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Asynchronous distributed control of biogas supply and multi-energy demand

Desti Alkano, Jacquélien M.A. Scherpen, and Younes Chorfi

Abstract—In this paper we study the coordination between biogas producers who can either use the biogas themselves, exchange biogas with their neighbors, or deliver it to the various energy grids, such as the low pressure gas grid or the power grid. These producers are called prosumers. In this setting gas storage, fuel cells, micro combined heat power systems, and heat buffers are all part of the prosumers' node. We aim to optimize the imbalance, profit, and comfort levels per prosumer, while taking the constraints of the energy grids into account, and while allowing prosumers to exchange energy with each other. This results in a two-layer optimization problem formulation. In addition, in practice, communication between prosumers among each other and with grid operators is done in an asynchronous manner. In this paper we study the problem of two-layer optimization for biogas prosumers embedded in multiple energy grids, while the (bidirectional) communication between the various partners is done asynchronously. We prove the convergence of the asynchronous coordination algorithm that uses both the inputs and the states. We conduct simulations for the biogas prosumer setting, using realistic data to illustrate the convergence of the algorithm and to study its practical implementation.

Note to Practitioners—This paper is motivated and supported by a smart gas grid project of the Energy Delta Gas Research (EDGaR) consortium in the Netherlands. The project deals with investigating the capacity of smart grid technologies to facilitate the introduction of new gases into the distribution grids, with diverse gas qualities and multiple injection points. The gas distribution grid will have to move from a passive to an active distribution system that dynamically control bidirectional flows between end-users and the grid operators. As the end-users may be equipped with energy converters, other energy distribution grids also need to transform to active distribution systems. Existing approaches are distributed, where each end-user and energy grid operator can locally solve their optimal control problem. In this paper, we consider the fact that both end-users and grid operators do not have access to a common clock when solving their problem and when sharing their information. The information includes some of their states and controllable inputs. The asynchronous information exchange problem was pointed out by DNV GL Netherlands, Gasunie, and Gastera which are companies we collaborate with within the EDGaR consortium. It is highly relevant for practical implementation of our distributed algorithms. In future research, we will include practical control considerations due to on-off constraints of

energy converters and some flexible demand due to, e.g., the use of plug-in electric vehicles. These require some extension on the proposed algorithms.

Index Terms—Network congestion control, flexible demand, gradient algorithm, distributed asynchronous, multi-energy carrier, cooperative systems, decentralized energy storage.

I. INTRODUCTION

THERE have been a significant number of studies on congestion control mechanisms for large-scale energy networks. See, e.g., [1], [2], [3], for instance. The mechanisms deal with the problems of controlling the supply and demand levels of distributed generation and active demand units such that the associated utility functions are maximized under technical constraints. Equipped with energy storage devices, the optimization problem of the units take the dynamic states of the storage devices into account explicitly [4]–[5].

The impact of the integration of distributed generation and active demand units in the existing energy grids have been studied in, e.g., [4], [6], and [7]. Due to practical and computational limitations, most of the studies solve the aforementioned optimization problem in a distributed manner, with an assumption that all units and the energy grid operators update their systems, i.e. by calculating and (partly) communicating their controllable inputs, at the same time.

Practically, they do not have a common clock to update their systems. As a result, they solve their local optimization problem based on outdated information. For the problem of controlling the aggregated flow rates of sources such that their utilities are maximized under individual and grid capacity constraints, a distributed and asynchronous mechanism has been introduced in [8]. In addition, there has been a sustained effort to implement the mechanism for large-scale networks, e.g., in wireless networks [9], [10], and [11].

The purpose of this paper is to propose a two-layer distributed optimal control, where the control mechanism is derived as a means to optimize a global performance of a community consisting of dynamic agents, while obeying individual objectives and constraints. The associated setting is given in the subsection below. In this paper, we will present asynchronous algorithms and analytically prove the convergence in a dynamic environment. We will then implement the algorithms using realistic data for illustrating the convergence and studying the impact of individual characteristics on the optimality.

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A. Considered setting

Consider a micro grid consisting of agents. The agents are equipped with local anaerobic digesters, distributed generation units by means of micro Combined Heat and Power (μ -CHP) devices, heat buffers, fuel cells, and decentralized gas storage devices. The μ -CHP devices run on biogas generated from organic waste using local anaerobic digesters [12], [13], [14]. The biogas μ -CHP devices are switched on to satisfy the local power and heat demand. Equipped with smart space heating systems, some part of agents' heat demand can be shifted in time. Furthermore, the agents have capability to control the power and heat output from the μ -CHP devices. Hence, in what follows we call them (active) prosumers, as they are able to control both production and demand sides. A schematic illustration of such a prosumer is shown in Fig. 1.

Within the micro grid, the prosumers can contribute to balance between supply and demand hence increasing the value of the produced renewable energy, i.e. biogas, and reducing the losses due to energy transmission. When there is some amount of excess biogas after filling the demand in the micro grid, the prosumers may inject the biogas to a low-pressure (LP) gas grid and/or may sell it to a gas filling station. Moreover, the excess biogas can be converted to electrical energy using a μ -CHP or a fuel cell. When the prosumers have a lack of energy, i.e. due to the fact that the biogas production level highly depends on the agricultural seasons and weather conditions, we allow the prosumers to import energy from some external energy grids. Nevertheless, the capacities of the energy grids are limited thereby restricting the amount of energy injected and imported from the grids.

We use imbalance information proposed in [15] allowing the prosumers to keep track on the imbalance of their connected prosumers in the micro grid. In this way, the prosumers can coordinate their decisions in order to maintain the balance between supply and demand within the micro grid.

To handle technical constraints and to anticipate on the future situation of the energy grids and the local load profiles, we formulate the associated optimization problem in model predictive control (MPC) framework. We refer the reader to, e.g., [16], for the detailed explanation of the MPC approach. We solve the MPC problem in a distributed manner, because of practical and computational limitations. The stability and feasibility of the distributed MPC have been studied in [17].

We allow the prosumers and the energy grid operators to coordinate their decision variables to each other asynchronously, as they may not have access to a common clock when solving their optimization problems. Dynamic pricing mechanisms are here utilized to: *a)* coordinate the imbalance between local biogas supply and demand hence minimizing the imbalance within the micro grid and *b)* coordinate the supply levels of the excess biogas from the prosumers to the energy grid operators thus avoiding overloading grids while maximizing the prosumers' expected profit.

B. Related work

An extensive literature exists to extend the work presented in [8] to, e.g., end-to-end congestion control schemes [18],

simultaneous routing and resource allocation [19], and load balancing [20]. Some recent work has derived an improvement for the distributed mechanism proposed in [8], which uses a standard gradient projection method for the decomposition approach, by employing alternating direction method of multipliers (ADMM) in order to obtain faster and more robust computation. See, e.g., [21]. It however requires substantial communication overhead, i.e., besides requiring each agent to know the cost and benefit of all possible choices, the agents need to know their share of unallocated capacity at each connected grid [22].

Furthermore, in [23] we extend the distributed asynchronous mechanism presented in [8] to a setting where a number of agents are embedded in multiple grids. Equipped with (energy) converters, the agents are able to offer multiple types of products to the corresponding grids. Each product is characterized by a utility function of its supply rate and the agents' goal is to maximize the aggregated utility. Each product has a particular path to transfer to the corresponding grids, which in practice have limited capacities. In [23] however, there exists no local interaction between the agents. Only asynchronous updates of the supply levels, which are controllable inputs, shared between the agents and the connected grid operators are considered.

C. Contribution

Under the considered setting presented in Subsection I-A, we specifically extend the distributed mechanism proposed in [23] for *state and input asynchronous updates* for the coordination and communication done between the agents within a community (micro grid) and between the agents and the connected grid operators. The contributions of this paper are twofold, summarized as follows:

- 1) Two-layer distributed optimal control is proposed for achieving the community' goal while obeying the grid capacity constraints and individual technical constraints' of each prosumer, given the fact that the prosumers can fully control biogas supply from the decentralized gas storage device and partly control demand levels. Specifically, we propose distributed algorithms for the supply and demand coordination:
 - a) between the prosumers and their neighboring prosumers within the micro grid to minimize the community imbalance between supply and demand and
 - b) between the prosumers and the external energy grid operators to maximize the profit from their surplus energy without exceeding the grid capacity constraints.
- 2) An asynchronous algorithm for communicating the input and dynamic states on the distributed coordination is derived. Moreover, the convergence of the algorithm is analytically proved in this paper.

The rest of the paper is organized as follows. In Section II, we formulate the dynamics and technical constraints of prosumers equipped with anaerobic digesters, biogas μ -CHP devices, heat buffers, fuel cells, and gas storage devices. The

Fig. 1: The schematic illustration of a prosumer equipped with an anaerobic digester, decentralized gas storage device, fuel cell, micro-CHP device, and heat buffer. The prosumer is embedded in a low-pressure gas grid, low-voltage power grid, and coupled to a gas filling station. The produced biogas needs to be upgraded to green gas before injecting it to the low-pressure gas grid and the gas filling station, in order to satisfy the quality requirement of the injection. Additionally, the green gas being sold to the gas filling station needs to be compressed in an effort to use less space in transporting the green gas to the station.

constraints of external energy grids embedded in the micro grid of prosumers, including an LV power grid, an LP gas grid, and a gas filling station are described also in Section II. We define the optimal supply and demand control formulated in the MPC framework and propose the corresponding algorithms to solve the optimization problem in a distributed manner in Section III. In Section IV, we incorporate the asynchronous exchange information scheme on the algorithms and prove their convergence. Numerical results for various cases using realistic data are presented in Section V. Some discussion and future work are presented in Section VI.

II. SYSTEM MODEL

We first develop a model of a micro grid consisting of prosumers. The prosumers are equipped with a local anaerobic digester, a fuel cell, a biogas storage device, a μ -CHP device, and a heat buffer. They have local power and heat demands. They are embedded in a low-pressure (LP) gas grid, a low-voltage (LV) power grid, and coupled to a gas filling station. The schematic illustration of such a prosumer is presented in Fig. 1. The technical constraints and dynamics of the prosumers are described in Section II-A. We explain the role of the three aforementioned external energy grids in Section II-B. We assume that the strategy of the prosumers is to maximize their revenue, to minimize the associated costs, to minimize the overall biogas imbalance in the micro grid, and to maintain the heat comfort level of the prosumers, as described in Section II-C.

A. A micro grid of prosumers

A prosumer $i \in \{1, \dots, n\}$ has its local heat demand $h_{d,i}(k) \in \mathbb{R}_{\geq 0}$ and local power demand $p_{d,i}(k) \in \mathbb{R}_{\geq 0}$ at time $k \in \{1, \dots, K\}$. The prosumers aim at locally fulfilling the heat and power demands by producing the energy using their μ -CHP devices running on biogas. The biogas is generated from prosumers' organic waste using the local anaerobic digester. See [12] for the detailed anaerobic digestion process. In this way, the value of the produced biogas is increased and some losses due to the energy transmission can be reduced.

Produced from the organic waste, it can be argued that the biogas production level varies depending on weather conditions and agricultural seasons. Equipped with a decentralized gas storage device, the prosumers can better cover their local energy demands against the uncertain biogas production $p_i(k) \in \mathbb{R}_{\geq 0}$. Define the amounts of biogas taken from the storage device as $u_i(k) \in \mathbb{R}_{\geq 0}$. Given the initial value of the stored biogas in the storage device denoted by $z_{g,i}(1)$, the dynamics of the available biogas in the device $z_{g,i}(k) \in \mathbb{R}_{\geq 0}$ is specified by

$$z_{g,i}(k+1) = z_{g,i}(k) + p_i(k) - u_i(k). \quad (1)$$

Suppose that S_i is the maximum capacity of the biogas storage device. Then the available biogas in the device is bounded by

$$0 \leq z_{g,i}(k) \leq S_i. \quad (2)$$

The biogas available in the storage device can be used for

the following purposes.

- It can be used as a fuel for running the μ -CHP devices in order to satisfy the prosumers' local heat and power demands [14]. It can also be used to help the connected prosumers to satisfy their local demands. Define the amounts of biogas needed to run the μ -CHP device and to help the connected prosumers as $q_i(k) \in \mathbb{R}_{\geq 0}$.
- The stored biogas can be sold to the gas filling station at $g_i(k) \in \mathbb{R}_{\geq 0}$ using a lorry and/or to the LP gas grid at $f_i(k) \in \mathbb{R}_{\geq 0}$. The station and grid may have different capacities, selling price patterns, and associated costs.
- The stored biogas can be converted into electrical energy using a fuel cell. The produced electricity can then be sold to an LV power grid. The amount of biogas is denoted by $e_i(k) \in \mathbb{R}_{\geq 0}$.

With the aforementioned purposes of the stored biogas in the storage device, we have

$$u_i(k) = q_i(k) + f_i(k) + g_i(k) + e_i(k). \quad (3)$$

Due to gas quality requirements for the grid injections, the biogas being sold to the gas filling station and to the LP gas grid needs to be upgraded to green gas. Because of high investment cost, the prosumers in a micro grid may build a central biogas upgrader [32]. As the upgrader has a production capacity of $F_g \in \mathbb{R}_{\geq 0}$, we have

$$\sum_{i=1}^n f_i(k) + g_i(k) \leq F_g. \quad (4)$$

Remark 1: Besides using the fuel cell, the conversion from biogas into electrical energy can also be done by the μ -CHP devices, which will be described shortly in the next paragraph. Nevertheless, we here assume that the power output from the μ -CHP device is only for satisfying the local power demand and the power output from the fuel cell is only for creating some profit by selling the produced electrical energy to the LV power grid.

A μ -CHP device mainly consists of a prime mover and an auxiliary burner. Among the prime mover technologies, a proton exchange membrane fuel cell (PEMFC) prime mover is commonly chosen as it has the highest electric efficiency among other prime mover technologies and provides low emissions [30]. The prime mover can produce heat and electricity at the same time k . When the prime mover is controlled based on power demand, the heat output at a level of $h_{pm,i}(k) \in \mathbb{R}_{\geq 0}$ fluctuates accordingly. Otherwise, when the prime mover is controlled based on heat demand, the power output at a level of $p_{pm,i}(k) \in \mathbb{R}_{\geq 0}$ from the prime mover fluctuates accordingly. As the power output and heat output from the prime mover are coupled, we have

$$h_{pm,i}(k) = \frac{\eta_{h,i}}{\eta_{p,i}} p_{pm,i}(k), \quad (5)$$

where $\eta_{p,i} \in (0, 1]$ and $\eta_{h,i} \in (0, 1]$ are the power and heat output efficiencies, respectively. The power output should satisfy its lower limit $p_{pm,i}^{min} \in \mathbb{R}_{\geq 0}$ and its upper limit $p_{pm,i}^{max} \in \mathbb{R}_{\geq 0}$, as the prime mover has a production capacity. Hence,

$$p_{pm,i}^{min} \leq p_{pm,i}(k) \leq p_{pm,i}^{max}. \quad (6)$$

When the power output from the prime mover cannot satisfy the local power demand, the prosumers import some amounts of power from the external power grid at $p_{imp,i}(k) \in \mathbb{R}_{\geq 0}$, i.e.

$$p_{d,i}(k) = p_{pm,i}(k) + p_{imp,i}(k). \quad (7)$$

The parameters in (7) are in kWh. As the local power demand $p_{d,i}(k)$ is a given value at each time k , we can choose either the amount $p_{imp,i}(k)$ or $p_{pm,i}(k)$ to be the controllable input.

In contrast to the prime mover, the auxiliary burner of the μ -CHP device only generates the heat output at $h_{aux,i}(k) \in \mathbb{R}_{\geq 0}$. The heat output can vary between the minimum capacity $h_{aux,i}^{min}$ and the maximum capacity $h_{aux,i}^{max}$, thus

$$h_{aux,i}^{min} \leq h_{aux,i}(k) \leq h_{aux,i}^{max}. \quad (8)$$

Prosumers turn on the auxiliary burner in the case of high heat demand. In this way, undesirable fluctuating power output from the prime mover is avoidable.

To mitigate the fluctuating heat output from the prime mover due to the power-led control, prosumers can utilize local heat buffers. A hot water tank can be a realistic heat buffer for residential buildings. Given the initial value of stored heat in the heat buffer $z_{h,i}(1)$, we define the dynamics of available heat $z_{h,i}(k) \in \mathbb{R}_{\geq 0}$ in the heat buffer of prosumer i by a linear model specified by

$$z_{h,i}(k+1) = z_{h,i}(k) + h_{pm,i}(k) + h_{aux,i}(k) - h_{d,i}(k). \quad (9)$$

The heat buffer has its lower bound $z_{h,i}^{min}$ and upper bound $z_{h,i}^{max}$ thereby

$$z_{h,i}^{min} \leq z_{h,i}(k) \leq z_{h,i}^{max}, \quad (10)$$

where $z_{h,i}^{min(max)} = m_i c_p \Delta T_i^{min(max)}$ with m_i is the mass of water, c_p is heat constant, and $\Delta T_i^{min(max)}$ is the difference between the inside room temperature and the minimum (maximum) temperature of the water.

In the case that there exists excess heat due to the power-led control of the prime mover and there is no remaining space in the heat buffer, the excess heat must be disposed in, e.g., some district heating systems. We do not associate the disposal with any costs.

Here, we refer to the heat consumption for space heating systems as the local heat demand $h_{d,i}(k)$. The systems are designed to guarantee a comfortable room temperature range. Equipped with these systems, the prosumers can contribute to minimize the mismatch between the available biogas and the heat demand by means of demand response [33]-[34]. Define $T_{in,i}(k) \in \mathbb{R}_{\geq 0}$ and $T_{out,i}(k) \in \mathbb{R}_{\geq 0}$ as the temperature at time k inside and outside a room of prosumer i , respectively. As in [27] and [34], the dynamics of the inside temperature $T_{in,i}(k) \in \mathbb{R}_{\geq 0}$ is defined by

$$T_{in,i}(k+1) = T_{in,i}(k) + \alpha (T_{out,i}(k) - T_{in,i}(k)) + \beta h_{d,i}(k), \quad (11)$$

given the initial inside temperature $T_{in,i}(1)$, with $\alpha > 0$ and $\beta > 0$ represent the thermal characteristics of the heater and prosumer i 's room, respectively. Each prosumer has its comfortable range of inside temperature specified by $[T_{in,i}^{min}, T_{in,i}^{max}]$.

Consequently,

$$T_{in,i}^{min} \leq T_{in,i}(k) \leq T_{in,i}^{max}. \quad (12)$$

As stated earlier, one of the prosumers' control goals is to minimize the biogas imbalance within the micro grid. It corresponds to adjusting the biogas supply and demand. When calculating the imbalance dynamics $\tilde{x}_i(k)$ of prosumer i , we only consider the amounts of biogas needed to satisfy the local heat and power demands and we exclude the amounts of biogas sold to the LP gas grid, to the gas filling station, and to the LV power grid. Given the initial value of the imbalance $\tilde{x}_i(1)$, we specify the dynamics of prosumer's biogas imbalance as the difference equation given by

$$\tilde{x}_i(k+1) = \tilde{x}_i(k) + q_i(k) - \frac{1}{\eta_{p,i}} p_{pm,i}(k) - \frac{1}{\eta_{aux,i}} h_{aux,i}(k), \quad (13)$$

where $\eta_{aux,i} \in (0, 1]$ is the efficiency of heat output produced from the auxiliary burner of the μ -CHP device of prosumer i . Eq. (13) shows that prosumer i keeps track on its own imbalance $\tilde{x}_i(k)$. However, as the prosumers aim at minimizing the overall biogas imbalance within the micro grid, they need to keep track on other prosumers' imbalance as well.

Now define the imbalance information of prosumer i as $x_i(k)$. This information depends on other prosumers' imbalance information. Then by referring to [29], given the initial value of the imbalance information $x_i(1)$, the dynamics of the imbalance information $x_i(k)$ is given by

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \neq i} A_{ij}x_j(k) + q_i(k) - \frac{1}{\eta_{p,i}} p_{pm,i}(k) - \frac{1}{\eta_{aux,i}} h_{aux,i}(k), \quad (14)$$

where A_{ii} and A_{ij} weight, respectively, the biogas imbalance information of prosumer i itself and the information obtained from its neighboring prosumers j . The value of $q_i(k)$ may be higher than the summation of $\frac{1}{\eta_{p,i}} p_{pm,i}(k)$ and $\frac{1}{\eta_{aux,i}} h_{aux,i}(k)$, when prosumer i aims at providing some amount of biogas to its neighboring prosumers.

Define the vectors $x(k) = [x_1(k), \dots, x_n(k)]^T$, $q(k) = [q_1(k), \dots, q_n(k)]^T$, $p_{pm}(k) = [p_{pm,1}(k), \dots, p_{pm,n}(k)]^T$, $h_{aux}(k) = [h_{aux,1}(k), \dots, h_{aux,n}(k)]^T$. We can rewrite the dynamics (14) in a more compact form as

$$x(k+1) = Ax(k) + q(k) - Bp_{pm}(k) - Ch_{aux}(k), \quad (15)$$

where the weights A_{ii} and A_{ij} are the elements of imbalance information matrix $A \in \mathbb{R}^{n \times n}$ and B and C are the appropriate time-invariant matrices of vectors $p_{pm}(k)$ and $h_{aux}(k)$, respectively. According to [15], there are four restrictions in designing the entries of the imbalance information matrix A , given by

- R_1 : $A_{ij} \neq 0$ if and only if there is imbalance information exchanged from prosumer j to i .
- R_2 : $A_{ij} \geq 0, i, j = 1, \dots, N$.
- R_3 : $\sum_j A_{ij} = 1, j = 1, \dots, N$.
- R_4 : The graph corresponding to the A matrix is strongly connected.

Given that the initial value of the imbalance information is

equal to the initial value of the physical imbalance, i.e. $x(1) = \tilde{x}(1)$, and suppose that restrictions R1-R4 hold, the total imbalance information in the micro grid is equal to the total physical imbalance in the micro grid, i.e. $\sum_{i=1}^n x_i(k) = \sum_{i=1}^n \tilde{x}_i(k)$ [15]. It is however not necessary to have $x_i(k) = \tilde{x}_i(k)$ at each time k .

B. External energy grids

To support a sustainable and environmental-friendly energy sources, it is desirable that the biogas prosumers are connected to an LP gas grid. One can see that the gas grid acts as sink for prosumers to create revenue from excess biogas. As mentioned earlier, the biogas needs to be upgraded to green gas with an efficiency $\eta_u \in (0, 1]$ before injecting it into the gas grid. Due to a limited capacity of the gas grid which is denoted by $F(k)$, the aggregated green gas supplied by prosumers is limited by

$$\sum_{i=1}^n \eta_u f_i(k) \leq F(k). \quad (16)$$

The prosumers may also create some revenue by transporting the green gas using a lorry to the gas filling station. However, the gas filling station has a maximum capacity which is specified by $G(k)$. Thus, we have the following inequality constraint given by

$$\sum_{i=1}^n \eta_u g_i(k) \leq G(k). \quad (17)$$

Remark 2: It is worthwhile to note that the produced green gas needs to be compressed in order to use less space for transporting green gas to the gas filling station. However the compressor has a limited capacity, denoted by L_c . As the prosumers aim at creating as much revenue as possible by injecting the produced green gas to both the LP gas grid and to the gas filling station, sufficiently large capacities of the central biogas upgrader and green gas compressor are therefore chosen. The bottleneck for injecting the green gas is then constraints (16)-(17), not constraints (4) and the capacity L_c .

Due to the decrease of conventional energy production coupled with the drive towards a low-carbon economy, it is desirable that the renewable prosumers are not only connected to the gas grid, but also to the power grid. When the selling price of the power grid is appealing, the prosumers may produce power output by switching on their fuel cells. The produced power output is then sold to the LV power grid hence creating some profit. Nevertheless, the power grid has a time-varying limited capacity which is denoted by $E(k)$. It thus restricts the aggregated power supply level from the prosumers, given by

$$\sum_{i=1}^n \eta_{e,i} e_i(k) \leq E(k), \quad (18)$$

where $\eta_{e,i} \in (0, 1]$ is an efficiency of the fuel cell owned by prosumer i .

The constraints (18) are in fact nonlinear, dependent on the distribution factor and nodal injection [35]-[36]. We however assume these to be external signals, thus allowing us to write $E(k)$ as a time-varying bound.

C. Objectives

Each prosumer may have different choices on how to utilize their biogas and on how to cover its local power and heat demands. In particular, the prosumers can use their biogas to turn the μ -CHP devices on, to create revenue by upgrading and selling biogas to the LP gas grid and to the gas filling station, and/or to convert it into electrical energy and then sell it to the LV power grid. They can locally cover their power demand from the prime mover of their μ -CHP devices and/or from the external power grid. They can satisfy their local heat demand from the heat output of the auxiliary burner and/or from the stored heat in the heat buffers. See again Fig. 1.

With those aforementioned choices, the prosumers may have a multi-variable optimization problem. Define the gas filling station as grid g , the LP gas grid as grid f , and the LV power grid as grid e . As in [24], [25], [26], [27], [28], [29], and [30], for $k \in \{1, \dots, K\}$ and $i \in \{1, \dots, n\}$, define the following quantities:

- $U_{1,i} = -c_{x,i}x_i(k)^2$, where $c_{x,i} > 0$ indicates the relative importance of minimizing the imbalance compared to other control goals. When the imbalance minimization is the most important consideration, one can set the associated weight higher than other weights.
- $U_{2,i} = \sum_{m=f,g,e} (r_m(k) - c_{m,i}(k))\eta_{m,i}m_i(k) - c_{tm,i}(k)(\eta_{m,i}m_i(k))^2$, where $r_m(k) \in \mathbb{R}_{\geq 0}$ is the selling price on the grid m , $c_{m,i}(k) \in \mathbb{R}_{\geq 0}$ is the cost producing energy output at a level of $m_i(k)$, $c_{tm,i}(k) \in \mathbb{R}_{\geq 0}$ is the cost associated with transmission losses of $m_i(k)$, and η_m is the efficiency of the associated converters, upgraders, and compressors. As stated in Subsection II-B, we have $\eta_{g,i} = \eta_{f,i} = \eta_u$.
- $U_{3,i}(k) = -h_i h_{aux,i}(k)^2$, where $h_i > 0$ represents the relative importance of minimizing the cost related to turning on the auxiliary burner compared to other control goals. The quantity $U_{3,i}(k)$ will be included as one of prosumers' objectives to ensure that the auxiliary burner of their μ -CHP devices is only switched on to meet the heat storage constraints.
- $U_{4,i}(k) = -(c_{p,i}(k)(p_{d,i}(k) - p_{pm,i}(k)) + c_{tp,i}(k)(p_{d,i}(k) - p_{pm,i}(k))^2)$, where $c_{p,i}(k) \in \mathbb{R}_{\geq 0}$ indicate the prices for buying power from the power grid, whereas $c_{tp,i}(k) \in \mathbb{R}_{\geq 0}$ specify the costs of transmission losses for importing some amount of power from the LV power grid.
- $U_{5,i}(k) = -(c_{zgg,i}(k)z_{g,i}(k) + c_{zg,i}(k)z_{g,i}(k)^2 + c_{zh,i}(k)z_{h,i}(k) + c_{zh,i}(k)z_{h,i}(k)^2)$, representing opportunity cost as a function of $z_{g,i}(k)$ and $z_{h,i}(k)$, where $c_{zgg,i}(k) \in \mathbb{R}_+$ and $c_{zh,i}(k) \in \mathbb{R}_+$.
- $U_{6,i}(k) = -(t_a(T_{in}(k) - T_a(k))^2)$, $t_a > 0$ defines the relative importance of maintaining the inside temperature close to the given target $T_a(k)$ compared to other control goals.
- $U_{7,i}(k) = -(c_{hd,i}h_{d,i}(k)^2 + c_{qi}q_i(k)^2)$, where $c_{hd,i}$ and c_{qi} are the costs associated with utilizing heat from the heat buffer and the produced biogas for running μ -CHP devices, respectively.

With a given time horizon K , the multi-variable optimization problems are subject to all constraints (1)-(3), (5)-(12), (14), and (16)-(18), i.e.

Problem 1:

$$\begin{aligned} & \text{maximize} && \sum_{k=1}^K \sum_{i=1}^n \sum_{l=1}^7 U_{l,i}(k) \\ & \text{subject to} && (1) - (3), (5) - (12), (14), \text{ and } (16) - (18). \end{aligned}$$

In order to write Problem 1 in a compact form, define the vectors

$$\begin{aligned} z_i(k) &= [z_{g,i}(k), z_{h,i}(k), T_{in,i}(k), \tilde{x}_i(k)]^T, \\ y_i(k) &= [p_{pm,i}(k), h_{aux,i}(k), f_i(k), g_i(k), e_i(k), h_{d,i}(k), q_i(k)]^T, \\ w_i(k) &= [p_i(k), 0, \alpha T_{out,i}(k), \sum_{j \neq i} A_{ij} \tilde{x}_j(k)]^T, \\ b_i(k) &= [p_{d,i}(k) - p_{imp,i}(k), h_{imp,i}(k)]^T, \\ h(k) &= [F(k), G(k), E(k)]^T, \\ D_i &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & (1-\alpha) & 0 \\ 0 & 0 & 0 & A_{ii} \end{bmatrix}, \\ J_i &= \begin{bmatrix} 0 & 0 & -1 & -1 & -1 & 0 & -1 \\ \frac{\eta_{h,i}}{\eta_{p,i}} & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ -\frac{1}{\eta_{p,i}} & -\frac{1}{\eta_{aux,i}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\ F_i &= [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0], \\ H_i &= \begin{bmatrix} 0 & 0 & \eta_u & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_u & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_{e,i} & 0 & 0 \end{bmatrix}. \end{aligned}$$

We then have a compact form of Problem 1 given by

$$\begin{aligned} & \max_{z_i(k), y_i(k)} && \sum_{k=1}^K \sum_{i=1}^n U_i(z_i(k), y_i(k)) \\ & \text{subject to} && z_i(k+1) = D_i z_i(k) + J_i y_i(k) + w_i(k), \\ & && \sum_{i=1}^n H_i y_i(k) \leq h(k), \quad F_i y_i(k) = b_i(k). \end{aligned} \quad (19)$$

and boundary conditions on $z_i(k) \in \mathcal{Z}_i$ and $y_i(k) \in \mathcal{Y}_i$, with $U_i(z_i(k), y_i(k)) = \sum_{l=1}^7 U_{l,i}(z_i(k), y_i(k))$, where $\mathcal{Z}_i$ and $\mathcal{Y}_i$ are intervals, i.e. $\mathcal{Z}_i = [\underline{z}_i, \bar{z}_i]$ and $\mathcal{Y}_i = [\underline{y}_i, \bar{y}_i]$, respectively. One can consider $w_i(k)$ as an external signal of prosumer i .

Inspired by [37], the problem stated in (19) can be rewritten in a more compact form for each time k as

$$\begin{aligned} & \max_{a_i \in \mathcal{A}_i} && \sum_{i=1}^n U_i(a_i) \\ & \text{subject to} && C_i a_i = c_i, \\ & && \sum_{i=1}^n N_i a_i \leq d, \end{aligned} \quad (20)$$

where we stack the variables over time in the vector a_i , i.e.,

in the case of, e.g., $K = 3$, we have

$$\begin{aligned} a_i &= [y_i(1), z_i(2), y_i(2), z_i(3)]^T, \\ c_i &= [b_i(1), D_i z_i(1) + w_i(1), b_i(2), w_i(2)]^T, \\ d &= [h(1), h(2)]^T, C_i = \begin{bmatrix} F_i & 0 & 0 & 0 \\ -E_i & I & 0 & 0 \\ 0 & 0 & F_i & 0 \\ 0 & -D_i & -E_i & I \end{bmatrix}, \\ N_i &= \begin{bmatrix} H_i & 0 & 0 & 0 \\ 0 & 0 & H_i & 0 \end{bmatrix}, \end{aligned}$$

with $z_i(1)$ is the initial value of available energy in the storage devices. This is a convenient form for notational purposes further on in the paper.

When the utility function $U_i(a_i)$ is a concave function and the constraints in the problem (20) as well as the boundary conditions on $a_i \in \mathcal{A}_i$ are compact and convex, we obtain a unique maximizer a_i at most [38].

III. OPTIMAL SUPPLY AND DEMAND CONTROL

Here we formulate our optimal control problems in model predictive control (MPC) framework. In this section, we also propose the corresponding algorithms to solve the problem (20) in a distributed fashion.

A. Model predictive control

To handle all technical constraints and to anticipate on the future situation of the external energy grids and local load profiles, problem (20) is solved using an MPC approach. See, e.g., [16], for the detailed explanation of the approach. With this approach, the utility is maximized over a prediction horizon T given the estimates of future conditions in the LV power grid, the LP gas grid, the gas filling station, and the local heat and power demands. From the sequence of optimal solutions over prediction horizon T , only the optimal solution of the first step is applied. At the next time step the optimization problem is re-solved and, again, only the solution of the first step is implemented. In what follows, we call the utility function $U_i(a_i)$ as the total profit.

Let $\hat{U}_i(\hat{a}_i)$ be the predicted profit of prosumer i . Over a given prediction horizon T , the MPC problem is given by

$$\underset{\hat{a}_i(k) \in \mathcal{A}_i}{\text{maximize}} \quad \sum_{\tau=k}^{k+T} \sum_{i=1}^n \hat{U}_i(\hat{a}_i(\tau)) \quad (21)$$

subject to

$$\sum_{i=1}^n N_i(\tau) \hat{a}_i(\tau) \leq \hat{d}(\tau), \quad (22)$$

$$C_i(\tau) \hat{a}_i(\tau) = \hat{c}_i(\tau), \quad (23)$$

and boundary conditions $\hat{a}_i(\tau) \in \mathcal{A}_i$ for all $\tau = k, \dots, k+T$. The hat notations are defined to distinguish the prediction parameters from the system model parameters and $\tau = k, \dots, k+T$ is a new time variable introduced to distinguish between the system time k and the prediction time τ .

It is foreseen that the future energy grids become highly complex systems as they have bidirectional energy flows due

to the integration of the prosumers in the existing energy grids. Solving such a large optimization problems for a huge number of producers and consumers in a centralized manner is time consuming and does not scale well due to the computational complexity [39]. Moreover, the nature of the problem requires a distributed controller, that each prosumer i locally decides its supply and demand levels based on its own local information, yet some coordination with the energy grid operators and some communication with its neighboring prosumers are still necessary to avoid overloading grids and to keep track on the overall imbalance in the micro grid, respectively. It is therefore impractical to solve problem (21) in a centralized manner. In what follows, we solve problem (21) in a distributed fashion, thereby allowing the prosumers control their supply and demand levels locally.

B. Distributed MPC problem

As proposed in [17] and [31], we combine a dual decomposition approach with the gradient projection method to decouple problem (21). The feasibility and stability of the distributed MPC approach have been studied in [17] and [31] as well. Let $\hat{v}_i(\tau)$ be the influence prosumer i expects to receive from its neighboring prosumers in the micro grid, given by

$$\hat{v}_i(\tau) = \sum_{j \neq i} A_{ij} \hat{x}_j(\tau). \quad (24)$$

Hence, the imbalance information (14) becomes

$$\begin{aligned} \hat{x}_i(\tau+1) = & A_{ii} \hat{x}_i(\tau) + \hat{v}_i(\tau) + \hat{q}_i(\tau) - \frac{1}{\eta_{p,i}} \hat{p}_{pm,i}(\tau) - \frac{1}{\eta_{aux,i}} \hat{h}_{aux,i}(\tau). \end{aligned}$$

Now, compile the variables $\hat{a}_i(\tau)$ and $\hat{v}_i(\tau)$ together in $\hat{s}_i(\tau)$, i.e. $\hat{s}_i(\tau) = [\hat{a}_i(\tau), \hat{v}_i(\tau)]^T$.

Remark 3: In the objective function of problem (21), we put an additional term, i.e. $w_{v,i}(\hat{v}_i(\tau) - \sum_{j \neq i} A_{ij} \hat{x}_j(\tau))^2$ where $w_{v,i}$ is a weighting factor. Hence, it results in new profit functions given by $\hat{V}_i(\hat{s}_i(\tau))$. With this additional term, we ensure that the convexity and differentiability arguments of the new variable stated in (24) hold. Adding the additional term $w_{v,i}(\hat{v}_i(\tau) - \sum_{j \neq i} A_{ij} \hat{x}_j(\tau))^2$ in the original problem (Problem 1) will result in no difference in the optimal solutions, as at optimality $\hat{v}_i(\tau) = \sum_{j \neq i} A_{ij} \hat{x}_j(\tau)$ due to the constraint (24).

We then define a Lagrangian function L associated with the coupling constraints (22) and (24) with dual variables $\hat{\lambda}_a(\tau)$ and $\hat{\lambda}_i(\tau)$, respectively, as

$$\begin{aligned} L = & \sum_{\tau=k}^{k+T} \sum_{i=1}^n \hat{V}_i(\hat{s}_i(\tau)) - \hat{\lambda}_a^T(\tau) \left(\sum_{i=1}^n N_i(\tau) \hat{a}_i(\tau) - \hat{d}(\tau) \right) \\ & + \hat{\lambda}_i(\tau) \left(\hat{v}_i(\tau) - \sum_{j \neq i} A_{ij} \hat{x}_j(\tau) \right) \\ = & \sum_{\tau=k}^{k+T} \sum_{i=1}^n \hat{V}_i(\hat{s}_i(\tau)) - \hat{\lambda}_a^T(\tau) \left(\sum_{i=1}^n N_i(\tau) \hat{a}_i(\tau) - \hat{d}(\tau) \right) \\ & + \hat{\lambda}_i(\tau) \hat{v}_i(\tau) - \sum_{j \neq i} \hat{\lambda}_j(\tau) A_{ij} \hat{x}_i(\tau). \quad (25) \end{aligned}$$

Note that $\hat{a}_i$ and the dual variable $\hat{\lambda}_a$ are column vectors.

The objective function of the dual problem is given by

$$\begin{aligned} D &= \max_{\hat{s}_i \in \mathcal{S}_i} L \\ &= \sum_{i=1}^n W_i + \hat{\lambda}_a^T(\tau) \hat{d}(\tau), \end{aligned} \quad (26)$$

where

$$\begin{aligned} W_i &= \max_{\hat{s}_i \in \mathcal{S}_i} \hat{V}_i(\hat{s}_i(\tau)) - \hat{\lambda}_a(\tau)^T N_i(\tau) \hat{a}_i(\tau) \\ &\quad + \hat{\lambda}_i(\tau) \hat{v}_i(\tau) - \sum_{j \neq i} \hat{\lambda}_j(\tau) A_{ij} \hat{x}_j(\tau), \end{aligned} \quad (27)$$

representing the exact profit function of prosumer i , given the dual variables $\hat{\lambda}_a(\tau) = [\lambda_f(\tau), \lambda_g(\tau), \lambda_e(\tau)]^T$ from the associated grid operators f, g, e , the dual variable $\hat{\lambda}_j$, and influence $\hat{x}_j$ from the neighboring prosumers. From (26), we obtain the dual problem defined by

Problem 2:

$$\begin{aligned} &\text{minimize } D \\ &\quad \hat{\lambda}_a, \hat{\lambda}_i \end{aligned}$$

subject to the constraints (23) and boundary conditions $\hat{s}_i(\tau) \in \mathcal{S}_i$, where $\mathcal{S}_i = [\underline{s}_i, \bar{s}_i]$ and $\hat{\lambda}_a$ is a non-negative column vector. The optimality conditions of the dual problem can be directly obtained from the Karush-Kuhn-Tucker theorem [38].

Remark 4: The dual variable $\hat{\lambda}_m$ from the associated grid operator $m \in \{f, g, e\}$ can be interpreted as the distribution charge for the transport and system services which are utilized by prosumers when injecting their energy to the associated grid. In what follows, we call the dual variables as the distribution charges. Since the changes are functions of excess supply, the distribution charges might be measured in monetary unit per a unit of energy flow. The distribution charges are modified by associated energy grid operators and are uniform for all prosumers connected. When overloading, the distribution charges increase from their initial values. Otherwise, they decrease with zero as a lower bound.

Remark 5: As in [29], the dual variable $\hat{\lambda}_i$ can be interpreted as shadow prices which are functions of the deviation between the expected and real influence prosumer i expects from its neighboring prosumers.

To solve Problem 2, we use a gradient projection method. The distribution charges $\hat{\lambda}_m(\tau)$ are initially set at some non-negative value. Define the index r is internal index iteration between times τ and $\tau + 1$. For all $\tau = k, \dots, k + T$, the distribution charges and the shadow prices are updated based on

$$\hat{\lambda}_m^\tau(r+1) = \hat{\lambda}_m^\tau(r) + \gamma_m^\tau \left[\sum_{i=1}^n \eta_m \hat{m}_i^\tau(r) - \hat{M}_i^\tau \right], \quad (28)$$

$$\hat{\lambda}_i^\tau(r+1) = \hat{\lambda}_i^\tau(r) + \gamma_i^\tau \left[\hat{v}_i^\tau(r) - \sum_{j \neq i} A_{ij} \hat{x}_j^\tau(r) \right], \quad (29)$$

for each grid operator $m \in \{f, g, e\}$ and each prosumer $i \in \{1, \dots, n\}$. Note that, e.g., for the gas grid operator $m = f$, we have $M = F$ which is the limited amount of green gas that can be injected in the gas grid (as in (16)). The parameters $\gamma_m^\tau, \gamma_i^\tau > 0$ are time-varying step sizes. The choices on the step

sizes are given in [38]. The iterations are terminated when the successive updates of the dual variables are smaller than some small bounds and the grid capacity constraints are satisfied.

When the prosumers and the energy grid operators have a common clock, the updates (28)-(29) can be done synchronously at each iteration step r . In practice, they may not update their systems synchronously due to a time delay, or simply because they do not have access to a common clock. Hence, we implement a scheme of asynchronous exchange information in solving Problem 2 and provide the corresponding algorithm in the following section.

IV. ASYNCHRONOUS DISTRIBUTED OPTIMAL SUPPLY AND DEMAND CONTROL

Inspired by [8], here we incorporate the asynchronous setting in the distributed optimal supply and demand control proposed in Subsection III-B. Assumptions on the asynchronous setting are stated throughout this section. The detailed algorithms for each prosumer and energy grid operator as well as their convergence proofs are provided in this section.

Consider the fact that the prosumers and the energy grid operators have diverse clocks to update their systems. For notational convenience, consider the case at time τ . In the asynchronous setting, it is required for

- each energy operator $m \in \{f, g, e\}$ to estimate the aggregated supply bids from all prosumers by

$$\sum_{i=1}^n \check{m}_i(r) = \sum_{i=1}^n \sum_{r'=r-r_0}^r bb(r', r) \cdot \hat{m}_i(r'), \quad (30)$$

with $\sum_{r'=r-r_0}^r bb(r', r) = 1$ denoting the weighting factor of the aggregated supply bids received by the grid operators m and $\hat{m}_i(r')$ is the received supply bids at time $r' \in [r - r_0, r]$ from prosumer i ,

- each prosumer i to estimate a) the distribution charges by

$$\check{\lambda}_m(r) = \sum_{r'=r-r_0}^r ee_i(r', r) \cdot \hat{\lambda}_m(r'), \quad (31)$$

for all $m = f, g, e$, b) the shadow price from the neighboring prosumers by

$$\check{\lambda}_j(r) = \sum_{r'=r-r_0}^r oo_i(r', r) \cdot \hat{\lambda}_j(r'), \quad (32)$$

and c) the connected prosumers' imbalance information by

$$\check{x}_j(r) = \sum_{r'=r-r_0}^r uu_i(r', r) \cdot \hat{x}_j(r'), \quad (33)$$

based on their current knowledge. $\hat{m}_i(r')$ is the received supply bids at time $r' \in [r - r_0, r]$ from prosumer i . $\hat{\lambda}_m(r')$, $\hat{\lambda}_j(r')$ and $\hat{x}_j(r')$ are the distribution charge, the shadow price from the neighboring prosumer, and the connected prosumers' imbalance information received by prosumer i at time $r' \in [r - r_0, r]$, respectively. $\sum_{r'=r-r_0}^r ee_i(r', r) = 1$, $\sum_{r'=r-r_0}^r oo_i(r', r) = 1$, and $\sum_{r'=r-r_0}^r uu_i(r', r) = 1$ weight the received distribution charge, the obtained shadow price, and the received connected prosumers' imbalance information at time $r' \in [r - r_0, r]$, respectively.

Let $R_i \in \{1, 2, \dots\}$ be the time at which prosumer i performs its optimization. After calculating the estimates (31) and (32), at times $r \in R_i$ the agents solve their own optimization problem (27), communicate their states $\hat{x}_i^\tau(r)$ to their connected prosumers and coordinate $\hat{m}_i^\tau(r)$ to corresponding grid operators $m \in \{f, g, e\}$. Afterwards, using the estimates (33), the agents then update their prices according to (29). Next, they share the prices to the connected prosumers. If times $r \notin R_i$, the variables $\hat{s}_i(r)$ and $\hat{\lambda}_i(r)$ remain the same. The whole procedure for the prosumers is shown in Algorithm 1.

Algorithm 1: Asynchronous input and state exchange information by each prosumer i

Result: Find y_i at each time k of the distributed MPC scheme.

Let $R_i \in \{1, 2, \dots\}$ be the time at which prosumer i performs its optimization.

for $k = 1, \dots, K$ **do**

Each agent i measures $\hat{z}_i(k), w_i(k), b_i(k)$;

while $|\hat{\lambda}_i^\tau(r) - \hat{\lambda}_i^\tau(r-1)| > \xi$ **and**
 $|\hat{m}_i^\tau(r) - \hat{m}_i^\tau(r-1)| > \xi$ **for all** $m = g, f, e$ **do**

if $r \in R_i$ **then**

For all $\tau = k, \dots, k+T$,

- Calculate the estimates $\hat{\lambda}_a^\tau(r) = [\hat{\lambda}_g^\tau(r), \hat{\lambda}_f^\tau(r), \hat{\lambda}_e^\tau(r)]^T$ based on (31) and the estimate $\hat{\lambda}_j^\tau(r)$ based on (32).
- Solve problem (27).
- Communicate $\hat{x}_i^\tau(r)$ to the connected prosumers.
- Coordinate $\hat{m}_i^\tau(r)$ to corresponding energy grid operator m for all $m = f, g, e$.
- Calculate the estimate $\hat{x}_j^\tau(r)$ according to (33).
- Update the shadow price $\hat{\lambda}_i^\tau(r)$ based on (29).
- Share $\hat{\lambda}_i^\tau(r)$ to the connected prosumers.

end

if $r \notin R_i$ **then**

$\hat{s}_i(r+1) = \hat{s}_i(r)$ and $\hat{\lambda}_i(r+1) = \hat{\lambda}_i(r)$.

end

end

Implement y_i which includes g_i, f_i, e_i only for $\tau = k$.

end

Let $R_m \in \{1, 2, \dots\}$ be the time at which energy grid operator m performs its optimization. After calculating the estimates (30), at times $r \in R_m$ the operators update their distribution charges according to (28). Next, they share the charges to the connected prosumers. If times $r \notin R_m$, the distribution charges remain the same. The whole procedure for the prosumers is shown in Algorithm 2. Furthermore, the illustrations of the algorithms 1 and 2 are shown in Fig. 2.

We consider the following assumptions on the profit functions $\hat{V}_i(\hat{s}_i)$.

Assumption 1: On intervals $\hat{s}_i \in \mathcal{S}_i$, profit functions $\hat{V}_i(\hat{s}_i)$ are increasing, strictly concave, and twice continuously differentiable.

Fig. 2: Interaction between prosumer 1 and prosumer 2 and between prosumer 1 and the gas grid operator, with asynchronous updates. The distribution charge $\hat{\lambda}_g(r)$ is an element of $\hat{\lambda}_a(r)$, whereas the gas supply bid being injected to the gas grid $\hat{s}_i(r)$ is an element of $\hat{s}_i(r)$.

Algorithm 2: Asynchronous exchange information by each grid operator m

Result: Find the aggregated supply bids $\sum_{i=1}^n m_i$ at each time k of the distributed MPC scheme.

Let $R_m \in \{1, 2, \dots\}$ be the time at which grid operator m updates its distribution charge.

for $k = 1, \dots, K$ **do**

Each grid operator m measures its grid capacity M ;

while $|\hat{\lambda}_m^\tau(r) - \hat{\lambda}_m^\tau(r-1)| > \xi$ **and** $\sum_{i=1}^n \eta_m \hat{m}_i^\tau(r) \geq M$ **do**

if $r \in R_m$ **then**

For all $\tau = k, \dots, k+T$,

- Calculate the estimate of the aggregated supply bids $\sum_{i=1}^n \hat{m}_i^\tau(r)$ based on (30).
- Update the distribution charge $\hat{\lambda}_m^\tau(r)$ based on (28).
- Share $\hat{\lambda}_m^\tau(r)$ to the connected prosumers.

end

if $r \notin R_m$ **then**

$\hat{\lambda}_m(r+1) = \hat{\lambda}_m(r)$.

end

end

Implement $\sum_{i=1}^n m_i$ only for $\tau = k$.

end

Assumption 2: The curvatures of $\hat{V}_i(\hat{s}_i)$ are bounded away from zero for all $\hat{s}_i \in \mathcal{S}_i$: $-\hat{V}_i''(\hat{s}_i) \geq T_i > 0$ with T_i is a 12×12 diagonal matrix in which the diagonal entries are $1/\alpha$ for all $\alpha = \alpha_{p_{pm}}, \alpha_{h_{aux}}, \alpha_f, \alpha_g, \alpha_e, \alpha_{h_d}, \alpha_q, \alpha_{z_g}, \alpha_{z_h}, \alpha_{T_{in}}, \alpha_{\tilde{x}}, \alpha_{\tilde{v}} > 0$, and in which the entries outside the main diagonal are all zero values.

The time between the successive updates are assumed to obey the following condition.

Assumption 3: For the energy grid operators g, f, e and prosumer $i = 1, \dots, n$, the difference between successive elements of R_g, R_f, R_j , and R_i is bounded by r_0 .

For each time τ , define the error in distribution charge estimation as $\Delta \hat{\lambda}_a(r) = [\Delta \hat{\lambda}_f(r), \Delta \hat{\lambda}_g(r), \Delta \hat{\lambda}_e(r)]^T$ where $\Delta \hat{\lambda}_m^i(r) =$

$|\hat{\lambda}_m^i(r) - \check{\lambda}_m^i(r)|$ for all $m = f, g, e$, the deviation in estimations of the decision variables $\Delta\hat{f}_i(r), \Delta\hat{g}_i(r), \Delta\hat{e}_i(r), \Delta\hat{v}_i(r)$, the deviation in distribution charge estimation as $\Delta\hat{\lambda}_m(r) = |\hat{\lambda}_m^i(r) - \check{\lambda}_m^i(r)|$ for all $m = f, g, e$, the deviation in shadow price estimation as $\Delta\hat{\lambda}_j(r) = |\hat{\lambda}_j(r) - \check{\lambda}_j(r)|$ and the error in gradient estimation as $\Delta\hat{l}(r) = [\Delta\hat{l}_f(r), \Delta\hat{l}_g(r), \Delta\hat{l}_e(r)]^T$ where $\Delta\hat{l}_m(r) = |\hat{l}_m(r) - \frac{\partial D}{\partial \lambda_m} \check{\lambda}(r)|$ for all $m = f, g, e$.

We summarize the main results on the convergence of the asynchronous distributed supply and demand control in the following theorem.

Theorem 1: Given knowledge of the dynamic stored energy in the storage device $\hat{z}_i(r)$, initial distribution charge $\hat{\lambda}_m(1) \geq 0$, and suppose that assumptions 1, 2, and 3 hold, then the error in the distribution charge estimation $\Delta\hat{\lambda}_a(r)$, the deviation in shadow price estimation $\Delta\hat{\lambda}_j(r)$, the deviation in estimations of the decision variables $\Delta\hat{s}_i(r)$ and the error in the gradient estimation $\Delta\hat{l}(r)$ converge to zero as $r \rightarrow \infty$ for all $i \in \{1, 2, \dots, n\}$ and all $m = f, g, e$.

Proof: We follow similar steps as in Theorem 1 in [23], where only some controllable inputs are asynchronously exchanged. Firstly, we formulate our problem in a static optimization form, as in 21, so that the framework of [23] is similar. Then we add asynchronous state coordination. To use the same steps as in proofing Theorem 1 in [23], we have the assumptions (1) and (2) stating that the profit functions $\hat{V}_i(\hat{s}_i)$ have quadratic functions of all states and controllable inputs, which are denoted by $\hat{s}_i$. Therefore, we add an additional term in the objective function of problem (21), as stated in Remark 3 in Subsection III-B. The rest of the proof then follows similar to [23]. $\square$

V. SIMULATION RESULTS

In this section, we conduct simulations to: *a)* assess the impact of different information topologies of the community shown in the A matrix at (15), *b)* study the impact of owning flexible heat demand, *c)* interpret shadow prices, and *d)* illustrate the convergence of the proposed asynchronous coordination shown in Section IV. All dynamics and technical constraints of the prosumers in Subsection II-A and the constraints on the external energy grids in Subsection II-B are taken into account when solving the distributed optimization problem synchronously and asynchronously, as stated in Subsections III and IV, respectively. We use the quadratic programming solver from Gurobi 6.0.5. with YALMIP R20150626 embedded in MATLAB 2015a to find the optimal solutions.

A. Simulation setup

We consider a micro grid consisting of 2 prosumers and 2 consumers. The prosumers are equipped with an anaerobic digester to produce biogas, a fuel cell and a decentralized gas storage device. Their biogas production level is set at $25 \text{ Nm}^3/15 \text{ min}$, which is equivalent to $150 \text{ kWh}/15 \text{ min}$ per time step k . We initially set the available biogas at the prosumers' gas storage devices at zero, i.e. $z_{g,i}(1) = 0$. Both prosumers and the consumers have μ -CHP devices, heat buffers and smart air conditioners which have a comfortable range for the inside

temperature. We use the characteristics of μ -CHP devices and heat buffers as presented in [25]. We choose the same fuel cell, i.e. Proton Exchange Membrane (PEM) fuel cell, as what we have for the prime mover of μ -CHP devices. The above-ground low-pressure vessels with a capacity of 2000 m^3 (equivalent to 12000 kWh [40]) is chosen for the decentralized gas storage devices [41].

The outside temperature $t_{out,i}(k)$ and the target of the inside temperature $t_{a,i}(k)$ for all prosumers and consumers are identical and provided in Fig. 3. The power consumption patterns of the prosumers and consumers are shown in Fig. 4. The patterns in the figure represent the data for 21 November 2012. The time horizon of that day is divided into 15-minute samples, resulting in 96 samples. We assume that the prosumers and consumers have capability to well predict their own local energy demands for the next hour, i.e. $T = 4$. We therefore provide 100 samples of the power demand profiles (in Fig. 4), the outside temperature and the target of inside temperature (in Fig. 3).

Fig. 3: Outside temperature profile for the whole day of 21 November 2012 in the Netherlands [44] and the target of inside temperature of all prosumers and consumers.

The selling prices of the LP gas grid and the gas filling station are gathered from [42], whereas the selling prices of the LV power grid are obtained from [43]. We assume that the capacity of the gas grid and power grid are equal to the aggregated demand of 600 gas consumers and 100 power consumers, respectively. The average consumption patterns of a gas consumer and power consumer are shown in [24] and in the red line depicted in Fig. 4, respectively. We assume that the lorry capacity picking up the green gas from the micro grid to bring it to the gas filling station is sufficiently high. Table I presents the values of the parameters involved in the objective function of Problem 1, which is stated in Section III. The upgrading efficiency is set at 100%, whereas the efficiency of the fuel cell is set at 30%. We set the non-summable diminishing step sizes at $\gamma_i(r) = \gamma_g(r) = \gamma_f(r) = \gamma_e(r) = \frac{0.4}{\sqrt{r}}$. To terminate the internal iteration r , we use the small bounds $\xi_i = \xi_g = \xi_f = \xi_e = 0.001$.

Fig. 4: Power consumption patterns of 4 prosumers.

TABLE I: The parameter values in the objective function of Problem 1 stated in Section III

Parameter	Value	Parameter	Value
$c_{x,i}$	1	$c_{zh,i}$	1e-3
$c_{tf,i}, c_{tg,i}, c_{te}$	1e-6	$t_{a,i}$	1e-6
$c_{zg,i}$	1e-6	c_{aux}	1e-6
$c_{tp,i}$	10000	$c_{hd,i}, c_{q,i}$	1e-6

B. The impact of different information topology of the community shown in A matrix

In order to assess the impact of different information topologies of the community shown in the A matrix at (15), we use the following three information matrices A given by

$$A_1 = \begin{bmatrix} 0.2 & 0.4 & 0 & 0.4 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.4 & 0.2 & 0.4 \\ 0.4 & 0 & 0.4 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.6 & 0.2 & 0 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0.2 & 0.6 & 0.2 \\ 0.2 & 0 & 0.2 & 0.6 \end{bmatrix},$$

and A_3 whose entries are all set at 0.25. Utilizing matrix A_1 , the prosumers weigh their own imbalance information with 0.2 and their neighboring prosumers' imbalance information with 0.4. Hence, using matrix A_1 , the prosumers weigh their imbalance information less important than their neighboring prosumers. For A_2 it is precisely the other way around. In contrast, the prosumers weigh their imbalance information equally to their neighboring prosumers when using matrix A_3 .

The total biogas imbalance patterns with those three information topologies are shown in Fig. 5. As can be seen in the figure, they have similar pattern. However, information topology A_1 results in better balancing between biogas supply and demand in the micro grid than the other information topologies. It is due to the fact that using information topology A_1 the prosumers weight their neighboring prosumers' imbalance more important than their own imbalance. In this way, the prosumers take better care on what is happening in the micro grid than when they use information topologies A_2 and

Fig. 5: Total biogas imbalance in the micro grid with different A_1 , A_2 , and A_3 matrices

A_3 .

The negative imbalance levels shown in Fig. 5 depict the moments when the consumers discharge some amount of biogas from the pipeline of the micro grid thereby decreasing the pressure of the micro grid. In contrast, the positive imbalance levels represent the moments when the prosumers charge some amount of biogas to the pipeline of the micro grid as they have capabilities to predict the load profiles in the micro grid using the imbalance information formulated in Subsection II-B.

We then use information topology A_1 for different weighting factors on the imported power, i.e. $c_{tp,i} = 10, 100, 10000$. The corresponding total biogas imbalance patterns are shown in Fig. 6. With $c_{tp,i} = 10000$, where it is immensely expensive to import the power, the prosumers contribute more to help the consumers in the micro grid to minimize the biogas imbalance within the micro grid. It can be verified by looking at the total amount of imported power from the external power grid to satisfy the power demand in the micro grid, as depicted in Fig. 7. The total imported power level with $c_{tp,i} = 10000$ is always the lowest level, i.e. $p_{imp}(k) \approx 0$, in comparison to other values of $c_{tp,i}$. It means that the consumers get biogas from their neighboring prosumers to fulfill their local demand.

C. Flexible and fixed heat demand

Our heat demand cannot be shifted in time when we have the constraints given by $T_{in,i}(k) = T_{a,i}(k)$ at each time k . Instead, the prosumers and consumers in our study aim at participating in demand response by controlling the inside temperature of their room within a certain temperature range. They have targets of the inside temperature, which are set in the middle of the lower and upper bounds presented respectively in the dashed and solid black lines in Fig. 8. The outside temperature pattern can be seen in Fig. 3. The prosumers and consumers allow their inside temperature to deviate from the target with some boundaries, depending on the available heat in their systems. As mentioned earlier, the

Fig. 6: Total biogas imbalance with different weighting factors on the imported power ($c_{tp,i}$).

Fig. 7: Total imported power from the external power grid to satisfy local power demand in the micro grid, with different weighting factors on the imported power ($c_{tp,i}$).

lower and upper bounds of the inside temperature are depicted by the dashed and solid black lines in Fig. 8.

We here conduct 5 scenarios with diverse weighting factors on the available heat stored in the water tank, denoted by $c_{zh,i}$, and with different weighting factors on the deviation from the targets, specified by $t_{a,i}$, as is presented in Table II. We set the rest of the parameters involved in the objective function of Problem 1 as in Table I. The total available heat for all prosumers and consumers for the whole day on 21 November 2012, i.e. $\sum_{k=1}^{96} \sum_{i=1}^4 (z_{h,i}(k) - 8.1)$ and the total deviation of inside temperature from the targets, i.e. $\sum_{k=1}^{96} \sum_{i=1}^4 |T_{in,i}(k) - T_a(k)|$, can be found in Table III. The value 8.1 is the minimum heat available in the water tank, calculated using (10), where $m = 200\text{kg}$ and $c_p = 4.05\text{kJ}/(\text{kg}\cdot\text{K})$.

From cases 1, 4, and 5 in Table III, we see that the higher weighting factor of $t_{a,i}$, the lower total of deviation between the

inside temperature and its target. Moreover, we can conclude from cases 1, 2, and 3 in the table that the higher weighting factor of $c_{zh,i}$, the lower total of heat available in the water tank. These two facts are also shown in Fig. 8 and Fig. 9, respectively. From Fig. 9, the available heat in the water tank during $k = 0, \dots, 43$ is close to zero, as the power demand is low at the moments. Hence, the heat output from the prime mover is directly used to satisfy the lower bound of the inside room temperature, as shown in Fig. 8.

TABLE II: Five cases with diverse values of $c_{zh,i}$ and $t_{a,i}$

Case	$c_{zh,i}$	$t_{a,i}$
1	1e-3	1e-3
2	5e-3	1e-3
3	1e-2	1e-3
4	1e-3	5e-3
5	1e-3	1e-1

TABLE III: Results of five cases stated in Table II

Case	$\sum_{k=1}^{96} \sum_{i=1}^4 (z_{h,i}(k) - 8.1)$	$\sum_{k=1}^{96} \sum_{i=1}^4 T_{in,i}(k) - T_a(k) $
1	198.0	197.4
2	165.6	214.8
3	163.0	210.4
4	235.1	109.9
5	256.1	70.6

Fig. 8: The inside temperature of consumer 1 for cases 1 and 5

D. Interpretation of shadow price

Here we examine the interpretation of shadow prices. The interpretation of distribution charges has been briefly discussed in Remark 4. We show the shadow price $\lambda_1'(1)$ of consumer 1 at time step $k = 1$ and its deviation between the expected and real influence from its neighboring prosumers, i.e. $\Delta_{v_1}'(1) = v_1'(1) - \sum_{j \neq 1} A_{1j} x_j'(1)$ for all $j = 2, 3, 4$ in Fig. 10. The shadow

Fig. 9: Available heat in the heat tank of consumer 1 for cases 1 and 3

price is initially set at 1, i.e. $\lambda^{r=0}(1) = 1$. As seen in the figure, when $\Delta_{v_1}^r(1) < 0$ the consumer decreases its shadow price. Otherwise, it increases its shadow price up till $\Delta_{v_2}^r(1) = 0$. This phenomenon also holds for the shadow price of prosumers.

Fig. 10: The shadow price $\lambda_1^r(1)$ of consumer 1 and its deviation between the expected and real influence from its neighboring prosumers, i.e. $\Delta_{v_1}^r(1) = v_1^r(1) - \sum_{j \neq 1} A_{2j} x_j^r(1)$ for all $j = 2, 3, 4$

Moreover, we simulate another case where the shadow price is initially set at zero. The evolution of the shadow price in comparison to the evolution of the shadow price when using the initial value of one is shown in Fig. 11. It shows us that with different initializations of the shadow prices, we converge to the same optimal values of shadow price, but with different number of iterations.

Fig. 11: Shadow price evolution of consumer 1 when its initial shadow price is set at 1 and 0

E. Convergence of synchronous and asynchronous coordination

Here we illustrate the convergence of the synchronous and asynchronous coordination. To do so, we use the following simple setup to implement the asynchronous coordination proposed in Subsection IV. The micro grid is divided into two groups, namely a group with consumers and a group with prosumers. The consumers perform their optimization when the iteration number r is uneven. The biogas prosumers perform their optimization when the iteration number r is even. Once the termination criteria stated in Algorithm 1 hold, they implement the first input sequence.

The evolution of shadow prices when implementing synchronous and asynchronous exchange information can be found in Fig. 12. As expected, it requires more number of internal iterations to converge to the optimal shadow prices when implementing the asynchronous updates than when implementing the synchronous updates. They however converge to the same optimal solution.

VI. DISCUSSION

We have proposed a utility-based asynchronous state and input coordination algorithm for dynamic prosumers embedded in multiple energy grids. The coordination examines bidirectional communication between a prosumer and connected energy grid operators and between a prosumer and its neighboring prosumers. We have proved theoretically and illustrate the convergence of the algorithm. We have shown that the distribution charges help the energy grid operators to optimally decide the maximum allowable energy injected by each prosumer connected, hence avoiding overloading energy grids. The distribution charges modified by the energy grid operators are uniform for all connected prosumers. We also have shown that the shadow prices help the prosumers to reach a consensus, i.e. through information topology shown in the A matrix, with their neighboring prosumers on how much they

Fig. 12: The convergence of synchronous and asynchronous coordination

influence each other. The shadow prices are unique for each prosumer.

In future research, we will include practical control considerations due to on-off constraints of the energy converters hence incorporating non-convex constraints when iteratively and asynchronously solving the prosumers' optimal control problem. Additionally, we will extend the problem by incorporating shiftable power demand due to, e.g., the use of plug-in electric vehicles. Hence, the prosumers can participate in the demand response more actively.

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