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Transforming intraclass correlation coefficients with the Spearman–Brown formula

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Abstract

Objectives: De Vet, Mokkink, Mosmuller, and Terwee (2017) discovered that the Spearman–Brown formula (SB formula) can be used to transform certain intraclass correlation coefficients (ICCs) for single measurements into the corresponding ICCs for average measurements, without knowledge of the variance components.

Results: A formal proof of the discovery by De Vet et al. is presented. It is also proved that, vice versa, the SB formula can be used to transform certain ICCs for average measurements into the corresponding ICCs for single measurements. Furthermore, it is specified to which type of ICC the transformations can be applied. Finally, several illustrations of the transformations are presented.

Conclusion: The discovery by De Vet et al. is quite remarkable because ICCs and the SB formula come from quite different scientific disciplines. © 2017 Elsevier Inc. All rights reserved.

Keywords: Intraclass correlation coefficients; Spearman–Brown formula; Single measurements; Average measurements; Agreement; Consistency

In behavioral and biomedical sciences, intraclass correlation coefficients (ICCs) are used to quantify the proportion of variance (variously defined) that is attributable to objects of measurement [1]. ICCs are not limited to two measurements per object but can be used if there are three or more observations per object of measurement. Examples are, CT scans scored by multiple physicians, students filling in a number of test items, or subjects or products being rated by different observers.

Data may have different sources of variance. Depending on the type and number of sources, various analysis of variance models can be specified for the sample data. McGraw and Wong [1] discuss five models and review various ICCs that can be defined for the models. Let σ_t^2 denotes the variance attributable to the objects of measurement (scans, students, subjects rated by observers) called targets in Shrout and Fleiss [2]. Furthermore, let m be a positive real number, and let $\sigma_1^2 + \dots + \sigma_q^2$ be a summation of q variance

components associated with other sources than the object of measurements, for example, σ_w^2 , σ_c^2 , σ_{rc}^2 or σ_e^2 defined in McGraw and Wong [1]. A general formula for an ICC is given by

$$\text{ICC}(m) = \frac{\sigma_t^2}{\sigma_t^2 + (\sigma_1^2 + \dots + \sigma_q^2)/m} \quad (1)$$

Many ICCs only differ in how the summation $\sigma_1^2 + \dots + \sigma_q^2$ is defined. Furthermore, McGraw and Wong [1] distinguish between ICCs for single measurements, for example, ratings of observers or individual item scores, and ICCs for average measurements, for example, the average rating for k observers or the average score for a k -item test. For $m = 1$, the ICC in Eq. (1) is a general formula for single measurements. For $m = k$, the ICC in Eq. (1) is a general formula for average measurements.

The Spearman–Brown prophecy formula (SB formula) is a formula from classical test theory that relates the reliability of a test to its length [3–6]. It can be used to predict the reliability of an extended or shortened test. Let ρ^* denote the reliability of the extended test, ρ the reliability of the original test, and n the ratio of the length of the extended test to the length of the original test, for example, we have $n = 2$ if the test length is doubled. The SB formula is given by

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$$\rho^* = \frac{np}{1 + (n-1)\rho} \quad (2)$$

De Vet et al. [7] discovered that the SB formula can be used to transform the value of an ICC for single measurements into the corresponding ICC for average measurements. The transformation only requires the value of the ICC for single measurements, together with the SB formula. The values of the variance components are not required. The discovery is remarkable because ICCs and the SB formula come from quite different scientific disciplines.

De Vet et al. [7] presented examples to illustrate the connection between ICCs and the SB formula. Here, we present a formal proof of the transformation. Furthermore, it turns out that the SB formula can also be used to transform the value of an ICC for average measurements into the corresponding ICC for single measurements. More generally, any $\text{ICC}(m)$ can be transformed into $\text{ICC}(u)$ of the same general form, that is, consisting of the same variance components, where m and u can be any positive numbers, using the SB formula and $n = u/m$. A proof is as follows.

Inserting $\rho = \text{ICC}(m)$ and $n = u/m$ in Eq. (2) we obtain, using Eq. (1),

$$\rho^* = \frac{\frac{u\sigma_i^2}{m\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2)}}{1 + \left(\frac{u}{m} - 1\right) \frac{\sigma_i^2}{\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2)/m}} \quad (3)$$

Since $u/m - 1 = (u - m)/m$, Eq. (3) is equivalent to

$$\rho^* = \frac{\frac{u\sigma_i^2}{m\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2)}}{1 + \frac{(u-m)\sigma_i^2}{m\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2)}} \quad (4)$$

Multiplying all terms on the right-hand side of Eq. (4) by $m\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2)$ yields

$$\begin{aligned} \rho^* &= \frac{u\sigma_i^2}{m\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2) + (u-m)\sigma_i^2} \\ &= \frac{u\sigma_i^2}{u\sigma_i^2 + (\sigma_1^2 + \dots + \sigma_q^2)} \end{aligned} \quad (5)$$

Dividing all terms on the right-hand side of Eq. (5) by u yields $\text{ICC}(u)$. Hence, any $\text{ICC}(m)$ can be transformed into $\text{ICC}(u)$ of the same general form using the SB formula and $n = u/m$. The result does not depend on how $\sigma_1^2 + \dots + \sigma_q^2$ is defined. Examples of ICCs from McGraw and Wong [1] are the ICCs corresponding to the one-way random effects model (case 1), the two-way random model with interaction (case 2), the two-way random model without interaction (case 2A), and the two-way mixed model without interaction (case 3A). The transformation does not work for the ICCs corresponding to the two-way mixed model with interaction (case 3).

The above result is partially illustrated in De Vet et al. [7]. Here, we illustrate the result using the data presented

Table 1. ICC values of the data in Table 2 in the study by Shrout and Fleiss [2]

Model	One way	Two way	
		Agreement	Consistency
McGraw and Wong [1]	$\text{ICC}(m)$	$\text{ICC}(A,m)$	$\text{ICC}(C,m)$
Shrout and Fleiss [2]	$\text{ICC}(1,m)$	$\text{ICC}(2,m)$	$\text{ICC}(3,m)$
$m = 1$	0.166	0.290	0.715
$m = 2$	0.284	0.449	0.834
$m = 3$	0.373	0.550	0.883
$m = 4$	0.443	0.620	0.909
$m = 5$	0.498	0.671	0.926
$m = 6$	0.544	0.710	0.938

in Table 2 on page 423 of Shrout and Fleiss [2]. These data consist of ratings by four judges on six targets. The corresponding ICC values are presented in Table 1. The second, third, and fourth columns present, respectively, the values of the ICCs based on the one-way model, absolute agreement, and consistency. The third and fourth rows of Table 1 present the symbols used in, respectively, McGraw and Wong [1] and Shrout and Fleiss [2].

The fifth row of Table 1 ($m = 1$) contains the ICC values for single measurements. Since there are four judges ($k = 4$), the eighth row ($m = 4$) contains the ICC values for average measurements. The SB formula can be used to transform the value of an ICC for single measurements into the corresponding ICC (in the same column) for average measurements. Using $\rho = 0.166$, 0.290, and 0.715 together with $n = 4$ in Eq. (2), we obtain, respectively, 0.443, 0.620, and 0.909. Vice versa, we can transform the value of an ICC for average measurements into the corresponding ICC for single measurements. Using $\rho = 0.443$, 0.620, and 0.909 together with $n = 1/4 = 0.25$ in Eq. (2), we obtain, respectively, 0.166, 0.290, and 0.715.

The values in the sixth ($m = 2$), seventh ($m = 3$), ninth ($m = 5$), and tenth ($m = 6$) rows of Table 1 can be interpreted as estimates of ICC values for average measurements based on, respectively, 2, 3, 5, and 6 judges [7]. Here, the estimates incorporate variance components based on four judges (Table 2 in the study by Shrout and Fleiss [2]). As noted in De Vet et al. [7], the values in these four rows of Table 1 can be calculated using $\rho = 0.166$, 0.290, and 0.715 (the ICC values for single measurements) together with, respectively, $n = 2$, $n = 3$, $n = 5$, and $n = 6$ in Eq. (2). Alternatively, the above proof shows that the values in these four rows can also be obtained using $\rho = 0.443$, 0.620, and 0.909 (the ICC values for average measurements) together with, respectively, $n = 2/4 = 0.5$, $n = 3/4 = 0.75$, $n = 5/4 = 1.2$, and $n = 6/4 = 1.5$ in Eq. (2).

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