspondingly, maintenance is always performed directly when deterioration is noted upon inspection, and in this respect the system configuration is not utilized to optimize maintenance decisions. Zhang et al. (2014) do consider optimizing maintenance decisions under economic dependence (through a shared maintenance

setup cost) for multi-component systems, where components can be in parallel. However, different from a system with redundancy, they do not include a cost for a system failure if all components fail. Rasmekomen and Parlikad (2016) consider a system with two components (which are gas tubes), where the system is still operative but performing at a lower level when one of the components fails. So, the redundancy is only partial. Correspondingly, they assume that failed components must be repaired immediately, which does not apply to a system with redundancy.

Besides the before-mentioned contributions of Zhang et al. (2014) and Do et al. (2015), many others have considered economically dependent maintenance decisions for multi-component systems, considering series systems, parallel systems and mixtures. Barbera et al. (1999) and Castanier et al. (2005), for instance, find optimal decisions for two-component series system. As explained above, however, such series systems do not deal with redundancy at all since all components need to function in order for the asset to function. Li et al. (2016) and Olde Keizer and Teunter (2015) do consider parallel systems, and Zhou et al. (2013) study a system with components both in series and in parallel. So they do consider (1-out-of- N) redundancy for systems with economic dependence, but they do not include load sharing. The same holds, to the best of our knowledge, for all other studies on CBM systems with economic dependence, where we refer interested readers to Olde Keizer et al. (2017a) for a recent review of such studies.

As our review of redundancy, load sharing and economic dependence for CBM systems indicates, the separate elements have been studied well, but combinations much less so. In particular, although load sharing and economic dependence have been considered for complex multi-component systems with some components in parallel, those systems did not have (completely) redundant components. The main objective of this research is to generate insights into the structure of the optimal CBM policy for systems with redundancy, load sharing and economic dependence. As this research is about exploring the structure of the optimal maintenance policy, we do not restrict our attention to policies of a certain threshold type in our analysis. Instead, any possible maintenance action, i.e. replacing one or more components, is allowed for each possible state of the system. In the next section, we explain how this is done by formulating the system as a Markov Decision Process. This leads to several insights into the structure of the optimal policy. In particular, we show that intuitively appealing threshold-type policies are not necessarily optimal. Moreover, we show that such threshold-type policies can be far (more than 10%) from optimal. Further insights are provided into the circumstances for which the optimality gap is particularly large.

The remainder of this paper is organized as follows. The system is described in Section 2. Section 3 provides the Markov Decision Process formulation of our model, after which numerical experiments (including a sensitivity analysis) are performed in Section 4. Section 5 concludes the paper.

2. System description

2.1. Deterioration model

We consider a discrete-time system consisting of N identical components, which are subject to economic dependence and load sharing. The components are placed in a parallel setting, which means that the system functions as long as at least one component functions (properly). We consider warm-standby redundancy, i.e., all non-failed components are functioning and subject to deterioration. Let x_j denote the (discrete) state of component j , and let L denote the fixed failure level of a component. If component j is in state 0, i.e., $x_j = 0$, component j is as-good-as-new, while component j has failed if $x_j \geq L$. A replacement, which is assumed to be instantaneous, can be performed at the start of any time unit. In our numerical investigation in the next section, we will mainly use the Poisson distribution to model the deterioration processes as is

done in, e.g., Barbra et al. (1999) and Olde Keizer et al. (2016). However, we will also consider the binomial distribution as this allows us to consider degradation processes with varying degrees of variation.

Neglecting the duration of maintenance operations is justified if maintenance actions take days (weeks) whereas there are months (years) between successive maintenance actions. This applies to many real-life cases, such as our motivating case (discussed in the introduction), where it takes a few weeks to maintain a gas pump and the maintenance frequency is less than once per year.

2.2. Maintenance actions and corresponding costs

The state of each component is known at the start of each time unit. The condition may be monitored based on visual inspection or using a hand-held device that reads input from built-in sensors. Many companies still use such devices rather than monitoring the condition continuously using WiFi sensors, as the electronic currents of WiFi sensors may interfere with operations (and not be allowed for safety reasons in e.g. the process industries). In case all components have failed, the system is shut down and a penalty cost p is incurred. Next, replacements can be performed. A preventive replacement, on a component that still functions, costs c_p , while a corrective replacement, on a failed component, incurs a cost c_c . Generally, replacing a failed component is more expensive than performing a preventive replacement, i.e., $c_c \geq c_p$. We note that the purchase value of the new part is included in both c_c and c_p . Although this value is the same for both types of maintenance actions, it should be included since the maintenance strategy affects the number of maintenance actions (per time unit). The economic dependence is included in the form of a fixed set-up cost for maintenance c_s . This set-up cost needs to be paid once if at least one component is replaced (either preventively or correctively). This means that the higher this set-up cost for maintenance, the stronger the economic dependence.

As this research is about exploring the structure of the optimal maintenance policy, we do not restrict our attention to policies of a certain (threshold) type. Instead, any possible maintenance action (i.e. replacing one or more components) is allowed for each possible state of the system. In the next section, we will explain how this is done by formulating the system as a Markov Decision Process.

2.3. Load sharing

If the system consists of a single functioning component, then we assume that this component deteriorates with rate μ . However, we incorporate redundancy in our model by including extra components. Besides reducing the probability of a system failure, these components allow the total system load to be shared to a certain extent, thus leading to slower deterioration. To incorporate this, we apply the so-called redundant dependency as defined in Yu et al. (2007), by setting the failure rate with k functioning components to $\mu \cdot g(k)$, where $g(k)$ is defined as follows:

$$g(k) = \left(\frac{1}{k}\right)^c, \quad \text{for } k \geq 1,$$

where c can be interpreted as the load sharing factor. Remark that Yu et al. (2007) use the term failure dependence or redundant dependence rather than load sharing. Note that if $k = 1$, only one component is functioning, which is subject to its nominal failure rate μ as $g(1) = 1$. A higher value of c means that redundant components benefit more from sharing the load, and thus are subject to a lower deterioration rate (if $k \geq 2$). According to Yu et al. (2007), some special cases exist:

- $c = 0$: (no load sharing)
No load sharing exists between the components, so they are always subject to deterioration rate μ (i.e., $g(k) = 1$ for any k).
- $0 < c < 1$: (weak load sharing)

The system load is shared, but less than proportional to the number of components (i.e., $1/k < g(k) < 1$).

- $c = 1$: (proportional load sharing)

The load is proportional to the number of components (i.e., $g(k) = 1/k$).

- $c > 1$: (strong load sharing)

Including an extra component has a strong influence on the failure rates (i.e., $g(k) < 1/k$).

In Fig. 1, we show $g(k)$ for different realizations of the number of functioning components k and degrees of load sharing c . Of course, when only one component is functioning, it is subject to a deterioration rate of μ , independent of the degree of load sharing. When at least two components are functioning, however, we observe that a positive degree of load sharing c greatly influences the failure rates (given by $\mu \cdot g(k)$).

We remark that load sharing can be modeled in different ways, with more complex formulations than ours that are possibly more realistic. However, as this research is the first to consider redundancy and load sharing for a system subject to condition-based maintenance, we feel that it is appropriate to use a simple load sharing model that has the advantage of (its parameters) being easy to interpret, whilst still being flexible as illustrated in Fig. 1.

2.3.1. Deterioration process

Let X_k be the stochastic amount of deterioration of component k during a time unit. Letting x and y present the levels of deterioration at the start and end of this time unit, respectively, then component k moves from state x to state y with probability $P(X_k = y - x)$. Recall that the failure rate is equal to $\mu \cdot g(k)$. So if we model deterioration using a Poisson process, as we will do in our numerical base case, then we have $X_k \sim \text{Poisson}(\mu \cdot g(k))$ for $k = 1, 2, \dots, N$. As part of a sensitivity study, we will also discuss the case with a binomial degradation process.

3. Markov Decision Process formulation

In this section, we provide the Markov Decision Process (MDP) formulation of our model. At the start of each time unit, the system can be in a set of states $\mathcal{S}$. Depending on the current state $i \in \mathcal{S}$, a set of actions $\mathcal{A}_{\{i\}}$ can be performed, including the option to not perform any maintenance. The system then moves from state $i \in \mathcal{S}$ to some state $\bar{i} \in \mathcal{S}$ under action $a \in \mathcal{A}_{\{i\}}$ with a certain transition probability $p^a(i, \bar{i})$, while a cost of $c^a(i)$ is incurred. Below, we define this state space and action space, and provide an expression for the transition probabilities and the expected cost function.

Fig. 1. The dependence function $g(k)$ for different values of k and c .

3.1. State space

The CBM replacement decisions are based on the complete system state. For that reason, we keep track of the states of all N components in our state space, i.e.,

$$\mathcal{S} = \{(x_1, x_2, \dots, x_N)\},$$

where $x_j \in \{0, 1, \dots, L-1, L\}$ denotes the state of component j , for $j = 1, 2, \dots, N$. Since L denotes the failure level of each component, we assume that state L denotes the failed state of a component, i.e., where $x_j \geq L$. In this way, we truncate the state space.

3.2. Action space

At the start of each time unit, we need to decide which components will be replaced. Thus, the action space is defined as

$$\mathcal{A} = \{(\delta_1, \delta_2, \dots, \delta_N)\},$$

where, for $j = 1, 2, \dots, N$,

$$\delta_j = \begin{cases} 1, & \text{if component } j \text{ is replaced,} \\ 0, & \text{otherwise.} \end{cases}$$

At each state, any component can be replaced. However, as we are dealing with identical components, we can limit the set of possible actions without affecting the results. If two (or more) components are in the same state, replacing any one of these components will have the same effect on the system state. We do not need to decide which of the components will be replaced, but can instead assume that, if component i and j are in the same state, and only one of them is replaced, then that will be component i if $i < j$, i.e.,

$$\mathcal{A}_{\{(x_1, x_2, \dots, x_N)\}} = \{(\delta_1, \delta_2, \dots, \delta_N) : \delta_i \geq \delta_j \text{ if } x_i = x_j, \forall i, j \in \{1, 2, \dots, N\} \text{ with } i < j\}.$$

This assumption will also prove useful for deciding whether it is beneficial to include an additional redundant component to the system, as this component may or may not be present in the failed state. Furthermore, this assumption is of use for solving the MDP, as explained later in this section. Note that, immediately following possible replacements, component j will be in state $(1 - \delta_j) \cdot x_j$, provided that the component was in state x_j at the start of the time unit.

Before we define the transition probabilities and the expected costs, we first introduce the function R_j , which equals one if component j is functioning, and zero otherwise, i.e.,

$$R_j(x_j) = \begin{cases} 1, & \text{if } x_j < L, \\ 0, & \text{if } x_j = L, \end{cases}$$

for $j = 1, 2, \dots, N$. In addition, we define R as the system availability, which indicates whether the complete system is functioning or not, i.e.,

$$R(x_1, x_2, \dots, x_N) = \begin{cases} 1, & \text{if } \sum_{j=1}^N R_j(x_j) \geq 1, \\ 0, & \text{if } \sum_{j=1}^N R_j(x_j) = 0. \end{cases}$$

Note that if the system is in state $(x_1, x_2, \dots, x_N)$, then the number of functioning components is equal to $\sum_{j=1}^N R_j(x_j)$.

3.3. Transition probabilities

Let $p_j(x; y|k)$ denote the probability that component j moves from state x to state y , provided that k components are functioning. Because we truncated the state space (by assuming that $x_j = L$ is equivalent to $x_j \geq L$),

it follows that $p_j(x; y|k)$ is defined as

$$p_j(x; y|k) = \begin{cases} P(X_k = y - x), & \text{if } x \leq y < L, \\ P(X_k \geq y - x), & \text{if } x < y = L, \\ 1, & \text{if } x = y = L, \\ 0, & \text{if } x > y, \end{cases}$$

for $j = 1, 2, \dots, N$. Observe that, as we consider identical components, this expression is independent of j . The transition probabilities (of moving from state $(x_1, x_2, \dots, x_N)$ to state $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_N)$ under action $(\delta_1, \delta_2, \dots, \delta_N)$) are then given by

$$p^{(\delta_1, \delta_2, \dots, \delta_N)}((x_1, x_2, \dots, x_N); (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_N)) = \prod_{j=1}^N p_j \left((1 - \delta_j) \cdot x_j; \bar{x}_j \mid \sum_{m=1}^N R_m((1 - \delta_m) \cdot x_m) \right).$$

3.4. Expected costs

The expected cost per time unit of performing action $(\delta_1, \delta_2, \dots, \delta_N)$ in state $(x_1, x_2, \dots, x_N)$ consists of the penalty cost for a system failure, the set-up cost for maintenance, and the preventive and corrective replacement costs as follows.

$$c^{(\delta_1, \delta_2, \dots, \delta_N)}(x_1, x_2, \dots, x_N) = p \cdot (1 - R(x_1, x_2, \dots, x_N)) + c_s \cdot \left(1 - \prod_{j=1}^N (1 - \delta_j) \right) + c_p \cdot \sum_{j=1}^N \delta_j \cdot R_j(x_j) + c_c \cdot \sum_{j=1}^N \delta_j \cdot (1 - R_j(x_j))$$

3.5. Performance criterion

As a performance criterion, we are interested in minimizing the long-run average cost per time unit. In this way, we can quantify the impact of adding an additional component, and find the most cost-efficient maintenance policy. We are dealing with a finite-sized state space and action space. Moreover, our cost function is bounded by definition. Due to our restriction of the set of possible actions, our MDP model does satisfy the Weak Unichain Assumption² as defined in Tijms (1994). Olde Keizer et al. (2017b) also apply this assumption to a CBM setting, where both the spare parts and maintenance decisions are condition-based and optimized simultaneously. We therefore choose to solve our MDP by applying the Value Iteration algorithm (see Puterman (2005) and Olde Keizer et al. (2017b)).

4. Numerical analysis

From Fig. 1, observe that adding one component to a single-component system has a relatively large impact on the effect of load sharing, while the effect of adding a large component decreases as the number of functioning components increases. For that reason, we start our analysis with a system consisting of two components, i.e., we set $N = 2$. Similar to Olde Keizer et al. (2016), we select a penalty cost of $p = 300$, a set-up cost of $c_s = 4$, a preventive replacement cost of $c_p = 5$, and a corrective replacement cost of $c_c = 11$, while we select a nominal failure rate of $\mu = 0.7$ and a failure level of $L = 5$. We consider four different values for the degree of load sharing (c): 0 (no load sharing), 0.5 (weak load sharing), 1.0 (proportional load sharing), and 1.5 (strong load sharing). We assume for this ‘base case’ that the degradation per time unit is Poisson distributed. The Poisson distribution has also been considered by others (as discussed in the introduction) and has the

² For each average cost optimal stationary policy, the associated Markov chain has no two disjoint closed sets (Tijms, 1994).

Table 1

Minimal long-run average cost per time unit for a system with $N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c .

Degree of load sharing	Minimal average costs
$c = 0.0$	3.42
$c = 0.5$	2.33
$c = 1.0$	1.60
$c = 1.5$	1.10

following clear interpretation. If a system deteriorates at random moments (by a fixed amount), where the intervals between successive moments follows an exponential distribution, then the total deterioration per time unit follows a Poisson distribution.

The resulting minimal costs are shown in Table 1. Recall that a high degree of load sharing c corresponds to a lower deterioration rate in case both components are functioning. Fig. 2a–d shows the optimal replacement policies for the different degrees of load sharing. In these figures, we represent the optimal replacement decisions as $\delta_1\delta_2$ for every possible system state (x_1, x_2) , so 01, for example, means that only component 2 is replaced. First, consider the case with no load sharing ($c = 0$). As in Olde Keizer et al. (2016), we observe that corrective maintenance on a failed component is postponed in some cases to allow for clustering (consider e.g. the case where $x_1 = 0$ or 1 and $x_2 = 5$). As we introduce load sharing, we introduce an incentive to perform corrective replacements immediately. Indeed, in case $c = 0.5$, we observe that corrective maintenance is only postponed if the other component is as-good-as-new (consider e.g. $x_1 = 0$ and $x_2 = 5$), while corrective replacements are no longer postponed for higher degrees of load sharing c .

4.1. Adding additional components

If we include an additional component in our two-component system, we benefit on the one hand from the decreased probability of a system failure and the increased load sharing between components, but on the other hand incur an additional cost from maintaining this extra

component. To investigate this trade-off, we now consider a similar system as before, but with $N = 3$ rather than $N = 2$. The results are shown in Table 2. From this table, we observe that including a third component is not beneficial for low degrees of load sharing ($c = 0$ and $c = 0.5$), as the optimal policy is to keep the third component in the failed state. The benefit from load sharing does outweigh the increase in maintenance costs for $c = 1.0$ and $c = 1.5$, for which the average cost decreases with 9 and 24 percent, respectively, to 1.45 and 0.84 per time unit. The optimal replacement policy for a three-component system with $c = 1.0$ is shown in Fig. 3. The optimal replacement decisions are presented as $\delta_1\delta_2\delta_3$ for each system state (x_1, x_2, x_3) . In Fig. 3, we observe that corrective maintenance is postponed regularly for a system with three components. If for example component 1 has failed, i.e., $x_1 = 5$, it will not be replaced until at least one other component is in state 4 or 5.

It is important to keep in mind that we do not consider the purchase or investment cost of including additional components, as our focus is on the structure of the optimal maintenance policy rather than system design. In cases where including an additional component reduces operational costs, that reduction should be traded off against the increased investment (in a net present value analysis), which is beyond the scope of this research.

4.2. Comparison to a threshold policy

In the literature, CBM is often implemented in the form of a deterioration threshold that is used to schedule maintenance (see e.g. Barata et al. (2002), Marsegua et al. (2002), Hong et al. (2014), and Mercier and Pham (2014)). Such a threshold policy is, however, not necessarily optimal for a system with economic dependence and redundancy. To investigate this, we compare the performance of the optimal policy to that of the best threshold policy. We define T_R as the threshold for the state of a component at or above which a maintenance action is triggered. Table 3 shows the costs corresponding to the best threshold policy, obtained through simulation and by applying a full grid search, along with the optimal threshold and the percentage increase in cost compared with

Fig. 2. Optimal replacement policy for a system with $N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c .

Table 2

Minimal long-run average cost per time unit (decrease compared to $N = 2$) for a system with $N = 3$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c .

Degree of load sharing	Minimal average costs
$c = 0.0$	3.42 (−0%)
$c = 0.5$	2.33 (−0%)
$c = 1.0$	1.45 (−9%)
$c = 1.5$	0.84 (−24%)

our optimal CBM policy, for $N = 2$ and $N = 3$. We observe that significant cost savings can be obtained by applying our optimal CBM policy; the costs corresponding to the threshold policy are about 10 and 8 percent higher for $c = 0.0$ and $c = 0.5$, respectively, for $N = 2$, while for $N = 3$, the threshold policy is about 10 percent more expensive than our optimal policy for both $c = 1.0$ and $c = 1.5$.

4.3. Sensitivity analysis with respect to maintenance set-up cost

So far, we considered a maintenance set-up cost of $c_s = 4$. For a larger set-up cost, we expect replacements to be clustered more often. To investigate this, we consider the same case as before, but increase the maintenance set-up cost to $c_s = 8$. In Table 4, we show the minimal long-run average cost corresponding to the optimal policy for $N = 2$ and $N = 3$, for different degrees of load sharing c . We observe that adding a third component to the two-component system results in a cost decrease of 0, 3, 18, and 31 percent for $c = 0.0, 0.5, 1.0$, and 1.5 , respectively. These cost decreases are larger than those we observed in Section 4.1 for a set-up cost of $c_s = 4$. We thus observe that adding a third component is more rewarding for a higher maintenance set-up cost. Indeed, maintenance actions are clustered more often for larger systems, and the high set-up cost is paid less often.

In addition, Fig. 4 shows the optimal policy for $N = 2$ for different values of c . When increasing the set-up cost for maintenance, the economic dependence between the components becomes stronger. Clustering maintenance thus becomes much more rewarding. From Fig. 4, observe that all maintenance actions are indeed clustered for $c = 0$, i.e., without load sharing. Preventive replacements are no longer performed solely to prevent corrective replacements, as clustering maintenance has become more rewarding ($c_s + c_p > c_c$). When we do include load sharing (i.e., when we select $c > 0$), preventive replacement becomes more rewarding, as deterioration is slowed down when both components are

Table 3

Minimal long-run average cost per time unit of the threshold policy (increase compared to the optimal policy) for different values of c .

Degree of load sharing	Average cost	Optimal threshold
(a) $N = 2$		
$c = 0.0$	3.77 (+10%)	$T_R^* = 3$
$c = 0.5$	2.52 (+ 8%)	$T_R^* = 4$
$c = 1.0$	1.69 (+ 6%)	$T_R^* = 4$
$c = 1.5$	1.16 (+ 5%)	$T_R^* = 4$
(b) $N = 3$		
$c = 0.0$	4.89 (+43%)	$T_R^* = 4$
$c = 0.5$	2.80 (+20%)	$T_R^* = 4$
$c = 1.0$	1.60 (+10%)	$T_R^* = 4$
$c = 1.5$	0.92 (+10%)	$T_R^* = 4$

functioning. We thus observe that not all replacement actions are clustered for $c > 0$, though it does happen more often than in Fig. 2, for a set-up cost of $c_s = 4$.

Table 5 shows the long-run average cost per time unit obtained with the threshold maintenance policy for $N = 2$ and $N = 3$, along with the optimal threshold values, and the percentage increase in costs compared with our optimal policy. We observe that the threshold policy results in significantly higher costs than our optimal CBM policy for all considered values of c . Whereas two components are sufficient for $c = 0.0$ and $c = 0.5$, a cost decrease is observed for $c = 1.0$ and $c = 1.5$ when adding a third component. For these cases, the threshold policy is 30 percent more expensive than the optimal policy. Compared with Section 4.2, where we considered a maintenance set-up cost of $c_s = 4$, we observe that the threshold policy performs worse for a higher set-up cost. Indeed, in the case of strong economic dependence, clustering maintenance actions becomes more rewarding. The threshold policy is not able to capture these benefits.

4.4. Ignoring the failure dependence through load sharing

In practice, observing load sharing effects between multiple components can be challenging. Even if such failure dependence within a system is recognized, further difficulty arises in estimating the actual degree of load sharing. Failure data are often lacking or incomplete, preventing the maintenance managers from justifying their assumptions. To investigate the consequences of ignoring or misinterpreting the load sharing effects between components, we apply the optimal policy for independent components ($c = 0$) to the cases where load sharing exists ($c > 0$).

$x_1 = 0$							$x_1 = 1$							$x_1 = 2$									
		x_3							x_3							x_3							
		0	1	2	3	4	5			0	1	2	3	4	5			0	1	2	3	4	5
x_2	0	000	000	000	000	001	000	x_2	0	000	000	000	000	001	000	x_2	0	000	000	000	000	000	000
	1	000	000	000	000	001	000		1	000	000	000	000	001	000		1	000	000	000	000	000	000
	2	000	000	000	000	000	000		2	000	000	000	000	000	000		2	000	000	000	000	000	000
	3	000	000	000	000	011	000		3	000	000	000	000	011	000		3	000	000	000	000	000	000
	4	010	010	000	011	011	011		4	010	010	000	011	011	011		4	000	000	000	000	011	011
5	000	000	000	000	011	011	5	000	000	000	000	011	011	5	000	000	000	000	011	011			

$x_1 = 3$							$x_1 = 4$							$x_1 = 5$									
		x_3							x_3							x_3							
		0	1	2	3	4	5			0	1	2	3	4	5			0	1	2	3	4	5
x_2	0	000	000	000	000	101	000	x_2	0	100	100	000	101	101	101	x_2	0	000	000	000	000	101	101
	1	000	000	000	000	101	000		1	100	100	000	101	101	101		1	000	000	000	000	101	101
	2	000	000	000	000	000	000		2	000	000	000	000	101	101		2	000	000	000	000	101	101
	3	000	000	000	000	111	000		3	110	110	000	111	111	111		3	000	000	000	000	111	111
	4	110	110	000	111	111	111		4	110	110	110	111	111	111		4	110	110	110	111	111	111
5	000	000	000	000	111	111	5	110	110	110	111	111	111	5	110	110	110	111	111	111			

Fig. 3. Optimal replacement policy for a system with $N = 3$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, $L = 5$, and $c = 1.0$.

Table 4

Minimal long-run average cost per time unit (decrease compared to $N = 2$) for different values of c and N , for $c_s = 8$.

Degree of load sharing	$N = 2$	$N = 3$
$c = 0.0$	4.29	4.29 (−0%)
$c = 0.5$	3.02	2.94 (−3%)
$c = 1.0$	2.09	1.72 (−18%)
$c = 1.5$	1.46	1.01 (−31%)

We consider the same cases as before, and use simulation to find the corresponding costs. The results are summarized in Table 6. For a weak degree of load sharing ($c = 0.5$) and a low degree of economic dependence ($c_s = 4$), we observe that ignoring the load sharing will result in a 1 percent more expensive policy, while for $c_s = 8$ this difference increases to 3 percent for $N = 3$. For stronger degrees of load sharing, however, we find that ignoring the failure dependence will increase costs by 3–8 percent for $N = 2$. For $c = 1.0$ and $c = 1.5$, we found in Section 4.1 that adding a third redundant component can reduce costs significantly. When ignoring the load sharing effects in these cases, this extra component will be kept in the failed state, which increases the costs by up to 54 percent. We can thus conclude that weak load sharing can be ignored, but a stronger degree of load sharing has to be taken into account. This holds in particular for systems with a strong degree of economic dependence.

4.5. Binomial degradation process

So far, we have assumed that the degradation is driven by a Poisson process (with average degradation of $\mu g(k)$ per time unit if k components are functioning, where $\mu = 0.7$). A limitation of the Poisson process is that the variance is equal to the mean, and so we cannot explore the effects of varying the variance or its standard deviation relative to the mean. In this section, we therefore model the degradation per time unit using a Binomial process with m repetitions and probability of ‘success’ (i.e., degradation occurs) p . Like the Poisson distribution, the binomial

Table 5

Minimal long-run average cost of the threshold policy (increase compared to the optimal policy) for different values of c , for $c_s = 8$.

Degree of load sharing	Average cost	Optimal threshold
(a) $N = 2$		
$c = 0.0$	5.15 (+20%)	$T_R^* = 4$
$c = 0.5$	3.40 (+13%)	$T_R^* = 4$
$c = 1.0$	2.34 (+12%)	$T_R^* = 4$
$c = 1.5$	1.62 (+11%)	$T_R^* = 4$
(b) $N = 3$		
$c = 0.0$	6.52 (+52%)	$T_R^* = 4$
$c = 0.5$	3.84 (+31%)	$T_R^* = 4$
$c = 1.0$	2.24 (+30%)	$T_R^* = 4$
$c = 1.5$	1.31 (+30%)	$T_R^* = 4$

distribution has a clear interpretation as it cuts up a time unit into multiple segments (the number of trials) and then assumes that deterioration occurs in a segment with some fixed (success) probability. We keep the average degradation rate $m p g(k)$ unchanged at $0.7 g(k)$ by varying m and setting $p = 0.7 g(k)/m$. We remark that the degradation variance per time unit is then equal to $m p [1 - p] = 0.7 g(k) [1 - 0.7 g(k)/m]$, and thus increases with m , and that the Binomial process approaches the Poisson process for very large values of m .

The resulting minimal costs for different values of m and c are shown in Table 7. It appears that the minimal cost per time unit for the optimal policy increases with m for all considered values of the load sharing factor c . This is expected, since an increase in m leads to a larger degradation variance and so results in a less predictable degradation process. Indeed, a larger value for m leads to a more conservative maintenance policy, as shown in Fig. 5 for $c = 0.5$.

Note that Fig. 5(d) is the same as Fig. 2(b). Apparently, the Binomial degradation process for $m = 20$ is close enough to the Poisson process for the optimal policies to be identical. Note also from a general comparison between Fig. 2 and 5 that the general structure of the optimal policies is unchanged. In particular, the optimal strategy is not always monotone in the level of degradation and so a simple threshold strategy is generally

Fig. 4. Optimal replacement policy for a system with $N = 2$, $p = 300$, $c_s = 8$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c .

Table 6

Costs corresponding to the optimal policy for $c = 0.0$ applied to different degrees of load sharing c , for $p = 300$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$.

Degree of load sharing	Optimal policy		Ignoring load sharing	
	$N = 2$	$N = 3$	$N = 2$	$N = 3$
(a) $c_s = 4$				
$c = 0.0$	3.42	3.42	3.42 (+0%)	3.42 (+0%)
$c = 0.5$	2.33	2.33	2.35 (+1%)	2.35 (+1%)
$c = 1.0$	1.60	1.45	1.64 (+3%)	1.64 (+13%)
$c = 1.5$	1.10	0.84	1.16 (+5%)	1.15 (+37%)
(b) $c_s = 8$				
$c = 0.0$	4.29	4.29	4.29 (+0%)	4.29 (+0%)
$c = 0.5$	3.02	2.94	3.03 (+0%)	3.03 (+3%)
$c = 1.0$	2.09	1.72	2.17 (+4%)	2.18 (+27%)
$c = 1.5$	1.46	1.01	1.58 (+8%)	1.56 (+54%)

Table 7

Minimal long-run average cost per time unit for a system with $N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, $L = 5$ for different values of the load sharing factor c and of the Binomial distribution parameter m .

m	$c = 0.0$	$c = 0.5$	$c = 1.0$	$c = 1.5$
1	1.68	1.57	1.45	1.04
2	1.98	1.98	1.52	1.07
5	2.99	2.28	1.58	1.09
20	3.38	2.32	1.60	1.10

Fig. 5. Optimal replacement policy for a system with $N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, $L = 5$ and $c = 0.5$, for different values of the Binomial distribution parameter m .

Table 8

Costs of optimal and threshold policies for different values of c and m ($N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, $L = 5$).

	Cost for optimal policy				Best threshold policy and cost gap							
	$c = 0.0$	$c = 0.5$	$c = 1.0$	$c = 1.5$	$c = 0.0$		$c = 0.5$		$c = 1.0$		$c = 1.5$	
					T_R^*	Gap	T_R^*	Gap	T_R^*	Gap	T_R^*	Gap
$m = 1$	1.68	1.57	1.45	1.04	4	92%	4	38%	4	6%	4	6%
$m = 2$	1.98	1.98	1.52	1.07	4	77%	4	18%	4	6%	4	5%
$m = 5$	2.99	2.28	1.58	1.09	3	26%	4	7%	4	5%	4	5%
$m = 20$	3.38	2.32	1.60	1.10	3	11%	4	7%	4	5%	4	5%

again suboptimal. Table 8 shows how the optimality gap of the threshold policy depends on m for the previously considered two-component system. The cost gap is particularly large for situations where the degradation process is highly predictable (m equals 1 or 2) and there is a low degree of load sharing (c equals 0 or 0.5). Under such circumstances, it is relatively risk free to postpone maintenance activities on an (nearly) failed component in order to decrease the maintenance frequency, as long as the other component is still in a good state. Similar results are obtained for the three-component system considered before if the Poisson degradation process is replaced by a Binomial process.

5. Conclusion

In this paper, we considered a multi-component, parallel condition-based maintenance (CBM) system with economic dependence and load sharing. On the one hand, the redundancy resulting from the parallel setting allows corrective replacements to be postponed without affecting the system performance. In this way, a corrective replacement can be combined with a preventive replacement of other components to reduce the maintenance frequency and reduce the maintenance set-up cost. On the other hand, a failure of a component implies an increased load on the remaining components, thus leading to faster deterioration. This provides an incentive to perform corrective replacements as soon as possible. We are the first to explore this trade-off under a condition-based maintenance (CBM) strategy. We formulated our system as a Markov Decision

Process and determined cost-minimizing CBM strategies. Through a numerical study, we discovered interesting properties of the optimal policy structure, for different degrees of load sharing and economic dependence. We observed that maintenance clustering is especially beneficial for systems with a strong economic dependence and a relatively low degree of load sharing. In line with this, corrective replacements can be postponed to allow for maintenance clustering and to save on set-up costs. Furthermore, we observed that preventive replacements can best be performed at a relatively early stage for a high degree of load sharing and weak economic dependence, as more cost savings can be obtained from the load sharing than through maintenance clustering.

By comparing the performances of our optimal policy with those of a 'standard' threshold CBM policy, popular in both theory and practice, we observed that significant cost savings can be obtained by basing the replacement decisions on the complete system state. The inability of the threshold policy to cluster maintenance actions by postponing corrective replacements of redundant components results in up to 30 per cent more expensive maintenance strategies. Especially for systems with a strong economic dependence and a low degree of load sharing, the threshold policy can be far from optimal.

In practice, the presence of load sharing interactions can be difficult to recognize and quantify. We therefore investigated the effects of ignoring the failure dependence through load sharing (even though load sharing is actually present), and found that this can lead to sub-optimal policies that are significantly more expensive. This holds in particular for systems with a strong degree of load sharing and a strong degree of economic dependence, thus stressing the importance of applying a suitable policy. Weak degrees of load sharing, on the other hand, can be ignored at a relatively small cost.

There are a number of limitations to our research, many related to its exploratory nature with the objective to obtain insights into the optimal policy for relatively simple systems. We considered Poisson and Binomial deterioration processes, where future research could also consider more complex Gamma and Weibull processes. Our analysis was restricted to load sharing functions of a specific 'power type', but more complex functions might better describe real life applications. We considered 1-out-of-N systems with at most three components, and larger and more general k-out-of-N systems also deserve attention. We assumed that maintenance times are negligible and also did not consider time to plan maintenance activities, i.e. to get engineers, parts and tools ready.

It is certainly worthwhile to consider the optimal policy structure for more complex settings in future research, as our research has revealed the need for a custom-fit maintenance policy for systems with inter-component dependencies. However, the exact optimal maintenance policy may be too difficult to interpret and apply in practice. Therefore, further research could also focus on developing heuristics that incorporate key policy properties but are still easy to implement. Furthermore, we assume in our current model that the state of each component is known at the start of each time unit. Inspections are thus performed with a given periodicity. For future research, it could be interesting to consider a system in which the inspection interval (either periodic or aperiodic) needs to be optimized along with the replacement decisions at such an inspection. Continuous monitoring is also an interesting field of research for systems subject to both redundancy and load sharing.

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