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Distributed formation tracking using local coordinate systems

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Abstract

This paper studies the formation tracking problem for multi-agent systems, for which a distributed estimator-controller scheme is designed relying only on the agents' local coordinate systems such that the centroid of the controlled formation tracks a given trajectory. By introducing a gradient descent term into the estimator, the explicit knowledge of the bound of the agents' speed is not necessary in contrast to existing works, and each agent is able to compute the centroid of the whole formation in finite time. Then, based on the centroid estimation, a distributed control algorithm is proposed to render the formation tracking and stabilization errors to converge to zero, respectively. Finally, numerical simulations are carried to validate our proposed framework for solving the formation tracking problem.

Keywords: Formation control, Cooperative control, Rigidity graph theory, Multi-agent systems

1. Introduction

Formation control for multi-agent systems has attracted increasing attention from control scientists and engineers due to its broad applications [1, 2]. A central problem is to drive the agents to realize some prescribed formation shape, and such a problem is usually referred to as the formation stabilization problem. In this line of research, formation stabilization for those with different shapes has been investigated, see, for example, circular formation [3, 4], acyclic formation [5], and formations associated with tree graphs [6], minimally rigid graphs [7, 8], and more general rigid graphs [9]. Time-varying formation control problems for linear multi-agent systems under switching directed topologies are also investigated in [10]. In addition, the effects of the measurement inconsistency between neighboring agents on the formation's stability are addressed in [11], where it is shown the resulted distorted formation will move following a closed circular orbit in the plane for any rigid, undirected formation consisting of more than two agents. In [12], the steady-state rigid formation is achieved using an estimator-based gradient control law; in addition, both the static and time-varying mismatched compasses are studied in [13].

Another key problem concerned with formation control for multi-agent systems is formation tracking, which requires to stabilize the prescribed formation, and, additionally, requires that the whole formation follows a given reference trajectory. One commonly reported approach to deal with the formation tracking problem is to use the virtual structure strategy. This technique is built upon assigning a virtual leader to the centroid of the formation to be tracked while achieving the prescribed formation shape [14]. Under this framework, it is shown that the formation tracking can be achieved in finite time by employing the signum function if the virtual leader has directed paths to all the followers [15]. The virtual structure approach is also reported in [16], in which the control and estimation on a common virtual leader is addressed using a consensus algorithm. Integrating the techniques from nonsmooth analysis, collective potential functions and navigation feedback, a distributed algorithm for second-order systems is designed such that the velocity consensus to the virtual leader is achieved [17]. The formation tracking problem can also be solved using the distributed receding horizon control (RHC), for a group of nonholonomic multi-vehicle systems [18].

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By applying RHC, some additional tasks, e.g., collision avoidance and consistency, can be realized through adding constraints on allowed uncertain deviation.

Akin to the virtual structure approach, the leader-follower strategy has also been widely employed to solve formation tracking problems (e.g., [19, 20, 21, 22, 23]). In [19], the formation tracking problem is solved based on formation stabilization with one designated leader among the group. To deal with the intrinsic unknown parameters for a class of nonlinear systems, an adaptive control law using the backstepping technique is proposed in [20], such that all the subsystems' outputs are regulated to achieve consensus tracking. In [21], to compensate the unknown slippage effect of mobile robots, a distributed recursive design strategy involving the adaptive function approximation technique is developed. More recently, the formation tracking problem for second-order multi-agent systems under switching topologies is studied in [22], where one of the agents is set to be the leader to perform tracking tasks. The results therein are also feasible to the target enclosing problem for multi-quadrotor unmanned aerial vehicle systems. In [23], different from the one-leader tracking case, the formation tracking problem with multiple leaders is addressed. To drive the followers to the convex hull spanned by the leaders, a protocol is designed via solving an algebraic Riccati equation.

It should be noted that in the results discussed above, almost all the desired formations are specified by offset vectors with respect to the virtual/real leader or virtual centroid of the group. Those offset vectors are required to be set a priori in a common global coordinate system. In addition, each agent needs to know its corresponding desired offsets as well as its neighbors'. In particular, the agreement reached on the estimations of the virtual centroid is normally different from the real centroid of the group. However, it is sometimes meaningful to locate the real centroid when performing tasks like the transportation of objects. Furthermore, the approaches developed in these existing works are only applicable to the scenarios where the reference trajectory is an exogenous signal that is independent of the states of the system. To estimate the centroid of the formation, a consensus-based algorithm is proposed in [24], wherein the estimation of each agent is updated by averaging their projections and directions. However, the convergence can be ensured only when the underlying graph is complete. In [25], a tree-based algorithm is adopted to estimate the centroid, while, each agent is required to maintain a list of trees with constant size. Recently, the weighted-centroid tracking problem has been considered in [26, 27, 28]. Unlike the leader-follower structures in which the dynamics of the followers and leaders can be separated, the control objective therein is to track some globally assigned function which is implicitly related to all agents' dynamics. In [26], a controller-observer scheme is designed for the single integrator dynamics such that the weighted centroid of the whole formation follows some given trajectory. As an extension, one additional task function for the formation is introduced in [27]. In [28], a finite-time centroid observer is constructed, and the distance-based control laws are developed by employing rigidity graph theory.

In the present paper, we consider the formation tracking problem, in which the centroid of the formation moves as the agents move and is unknown to all of the agents. Under this case, the problem becomes more challenging due to the inner coupling and conflict between centroid estimation, formation stabilization and reference tracking. By adopting the feedback term from the gradient descent control, we design a new class of finite-time centroid estimator that is continuously differentiable. Based on the output of the estimator, the proposed distance-based control laws render the convergence to the prescribed formation shape while keeping its centroid following the reference. Compared with the previous work of using virtual/real leader structure, the proposed estimator-controller framework can be implemented in agents' local coordinate systems, which not only increases the robustness to the noises in the sensing signals but also reduces the equipment cost of the overall system. Moreover, the control law in this paper is more scalable and distributed in the sense that some constraints are removed, including the a priori knowledge of the position information of the reference trajectory [26, 27] and the agents' maximum speed [28]. In addition, the precise knowledge of the time-varying centroid can be obtained in finite time via the proposed smooth centroid estimator, which renders a faster convergence speed than that in [24, 25]. In addition, the centroid estimator in [24] is only valid under complete graphs whereas the one in this paper can be directly applied to any general undirected graphs.

The paper is organized as follows. Section 2 introduces the formation tracking problem and basic concepts of graph rigidity. In Section 3, the main results are presented including the estimator-controller scheme and the theoretical analysis. Section 4 extends the results to a more general case. The numerical simulations are presented in Section 5. Finally, we give the conclusions in Section 6.

2. Problem formulation

A team of $n > 1$ agents is considered, each of which is characterized by the single integrator dynamics

$$\dot{q}_i^g = u_i^g, \quad i = 1, \cdots, n, \tag{1}$$

where $q_i^g \in \mathbb{R}^d$ and $u_i^g \in \mathbb{R}^d$ are, respectively, the position and the control input of mobile agent i with respect to the global coordinate system ${}^g\Sigma$. Each agent i is also assigned with the local coordinate system ${}^i\Sigma$, whose origin is exactly the point q_i^g. In this paper, the local coordinate systems are assumed to share the same orientations. We use q_j^i to denote agent j's position with respect to ${}^i\Sigma$. This definition also applies to other variables. Note that the local variable q_j^i and the global one q_j^g have the following relationship

$$q_j^g = q_j^i + q_i^g.$$

Here, q_i^g is actually unknown to the agents, since the global coordinate system is introduced only for analysis purposes.

The neighboring relationships between the agents are defined by an undirected graph $\mathcal{G}$ with the vertex set $\mathcal{V} = \{1, 2, \cdots, n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ where there is an edge (i, j) if and only if agents i and j are neighbors of each other. We use $\mathcal{N}_i$ to denote the set of neighbors of agent i. The graph $\mathcal{G}$ is embedded in $\mathbb{R}^d$ when $q = [q_1^T, q_2^T, \cdots, q_n^T]^T$ is realizable and the pair $(\mathcal{G}, q)$ is called a *framework*. The *adjacency matrix* $A = [a_{ij} \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined as $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The interaction relationships among the agents and the reference signal is denoted by matrix $B = \text{diag}\,\{b_1, \cdots, b_n\}$, where $b_i = 1$ if agent i has access to the reference signal directly, and $b_i = 0$ otherwise. By assigning an arbitrary orientation to $\mathcal{G}$, the *incidence matrix* $H = [h_{ij}] \in \mathbb{R}^{|\mathcal{E}|\times n}$ is defined by

$$h_{ij} = \begin{cases} 1, & i\text{th edge enters node j}, \\ -1, & i\text{th edge leaves node j}, \\ 0, & \text{otherwise}, \end{cases}$$

where $|\mathcal{E}|$ represents the cardinality of the edge set $\mathcal{E}$, and it is taken to be m throughout the paper. The *Laplacian matrix* is then given by $L = H^T H \in \mathbb{R}^{n\times n}$.

Now, we formulate the problem to be investigated in this paper. On one hand, to achieve a desired shape of the formation, each agent i is required to keep some prescribed distance d_{ij}, $j \in \mathcal{N}_i$, namely, the agents are driven to the following target set

$$\mathcal{T}_d = \{q^g \in \mathbb{R}^{nd} | \, \|q_i^g - q_j^g\| = d_{ij}, \ \forall (i, j) \in \mathcal{E}\}. \tag{2}$$

On the other hand, at the same time, the stabilized formation is guided through the control law such that its centroid q_c^g tracks some smooth reference signal $q_d^g(t) : t \to \mathbb{R}^d$, where the centroid of the formation is defined by

$$q_c^g = \frac{1}{n} \sum_{i=1}^{n} q_i^g. \tag{3}$$

Equivalently, the tracking task can be written as

$$\lim_{t\to\infty} (q_c^g - q_d^g(t)) = \mathbf{0}. \tag{4}$$

To introduce the notion of graph rigidity, we firstly define a function

$$f_{\mathcal{G}}(q_1^g, \cdots, q_n^g) = [\cdots, \|q_i^g - q_j^g\|^2, \cdots]^T, \tag{5}$$

where $(i, j) \in \mathcal{E}$, and $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^n$. Then, graph rigidity is defined as follows.

Definition 1. *[29] A framework $(\mathcal{G}, q)$ is* rigid *if there exists a neighborhood U of q in $\mathbb{R}^{nd}$ such that $f_{\mathcal{G}}^{-1}(f_{\mathcal{G}}(q)) \cap U = f_{\mathcal{K}}^{-1}(f_{\mathcal{K}}(q) \cap U$, where $\mathcal{K}$ is the complete graph with the same vertices as $\mathcal{G}$.*

For a rigid framework, it means if one node moves, the rest also moves as a whole in order to satisfy the distance constraints. To characterize the rigidity of a framework, the rigidity matrix $R(q) \in \mathbb{R}^{m \times nd}$ is defined by

$$R(q) = \frac{1}{2} \frac{\partial f_{\mathcal{G}}(q)}{\partial q}. \tag{6}$$

The relationship between the rigidity matrix and the rigidity of a graph is as follows:

Lemma 1. *[30] A framework $(\mathcal{G}, q)$ is infinitesimally rigid in a d-dimensional space if*

$$rank(R(q)) = nd - d(d+1)/2.$$

In general, infinitesimal rigidity implies rigidity, but the converse is not true. Infinitesimal rigidity only allows the combinational motions of translation and rotation. In this paper, we consider the generic shapes which exclude all collinear (2D) or coplanar (3D) ones.

Definition 2. *[31] A framework is minimally rigid if it is rigid and no edge can be removed without losing rigidity.*

To be specific, a rigid framework $(\mathcal{G}, q)$ with n vertices in 2D or 3D is minimally rigid, if it has exactly $2n-3$ or $3n-6$ edges, respectively.

3. Formation tracking control

In this section, we first present the estimation algorithm for each agent to obtain the centroid information in finite time. Then, distributed control laws are proposed in local coordinate systems such that the formation tracking problem is solved.

Some useful lemmas are introduced as follows.

Lemma 2. *[32]. For an undirected connected graph, the following property holds,*

$$\min_{\substack{x \neq 0 \\ \mathbf{1}_n^T x = 0}} \frac{x^T L x}{\|x\|^2} = \lambda_2(L),$$

where λ_2 is the algebraic connectivity of the undirected graph, i.e., the smallest non-zero eigenvalue of the Laplacian matrix.

Lemma 3. *[33] Let $\xi_1, \cdots, \xi_n \geq 0$ and $0 < p \leq 1$, then*

$$\sum_{i=1}^{n} \xi_i^p \geq \left(\sum_{i=1}^{n} \xi_i \right)^p.$$

Lemma 4. *[33]. Suppose that the function $V(t) : [0, \infty) \to [0, \infty)$, is differentiable (the derivative of V(t) at 0 is in fact its right derivative) and*

$$\frac{dV(t)}{dt} \leq -KV(t)^{\alpha},$$

where $K > 0$ and $0 < \alpha < 1$. Then $V(t)$ will reach zero at some finite time $T_0 \leq V(0)^{1-\alpha}/(K(1-\alpha))$ and $V(t) = 0$ for all $t \geq T_0$.

Assumption 1. *The reference signal is bounded, as well as its first derivative, satisfying $\sup_{t>0} \|\dot{q}_d^g(t)\| \leq \sigma$. In addition, at least one of the n followers has access to the reference signal.*

Remark 1. *The reference signal is defined locally, namely, the information of the reference known by agent i is q_d^i if agent i has access to the reference signal. And, the local variable can be transformed to the global one through the following equation*

$$q_d^g = q_d^i + q_i^g.$$

We first introduce the vector $z^g = [(z_1^g)^T, \cdots, (z_m^g)^T]^T \in \mathbb{R}^{md}$ [34], defined as

$$z^g = (H \otimes I_d) q^g,$$

where $H \in \mathbb{R}^{m\times n}$ is the incidence matrix. Then, it is straightforward to check that z^g lies in the column space of $(H \otimes I_d)$, i.e., $z^g \in Im(H \otimes I_d)$. $z_k^g = q_j^g - q_i^g$ denotes the relative position of agents i and j connected by the kth edge. Note that $z_k^g = z_k^i, i = 1, \cdots, n$, owing to the fact that the local coordinate systems share the same orientation with the global one. Let $\hat{q}_{ci}^i$ be agent i's estimation of the centroid with respect to ${}^i\Sigma$, then

$$\hat{q}_{ci}^g = \hat{q}_{ci}^i + q_i^g, \tag{7}$$

where $\hat{q}_{ci}^g$ is agent i's estimation of the centroid with respect to ${}^g\Sigma$.

For controlling an infinitesimally rigid formation shape, we employ the standard quadratic potential function [11]

$$P(q^g) = \frac{1}{4}\sum_{k=1}^{m}(\|z_k^g\|^2 - d_k^2)^2. \tag{8}$$

Correspondingly, the gradient of $P(q)$ with respect to q_i^g, denoted by $\nabla_{q_i^g} P(q)$ is given by

$$\nabla_{q_i^g} P(q^g) = \sum_{j\in\mathcal{N}_i}(\|z_k^g\|^2 - d_k^2)(q_i^g - q_j^g) = -\sum_{j\in\mathcal{N}_i}(\|z_k^i\|^2 - d_k^2)z_k^i. \tag{9}$$

It can be aggregated as

$$\nabla P(q^g) = R(q^g)^T \phi(q^g), \tag{10}$$

where $R(q^g)$ is the rigidity matrix defined in (6) and $\phi(q^g)$ is as follows

$$\phi(q^g) = \left[\cdots, \|z_k^g\|^2 - d_k^2, \cdots\right]^T \in \mathbb{R}^m.$$

For achieving the tracking of the centroid to the reference with a prescribed formation shape, we propose the following control law for each agent i with respect to the reference q_d in ${}^i\Sigma$

$$u_i^d = \dot{q}_i^d = -k_p b_i \frac{\hat{q}_{ci}^g - q_d^g}{\delta + \|\hat{q}_{ci}^g - q_d^g\|} - k_s \nabla_{q_i^g} P(q^g), \tag{11}$$

where $\delta > 1$ is a constant scalar, and k_p and k_s are positive control gains. It also follows from (1) and (7) that

$$\hat{q}_{ci}^g - q_d^g = \hat{q}_{ci}^i + q_i^g - (q_d^i + q_i^g) = \hat{q}_{ci}^i - q_d^i.$$

Then, the control law u_i^d can be equivalently written as

$$u_i^d = -k_p b_i \frac{\hat{q}_{ci}^i - q_d^i}{\delta + \|\hat{q}_{ci}^i - q_d^i\|} + k_s \sum_{j\in\mathcal{N}_i}(\|z_k^i\|^2 - d_k^2)z_k^i. \tag{12}$$

The first term of the control law (12) is responsible for driving the centroid of the formation to track the reference signal, and the second one aims for stabilizing the desired formation. Note that not all the agents need to implement the first term but only those having access to the reference signal q_d^i, which is encoded in the binary variable $b_i \in \{0, 1\}$ as described in Section 2. However, all the agents are required to estimate the centroid of the formation through $\hat{q}_{ci}^i$ and to share this information with their neighbors. The dynamics of $\hat{q}_{ci}^i$ will be given later. It can be shown that the estimator can be implemented in a fully distributed manner. For the second term of (12), the relative position z_k^i and the distance $\|z_k^i\|$ between neighbors can be measured by sensors in the local coordinate system ${}^i\Sigma$.

The dynamics of $\hat{q}_{ci}^i$ is given by

$$\dot{\hat{q}}_{ci}^i = -k_1 \sum_{j\in\mathcal{N}_i} a_{ij} sig\left(\hat{q}_{ci}^i - \hat{q}_{cj}^i\right)^{\rho} - k_2 \sum_{j\in\mathcal{N}_i} a_{ij} \frac{\hat{q}_{ci}^i - \hat{q}_{cj}^i}{f_{ij}(\hat{q}_{ci}^i, \hat{q}_{cj}^i)} - k_s \sum_{j\in\mathcal{N}_i}(\|z_k^i\|^2 - d_k^2)z_k^i. \tag{13}$$

$f_{ij}(\hat{q}^i_{ci}, \hat{q}^i_{cj}) = \|\hat{q}^i_{ci} - \hat{q}^i_{cj}\| + \left(\sqrt{1 + \|\hat{q}^i_{ci} - \hat{q}^i_{cj}\|} - 1\right)$, and k_1 and k_2 are positive constants, and k_s is defined in (11). a_{ij} is the (i, j)th entry of the adjacency matrix A. $\hat{q}^i_{cj}$ is the centroid estimation of agent j with respect to ${}^i\Sigma$. For any $x \in \mathbb{R}$,

$$sig(x)^\rho = \left[sgn(x_1)|x_1|^\rho, \cdots, sgn(x_n)|x_n|^\rho\right]^T, \tag{14}$$

where $sgn(\cdot)$ is the signum function and $\rho \in (0, 1)$. For a vector $x \in \mathbb{R}^d$, the function $sig(x)$ is defined componentwise. It can be shown that the function $sig(\cdot)^\rho$ is continuous. The initial values for $\hat{q}^i_{ci}$ are chosen such that $\sum_{i=1}^n \hat{q}^i_{ci}(0) = \mathbf{0}$. Note that under the assumption that the orientation of the local coordinate systems are the same, the variable $\hat{q}^i_{cj}$ in (13) can be calculated by

$$\hat{q}^i_{cj} = \hat{q}^j_{cj} + q^i_{ji}. \tag{15}$$

where the neighbor's estimation $\hat{q}^j_{cj}$ is transmitted to agent i through communication. The relationship between $\hat{q}^j_{cj}$ and $\hat{q}^i_{cj}$ is shown in Fig. 1. Therefore, the estimator (13) can be implemented locally, and thus the proposed distributed control actions (12) and (13) can be implemented by only employing local information.

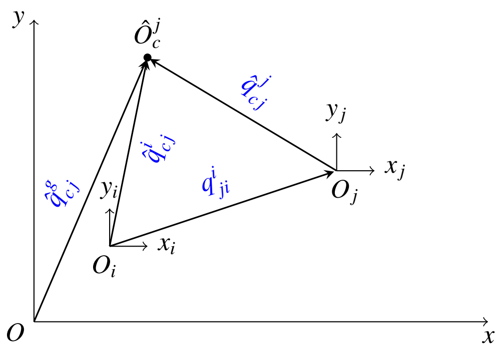

Figure 1: Relationship between $\hat{q}^j_{cj}$ and $\hat{q}^i_{cj}$

To precisely estimate the centroid, it is required that all the local coordinate systems share the same orientation with the global one. However, it will be shown in Section 4 that this constraint can be removed.

Now, we present the following main result.

Theorem 1. *Suppose the framework* $(\mathcal{G}, q)$ *is minimally and infinitesimally rigid. Under Assumption 1, the formation tracking task* (4) *can be achieved using the control law* (22) *for each agent i together with the estimator* (13)*, if the parameters are chosen such that*

$$k_2 \geq \frac{(k_p + \sigma)\sqrt{n}}{\epsilon\sqrt{1 - cos\frac{\pi}{n}}}, \tag{16}$$

and

$$k_s > \frac{k_p n}{2\delta}, \tag{17}$$

where ϵ *is a positive scalar satisfying* $\epsilon \in (0, 2/3]$*. Under an undirected connected graph, the estimation* $\hat{q}^g_{ci}, i = 1, \cdots, n$*, will converge to* q^g_c *in finite time.*

Proof. We carry out the proof in two steps. We first prove the estimation $\hat{q}^g_{ci}, i = 1, \cdots, n$, will converge to q^g_c in finite time. Consider the following equality

$$\hat{q}^i_{ci} - \hat{q}^i_{cj} = \hat{q}^i_{ci} - \hat{q}^j_{cj} - q^i_{ji} = -\left(\hat{q}^j_{cj} - \hat{q}^i_{ci} - q^i_{ij}\right).$$

In view of the definition (14), we have

$$sig\left(\hat{q}^i_{ci} - \hat{q}^j_{cj} - q^i_{ji}\right)^\rho = -sig\left(\hat{q}^j_{cj} - \hat{q}^i_{ci} - q^i_{ij}\right)^\rho.$$

Note that for an undirected graph, $a_{ij} = a_{ji}$, thus it follows

$$\sum_{i=1}^{n} \dot{\hat{q}}_{ci}^{i} = \mathbf{0}. \tag{18}$$

Define the estimation error with respect to the global coordinate system ${}^{g}\Sigma$ as

$$\tilde{q}_{ci}^{g} = \hat{q}_{ci}^{g} - q_{c}^{g}, \quad i = 1, \cdots, n.$$

Now, consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}\sum_{i=1}^{n} \|\hat{q}_{ci}^{g} - q_c^g\|^2 = \frac{1}{2}\sum_{i=1}^{n} (\tilde{q}_{ci}^{g})^T(\tilde{q}_{ci}^{g}), \tag{19}$$

where the centroid q_c^g is defined in (3). The time derivate of V_1 is given by

$$\dot{V}_1 = \sum_{i=1}^{n} (\tilde{q}_{ci}^{g})^T \left(\dot{\hat{q}}_{ci}^{i} + \dot{q}_i^g - \dot{q}_c^g\right). \tag{20}$$

By combining (18) and the initial conditions for the estimator, i.e., $\sum_{i=1}^{n} \hat{q}_{ci}^{i}(0) = \mathbf{0}$, it follows $\sum_{i=1}^{n} \hat{q}_{ci}^{i}(t) = \mathbf{0}, \forall t > 0$. Consequently, recalling (7), we have $\sum_{i=1}^{n} \hat{q}_{ci}^{g} = \sum_{i=1}^{n} q_i^g = nq_c^g$, and thus

$$\sum_{i=1}^{n} (\tilde{q}_{ci}^{g})^T \dot{q}_c^g = \sum_{i=1}^{n} \left(\hat{q}_{ci}^{g} - q_c^g\right)^T \dot{q}_c^g = \left(\sum_{i=1}^{n} \hat{q}_{ci}^{g} - \sum_{i=1}^{n} q_c^g\right)^T \dot{q}_c^g = 0. \tag{21}$$

From the geometrical relationship, we know $q_i^d = -q_d^i$, and $q_d^i = q_d^g - q_i^g$. Then, in view of the system model (1), the control input with respect to the global coordinate system ${}^{g}\Sigma$, i.e., u_i^g is

$$\dot{q}_i^g = u_i^g = \dot{q}_d^g - k_p b_i \frac{\hat{q}_{ci}^{i} - q_d^i}{\delta + \|\hat{q}_{ci}^{i} - q_d^i\|} + k_s \sum_{j\in\mathcal{N}_i} (\|z_k^i\|^2 - d_k^2) z_k^i. \tag{22}$$

Then substituting (21) and (22) into (20), together with the facts that $q_{ji}^i = q_j^g - q_i^g$ and $\hat{q}_{ci}^{g} = \hat{q}_{ci}^{i} + q_i^g$, we have

$$\begin{aligned}\dot{V}_1 = &- k_1 \sum_{i=1}^{n} (\tilde{q}_{ci}^{g})^T \sum_{j\in\mathcal{N}_i} a_{ij}\, sig\left(\hat{q}_{ci}^{g} - \hat{q}_{cj}^{g}\right)^{\rho} - k_2 \sum_{i=1}^{n} (\tilde{q}_{ci}^{g})^T \sum_{j\in\mathcal{N}_i} a_{ij} \frac{\hat{q}_{ci}^{g} - \hat{q}_{cj}^{g}}{f_{ij}(\hat{q}_{ci}^{g}, \hat{q}_{cj}^{g})} \\ &- k_p \sum_{i=1}^{n} b_i (\tilde{q}_{ci}^{g})^T \left(\frac{\hat{q}_{ci}^{i} - q_d^i}{\delta + \|\hat{q}_{ci}^{i} - q_d^i\|}\right) + \sum_{i=1}^{n} (\tilde{q}_{ci}^{g})^T \dot{q}_d^g,\end{aligned}$$

where $f_{ij}(\hat{q}_{ci}^{g}, \hat{q}_{cj}^{g}) = \|\hat{q}_{ci}^{g} - \hat{q}_{cj}^{g}\| + \left(\sqrt{1 + \|\hat{q}_{ci}^{g} - \hat{q}_{cj}^{g}\|} - 1\right)$.

Note that

$$\hat{q}_{ci}^{g} - \hat{q}_{cj}^{g} = \hat{q}_{ci}^{g} - q_c^g - (\hat{q}_{cj}^{g} - q_c^g) = \tilde{q}_{ci}^{g} - \tilde{q}_{cj}^{g}.$$

When $g(x_i - x_j)$ is an odd function, under an undirected graph, we have $\sum_{i,j} a_{ij} x_i g(x_i - x_j) = \frac{1}{2}\sum_{i,j} a_{ij}(x_i - x_j) g(x_i - x_j)$. Therefore, $\dot{V}_1$ satisfies

$$\begin{aligned}\dot{V}_1 \leq &- \frac{k_1}{2} \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij} \left(\sum_{k=1}^{d} \left|\tilde{q}_{ci(k)}^{g} - \tilde{q}_{cj(k)}^{g}\right|^{\rho+1}\right) - \frac{k_2}{2} \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij} \frac{\left(\tilde{q}_{ci}^{g} - \tilde{q}_{cj}^{g}\right)^T \left(\tilde{q}_{ci}^{g} - \tilde{q}_{cj}^{g}\right)}{f_{ij}(\tilde{q}_{ci}^{g} - \tilde{q}_{cj}^{g})} \\ &+ k_p \sum_{i=1}^{n} b_i \|\tilde{q}_{ci}^{g}\| \left(\frac{\|\hat{q}_{ci}^{i} - q_d^i\|}{\delta + \|\hat{q}_{ci}^{i} - q_d^i\|}\right) + \|(\tilde{q}_c^g)^T(\mathbf{1}_n \otimes \dot{q}_d^g)\|,\end{aligned} \tag{23}$$

where $f_{ij}(\tilde{q}^g_{ci}, \tilde{q}^g_{cj}) = \|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\| + \left(\sqrt{1 + \|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\|} - 1\right)$, and $\tilde{q}^g_{ci(k)}$ denotes the kth entry of the vector $\tilde{q}^g_{ci}$. In addition, we have

$$\frac{\left(\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\right)^T \left(\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\right)}{\|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\| + \left(\sqrt{1 + \|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\|} - 1\right)} \geq \epsilon \|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\|, \tag{24}$$

where $\epsilon \in (0, 2/3]$. The proof of (24) is given in Appendix. It is also straightforward to know

$$\frac{\|\hat{q}^i_{ci} - q^i_d\|}{\delta + \|\hat{q}^i_{ci} - q^i_d\|} < 1. \tag{25}$$

Substituting (24) and (25) into (23), we obtain

$$\dot{V}_1 \leq -\frac{k_1}{2} \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij} \sum_{k=1}^{d} \left|\tilde{q}^g_{ci(k)} - \tilde{q}^g_{cj(k)}\right|^{\rho+1} - \frac{k_2}{2}\epsilon \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij}\|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\| + k_p \sum_{i=1}^{n} b_i \|\tilde{q}^g_{ci}\| + \sqrt{n}\sigma\|\tilde{q}^g_c\|. \tag{26}$$

It is clear that

$$\sum_{i=1}^{n} b_i \|\tilde{q}^g_{ci}\| = \|(B\mathbf{1}_n)^T \tilde{q}^g_c\| \leq \|B\mathbf{1}_n\|\|\tilde{q}^g_c\| \leq \sqrt{n}\|\tilde{q}^g_c\|. \tag{27}$$

In light of Lemma 3 and Lemma 2, it yields

$$\sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij}\|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\| \geq \left(\sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a^2_{ij}\|\tilde{q}^g_{ci} - \tilde{q}^g_{cj}\|^2\right)^{\frac{1}{2}} \geq \sqrt{2\lambda_2(L_{A_s})}\|\tilde{q}^g_c\|, \tag{28}$$

where $A_s = [a^2_{ij}] \in \mathbb{R}^{n\times n}$ is an adjacency matrix and $\tilde{q}^g_c = [(\tilde{q}^g_{c1})^T, \cdots, (\tilde{q}^g_{cn})^T]^T$.

From [35], we know that $\lambda_2(L_{A_s}) \geq 2e(\mathcal{G})(1 - \cos\frac{\pi}{n})$, where $e(\mathcal{G})$ is the *edge connectivity* of the underlying graph $\mathcal{G}$, i.e., the minimal number of those edges whose removal would result in losing connectivity of the graph $\mathcal{G}$. Obviously, for an undirected connected graph, $e(\mathcal{G}) > 1$. Under the condition (16), and combining (27) and (28), we have

$$\begin{aligned}
&-\frac{k_2}{2}\epsilon \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij}\|\tilde{q}^g_{ci} - \tilde{q}_{cj}\| + k_p \sum_{i=1}^{n} b_i\|\tilde{q}^g_{ci}\| + \sqrt{n}\sigma\|\tilde{q}^g_c\| \\
\leq &-\frac{k_2}{\sqrt{2}}\epsilon\sqrt{\lambda_2(L_{A_s})}\|\tilde{q}^g_c\| + k_p\sqrt{n}\|\tilde{q}^g_c\| + \sqrt{n}\sigma\|\tilde{q}^g_c\| \leq 0.
\end{aligned} \tag{29}$$

By Substituting (29) into (26), and applying Lemma 3, it can be obtained that

$$\begin{aligned}
\dot{V}_1 \leq &-\frac{k_1}{2} \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} a_{ij} \left(\sum_{k=1}^{d} \left|\tilde{q}^g_{ci(k)} - \tilde{q}^g_{cj(k)}\right|^{\rho+1}\right) \\
\leq &-\frac{k_1}{2} \sum_{i,j} a_{ij} \left[\sum_{k=1}^{d} (\tilde{q}^g_{ci(k)} - \tilde{q}^g_{cj(k)})^2\right]^{\frac{\rho+1}{2}} \\
\leq &-\frac{k_1}{2} (2\tilde{q}^g_c L_{A_\rho} \tilde{q}^g_c)^{\frac{1+\rho}{2}},
\end{aligned}$$

where $A_\rho = [a^{\frac{2}{\rho+1}}_{ij}] \in \mathbb{R}^{n\times n}$. From Lemma 2, we have

$$\dot{V}_1(t) \leq -\frac{k_1}{2}\left[2\lambda_2(L_{A_\rho})\right]^{\frac{1+\rho}{2}} \left(\|\tilde{q}_c\|^2\right)^{\frac{1+\rho}{2}} \leq -k_1 2^\rho [\lambda_2(L_{A_\rho})]^{\frac{1+\rho}{2}} V_1(t)^{\frac{1+\rho}{2}}.$$

Consequently, we conclude from Lemma 4 that

$$\lim_{t\geq T_0}\left(\hat{q}^g_{ci}(t)-q^g_c(t)\right)=\mathbf{0}, \tag{30}$$

where $T_0 \leq V_1(0)/k_1(1-\rho)2^{\rho-1}\left[\lambda_2(L_{A_\rho})\right]^{\frac{1+\rho}{2}}$. This completes the proof that $\hat{q}^g_{ci}$ converge to q^g_c in finite time.

Now we prove that the tracking errors converge to zero.

We will prove in Appendix 7.2 that, by applying the proposed estimator and control algorithms, the state of the closed-loop system, i.e., $\tilde{q}^g_d$, is bounded in $(0, T_0]$. In addition, the states q^g_i, the control signal u^g_i and the estimation variable $\hat{q}^i_{ci}$ are also bounded in finite time given bounded initial states $q^g_i(0)$ and $\hat{q}^g_{ci}(0)$.

Now we are in the position to show the effectiveness of our control laws in achieving estimation based average tracking. Note that control laws (11) can be written in a stacked form as

$$u^g=\mathbf{1}_n\otimes\dot{q}^g_d-k_p\left(B\hat{Q}_\delta\otimes I_d\right)(\hat{q}^g_c-\mathbf{1}_n\otimes q^g_d)-k_s\nabla P(q^g), \tag{31}$$

where

$$\hat{Q}_\delta=\begin{bmatrix}\frac{1}{\delta+\|\hat{q}^g_{c1}-q^g_d\|} & \cdots & 0\\ \vdots & \ddots & \vdots\\ \cdots & \cdots & \frac{1}{\delta+\|\hat{q}^g_{cn}-q^g_d\|}\end{bmatrix}.$$

It is easy to show the matrix Q_δ is positive definite. From Theorem 1, when $t\geq T_0$, $\hat{q}^g_{ci}$ can be replaced by q^g_c. Then, u^g becomes

$$u^g=\mathbf{1}_n\otimes\dot{q}^g_d-k_p\left(BQ_\delta\otimes I_d\right)\left[\mathbf{1}_n\otimes(q^g_c-q^g_d)\right]-k_s\nabla P(q^g), \tag{32}$$

where

$$Q_\delta=\frac{1}{\delta+\|q^g_c-q^g_d\|}I_n.$$

Multiplying both sides of (32) by $(\mathbf{1}^T_n\otimes I_d)$, we have

$$(\mathbf{1}^T_n\otimes I_d)(u^g-\mathbf{1}_n\otimes\dot{q}^g_d)=-k_p[\left(\mathbf{1}^T_nBQ_\delta\otimes I_d\right)\left[\mathbf{1}_n\otimes(q^g_c-q^g_d)\right]-k_s(\mathbf{1}^T_n\otimes I_d)\nabla P(q^g). \tag{33}$$

When $t\geq T_0$, the Lyapunov function candidate is chosen as

$$V=\frac{1}{2}(\tilde{q}^g_d)^T(\tilde{q}^g_d)+P(q^g), \tag{34}$$

where $\tilde{q}^g_d\triangleq q^g_c-q^g_d$ is the centroid tracking error. The derivative of V is given by

$$\dot{V}=(\tilde{q}^g_d)^T(\dot{q}^g_c-\dot{q}^g_d)+\nabla P(q^g)^T\dot{q}^g. \tag{35}$$

Note that

$$\dot{q}^g_c=\frac{1}{n}\sum_{i=1}^n\dot{q}^g_i=\frac{1}{n}(\mathbf{1}^T_n\otimes I_d)\dot{q}^g=\frac{1}{n}(\mathbf{1}^T_n\otimes I_d)u^g. \tag{36}$$

Then it follows

$$\dot{q}^g_c-\dot{q}^g_d=-k_p\left(BQ_\delta\otimes I_d\right)\left[\mathbf{1}_n\otimes(q^g_c-q^g_d)\right]-k_s\nabla P(q^g).$$

Substituting (31), (33), and (36) into (35), we get

$$\begin{aligned}\dot{V}=&-\frac{k_p}{n}(\tilde{q}^g_d)^T\left(\mathbf{1}^T_nBQ_\delta\mathbf{1}_n\otimes I_d\right)\tilde{q}^g_d-\frac{k_s}{n}(\tilde{q}^g_d)^T\left(\mathbf{1}^T_n\otimes I_d\right)\nabla P(q^g)\\&-k_p\nabla P(q^g)^T\left(BQ_\delta\otimes I_d\right)\left(\mathbf{1}_n\otimes\tilde{q}^g_d\right)-k_s(\nabla P(q^g))^T\nabla P(q^g)+(\nabla P(q^g))^T(\mathbf{1}_n\otimes\dot{q}^g_d).\end{aligned}$$

From (10), we have $(\tilde{q}_d^g)^T \left(\mathbf{1}_n^T \otimes I_d\right) \nabla P(q^g) = \mathbf{0}$ and $(\nabla P(q^g))^T(\mathbf{1}_n \otimes \dot{q}_d^g) = \mathbf{0}$ due to the fact that $R(q^g)\left(\mathbf{1}_n^T \otimes I_d\right) = \mathbf{0}$. In light of (36), we obtain that

$$
\begin{aligned}
\dot{V} \leq & -\frac{k_p}{n}\sum_i^n \frac{b_i}{\delta + \|\tilde{q}_d^g\|}\|\tilde{q}_d^g\|^2 - k_s\|\nabla P(q^g)\|^2 + \frac{k_p\sqrt{n}}{\delta + \|\tilde{q}_d^g\|}\|\tilde{q}_d^g\|\|\nabla P(q^g)\| \\
\leq & -\begin{bmatrix} \|\tilde{q}_d^g\| \\ \|\nabla P(q^g)\| \end{bmatrix}^T Q \begin{bmatrix} \|\tilde{q}_d^g\| \\ \|\nabla P(q^g)\| \end{bmatrix},
\end{aligned} \tag{37}
$$

where

$$
Q = \begin{bmatrix} \frac{k_p}{\delta + \|\tilde{q}_d^g\|} & -\frac{k_p\sqrt{n}}{2(\delta+\|\tilde{q}_d^g\|)} \\ -\frac{k_p\sqrt{n}}{2(\delta+\|\tilde{q}_d^g\|)} & k_s \end{bmatrix}.
$$

It can be checked that the matrix Q is positive definite when the control gains k_p and k_s are chosen such that

$$
k_s > \frac{k_p n}{4(\delta + \|\tilde{q}_d^g\|)},
$$

which naturally holds if the condition (17) is satisfied.

Then, we know $\tilde{q}_d^g$ is bounded, which implies q_c^g, and thus q_i^g are bounded under Assumption 1. It follows from (9) that $\nabla P(q^g)$ is bounded. Hence, the control input (22), i.e., the velocity $\dot{q}_i^g$ is bounded. Together with Assumption 1, we know $\dot{\tilde{q}}_d^g$ and $\nabla\dot{P}(q^g)$ are bounded. Therefore, taking the time derivative of (37), we know $\ddot{V}$ is bounded. It can be concluded from the Barbalat's Lemma [36] that $\dot{V} \to 0$, as $t \to \infty$, i.e., $\tilde{q}_d^g \to \mathbf{0}$ and $R(q^g)^T\phi(q^g) \to \mathbf{0}$, as $t \to \infty$, which implies the tracking objective is achieved. For a minimally and infinitesimally rigid framework, the rigidity matrix $R(q^g)$ is full row rank. Hence, we have $\phi(q^g) \to \mathbf{0}$, namely, all the agents converge to the target set $\mathcal{T}_d$ in (2)

The proof of Theorem 1 is completed.

□

Remark 2. *It is worth noting that u_i^g is employed in* (22) *for purposes of theoretical analysis. While the control input to be implemented in practice is* (12) *and* (13).

Remark 3. *The assumption that the framework is minimally and infinitesimally rigid can be relaxed to that the framework is only infinitesimally rigid [9, 11]. In view of the developed techniques for analyzing non-minimally infinitesimally rigid frameworks in [11], the proof is omitted here for the sake of brevity.*

Remark 4. *In this paper, to implement the centroid estimator* (13)*, the underlying communication graph is only required to be a general undirected graph, which could be the same one as required for formation shape control. To explore whether the condition of an undirected graph is necessary for the convergence of the proposed estimator, we carried out a numerical example with three agents under directed graphs. The results show that all the estimation errors will reach a consensus, but not at zero, which implies the proposed estimator fails in directed graphs, even in the simplest case of three agents. Focusing on the second term of* (12)*, i.e., the distance-based formation controller, there has been progress for achieving such formations by employing directed graphs using the notion of persistency [37].*

4. Extension to more general scenarios

The results in Section 3 are obtained under the condition that the local coordinate systems ${}^i\Sigma, i = 1, \cdots, n$, have the same orientations with the global coordinate system ${}^g\Sigma$. However, this constraint may not be satisfied in some applications. In this section, we consider a more general case where the orientations of the local coordinate systems differ from the global one, which is depicted in Fig. 2.

From Fig. 2, we have

$$
\hat{q}_{ci}^g = R_i^g\hat{q}_{ci}^i + q_i^g, \tag{38}
$$

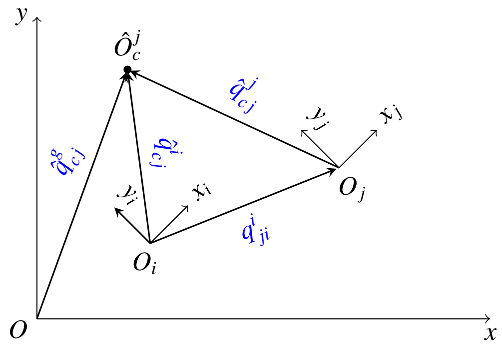

Figure 2: Different orientations between local coordinate systems and the global one.

where $R_i^g \in SO(d)$ is a constant rotation matrix. The centroid estimator is now given by

$$\dot{\hat{q}}_{ci}^i = -k_1 \sum_{j\in\mathcal{N}_i} a_{ij} sig\left(\hat{q}_{ci}^i - \hat{q}_{cj}^i\right)^{\rho} - k_2 \sum_{j\in\mathcal{N}_i} a_{ij} \frac{\hat{q}_{ci}^i - \hat{q}_{cj}^i}{f_{ij}(\hat{q}_{ci}^i, \hat{q}_{cj}^i)} + k_s \sum_{j\in\mathcal{N}_i} (\|z_k^i\|^2 - d_k^2) q_{ij}^i, \tag{39}$$

where k_1 and k_2 are chosen according to Theorem 1, and $f_{ij}(\hat{q}_{ci}^i, \hat{q}_{cj}^i) = \|\hat{q}_{ci}^i - \hat{q}_{cj}^i\| + \left(\sqrt{1 + \|\hat{q}_{ci}^i - \hat{q}_{cj}^i\|} - 1\right)$. Again, the variable $\hat{q}_{cj}^i$ is obtained through $\hat{q}_{cj}^i = \hat{q}_{cj}^j + q_{ji}^i$, where q_{ji}^i is the relative position between O_j and O_i with respect to ${}^i\Sigma$, which can be measured by agent i locally. It is worth noting that the variable q_{ji}^i employed in (39) is measured in the local coordinate system ${}^i\Sigma$, allowing the distinction of the orientations between the local coordinate systems and the global one, since the value of q_{ji}^i will not be altered in that case. Summing up both sides of (38), we have

$$\sum_{i=1}^{n} \hat{q}_{ci}^g = \sum_{i=1}^{n} R_i^g \hat{q}_{ci}^i + \sum_{i=1}^{n} q_i^g. \tag{40}$$

Since the local coordinate systems have the same orientation, we obtain that $R_i^g = R_j^g, i, j = 1, \cdots, n$. By denoting $R_l^g \triangleq R_i^g$, (40) can be written as

$$\sum_{i=1}^{n} \hat{q}_{ci}^g = R_l^g \sum_{i=1}^{n} \hat{q}_{ci}^i + \sum_{i=1}^{n} q_i^g.$$

Considering the estimator (39), we know $\sum_{i=1}^{n} \dot{\hat{q}}_{ci}^i = \mathbf{0}$. Then, in combination with the initial condition $\sum_{i=1}^{n} \hat{q}_{ci}^i(0) = \mathbf{0}$, it yields $\sum_{i=1}^{n} \hat{q}_{ci}^g = \sum_{i=1}^{n} q_i^g = nq_c^g$. Following the similar steps as in Section 3, it can be shown that $\hat{q}_{ci}^g$ converges to q_c^g in finite time.

In this scenario, the control law is designed as

$$u_i^d = \dot{q}_i^d = -k_p b_i \frac{\hat{q}_{ci}^i - q_d^i}{\delta' + \|\hat{q}_{ci}^i - q_d^i\|} - k_s \sum_{j\in\mathcal{N}_i} (\|z_k^i\|^2 - d_k^2) q_{ij}^i, \tag{41}$$

where $\delta' > 1$ is a constant scalar, and k_p and k_s are chosen such that (17) holds. It can be seen that (41) has the same form as that of (12), while the value of q_{ij}^i here differs from q_{ij}^g due to orientation difference between local and global coordinate systems.

Following the similar proof steps as in Section 3, the centroid of the formation can be proved to converge to the reference signal. The details of the proof is omitted in this section to avoid repetition.

Remark 5. *For the scenario where the orientations of the local coordinate systems are different from each other, it can be shown that the estimator and the control law remain to be the same as* (39) *and* (41) *without loss of stability.*

While the variable $\hat{q}^i_{cj}$ *in* (39) *is now calculated by* $\hat{q}^i_{cj} = R^i_j \hat{q}^j_{cj} + q^i_{ji}$, *where* R^i_j *is the rotation matrix with respect to frames i and j. Note that the rotation matrix depends only on the relative rotation angle between local coordinate systems* ${}^i\Sigma$ *and* ${}^j\Sigma$. *Therefore, with the sensing capability of rotation angles with respect to neighbors, the proposed control framework is still applicable to the case when the orientations of local systems are not necessarily equal to each other. For those systems without such sensing capability, estimation techniques are reported in recent works, e.g., [13, 38].*

5. Simulations

To validate the theoretical results, we consider the formation tracking problem for eight agents with dynamics (1), whose interaction relationship is given in Fig. 3.

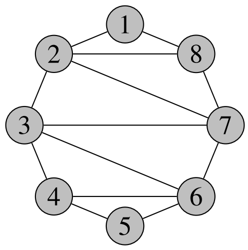

Figure 3: The prescribed framework of the eight agents–regular octagon

Take the initial positions for the eight agents as, respectively, $[1,3]^T$, $[-1,1]^T$, $[-3,0.2]^T$, $[-2.7,-0.2]^T$, $[0.2,-4]^T$, $[2,-2]^T$, $[1,-0.5]^T$, $[1,2]^T$. The reference signal is given by $\sigma_d(t) = [6*t, 5*\cos(t)]^T$. Let the initial values of the centroid estimation be $\hat{q}^i_{ci}(0) = [4.5-i, i-4.5]^T$, $i = 1,\cdots,8$, which satisfies the condition that $\sum_{i=1}^{8} \hat{q}^i_{ci}(0) = \mathbf{0}$. The control parameters are chosen as $\rho = 1/4$, $k_1 = 3$, $k_2 = 12$, $k_p = 9$ and $k_s = 13$.

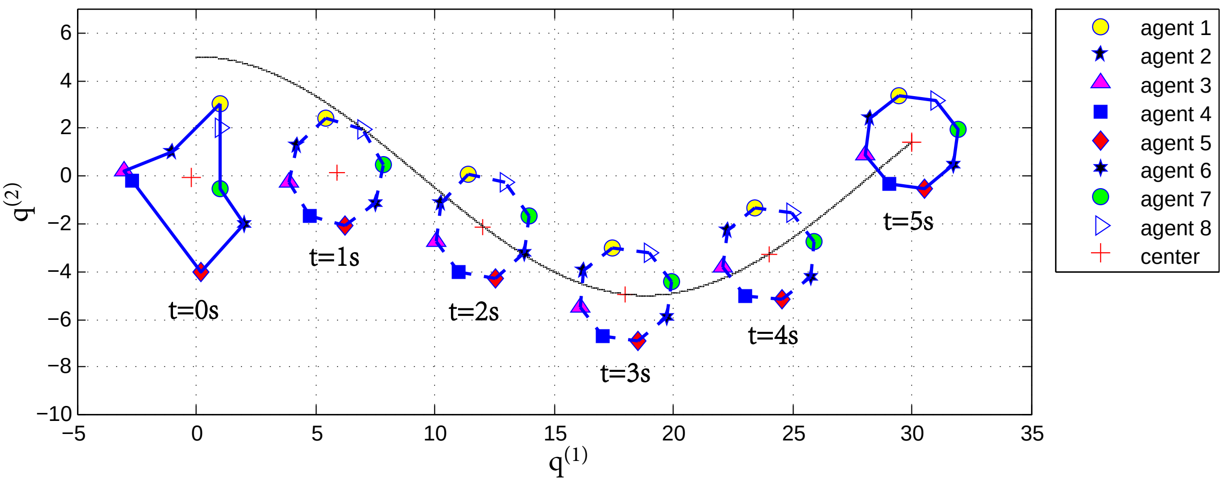

Figure 4: Formation shape evolution.

The simulation results are shown in Fig. 4 – 6, where we use $x^{(i)}$, $i = 1,2$, to denote the ith component of vector x. The formation geometries of the agents at $t \in \{0;1;2;3;4;5\}$s are shown in Fig. 4, where the red cross and the solid black line represent the centroid of the whole formation shape and the centroid's reference trajectory, respectively. From Fig. 4 we can see that the prescribed regular octagon is achived with its centroid converging to the reference trajectory. The convergence of the centroid tracking error is further shown in Fig. 5. Fig. 6 depicts the centroid estimation errors associated with agents 1, 3, 5, and 7 as representatives, which demonstrates the effectiveness of the proposed finite-time estimator.

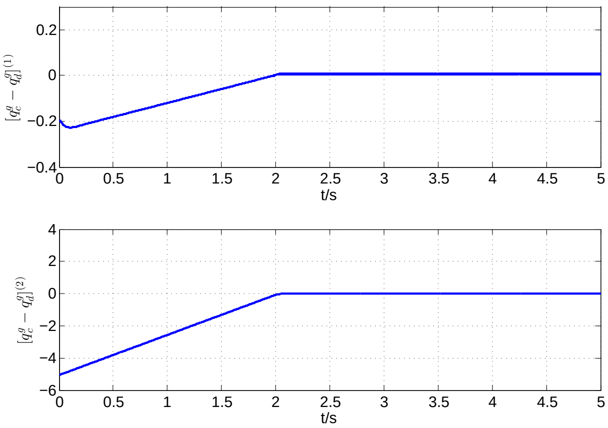

Figure 5: Centroid tracking error $q_c^g - q_d^g$.

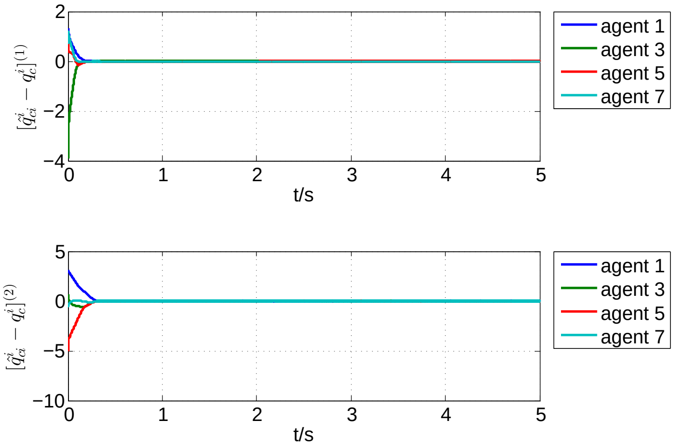

Figure 6: Centroid estimation error $\hat{q}_{ci}^i - q_c^i$.

6. Conclusion

In this paper, we have investigated the formation tracking problem using local coordinate systems. By introducing a new gradient descent term, an alternative estimator is designed for each agent such that they can obtain the precise knowledge of the formation's centroid in finite time. Moreover, we propose a distributed estimator-controller strategy, which can be implemented using only agents' local coordinate systems. One future study is to extend the current results to the case that the orientations of the local coordinate systems are inconsistent. Another possible exploration is to consider the time-varying formation tracking problems, e.g. formation spinning and formation scaling.

7. Appendix

7.1. Proof of (24)

Suppose $x \in \mathbb{R}^d$ and $\|x\| \neq 0$, when ϵ is chosen such that $0 < \epsilon \leq \frac{2}{3}$, we have

$$(1-\epsilon)^2\|x\|^2 + \epsilon(2-3\epsilon)\|x\| \geq 0.$$

Equivalently,

$$(1-\epsilon)^2\|x\|^2 + 2\epsilon\|x\| - 2\epsilon^2\|x\| \geq \epsilon^2\|x\|. \tag{42}$$

Then, adding ϵ^2 to both sides of (42), we obtain that

$$(1-\epsilon)^2\|x\|^2 + 2\epsilon(1-\epsilon)\|x\| + \epsilon^2 \geq \epsilon^2\|x\| + \epsilon^2,$$

which is can be written as

$$[(1-\epsilon)\|x\| + \epsilon]^2 \geq \epsilon^2(1+\|x\|). \tag{43}$$

By taking a square root of (43), it follows

$$(1-\epsilon)\|x\| + \epsilon \geq \epsilon\sqrt{1+\|x\|}.$$

After simple calculation, we get

$$\|x\| \geq \epsilon\|x\| + \epsilon\left(\sqrt{1+|x|} - 1\right). \tag{44}$$

Multiplying both sides of (44) by $|x|$, we have

$$|x|^2 \geq \epsilon|x|^2 + \epsilon\|x\|\left(\sqrt{1+\|x\|} - 1\right).$$

Since $\|x\| + \|x\|\left(\sqrt{1+\|x\|}-1\right) > 0$, it is straightforward to know

$$\frac{\|x\|^2}{\|x\| + \left(\sqrt{1+\|x\|}-1\right)} \geq \epsilon\|x\|. \tag{45}$$

When $\|x\| \to 0$, we have

$$\lim_{\|x\|\to 0} \frac{\|x\|^2}{\epsilon\|x\|\left(\|x\| + \left(\sqrt{1+\|x\|}-1\right)\right)} = \lim_{\|x\|\to 0} \frac{\|x\|^2}{\epsilon\|x\|^2 + \frac{1}{2}\epsilon\|x\|^2} = \frac{2}{3\epsilon},$$

where we have used the equivalent infinitesimal $\left(\sqrt{1+\|x\|}-1\right) \sim \frac{1}{2}\|x\|$. In view of the condition that $0 < \epsilon \leq \frac{2}{3}$, we further obtain that

$$\lim_{\|x\|\to 0} \frac{\|x\|^2}{\epsilon\|x\|\left(\|x\| + \left(\sqrt{1+\|x\|}-1\right)\right)} \geq 1.$$

In addition, it holds that

$$\lim_{\|x\|\to 0} \frac{\|x\|^2}{\|x\| + \left(\sqrt{1+\|x\|}-1\right)} = \lim_{\|x\|\to 0} \frac{\|x\|^2}{\|x\| + \frac{1}{2}\|x\|} = 0.$$

Hence, $\forall x \in \mathbb{R}^n, 0 < \epsilon \leq \frac{2}{3}$, we have

$$\frac{\|x\|^2}{\|x\| + \left(\sqrt{1+\|x\|}-1\right)} \geq \epsilon\|x\|.$$

7.2. Proof of the boundedness of $\tilde{q}_d^g$ in $(0, T_0]$

Now we consider the system dynamics during $t \in (0, T_0]$. Then (31) can be equivalently written as

$$\begin{aligned} u^g =& \mathbf{1}_n \otimes \dot{q}_d^g - k_p\left(B\hat{Q}_\delta \otimes I_d\right)\left(\hat{q}_c^g - \mathbf{1}_n \otimes q_c^g + \mathbf{1}_n \otimes q_c^g - \mathbf{1}_n \otimes q_d^g\right) - k_s\nabla P(q^g) \\ =& \mathbf{1}_n \otimes \dot{q}_d^g - k_p\left(B\hat{Q}_\delta \otimes I_d\right)\left(\mathbf{1}_n \otimes q_c^g - \mathbf{1}_n \otimes q_d^g\right) - k_s\nabla P(q^g) \\ & - k_p\left(B\hat{Q}_\delta \otimes I_d\right)\left(\hat{q}_c^g - \mathbf{1}_n \otimes q_c^g\right) \end{aligned} \tag{46}$$

Note that the first line after the second equality sign in (46) is exactly (32). In addition, we know

$$\begin{aligned} -\frac{k_p}{n}(\tilde{q}_d^g)^T(\mathbf{1}_n^T B\hat{Q}_\delta \otimes I_d)\tilde{q}_c^g \leq& \frac{k_p}{2n}\sum_{i=1}^n \frac{b_i}{\delta + \|\hat{q}_{ci}^g - q_d^g\|}(\|\tilde{q}_d^g\|^2 + \|\tilde{q}_{ci}^g\|^2) \\ \leq& \frac{k_p}{2n}\sum_{i=1}^n \frac{b_i}{\delta + \|\hat{q}_{ci}^g - q_d^g\|}\|\tilde{q}_d^g\|^2 + \frac{k_p}{2n\delta}\|\tilde{q}_c^g\|^2 \end{aligned}$$

Then in view of (37), we have

$$\begin{aligned} \dot{V} \leq& -\frac{k_p}{2n}\sum_{i}^n \frac{b_i}{\delta + \|\hat{q}_{ci}^g - q_d^g\|}\|\tilde{q}_d^g\|^2 - k_s\|\nabla P(q^g)\|^2 + \frac{k_p\sqrt{n}}{\delta + \|\hat{q}_{ci}^g - q_d^g\|}\|\tilde{q}_d^g\|\|\nabla P(q^g)\| \\ & + \frac{k_p}{2n}\sum_{i=1}^n \frac{b_i}{\delta + \|\hat{q}_{ci}^g - q_d^g\|}\|\tilde{q}_{ci}^g\|^2 \\ \leq& -\begin{bmatrix} \|\tilde{q}_d^g\| \\ \|\nabla P(q^g)\| \end{bmatrix}^T Q' \begin{bmatrix} \|\tilde{q}_d^g\| \\ \|\nabla P(q^g)\| \end{bmatrix} + \frac{k_p}{2n\delta}\|\tilde{q}_c^g\|^2, \end{aligned} \tag{47}$$

where

$$Q' = \begin{bmatrix} \frac{k_p}{2\left(\delta + \sup_{t\in(0,T]}\|\hat{q}_{ci}^g - q_d^g\|\right)} & -\frac{k_p\sqrt{n}}{2\left(\delta + \sup_{t\in(0,T]}\|\hat{q}_{ci}^g - q_d^g\|\right)} \\ -\frac{k_p\sqrt{n}}{2\left(\delta + \sup_{t\in(0,T]}\|\hat{q}_{ci}^g - q_d^g\|\right)} & k_s \end{bmatrix}.$$

Then, Q' is positive definite if k_s is chosen such that

$$k_s > \frac{k_p n}{2\left(\delta + \sup_{t\in(0,T])} \|\hat{q}^g_{ci} - q^g_d\|\right)},$$

which automatically holds under the condition (17). It follows from (47) that

$$V(T_0) = V(0) - \int_0^{T_0} \begin{bmatrix} \|\tilde{q}^g_d\| \\ \|\nabla P(q^g)\| \end{bmatrix}^T Q' \begin{bmatrix} \|\tilde{q}^g_d\| \\ \|\nabla P(q^g)\| \end{bmatrix} dt + \int_0^{T_0} \frac{k_p}{2n\delta} \|\tilde{q}^g_c\|^2 dt$$

Recalling the convergence of $\|\tilde{q}^g_c\|$ from (30), we know $\int_0^{T_0} \frac{k_p}{2n\delta}\|\tilde{q}^g_c\|^2 dt$ is bounded for finite number T_0. It thus follows from the formula of V in (34) that $V(T_0)$ is bounded. Hence, during $t \in (0, T_0]$, $\tilde{q}^g_d$ and $P(q^g)$ are both bounded.

In addition, we can also infer $\nabla P(q^g)$ is bounded from the boundedness of $P(q^g)$, and thus the control input u^g_i in (22) is bounded. Hence, the position variable q^g_i becomes bounded in finite time. It can also be obtained from (13) that $\hat{q}^i_{ci}$ is bounded in finite time.

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