Industry structure and collusion with uniform yardstick competition: Theory and experiments

Peter T. Dijkstra\textsuperscript{a,b,\textasteriskcommand}, Marco A. Haan\textsuperscript{a}, Machiel Mulder\textsuperscript{a,b}

\textsuperscript{a} Department of Economics, Econometrics and Finance, University of Groningen, PO Box 800, Groningen 9700 AV, The Netherlands
\textsuperscript{b} Netherlands Authority for Consumers & Markets (ACM), PO Box 16326, The Hague 2500 BH, The Netherlands

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For an industry that is subject to uniform yardstick regulation, we study cartel stability and the impact of cartels on the regulated price. In a theoretical model, an increase in the number of symmetric firms may facilitate collusion. Our laboratory experiment suggests that this effect is even stronger than what theory predicts. Theory predicts that firm-size heterogeneity hinders collusion, but leads to higher regulated prices if firms do not collude. In a laboratory experiment we find that the first effect is stronger, implying that in a more heterogeneous industry regulated prices are lower.

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\textsuperscript{\footnotesize*} Corresponding author at: Netherlands Authority for Consumers & Markets (ACM), PO Box 16326, 2500 BH The Hague, The Netherlands.

E-mail addresses: Peter.Dijkstra@acm.nl (P.T. Dijkstra), m.a.haan@rug.nl (M.A. Haan), machiel.mulder@rug.nl (M. Mulder).

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1. Introduction

It is widely accepted that in unregulated markets, an increase in the number of firms hinders collusion: The higher the number of firms, the higher the potential benefits of deviating from a collusive agreement (see e.g. Motta, 2004). The same holds for firm heterogeneity: the more firms differ, for example in terms of market share, the harder it is to collude. Smaller firms have stronger incentives to deviate from collusive agreements (see e.g. Ivaldi et al., 2003). Experimental studies, such as Abbink and Brandts (2005) or Fonseca and Normann (2008), confirm these insights.

Yet, much less is known about the incentives to collude in an industry that is subject to yardstick competition. With yardstick competition, prices for regional monopolies are set by a regulator on the basis of actual costs of similar firms. This gives such firms an incentive to collude to keep their costs high (Tangerås, 2002). Many industries are subject to yardstick regulation. For example, in the US, the payment a hospital receives for a treatment under Medicare, depends on the average cost of that treatment in other hospitals (Shleifer, 1985). But yardstick competition is also used in industries as diverse as water supply and sewerage, electricity networks, railways and bus transport services, to name but a few.

More often than not, industries with yardstick competition are subject to consolidation. Regulators and antitrust authorities then face an extra challenge: to decide whether to allow mergers. To be able to do so, it is essential to have a thorough understanding of how industry structure affects the incentives to collude. Insights from unregulated markets do not necessarily carry over to these environments. This paper aims to contribute to that understanding.

In the Dutch energy-distribution industry, for example, the number of firms was about 350 in 1950. Due to mergers and acquisitions, it had decreased to 25 in 1998 (Arentsen and Künneke, 2003). In 2014, there were only 8 companies left. The industry now consists of a few large players, and some small ones (see Haffner et al., 2010). Over the years, firm-size heterogeneity has increased. These changes influence the effectiveness of yardstick competition if they affect the incentives for firms to collude. Understanding the relationship between industry structure and collusion is thus essential for understanding the effectiveness of regulation through yardstick competition.

In this paper, we study to what extent the number of firms and firm-size heterogeneity in an industry subject to uniform yardstick competition affects collusion. We first do so in a theoretical framework. A theoretical model is an important tool in helping to understand the fundamental mechanisms that are at play, and the trade-offs that are involved. However, repeated game models of the type we study do suffer from a multiplicity of equilibria, and remain silent on how and to what extent players in the real world may

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1 Dassler et al. (2006).
3 Mizutani et al. (2009).
be able to coordinate on one of those equilibria. Therefore, we also test our model in a laboratory experiment.

Ever since Shleifer (1985) it is well known that yardstick competition is most effective with a discriminatory yardstick. The price that one firm can charge then depends on the cost levels of all other firms. In our analysis, however, we focus on a uniform yardstick, where the price that one firm can charge depends on the weighted average cost levels of all firms involved. Despite the superior theoretical properties of a discriminatory yardstick, a uniform yardstick is more often used in practice.\(^5\)

We use a simple model, loosely based on Shleifer (1985), where firms have to exert costly effort to lower their costs.\(^6\) Managerial benefits are a concave function of a firm’s marginal costs. Managers aim to maximize the sum of profits and managerial benefits, and do so in a repeated game. Surprisingly, we find that with symmetric firms the effect of the number of firms on cartel stability is ambiguous. Hence, having more firms does not necessarily make a cartel harder to sustain, and may even make it easier. Moreover, we find that firm-size heterogeneity hinders collusion.

We also test our model in a lab experiment. To be able to do so, we have to impose some additional structure. In particular, we assume that managerial benefits are quadratic in a firm’s marginal costs. The theoretical model then predicts that the number of symmetric firms has no effect on cartel stability, while the effect of mergers is still ambiguous. Mergers that lead to symmetric market shares facilitate collusion. Mergers that do not involve the smallest firm in the industry hinder collusion as they improve the competitive outcome for the smallest firm, making it more attractive for the manager of that firm to defect from a collusive agreement. If two small firms merge, then such a merger usually facilitates collusion.

Our experimental implementation is loosely based on Potters et al. (2004). To facilitate coordination we allow subjects to communicate in each round, before setting their cost level. We look at five treatments that differ in the number of firms (2 or 3) and the extent of firm-size heterogeneity. In each treatment, we study how successful experimental subjects are in establishing collusion. To do so, we look at three measures of collusion: the incidence of full collusion, a collusion index, and the resulting price.

Our experimental findings suggest that industries with 3 symmetric firms are more collusive than industries with 2 such firms. We find some evidence that mergers that lead to symmetric firms indeed facilitate collusion. Relative to what theory predicts, we

\(^5\) One example being the Dutch energy-distribution industry discussed above Haffner et al. (2010).

\(^6\) In Shleifer (1985), firms invest \(R(c)\) in a cost-reducing technology that lowers marginal cost by \(c\). With downward sloping demand, the monopolist cannot fully capture the social welfare generated by lower costs. Hence it will underinvest from a welfare perspective. This is no longer true, however, if demand is inelastic. In that case, the monopolist does capture the entire surplus and hence there is no underinvestment. For experimental simplicity, we do assume inelastic demand. We therefore use a slightly different set-up. We effectively assume that the manager of the firm has to exert some costly effort to lower costs. The discrepancy between the private and social optimum is then caused by the fact that the manager’s costly effort is not taken into account when maximizing social welfare. This allows us to generate the same theoretical prediction (that firms underinvest), but in a simpler set-up. Note that our interpretation closely follows Potters et al. (2004).
find that an increase in the number of firms makes it easier for experimental subjects to collude if they are symmetric, but harder if they are asymmetric. The theoretical effects of an increase in heterogeneity are ambiguous. On the one hand, such an increase raises prices if firms do not collude; on the other hand, it makes collusion harder. Our experiment suggests that the net effect for consumers is positive. Summing up, we find that a heterogeneous industry structure is of key importance for having relatively little collusion under yardstick competition.

The remainder of this paper is organized as follows. Section 2.1 provides more background on yardstick competition, while Section 2.2 discusses other experiments studying the impact of industry structure on the incidence of collusion. Section 3 describes our theoretical model. Section 4 presents the experimental design, while Section 5 describes the results of our experiment. Section 6 concludes.

2. Background

2.1. Yardstick competition

The last decades saw the liberalization of several network industries that used to be state-owned vertically-integrated monopolies. Examples include telecommunications, electricity, gas, water and sewerage. Parts of these industries are natural monopolies characterized by subadditivity of costs, which calls for regulatory supervision (Viscusi et al., 2005). A key component of such supervision is the regulation of tariffs. Historically, regulated tariffs were either based on actual costs or an allowed rate of return. However, such schemes give little incentive to increase productive efficiency as lower costs directly translate to lower revenues. Hence, the incentive power is low, as a change in costs hardly affect firms’ profits. Introducing price-cap regulation solves this problem as any cost decrease then fully benefits the firm. But this method also implies the risk of significant rents or losses for regulated firms. To overcome both problems, Shleifer (1985) proposed to impose tariffs based on the actual costs of a group of similar (benchmark) firms. This type of tariff regulation is called yardstick regulation. As tariffs are based on the relative performance of a firm, yardstick regulation is generally seen as a powerful tool both for giving incentives for productive efficiency as well as for rent extraction (Tangerás, 2002; Burns et al., 2005).

The incentive power of a yardstick partly depends on the exact manner in which costs of the benchmark firms determine tariffs. Essentially, there are two types of yardsticks. With a uniform yardstick every firm faces the same cap on the tariffs it may charge, based on cost information of all firms in the benchmark group. With a discriminatory yardstick, every firm faces a specific cap based on the costs of all other firms in the benchmark group. The latter scheme gives a stronger incentive to improve efficiency. For both schemes, numerous methods exist to determine the yardstick, including the (weighted) average costs, the median costs, the 25th percentile of costs or just the best
practice of all benchmark firms (Yatchew, 2001). In the Dutch regulation of electricity distribution networks, for instance, the yardstick is based on weighted average costs.

Experiences with yardstick regulation exist in several industries, including water supply and sewerage in the United Kingdom (Dassler et al., 2006), electricity networks in the Netherlands (Haffner et al., 2010), Spain (Blázquez-Gómez and Grifell-Tatjé, 2011) and the United Kingdom (Jamasb and Pollitt, 2007), railways in Japan (Mizutani et al., 2009) and bus transport services in Norway (Dalen and Gomez-Lobo, 2002). In several cases, the introduction of yardstick regulation improved productive efficiency or resulted in lower consumer prices. In other cases, the experience appeared to be less successful.

What these experiences have taught us is that a number of conditions have to be met before yardstick regulation can be effective. First, we need a sufficiently large number of benchmark firms, using similar techniques operating within a similar environment. Second, revenues of regulated firms should be fully based on the yardstick. No other concerns should be taken into account, such as the impact yardstick regulation may have on the risk of bankruptcy. Finally, firms should operate independently from each other. Any cooperation would reduce the effectiveness of yardstick competition (Jamasb et al., 2004).

In this paper we therefore assess the risk of collusion with yardstick regulation under different industry structures. Tangerás (2002) observes that yardstick competition is near useless if firms are able to collude. In his model, collusion takes place through joint manipulation of productivity reports that the regulator uses to determine tariffs. In our paper, firms that collude jointly refrain from exerting effort to lower actual costs. In Tangerás (2002), collusion becomes less likely if the number of firms increases, which is in line with the literature on collusion in unregulated markets.

In unregulated markets, the feasibility of collusion also depends on the heterogeneity of firms. The more firms differ in terms of market shares, the harder it is to reach an agreement (see e.g. Motta, 2004). In empirical work this is found for the US airline industry, where the extent of competition appears to be positively related to firm-size heterogeneity (Barla, 2000). In our paper, we study whether the same holds for industries with yardstick competition.

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7 Mizutani et al. (2009), for instance, find that the efficiency of Japanese railway companies, measured by the variable costs per unit of output, improved significantly after the introduction of yardstick regulation. Jamasb et al. (2004) report that the British model of incentive regulation in the electricity distribution industry has brought significant price reductions to consumers. Haffner et al. (2010) find that yardstick regulation of the Dutch electricity and gas networks resulted in lower network tariffs without adversely affecting network investments.

8 Dalen and Gomez-Lobo (2002) for instance do not find an effect on the cost efficiency of Norwegian bus companies, for which the authors blame the bargaining power of the regulated firms to reduce the incentive power of the regulatory scheme. For the incentive regulation in the Spanish electricity-distribution industry, negative effects on consumer welfare were found by Blázquez-Gómez and Grifell-Tatjé (2011), which resulted from the fact that inefficient firms received financial compensation afterward in order to prevent bankruptcies.
2.2. Experiments on collusion

A number of economic experiments have analyzed the impact of industry structure on collusion in unregulated markets. Most studies focus on the effect of the number of firms. Huck et al. (2004) study homogeneous-product quantity-setting oligopolies. They find some collusion in markets with 2 firms, little collusion in markets with 3 firms, and no collusion in markets with 4 or 5 firms. Other studies look at homogeneous-product price-setting oligopolies. Abbink and Brandts (2005) find that collusion decreases with the number of firms when costs are private information. Abbink and Brandts (2008) find the same result with increasing marginal costs, and Fonseca and Normann (2008) with capacity constraints. Fonseca and Normann (2012) also find more collusion with fewer firms. Moreover, they find that the ability to communicate facilitates collusion, the effect being strongest for industries with an intermediate number of firms.

Other experiments focus on asymmetries between firms. With quantity setting, Mason et al. (1992) find more collusion if firms have equal rather than different marginal costs. Phillips et al. (2011) confirm this result. With price competition, Fonseca and Normann (2008) find more collusion if firms have identical capacity constraints, and Dugar and Mitra (2009) find more collusion if the range of possible firm-specific marginal costs is smaller. Dugar and Mitra (2013) confirm the latter result in a slightly different context. Argentor and Müller (2012) however, find that collusion is unaffected if firms have different rather than identical cost structures. Still, overwhelmingly, experiments on unregulated markets find that asymmetries do hinder collusion.

To the best of our knowledge, the only economic experiment that deals with yardstick competition is Potters et al. (2004), on which our experiment is loosely based. Potters et al. (2004) study the effect of yardstick design on collusion in a duopoly with symmetric firms. They compare a uniform yardstick to a discriminatory one. When firms behave non-cooperatively, the discriminatory yardstick yields lower prices and lower profits, making it more prone to collusion. In their experiment, the authors indeed find higher cost levels with a discriminatory yardstick. Of course, our research question is very different from that addressed in Potters et al. (2004). Also, the experiment in that paper includes a stage in which firms set prices, while we simply impose prices to equal the price cap imposed by the regulator. This simplifies the experiment. Anyhow, Potters et al. (2004) find that subjects choose to set prices equal to the price cap in about 99% of the cases.

3. The model

3.1. Setup

There are \( n \) firms that play an infinitely repeated game. Each firm acts as a local monopolist. For simplicity, we assume that firm \( i \) faces demand that is completely inelastic and exogenously given by mass \( \alpha_i \in (0, 1) \). We normalize total demand, so \( \sum_{i=1}^{n} \alpha_i = 1 \) and \( \alpha_i \) reflects firm \( i \)'s share of total demand. For ease of exposition, we will refer to \( \alpha_i \) as
the market share of firm $i$. In each round, firm $i$ decides on its constant marginal cost $c_i$ for that round. Firm $i$’s profits then equal $\pi_i(c_i, c_{-i}) = (p - c_i)\alpha_i$, where $p$ is determined by regulation. The vector $c_{-i}$ consists of the cost levels chosen by other firms; these affect $\pi_i$ through the regulated price $p$.

We assume that the manager of firm $i$ maximizes

$$u_i(c_i) = \pi_i(c_i, c_{-i}) + r(c_i)\alpha_i,$$

with $r$ a managerial benefit that is concave: $r'' < 0$ and $r(0) = 0$. This specification is equivalent to Shleifer (1985), where managers invest in a cost-reducing technology. Crucially, we assume that total managerial benefit $r(c_i)\alpha_i$ is proportional to market share. With equal marginal costs, the total amount of managerial benefits in an industry is then independent of the size distribution of firms: $\sum \alpha_i r(c) = r(c) \sum \alpha_i = r(c)$. In a set-up similar to Laffont and Tirole (1993) for example, a firm would earn $\alpha_i(p - c_i) + r(c_i)$. The firm’s problem is then to find the optimal mix of marginal production cost and fixed cost, i.e. the efficient scale. Our assumption implies that the optimal mix does not depend on the size of the firm, and hence that there are constant returns to scale in the relevant interval.\(^9\) Hence, when two firms merge, the optimal mix is unaffected. This allows us to isolate the effect of mergers on market performance, without the confounding effect of already having lower average costs due to economies of scale. Our results should also be interpreted as such. In addition, our assumption is consistent with the available evidence: according to the empirical literature, economies of scale in electricity distribution are already exhausted with some 20,000 customers, far below the size that is relevant on e.g. the Dutch market (see Yatchew, 2000 and the references therein).

We assume that there is a unique $c^m$ that maximizes managerial benefit, so $r'(c^m) = 0$. We thus allow $r$ to be decreasing in $c_i$ for large enough $c_i$. In electricity networks for example, high marginal costs are often associated with a lack of maintenance. Such networks are prone to outages, compensation claims and political pressure, all factors that do not exactly contribute to a quiet life for the manager.

The regulator uses a uniform weighted yardstick, where the price a firm is allowed to charge equals the weighted average of all cost levels in the industry:

$$p(c_i, c_{-i}) = \sum_{j=1}^n \alpha_j c_j.$$

This implies that all firms get to charge the same price. Total utility to manager $i$ is thus given by

$$u_i(c_i, c_{-i}) = \left( \sum_{j=1}^n \alpha_j c_j - c_i \right) \alpha_i + r(c_i)\alpha_i.$$

\(^9\) We thank an anonymous referee for pointing this out.
3.2. Firm-size heterogeneity

We are interested in how the extent of firm-size heterogeneity affects competition and collusion. We thus need a measure of firm-size heterogeneity. For a fixed number of firms, the most natural measure to look at is the Herfindahl index $H \equiv \sum_{j=1}^{n} \alpha_j^2$. With $n$ equally-sized firms, this equals $1/n$, but it is easy to see that $H$ increases as firms differ more in size. To derive our analytical results, we will interpret an increase in firm heterogeneity as the transfer of market share from a (weakly) smaller to a (weakly) larger firm. Such an increase would necessarily increase the Herfindahl index.

3.3. Competition

Consider the case where managers unilaterally maximize their utility. For ease of exposition, we will refer to this as the competitive outcome. For given $c_{-i}$, maximizing (3) with respect to $c_i$ yields

$$r'(c_i^*) = 1 - \alpha_i. \tag{4}$$

This is a dominant strategy, as it does not depend on the costs level of other firms. Also note that $c_i^*$ is increasing in market share $\alpha_i^{10}$: the higher $\alpha_i$, the stronger the influence this firm has on the regulated price, and the more attractive it is to choose a higher cost level. Finally, note that $c_i^*$ is strictly lower than the cost level that maximizes managerial benefits.\footnote{With a similar argument as in the previous footnote: maximizing managerial benefits requires setting $r'(c^*) = 0$, and is thus equivalent to having $\alpha_i = 1$ in (4).} Lower costs increase profits, giving managers an incentive to be more efficient.

If consumers value the product at $v$, total welfare equals

$$W = (v - p) + \left( p - \sum_i \alpha_i c_i \right) + \sum_i r(c_i)\alpha_i \tag{5}$$

where the first term is consumer surplus, and the other two terms reflect total managerial utility. Maximizing with respect to $c_i$ a socially optimal cost level $c^W$ for all firms that is implicitly defined by

$$r'(c^W) = 1. \tag{6}$$

From (4), the social optimum is reached if all firms are vanishingly small, so $\alpha_i = 0 \forall i.$

In terms of comparative statics, we can now establish

**Theorem 1.** **In the competitive outcome, we have the following:**

(a) An increase in firm-size heterogeneity implies an increase in the regulated price, provided that $r'$ is not too concave, i.e. provided $r'''' > 2r''/\alpha$ for all $\alpha$'s that occur in the two situations we compare.

\footnote{A higher $\alpha_i$ implies a lower equilibrium value of $r'(c_i^*)$, concavity of $r$ then implies a higher $c_i^*$.}
(b) With symmetric firms, an increase in the number of firms implies a decrease in the regulated price.
(c) A merger leads to higher prices.

Proof. In Appendix. □

Hence, if firms choose to compete, having more equally-sized firms leads to lower prices, but a merger leads to higher prices. Under an additional weak technical condition, we also have that a more heterogeneous market structure leads to higher prices. That technical condition is satisfied for all functional forms that we consider in this paper.\(^{12}\) For simplicity, we assume that it is always satisfied.

Note that mergers always increase prices. Hence, there is no merger paradox in our model; provided that firms compete, a merger always makes their managers unambiguously better off. Admittedly, this may no longer be true if due to a merger, a cartel would no longer be stable. Still, with some potential abuse of terminology, in the remainder of this paper we will talk about “the effects of a merger” whenever we exogenously impose that two firms join forces.

3.4. Collusion

Following e.g. Friedman (1971), we assume grim trigger strategies and look for the critical discount factor \(\hat{\delta}\) such that all managers have an incentive to stick to the cartel agreement. With the usual arguments manager \(i\) does not defect if and only if

\[
\frac{u^K_i}{1-\delta} > u^D_i + \frac{\delta u^*_i}{1-\delta},
\]

with \(u^D_i\) the utility of manager \(i\) when she defects, \(u^K_i\) her utility in a cartel, and \(u^*_i\) her utility in the competitive outcome. In other words, cartel stability requires that the short-run benefits of defection are outweighed by the long-term losses due to a cartel breakdown. This implies that we require

\[
\delta > \hat{\delta}_i(\alpha_i) \equiv \frac{u^D_i - u^K_i}{u^*_i - u^*_i}.
\]

\(^{12}\) In particular, in Corollary 1, we use the following functional forms:

(a) Suppose \(r(c) = \sqrt{c} - bc\). We then have \(r'(c) = \frac{1}{2}c^{-1/2} - b\), so \(r''(c) = -\frac{1}{4}c^{-3/2} < 0\) and \(r'''(c) = \frac{3}{8}c^{-5/2} > 0 > 2r''/\alpha\), so the condition is satisfied.
(b) Suppose \(r(c) = bc - ac^2\). In that case \(r'''(c) = 0 > 2r''/\alpha\), so again the condition is satisfied.
(c) Suppose \(r(c) = bc - e^c\). In that case \(r'' = -e^c\) and \(r''' = -e^c\), so \(r'' > 2r''/\alpha\) if \(1 < 2/\alpha\), which is true for any \(\alpha \in (0, 1)\).
For collusion to be sustainable, we need that this condition is satisfied for all managers:

$$\delta > \hat{\delta} \equiv \max \{ \hat{\delta}_i(\alpha_1), \ldots, \hat{\delta}_i(\alpha_n) \}.$$  

(9)

We will focus on the cartel agreement in which all managers set the same cost level $c_k$ that maximizes their total utility. Arguably, this is the most obvious and focal agreement. We will refer to it as the perfect symmetric collusive agreement. Of course, if this would not yield a stable cartel, managers could still try to coordinate on a different agreement, either symmetric or asymmetric. In our theoretical analysis we rule out this option as we feel that it would be much harder to coordinate on such an alternative.\(^\dagger\) Of course, in the experimental implementation of our model, firms are free to try to coordinate on whatever they can agree upon.

If firms coordinate on a common cost level $c_k$, we immediately have $p = c_k$, so all profits are zero and the cartel simply maximizes joint managerial benefit $r(c)$, which implies

$$c_k = c^m.$$  

(10)

We now have:

**Lemma 1.** The perfect symmetric collusive agreement yields a stable cartel if, for all $i = 1, \ldots, n$,

$$\delta > \hat{\delta}(\alpha_i) \equiv \frac{(1 - \alpha_i)(c^m - c_i^*) + r(c_i^*) - r(c^m)}{(1 - \alpha_i)c^m - \sum_{j \neq i} \alpha_j c_j^*}. $$  

(11)

**Proof.** For a cartel to be stable, we need that (8) is satisfied for all $i$. To evaluate the values for $u_i^*$, $u_i^K$, and $u_i^D$, we proceed as follows. First, using (3), we have

$$u_i^K = \left( \sum_{j=1}^{n} \alpha_j c^m - c^m + r(c^m) \right) \alpha_i = r(c^m) \alpha_i.$$  

(12)

The optimal defection is simply to use your dominant strategy and set $c_i^*$. That yields, again using (3),

$$u_i^D = (r(c_i^*) - (1 - \alpha_i)(c_i^* - c^m)) \alpha_i.$$  

(13)

Moreover

$$u_i^* = u_i(c_i^*, c_{-i}^*) = \left( \sum_{j=1}^{n} \alpha_j c_j^* - c_i^* + r(c_i^*) \right) \alpha_i.$$  

(14)

Hence

$$\hat{\delta}(\alpha_i) = \frac{r(c_i^*) - (1 - \alpha_i)(c^m - c_i^*) - r(c^m)}{r(c_i^*) - (1 - \alpha_i)(c_i^* - c^m) - \left( \sum_{j \neq i} \alpha_j c_j^* - c_i^* + r(c_i^*) \right)},$$

which simplifies to (11). \(\square\)

\(^\dagger\) Indeed, in our experiment we also observe that groups, if anything, coordinate on the perfect symmetric collusive agreement, irrespective of the distribution of market shares.
3.5. Factors that facilitate collusion

We now study which factors facilitate or hinder collusion. Throughout the analysis, we assume that firm size heterogeneity increases competitive prices, i.e. that the condition in Theorem 1(a) is satisfied.\footnote{Although note that we do not need to make this assumption for all results that follow.} We can now show

**Theorem 2.** With symmetric firms, an increase in the number of firms facilitates collusion if and only if \( r(c_i^*) \) is sufficiently concave, more precisely if

\[
  r'' < -r'(c_i^*) \cdot \frac{r'(c_i^*)(c^m - c^s) - (r(c^m) - r(c^s))}{(r(c^m) - r(c^s))(c^m - c^s)}
\]

**Proof.** In Appendix. □

This can be understood as follows. Defection utility, cartel utility and competition utility all decrease in the number of firms.\footnote{For defection utility, note from (14)} That implies from (8) that the net effect on cartel stability is ambiguous. In particular, it depends on the curvature of \( r \). Indeed, we can find families of managerial benefit functions \( r \) such that an increase in the number of firms either increases, decreases or does not affect cartel stability:

**Corollary 1.**

(a) If \( r(c) = \sqrt{c} - bc \), with \( b > 0 \), then a cartel is less stable if the number of symmetric firms increases.

(b) If \( r(c) = bc - ac^2 \), with \( a, b > 0 \), then cartel stability is unaffected by the number of symmetric firms.

(c) If \( r(c) = bc - c^c \), with \( b > 2 \), then a cartel is more stable if the number of symmetric firms increases.

**Proof.** In Appendix. □
To analyze how exogenous factors affect the stability of collusion with heterogeneous firms, we need to know how these affect the manager with the highest critical discount factor. We can establish:

**Lemma 2.** If the perfect symmetric collusive agreement is stable, then the critical discount factor $\hat{\delta}(\alpha_i)$ is the highest for the smallest firm.

**Proof.** In Appendix. □

Lemma 2 immediately implies that a cartel is stable if and only if the smallest firm has no incentive to defect.

**Theorem 3.** An increase in firm-size heterogeneity increases the critical discount factor and hence makes a cartel less likely to be stable.

**Proof.** In Appendix. □

Comparing this result to Theorem 1, we thus have an interesting trade-off. On the one hand, a more homogeneous industry structure implies a lower price – provided that firms behave competitively. At the same time, however, it makes collusion more likely.

Now consider the effect of a merger. First, it is straightforward to establish:

**Lemma 3.** Denote the pre-merger industry structure as $A$, and the post-merger industry structure as $B$. The merger then facilitates collusion if and only if

$$\frac{(1 - \alpha^B)(c^m - c^* (\alpha^B)) + r(c^* (\alpha^B)) - r(c^m)}{(1 - \alpha^B)c^m + \alpha^B c^* (\alpha^B) - p^*_B} < \frac{(1 - \alpha^A)(c^m - c^* (\alpha^A)) + r(c^* (\alpha^A)) - r(c^m)}{(1 - \alpha^A)c^m + \alpha^A c^* (\alpha^A) - p^*_A},$$

with $\alpha^\kappa$ the market share of the smallest firm in industry structure $\kappa$, and $p^*_\kappa$ the competitive price in industry structure $\kappa$, $\kappa \in \{A, B\}$.

**Proof.** Follows directly from Lemma 1 and Lemma 2. □

Using this theorem, we can derive a number of scenarios in which a merger either always facilitates, or always hinders collusion:

**Theorem 4.** A merger affects cartel stability in the following manner:

(a) If it does not affect the market share of the smallest firm, then a merger hinders collusion.
(b) Suppose that we are in an environment where a cartel becomes weakly less stable if the number of symmetric firms increases. In that case, a merger that leads to symmetric firms facilitates collusion.

Proof. Part (a) is straightforward. From Lemma 2 the smallest firm has the highest critical discount factor. If (one of the) smallest firm(s) is not involved in a merger, its market share $\alpha_i$ is not affected, so after the merger it is still the smallest firm. However, $p^*$ will increase, which from (11) implies that $\hat{\delta}_i(\alpha_i)$ increases and collusion is less stable.

For part (b), note that, before the merger, there are $n > 2$ firms, that differ in size. As a first step, redistribute market shares between these firms in such a way that they are now equally sized. From Theorem 3, doing so facilitates collusion. As a second step, consider a change from $n$ equally sized firms to $n - 1$ equally sized firms. By assumption, this also facilitates collusion. This establishes the result. \qed

Result (a) is particularly strong: whenever the smallest firm (or one of the smallest firms) is not part of the merger, then the merger necessarily hinders collusion. The intuition is as follows. As the market share of the smallest firm is unaffected, the merger also has no effect on either the collusion utility or the defection utility of the manager of that firm. However, it does affect her utility in the competitive outcome. From the proof of Theorem 1 a more concentrated industry implies higher competitive prices, and hence higher competitive utilities. Thus, defection from a cartel becomes less costly, making it more attractive to defect.

Broadly speaking, the results suggest that mergers that lead to a more homogeneous industry structure (such as the scenario described in (b) of the Theorem) facilitate collusion, while mergers that lead to a less homogeneous industry structure (such as the scenario described in scenario (a)) hinder collusion – at least in an environment where having more equally sized firms makes it harder to collude.\footnote{If we are not in such an environment, the effect of a merger to equality is ambiguous. It is hard to derive general conditions under which such a merger would or would not facilitate collusion.}

4. Experiment

4.1. Design

In our experiment, subjects play the game described above for at least 20 rounds. From round 20 onward, the experiment ends with a probability of 20% in each round, to avoid possible end-game effects (see also Normann and Wallace, 2012). We use fixed matching: every subject plays with the same group members in all rounds.

Every round consists of three steps. First, subjects can communicate using a chat screen. This chat is completely anonymous.\footnote{We allowed subjects to chat for 150 s in round 1, for 120 s in round 3, for 60 s in rounds 3–10, and for 45 s in all remaining rounds.} Second, subjects unilaterally choose cost
levels $c_i$ from the set $\{1, 2, \ldots, 20\}$.\footnote{As saw in the theory section, both the competitive and the collusive strategy are independent of the competitors’ actions. In that sense, our game boils down to a relatively straightforward prisoners’ dilemma game with two strategies. Still, we choose not to represent it as such in our experiment, as that would get rid of much of the regulatory context that we feel is crucial for our purpose.} Third, prices are determined using (2). After each round, subjects learn their profits and managerial benefits, and the cost levels each subject has set.\footnote{Of course, in our set-up profit and managerial benefit are equally important in determining payoffs. This was also outlined in the experimental instructions. Subjects indeed seemed to understand this as they correctly answered control questions on this topic.} Although not very common in a cartel experiment, we feel that the unrestricted communication we allow for creates circumstances that are closest to the real world. The Dutch regulatory framework for the distribution-network operators, for example, does not forbid them to mutually exchange information on operational and strategic issues. The operators together have established a joint organization (Netbeheer Nederland) which facilitates the cooperation among the operators besides representing the group of operators in the political debate on energy policy in general and the design of regulation in particular. Without communication, it is also hard to sustain collusion in a cartel experiment with more than two players, see e.g. Haan et al. (2009).\footnote{Although illegal in practice, note that cartels cannot be prosecuted or fined in this experiment.}

We use the following quadratic specification for the managerial benefit function:

$$r(c_i) = bc_i - ac_i^2,$$  \hspace{1cm} (16)

where $a, b \in \mathbb{R}^+$ are parameters and $c_i \in [0, b/a]$. We assume that $a$ and $b$ are such that all expressions we derive are well defined. For this set-up, it is easy to drive that

$$c_i^* = \frac{b - 1 + \alpha_i}{2a};$$  \hspace{1cm} (17)

$$c^m = \frac{b}{2a}. $$  \hspace{1cm} (18)

We set $a = 1/24$ and $b = 1$. These choices assure that the competitive and collusive cost levels derived above are integers. We run 5 treatments that differ in the number of firms and the extent of firm-size heterogeneity. For ease of exposition, we normalize the total size of the market to 12 when naming our treatments, which allows us to represent market sizes of all firms in all treatments as integer numbers.\footnote{In the actual experiment, we always normalize the size of the smallest firm to 1, which makes it easier to explain the experiment to the subjects. See the instructions in the online appendix for an example.} The first 3 treatments have 3 firms and are denoted TRIOXYZ, with X, Y and Z the size of firms 1, 2 and 3, respectively. The other treatments have 2 firms and are denoted DUOXY, with X and Y the sizes of firms 1 and 2.

Table 1 provides information for each treatment. The first panel gives the market share for each firm and the resulting Herfindahl index. The second panel provides competitive cost levels and the resulting price. The third panel gives collusive cost levels and the
critical discount factor $\hat{\delta}$ of the cartel.\footnote{Strictly speaking, the critical discount factors we derive are only valid from round 20 onward. Still, it is easy to see that this does not affect our qualitative results. If a cartel is stable from round 20 onward, it is definitely stable in earlier rounds as well, since the incentives to defect become smaller as the first 19 rounds are played for sure. If a cartel is not stable from round 20 onward, then subjects know it will break down in that round and hence, by backward induction, are not willing to form a cartel in an earlier round.} From (10), collusive cost levels do not depend on industry structure; in all treatments, the perfect symmetric collusive cost level equals 12.

The 5 treatments allow us to evaluate the effect of the number of firms and firm-size heterogeneity on market performance in terms of cost levels and regulated prices. They also allow us to evaluate the effect of a merger. In our evaluation of the experimental results we do not interpret the critical discount factors as a strict prediction of the outcome of the experiment. Thus, we do not expect that there will always be collusion whenever the $\delta$ we impose is larger than the $\hat{\delta}$ we derived. Neither do we expect that there will never be collusion whenever the $\delta$ we impose is smaller than the $\hat{\delta}$ we derived. Rather, we interpret a higher value of $\delta$ as making a cartel less likely to occur. This is in line with other work (see e.g. Bigoni et al., 2012).

From Corollary 1(b) we expect that, with symmetric firms, the number of firms does not affect collusion:

**Hypothesis 1 (Number of Firms).** The amount of collusion in DUO66 is the same as that in TRIO444.

From Theorem 3 we expect that more heterogeneity implies less collusion:

**Hypothesis 2 (Heterogeneity).**

(a) There is more collusion in DUO66 than in DUO84;
(b) There is more collusion in TRIO444 than in TRIO633, and more collusion in TRIO633 than in TRIO642.

\begin{table}[h]
\centering
\caption{Overview of treatments.}
\begin{tabular}{lccccccc}
\hline
Treatment & Market shares & \multicolumn{3}{c}{Competition} & Collusion \\
 & $a_1$ (%) & $a_2$ (%) & $a_3$ (%) & HHI & $c_1^*$ & $c_2^*$ & $c_3^*$ & $p^*$ & $c_k = p_k$ & $\delta$ \\
\hline
Trio444 & 33.3 & 33.3 & 33.3 & 0.333 & 4 & 4 & 4 & 4.00 & 12 & 0.50 \\
Trio633 & 50.0 & 25.0 & 25.0 & 0.375 & 6 & 3 & 3 & 4.50 & 12 & 0.64 \\
Trio642 & 50.0 & 33.3 & 16.7 & 0.389 & 6 & 4 & 2 & 4.67 & 12 & 0.71 \\
Duo66 & 50.0 & 50.0 & & 0.500 & 6 & 6 & & 6.00 & 12 & 0.50 \\
Duo84 & 66.7 & 33.3 & & 0.556 & 8 & 4 & & 6.67 & 12 & 1.00 \\
\hline
\end{tabular}
\end{table}

Market shares give the market shares that we impose for each experimental firm in that treatment, HHI is the resulting Hirschman Herfindahl Index. $c_i^*$ is the competitive cost level of firm $i$ that our theory predicts, $p^*$ the resulting regulated price, $c_k^*$ and $p_k^*$ are the collusive cost levels and prices predicted by our theory, $\hat{\delta}$ the critical discount factor for collusion to be stable.
From Theorem 4(b) we expect that mergers that lead to symmetric firms, facilitate collusion:

**Hypothesis 3** (Merger to Symmetry).

(a) There is more collusion in DUO66 than in TRIO633;
(b) There is more collusion in DUO66 than in TRIO642.

Finally, we look at mergers that lead to asymmetric firms. At least in this case, these hinder collusion:

**Hypothesis 4** (Merger to Asymmetry).

(a) There is less collusion in DUO84 than in TRIO444;
(b) There is less collusion in DUO84 than in TRIO642.

Part (a) of this Hypothesis follows directly from Theorem 4(a). Part (b) does not follow directly from the results in the previous section, but can be shown to hold in our experimental implementation (see e.g. the last column of Table 1).

### 4.2. Implementation

The experiment was conducted at the Groningen Experimental Economics Laboratory (GrEELab) at the University of Groningen in February and March 2013. A total of 214 subjects participated, all students from the University of Groningen (80.4%) or the Hanze University of Applied Sciences (19.6%), most of them in the fields of economics and business (59.3%). Every session consisted of one treatment and lasted between 80 and 115 min. Subjects signed in for sessions, while treatments were randomly assigned to sessions.

Every treatment with 2 subjects was played in two sessions while every treatment with 3 subjects was played in three sessions. Between 14 and 18 subjects participated in a session, resulting in 16 to 17 groups per treatment. This is similar to other cartel experiments, see e.g. Bigoni et al. (2012), Dijkstra et al. (2014) and Hinloopen and Soetevent (2008). The experiment was programmed in z-Tree (Fischbacher, 2007). Printed instructions were provided and read aloud. Instructions for Trio633 are reproduced in the online appendix. Instructions for other treatments are similar and available upon request.

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23 Thanks to an anonymous referee for suggesting this hypothesis.

24 Instructions for Trio633 are reproduced in the online appendix. Instructions for other treatments are similar and available upon request.
full collusion.\textsuperscript{25} Furthermore, they received an initial endowment of € 4. Average earnings were € 16.80 and ranged from € 7.75 to € 24.00.

5. Results

5.1. Three measures of collusion

For comparison sake, we only include the first 20 rounds of each group in our analysis, as only these rounds are played by all groups. In this section, we introduce three measures to evaluate the extent of collusion: the incidence of full collusion; a collusion index; and price. We explain these measures in more detail below. For each measure, we give its development over time in each treatment, and compare averages between all treatments. In the next section, we confront our hypotheses with the results of the experiment.

As is common in experimental studies, we use non-parametric tests for differences between treatments.\textsuperscript{26} The unit of observation is always the average value over the first 20 rounds for each group.\textsuperscript{27} We use the Mann–Whitney $U$ test (MWU) to test for differences between two populations, and the Jonckheere–Terpstra test to test for an ordering of three populations.\textsuperscript{28} Whenever our hypothesis implies no difference between treatments we do a two-sided test. We also perform a two-sided test if the theoretical result is ambiguous. This may for example be the case with prices: when a treatment is expected to yield more collusion but also lower competitive prices, then the net effect on prices is ambiguous, so we do a two-sided test. Whenever our hypothesis implies a difference between treatments, we do a one-sided test.

First, we look at the incidence of full collusion. In all treatments, the perfect symmetric collusive agreement is for all firms to set a cost level of 12. For ease of exposition we refer to this as full collusion. The incidence of full collusion is thus defined as the percentage of markets where all firms set a cost level of 12. Fig. 1 shows how this measure develops over time, in all treatments.\textsuperscript{29} The number of markets with full collusion is substantial. From the figure, TRIO444 seems to be the most collusive, while the least collusion is found in TRIO642.

Table 2 gives the average incidence of full collusion for all treatments, and reports on pairwise comparisons between treatments. The entries in the right-hand panel indicate whether the row treatment has an incidence of collusion that is significantly higher (>)

\textsuperscript{25} This allows us to focus on the effect of asymmetries in industry structure without the possible interference of fairness issues.

\textsuperscript{26} Such tests do not impose a specific distribution on the data and are less sensitive to outliers than parametric tests. Still, we also ran OLS regression using panel data, which yielded the same qualitative results; see the online appendix.

\textsuperscript{27} Arguably there may be learning and/or end-game effects present in the data. Still, our qualitative results do not change if we would exclude first or last rounds from the analysis. Details are available from the authors upon request.

\textsuperscript{28} See Jonckheere (1954) or Terpstra (1952). The Kruskal–Wallis test is equivalent but has an unordered alternative hypothesis.

\textsuperscript{29} The development of costs over time for each individual market are given in the online appendix.
Fig. 1. Incidence of full collusion (across all groups).

Table 2
Incidence of full collusion (across all rounds and groups).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Average (%)</th>
<th>Trio633</th>
<th>Trio642</th>
<th>Duo66</th>
<th>Duo84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trio444</td>
<td>71.8</td>
<td>≈&lt;sub&gt;t&lt;/sub&gt;</td>
<td>&gt;*&lt;sup&gt;t&lt;/sup&gt;</td>
<td>≈&lt;sub&gt;t&lt;/sub&gt;</td>
<td>&gt;&lt;sup&gt;t&lt;/sup&gt;</td>
</tr>
<tr>
<td>Trio633</td>
<td>58.1</td>
<td></td>
<td>&gt;*&lt;sup&gt;t&lt;/sup&gt;</td>
<td>≈&lt;sub&gt;t&lt;/sub&gt;</td>
<td>≈&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>Trio642</td>
<td>34.4</td>
<td></td>
<td>≈&lt;sub&gt;t&lt;/sub&gt;</td>
<td>&lt;*&lt;sup&gt;t&lt;/sup&gt;</td>
<td>≈&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>Duo66</td>
<td>56.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duo84</td>
<td>43.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Entries in the right-hand panel indicate whether the row treatment yields rates of full collusion that are significantly lower (<), significantly higher (>), or that do not differ significantly (≈) from the rates in the column treatment. Differences between treatments are tested using the MWU test for equality; subscript t denotes a two-sided test, o a one-sided test. +: significant at 10% level; * at 5%; ** at 1%.

or significantly lower (<) than the column treatment, or whether the difference is not significant (≈). We use this convention throughout the remainder of this paper. From the table, we thus have for example that Trio444 leads to significantly more collusion than Trio642 and Duo84. The difference with Trio633 and Duo66 is not significant.

One drawback of this measure is that it only considers markets to be collusive if market participants succeed in achieving full collusion. Arguably, any market with prices higher than those in the competitive outcome is collusive to at least some extent. Also, the closer prices are to the fully collusive outcome, the more collusive that market is. We therefore study the **collusion index**, which is the relative premium that firms are able to achieve over and above the competitive outcome:
Collusion index (%)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Fig. 2. Average collusion index per round (across all groups).

Table 3
Collusion index (across all rounds and groups).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Average (%)</th>
<th>Trio633</th>
<th>Trio642</th>
<th>Duo66</th>
<th>Duo84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trio444</td>
<td>85.8</td>
<td>≈</td>
<td>&gt;**</td>
<td>&gt;t</td>
<td>&gt;o</td>
</tr>
<tr>
<td>Trio633</td>
<td>74.6</td>
<td>≈</td>
<td>≈</td>
<td>≈</td>
<td>≈</td>
</tr>
<tr>
<td>Trio642</td>
<td>65.9</td>
<td>≈</td>
<td>≈</td>
<td>&lt;+</td>
<td>≈</td>
</tr>
<tr>
<td>Duo66</td>
<td>73.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duo84</td>
<td>77.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Entries in the right-hand panel indicate whether the row treatment yields a collusion index that is significantly lower (<), significantly higher (>), or that does not differ significantly (≈) from the index in the column treatment. Differences between treatments are tested using the MWU test for equality; subscript t denotes a two-sided test, o a one-sided test. +: significant at 10%; * at 5%; ** at 1%.

\[
\text{collusion index} = \frac{\text{price} - p^*}{12 - p^*},
\]  

with \( p^* \) the competitive price. Note that this measure allows for the fact that the competitive price level differs across industry structures (see Table 1). If the competitive outcome is achieved, the collusion index equals 0. If the collusive outcome of 12 is achieved, it equals 1. The higher the index, the more successful firms are in colluding.

Fig. 2 shows how the collusion index develops over time, in all treatments. Qualitatively, the picture looks very similar to that for the incidence of full collusion. Trio444 seems the most collusive, and Trio642 the least collusive treatment. Table 3 gives treatment averages and pairwise comparisons.
Of course, a regulator is most interested in the price that consumers ultimately pay. Despite a slightly higher incidence of collusion, for example, an industry structure may still be preferable if it leads to lower competitive prices that more than outweigh the adverse effects of the occasional cartel. For that reason, we also look at prices. Average prices over time are given in Fig. 3. Consistent with our earlier measures, average prices are high, and come closest to the collusive outcome in Trio444. From Table 4, most pairwise comparisons are no longer significant.
Table 5
Comparison of treatments with symmetric firms (across all rounds and groups).

<table>
<thead>
<tr>
<th>Measure</th>
<th>Trio444</th>
<th>Duo66</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence of full collusion</td>
<td>71.8%</td>
<td>56.3%</td>
</tr>
<tr>
<td>Collusion index</td>
<td>85.8%</td>
<td>73.1%</td>
</tr>
<tr>
<td>Price</td>
<td>11.0</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Entries between values indicate whether the value to the left is significantly lower ($<$), higher ($>$), or does not differ significantly ($\approx$) from the value to the right. Differences between treatments are tested using the MWU test for equality; subscript $t$ denotes a two-sided test, $o$ a one-sided test. $^+$: significant at the 10% level.

Table 6
Comparison of treatments with 2 firms (across all rounds and groups).

<table>
<thead>
<tr>
<th>Measure</th>
<th>Duo66</th>
<th>Duo84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence of full collusion</td>
<td>56.3%</td>
<td>43.4%</td>
</tr>
<tr>
<td>Collusion index</td>
<td>73.1%</td>
<td>77.9%</td>
</tr>
<tr>
<td>Price</td>
<td>10.3</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Entries between values indicate that the value to the left is significantly lower ($<$), higher ($>$), or does not differ significantly ($\approx$) from the value to the right. Differences between treatments are tested using the MWU test for equality; subscript $t$ denotes a two-sided test, $o$ a one-sided test.

Table 7
Comparison of treatments with 3 firms (across all rounds and groups).

<table>
<thead>
<tr>
<th>Measure</th>
<th>Trio444</th>
<th>Trio633</th>
<th>Trio642</th>
<th>Trio444</th>
<th>JT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence of full collusion</td>
<td>71.8%</td>
<td>$\approx_o$</td>
<td>$&gt;^+_o$</td>
<td>34.4%</td>
<td>$&lt;^+_o$</td>
</tr>
<tr>
<td>Collusion index</td>
<td>85.8%</td>
<td>$\approx_o$</td>
<td>$\approx_o$</td>
<td>65.9%</td>
<td>$&lt;^+_o$</td>
</tr>
<tr>
<td>Price</td>
<td>11.0</td>
<td>$\approx_t$</td>
<td>$\approx_t$</td>
<td>9.5</td>
<td>$&lt;^+_t$</td>
</tr>
</tbody>
</table>

Entries between values indicate whether the value to the left is significantly lower ($<$), significantly higher ($>$), or does not differ significantly ($\approx$) from the value to the right. Differences between two treatments are tested using the MWU test for equality; differences between all treatments using the Jonckheere–Terpstra test (JT) for equality; subscript $t$ denotes a two-sided test, $o$ a one-sided test. $^+$: significant at 10%; $^*$ at 5%; ** at 1%.

5.2. Test of hypotheses

In the previous subsection, we gave an overview of the experimental results. We now discuss to what extent those results confirm our hypotheses.

Hypothesis 1 is concerned with the number of (symmetric) firms and argues that we expect no difference in collusion between both treatments. From Table 5, the collusion index is higher in Trio444 than in Duo66. The other measures are not significantly different between the two treatments. In Section 5.3 we further address the question as to why subjects may find it easier to coordinate on a collusive outcome with 3 rather than 2 players.

Hypothesis 2 is concerned with the effect of heterogeneity. For the case of 2 firms, Table 6 shows that the incidence of full collusion is higher in Duo66, but the average value of the collusion index is lower. Average prices are also lower. However, none of these differences is significant. Thus, Hypothesis 2(a) is not confirmed.
Table 8
Merger to symmetric firms (averages across all rounds and groups).

<table>
<thead>
<tr>
<th>Measure</th>
<th>Trio642</th>
<th>Duo66</th>
<th>Trio633</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence of full collision</td>
<td>34.4%</td>
<td>$&lt;^*$</td>
<td>56.3%</td>
</tr>
<tr>
<td>Collusion index</td>
<td>65.9%</td>
<td>$≈_o$</td>
<td>73.1%</td>
</tr>
<tr>
<td>Price</td>
<td>9.5</td>
<td>$≈_t$</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Entries between values indicate whether the value to the left is significantly lower ($<$), higher ($>$), or does not differ significantly ($≈$) from the value to the right. Differences between treatments are tested using the MWU test for equality; subscript $t$ denotes a two-sided test, $o$ a one-sided test. $^*$: significant at 5% level.

Table 9
Merger to asymmetric duopoly (averages across all rounds and groups).

<table>
<thead>
<tr>
<th>Measure</th>
<th>Trio642</th>
<th>Duo84</th>
<th>Trio444</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence of full collision</td>
<td>34.4%</td>
<td>$≈_o$</td>
<td>43.4%</td>
</tr>
<tr>
<td>Collusion index</td>
<td>65.9%</td>
<td>$&lt;^*$</td>
<td>77.9%</td>
</tr>
<tr>
<td>Price</td>
<td>9.5</td>
<td>$&lt;^*$</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Entries between values indicate whether the value to the left is significantly lower ($<$), higher ($>$), or does not differ significantly ($≈$) from the value to the right. Differences between treatments are tested using the MWU test for equality; subscript $t$ denotes a two-sided test, subscript $o$ a one-sided test. $^*$: significant at 10% level; $^*$ at 5%.

Table 7 looks at the results for three firms. Note that Trio444 is listed twice in this table, to facilitate all pairwise comparisons. All results reported in Table 7 are consistent with Hypothesis 2(b); for any measure, the extent of collusion is always higher if the industry structure is more homogeneous. Many of the pairwise comparisons are not significant, but results are highly significant in a Jonckheere–Terpstra test with the ordered alternative hypothesis that Trio444 yields more collusion than Trio633 which in turn yields more collusion than Trio642. Note that we argued in Section 3 that the effect of firm-size heterogeneity on price may be ambiguous, as our model predicts that more heterogeneity makes collusion harder, but also raises the competitive price. Our experimental results indicate that the effect on collusion is stronger, as prices do go down with more heterogeneity.

Summing up, in treatments with 2 firms we do not find evidence for an effect of the extent of firm-size heterogeneity on collusion. With 3 firms, however, we find strong support for Hypothesis 2(b): more heterogeneity leads to less collusion.

Table 8 evaluates Hypothesis 3 and looks at the extent to which a merger to symmetric firms facilitates collusion. We find that DUO66 is indeed more collusive than Trio642, when measured by the incidence of full collusion. On the other two measures, DUO66 also scores higher, but not significantly so. However, the differences between DUO66 and Trio633 are ambiguous and insignificant. Table 9 evaluates the evidence concerning Hypothesis 4 that predicts that a merger to asymmetric firms hinders collusion. Again, the evidence is mixed. On 2 out of 3 measures, a merger from Trio444 to DUO84 indeed hinders collusion. However, if anything, a merger from Trio642 to DUO84 only seems to facilitate collusion.
5.3. Chat analysis

To quantify the agreements subjects make during the chats, we use the methodology of content analysis (see e.g. Cooper and Kühn, 2014, and Cason and Mui, forthcoming). We employed two assistants to review the essence of each conversation and classify them according to a classification scheme. We asked them to judge whether an agreement was reached; whether this agreement was to “set all cost level 12” or a different agreement; whether no agreement was reached; or whether no proposal was made at all. Furthermore, we asked them to judge whether the possibility to all set cost level 12 was raised by any of the subjects. Coders coded the chat statements independently. They were unaware of the research questions addressed in this study. For each group, the full chat in a given round could only be assigned to one category.

Out of all markets where subjects did chat, we report the fraction of markets where the relevant issue is at least discussed once. Table 10 provides these results. From the table, most groups that chat, discuss cost levels at some point. In each treatment, some 90% of groups raised the possibility to coordinate on a cost level 12, with a slightly lower percentage in DUO84. The more heterogeneous the market shares, the lower the percentage of groups that actually agree to set cost levels of 12. Only few groups reach an alternative agreement, with the exception of DUO84.

Our results in Table 5 suggest that it might be easier to coordinate on a collusive outcome with 3 rather than 2 players. Table 11 takes a closer look at what might drive this result. The first thing to note is that in markets with 3 firms, there were only 5

---

Table 10
Chat content.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Discussed cost</th>
<th>Suggested agreement all set 12</th>
<th>Agreement made</th>
<th>Other than all set 12</th>
<th>Never all 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trio444 (%)</td>
<td>100.0</td>
<td>88.2</td>
<td>79.4</td>
<td>85.3</td>
<td>5.9</td>
</tr>
<tr>
<td>Trio633 (%)</td>
<td>100.0</td>
<td>96.9</td>
<td>62.5</td>
<td>68.8</td>
<td>25.0</td>
</tr>
<tr>
<td>Trio642 (%)</td>
<td>97.1</td>
<td>91.2</td>
<td>52.9</td>
<td>70.6</td>
<td>29.4</td>
</tr>
<tr>
<td>Duo66 (%)</td>
<td>93.8</td>
<td>87.5</td>
<td>78.1</td>
<td>78.1</td>
<td>25.0</td>
</tr>
<tr>
<td>Duo84 (%)</td>
<td>100.0</td>
<td>71.9</td>
<td>53.1</td>
<td>90.6</td>
<td>62.5</td>
</tr>
</tbody>
</table>

Entries reflect percentage of groups to which the statement applies at some point during the first 20 rounds of the experiment.

---

30 We define a conversation as the chat that takes place in a given market in a given round.
31 We classify instances in which coders did not agree as one half rather than one full observation.
32 In DUO84 the small firm earns the same with symmetric collusion as it does when both firms compete. Yet, there are alternative collusive agreements where it earns more. The large firm also has an incentive to make such agreements, as it implies a more stable cartel. The table suggests that this may be an issue, as we indeed see many alternative agreements in this treatment. There were 49 alternative agreements out of 320 total rounds. In 25 agreements the large firm set a higher cost level than the small firm, in 11 it set a lower cost level. 11 agreements implied setting different cost levels and rotating roles; 1 agreement setting a symmetric cost level different from 12; and 1 agreement involved randomization of cost levels. However, these alternative agreements were never long-lived, so they are more likely to reflect subjects trying different options to find the optimal solution, rather than that they made a conscious effort to coordinate on an asymmetric equilibrium.
instances in which subjects explicitly agreed on setting cost levels different from 12. In markets with 2 firms, there were 25 such instances.\textsuperscript{33} Hence, players in 2-player markets more readily agreed on alternative arrangements. Apparently, it is easier to reach an alternative agreement when only 2 rather than 3 players need to agree.

Moreover, groups that did figure out that it is a good idea to set cost level 12 took more time to do so in 2-firm markets than 3-firm markets. Arguably, it is more likely that one player will come up with that possibility if we have 3 players, rather than if we have 2. This can explain the numbers in Table 11 reasonably well.\textsuperscript{34} The differences cannot be explained by tacit agreements: in either treatment, there was not a single instance in which all subjects set costs equal to 12 without someone explicitly raising that possibility. Summing up, having 3 players makes it easier to agree on the focal equilibrium and less tempting to try other scenarios. The most likely explanation is simply that 3 people know more than 2.

6. Conclusion

In this paper we studied the effect of industry structure on collusion among economic agents subject to uniform yardstick competition, an issue particularly relevant for the debate on the effectiveness of tariff regulation. We did so both in a theoretical framework and in a laboratory experiment.

In our theoretical model, we find that firm-size heterogeneity hinders collusion. More surprisingly, in a symmetric industry, the effect of the number of firms on cartel stability is ambiguous, and an increase in the number of firms may even facilitate collusion. The

\textsuperscript{33} In the 3-firm case, these alternative agreements all consisted of setting different cost levels and rotating roles. In the 2-firm case, there were 11 such rotating agreements, 2 agreements to set a symmetric cost level different from 12, 11 asymmetric agreements, and 1 agreement that only involved 1 player.

\textsuperscript{34} Suppose for example that in each round each single player proposes with probability 8\% to set a common cost level of 12. In each round, the probability that at least one subject makes this suggestion then equals 15.4\% in a duopoly, and 22.1\% in a triopoly. The expected number of rounds before this proposal is made is then 6.51 in a duopoly and 4.52 in a triopoly. These numbers are similar to those in Table 11. We thank an anonymous referee for pointing this out.
theoretical effects of mergers on collusion are ambiguous. Mergers that do not involve the smallest firm in the industry hinder collusion, as they improve the competitive outcome for the smallest firm, making it more attractive for the manager of that firm to defect from a collusive agreement. Mergers that lead to symmetric market shares facilitate collusion.

Of course, no theoretical model can always predict what will happen in the real world. That is particularly true for models of collusion, as these assume that collusive agreements are always reached instantaneously and coordination is not an issue. In the real world, coordination problems may be an important obstacle to establishing a cartel. Therefore, we also conducted an experiment in which we allow for unrestricted communication between agents.

In our experiment we find that in triopolies, size heterogeneity indeed hinders collusion. We do not find evidence for this in duopolies. Also, we find weak evidence that mergers that lead to symmetric market shares indeed facilitate collusion, as theory predicts. Mergers to asymmetry hinder collusion, as theory predicts, but only if the market structure pre-merger consists of equally sized firms. Otherwise such a merger even facilitates collusion. Our findings also suggest that industries with 3 symmetric firms are more collusive than industries with 2 such firms.

Summing up, our experimental findings tentatively suggest the following. Relative to what theory predicts, an increase in the number of firms makes it easier for experimental subjects to collude if they are symmetric, but harder if they are asymmetric. This may be understood as follows. An increase in their number makes it harder for subjects to coordinate. With asymmetry that is bad news, as the lack of a focal outcome implies that it is now harder to reach any agreement. With symmetry, however, it is good news, as the focal outcome becomes more salient and subjects are less tempted to try to explore alternative agreements. We indeed find that 2-firm markets more often reach alternative agreements, and take more time before they start discussing the collusive outcome.

Hence, firm-size heterogeneity in an industry subject to yardstick competition strongly affects collusion in triopoly markets. Our theory suggests that an increase in heterogeneity increases the regulated price if firms do not collude, but also makes collusion harder, rendering the net effect ambiguous. Our experiment however suggests that the effect of collusion is stronger. Of course, it would be interesting to consider the effect on collusion of markets with even more firms (like e.g. Fonseca and Normann, 2012).

Our conclusions are relevant for the debate on the optimal industry structure. In a competitive market, a more homogeneous industry implies lower tariffs for network users. When also taking the effects on collusion into account, our results suggest that more homogeneity may imply higher tariffs.

We address the uniform weighted yardstick in our experiment. Potters et al. (2004) compare this yardstick with a discriminatory yardstick in which the price a firm might charge depends on the cost levels of all other firms. They observe a higher tendency to collude with a discriminatory yardstick than with a uniform yardstick in a duopoly with symmetric firms. They refrained from investigating the effect of different yardsticks
on collusion in different industry structures. We will address this question in future work.

Another interesting extension is to include multiple dimensions of regulation. We considered price regulation, but several industries also face quality regulation (Tangerås, 2009). This provides an interesting trade-off to firms, since higher quality is associated with higher costs while the incentive power tells them to reduce costs. In a theoretical model, Tangerås (2009) shows that larger firms need stronger incentives to improve quality and to reduce costs than smaller firms. It remains to be seen in which industry structures firms are able to collude on both dimensions and what effect this will have on social welfare.

Appendix A. Selected proofs

Proof of Theorem 1. For part (a), using (4) and (2) the regulated price with competition is given by

\[ p^* = \sum_{j=1}^{n} \alpha_j c_j^* = (1 - r'(c_j^*)) c_j^*. \]  

We look at an increase in firm heterogeneity. Needless to say, a large increase can always be written as the sum of many infinitesimally small increases. As noted, we will interpret an increase in firm size heterogeneity as the transfer of market share from a (weakly) smaller to a (weakly) larger firm. Suppose that we transfer market share \( \Delta \alpha \) from firm \( i \) to firm \( k \), with \( \alpha_i \leq \alpha_k \). This increases firm size heterogeneity. The resulting change in regulated price then equals

\[ \Delta p^* = (1 - r'(c^*(\alpha_k + \Delta \alpha)))c^*(\alpha_k + \Delta \alpha) - (1 - r'(c^*(\alpha_k)))c^*(\alpha_k) + (1 - r'(c^*(\alpha_i - \Delta \alpha)))c^*(\alpha_i - \Delta \alpha) - (1 - r'(c^*(\alpha_i)))c^*(\alpha_i), \]

with \( c^*(\alpha) \) the equilibrium cost level of a firm with market share \( \alpha \). Evaluating the effect on \( p^* \) of a change in \( \Delta \alpha \), we get

\[ \frac{\partial \Delta p^*}{\partial \Delta \alpha} = (1 - r'(c^*(\alpha_k + \Delta \alpha)))c''(\alpha_k + \Delta \alpha) - r''(c^*(\alpha_k + \Delta \alpha))c''(\alpha_k + \Delta \alpha) - (1 - r'(c^*(\alpha_i - \Delta \alpha)))c''(\alpha_i - \Delta \alpha) + r''(c^*(\alpha_i - \Delta \alpha))c''(\alpha_i - \Delta \alpha) \]
For infinitesimally small $\Delta_\alpha$, this implies
\[
\frac{\partial \Delta p^*}{\partial \Delta \alpha} \bigg|_{\Delta_\alpha \to 0} = (1 - r'(c^*(\alpha_k)))c^*(\alpha_k) - r''(c^*(\alpha_k))c^*(\alpha_k)c''(\alpha_k) - (1 - r'(c^*(\alpha_i)))c^*(\alpha_i) + r''(c^*(\alpha_i))c^*(\alpha_i)c''(\alpha_i)
\]

Note that this is indeed strictly positive for all $\alpha_k > \alpha_i$ if and only if the term on the first line is always larger than the term on the second line; in other words, if
\[
\frac{\partial}{\partial \alpha}([1 - r' - r''c^*]c^*) > 0,
\]
where we have dropped the arguments for ease of exposition. With $1 - r'(c^*(\alpha)) = \alpha$, this simplifies to the requirement that
\[
\frac{\partial}{\partial \alpha}(\alpha c^* - r'' c^* c^*) > 0.
\]

Hence we need
\[
c^* + \alpha c'' - \frac{\partial}{\partial \alpha}(r'' c^* - r'' c^* c^*) > 0. \quad (**)
\]
Note that $1 - r'(c^*(\alpha)) = \alpha$. Differentiating through twice with respect to $\alpha$, we obtain, respectively,
\[
\frac{\partial (1 - r')}{\partial \alpha} = r'' \cdot c^* = -1 \quad (***)
\]
\[
\frac{\partial (r'' \cdot c^*)}{\partial \alpha} = r''' \cdot c^* + r'' \cdot c'' = 0. \quad (****)
\]
Using (****), condition (***) simplifies to
\[
c^* + \alpha c'' - r'' c^* c^* > 0.
\]
From (**), this simplifies to
\[
\alpha c'' + 2c^* > 0
\]
Note that this is equivalent to the requirement that $\alpha c^*(\alpha)$ is strictly convex in $\alpha$. Using (****), we have
\[
\alpha c'' + 2c^* = \frac{-\alpha r''' c^*}{r''} + 2c^* = 2 - \frac{\alpha r'''}{r''} c^*.
\]
With $c^* > 0$ and $r'' < 0$, we thus need the condition in (a).
For part (b) note that with equal market shares, we have \( p^* = c^*(1/n) \). With \( 1 - r'(c(1/n)) = 1/n \) and \( r \) concave, this immediately implies the result.

For part (c), denote the price before the merger as \( p^* \), the price after the merger as \( p'' \). If firms \( i \) and \( k \) merge, we thus have from (A.1)

\[
p'' - p^* = (\alpha_i + \alpha_k)c^*(\alpha_i + \alpha_k) - \alpha_i c^*(\alpha_i) - \alpha_k c^*(\alpha_k).
\]

With \( c^* \) increasing, we have that \( \alpha_i c^*(\alpha_i + \alpha_k) > \alpha_i c^*(\alpha_i) \) and \( \alpha_k c^*(\alpha_i + \alpha_k) > \alpha_k c^*(\alpha_k) \). Hence \( p'' - p^* > 0 \).

**Proof of Theorem 2.** First note that with symmetric firms, \( \alpha_i = 1/n \) for all firms, whereas the competitive outcome has all firms setting \( c^* = c^*(1/n) \). From (11), we thus have

\[
\hat{\delta} = 1 - \frac{r(c^m) - r(c^*)}{(1-\alpha)(c^m - c^*)},
\]

which we can write as

\[
\hat{\delta} = 1 - \frac{r(c^m) - r(c^*)}{r'(c^*)(c^m - c^*)}.
\]

Consider this as a function of \( c^* \). For ease of exposition, denote \( \Delta_c \equiv c^m - c^* \) and \( \Delta_r \equiv r(c^m) - r(c^*) \). Note that \( \Delta_c, \Delta_r > 0 \). Dropping arguments, we then have

\[
\frac{\partial \hat{\delta}}{\partial c^*} = \frac{r'\Delta_cr' + \Delta_r r''\Delta_c - \Delta_r r'}{r'r'\Delta_c^2} = \frac{r'(\Delta_c - \Delta_r) + \Delta_r r''\Delta_c}{r'r'\Delta_c^2}.
\]

The denominator is obviously positive. With \( r'(c^*) > 0 \) concavity implies that \( r'(\Delta_c - \Delta_r) > 0 \). However, also due to concavity, we have \( \Delta_r r''\Delta_c < 0 \). Hence the numerator is ambiguous. It is positive if and only if

\[
r'' < \frac{-r'(\Delta_c - \Delta_r)}{\Delta_c \Delta_r},
\]

hence if \( r \) is sufficiently concave. In that case \( \delta \) is increasing in \( c^* \). With \( c^* \) decreasing in the number of firms, it implies that \( \delta \) is decreasing in the number of firms.

**Proof of Corollary 1.** First note that we look at equally sized firms throughout, so for each firm \( \alpha = 1/n \). We consider the three families of functions in turn.

(a) Suppose \( r(c) = \sqrt{c} - bc \). Note that \( r'(c) = \frac{1}{2\sqrt{c}} - b \) and \( r''(c) = -\frac{1}{4}c^{-3/2} < 0 \). Hence the function is concave. To find \( c^* \), equate \( r'(c) \) to \( 1 - \alpha \) to find

\[
c^* = \frac{1}{4(1 + b - \alpha)^2}.
\]

To find \( c^m \), we simply set \( \alpha = 1 \):

\[
c^m = \frac{1}{4b^2}.
\]
We also have

\[ r(c^m) = r \left( \frac{1}{4b^2} \right) = \frac{1}{2b} - \frac{1}{4b} = \frac{1}{4b} \]

\[ r(c^*) = \frac{1}{2(1 + b - \alpha)} - \frac{b}{4(1 + b - \alpha)^2} = \frac{1}{4} \left( 2 + b - 2\alpha \right) \]

Now

\[ \hat{\delta} = \frac{1 - r(c^m) - r(c^*)}{(1 - \alpha)(c^m - c^*)} = 1 - \frac{\frac{1}{4b} - \frac{1}{4} \frac{2 + b - 2\alpha}{(1 + b - \alpha)^2}}{(1 - \alpha)\left( \frac{1}{4b^2} - \frac{1}{4(1 + b - \alpha)^2} \right)} \]

\[ = 1 - \frac{b}{1 + 2b - \alpha}. \]

This critical discount factor is decreasing in \( \alpha \), hence increasing in \( n \).

(b) Suppose \( r(c) = bc - ac^2 \), with \( a, b > 0 \). Note that \( r'(c) = b - 2ac \) and \( r''(c) = -2a \).

Hence the function is concave. To find \( c^* \), equate \( r'(c) \) to \( 1 - \alpha \):

\[ c^* = \frac{b + \alpha - 1}{2a}. \]

To find \( c^m \), we simply set \( \alpha = 1 \):

\[ c^m = \frac{b}{2a}. \]

We thus have

\[ r(c^m) = \frac{b^2}{4a} \]

\[ r(c^*) = \frac{b^2 - (1 - \alpha)^2}{4a}. \]

Now

\[ \hat{\delta} = 1 - \frac{r(c^m) - r(c^*)}{(1 - \alpha)(c^m - c^*)} = 1 - \frac{\frac{b^2}{4a} - \frac{b^2 - (1 - \alpha)^2}{4a}}{(1 - \alpha)\left( \frac{b}{2a} - \frac{b + \alpha - 1}{2a} \right)} = \frac{1}{2}. \]

Hence, this is affected neither by \( \alpha \) nor by \( n \).

(c) Suppose \( r(c) = bc - e^c \), with \( b > 2 \). Note that \( r'(c) = b - e^c \) and \( r''(c) = -e^c < 0 \).

Hence the function is concave. To find \( c^* \), equate \( r'(c) \) to \( 1 - \alpha \):

\[ c^* = \ln (b + \alpha - 1). \]

To find \( c^m \), we simply set \( \alpha = 1 \):

\[ c^m = \ln b. \]
Note that the restriction $b > 2$ assures that both $c^*$ and $c^m$ are strictly positive. We thus have

\[
\begin{align*}
    r(c^m) &= b \ln b - b \\
    r(c^*) &= b \ln (b + \alpha - 1) - (b + \alpha - 1)
\end{align*}
\]

Now

\[
\begin{align*}
    \hat{\delta} &= 1 - \frac{r(c^m) - r(c^*)}{(1 - \alpha)(c^m - c^*)} \\
    &= 1 - \frac{b \ln b - b - (b \ln (b + \alpha - 1) - (b + \alpha - 1))}{(1 - \alpha)(\ln b - \ln (b + \alpha - 1))} \\
    &= 1 - \frac{b \ln \frac{b}{b + \alpha - 1} + (\alpha - 1)}{(1 - \alpha)\left(\ln \frac{b}{b + \alpha - 1}\right)} = 1 - \frac{b}{1 - \alpha} + \frac{1}{\ln \frac{b}{b + \alpha - 1}}
\end{align*}
\]

so

\[
\frac{\partial \hat{\delta}}{\partial \alpha} = -\frac{b}{(1 - \alpha)^2} + \frac{1}{\left(\ln \frac{b}{b + \alpha - 1}\right)^2 (b + \alpha - 1)}
\]

This is positive if

\[
\frac{b}{(1 - \alpha)^2} < \frac{1}{\left(\ln \frac{b}{b + \alpha - 1}\right)^2 (b + \alpha - 1)}
\]

hence

\[
b\left(\ln \frac{b}{b + \alpha - 1}\right)^2 (b + \alpha - 1) < (1 - \alpha)^2.
\]

Numerically, we can show that this is always the case. Therefore this critical discount factor is increasing in $\alpha$, hence decreasing in $n$.

**Proof of Lemma 2.** Note that, from (11), we can write

\[
\hat{\delta}(\alpha_i) = \frac{(1 - \alpha_i)(c^m - c_i^*) + r(c_i^*) - r(c^m)}{(1 - \alpha_i)c^m + \alpha_ic_i^* - p^*},
\]

with $p^* \equiv \sum_j \alpha_j c_j^*$ the competitive price. Take any two firms, $j$ and $k$, with $\alpha_j > \alpha_k$. Consider $\hat{\delta}(\alpha_j) - \hat{\delta}(\alpha_k)$, the difference between the critical discount factor of firm $j$ and that of firm $k$. For the lemma to hold, we need this difference to be strictly negative. Using (11), we can write

\[
\hat{\delta}(\alpha_j) - \hat{\delta}(\alpha_k) = \frac{1}{2} \int_{\alpha_k}^{\alpha_j} \left[ \frac{\partial \hat{\delta}(\alpha)}{\partial \alpha} \right] d\alpha.
\]
It is sufficient that $\partial \hat{\delta}(\alpha)/\partial \alpha < 0$ for all relevant $\alpha$. Denote the numerator of (A.5) as $\text{num}$, and the denominator as $\text{den}$. Hence $\partial \hat{\delta}(\alpha)/\partial \alpha < 0$ if

$$\text{den} \cdot \text{num}' - \text{num} \cdot \text{den}' < 0.$$  

(A.7)

If the perfect symmetric collusive agreement is stable, we necessarily have $\text{num} < \text{den}$ and $\text{num}', \text{den} > 0$. Furthermore, note that

$$\text{num}' = -(c^m - c_i^*) - (1 - \alpha_i)c_i^{\ast'} + r \cdot c_i^{\ast'} = -(c^m - c_i^*)$$
$$\text{den}' = -(c^m - c_i^*) + \alpha_i c_i^{\ast'}.$$

Hence $\text{num}' < 0$, and $\text{den}' > \text{num}'$. Combined with the fact that $\text{num} < \text{den}$, this necessarily implies that (A.7) is satisfied. This establishes the result.

**Proof of Theorem 3.** Consider a move from industry structure $A$ to industry structure $B$, where $B$ is more heterogeneous. Without loss of generality, $\alpha_i^B = \alpha_i^A - \Delta$ and $\alpha_k^B = \alpha_k^A + \Delta$, for some $\Delta > 0$ and $\alpha_i^A \leq \alpha_k^A$, whereas $\alpha_j^A = \alpha_j^B$ for all $j \neq i, k$. Denote the smallest market share of all other firms as $\alpha$, so $\alpha = \min_{j \neq i, k}\{\alpha_j\}$. From Lemma 2 the smallest firm determines the critical discount factor. We have three possibilities to consider: first, $\alpha_i^B > \alpha$; second, $\alpha_i^A < \alpha$ and third $\alpha_i^B < \alpha < \alpha_i^A$.

In case I, $\alpha_i^B > \alpha$, the critical discount factor under industry structure $\kappa \in \{A, B\}$ is given by

$$\hat{\delta}^\kappa \equiv \hat{\delta}\left(\alpha^\kappa\right) \equiv \frac{(1 - \alpha)(c^m - c^*(\alpha)) + r(c^*(\alpha)) - r(c^m)}{(1 - \alpha)c^m + \alpha c^*(\alpha) - p^*_\kappa},$$  

(A.8)

with $p^*_\kappa$ the competitive equilibrium price in market structure $\kappa$. Note that $c^*(\alpha)$ is not affected by the distribution of market shares over other firms. As market structure $B$ is more heterogeneous than market structure $A$, we have $p^*_B > p^*_A$, hence $\hat{\delta}^B > \hat{\delta}^A$.

In case II, $\alpha_i^A < \alpha$. Hence, the critical discount factor under industry structure $\kappa$ is

$$\hat{\delta}^\kappa \equiv \hat{\delta}(\alpha^\kappa) \equiv \frac{(1 - \alpha^\kappa)(c^m - c^*(\alpha^\kappa)) + r(c^*(\alpha^\kappa)) - r(c^m)}{(1 - \alpha^\kappa)c^m + \alpha_i c^*(\alpha^\kappa) - p^*_\kappa}. $$  

(A.9)

From the proof of Lemma 2, for fixed $p^*_\kappa$, $\hat{\delta}_i(\alpha_i)$ is decreasing in $\alpha_i$. Hence, as $\alpha_i^B < \alpha_i^A$, for fixed $p^*_\kappa$ we have $\hat{\delta}^B > \hat{\delta}^A$. The fact that $p^*_B > p^*_A$ only strengthens this result.

The proof for case III, where $\alpha_i^B < \alpha < \alpha_i^A$ follows from a combination of the above cases; first consider a decrease from $\alpha_i^A$ to $\alpha$. From the analysis for case I, this leads to an increase in $\hat{\delta}$. Next consider a decrease from $\alpha$ to $\alpha_i^B$. From the analysis for case II, this again leads to an increase in $\hat{\delta}$.

**Supplementary material**

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ijindorg.2016.10.001.
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