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INFORMATION AND ENDOGENOUS DELEGATION IN A RENT-SEEKING CONTEST

LAMBERT SCHOONBEEK*

We offer a new explanation for the occurrence of delegation in rent-seeking contests. We consider a two-player contest for a prize of common value. The players only know that the prize is high or low, with given probabilities. Each player can hire a delegate to act on his behalf. After a delegate is hired, she privately observes the true value of the prize. We derive the conditions under which, respectively, no player, only one player, or both players delegate in equilibrium. (JEL D7)

I. INTRODUCTION

We consider a rent-seeking contest where players can compete for a single prize by exerting nonrefundable efforts. The contest has been introduced in a seminal study by Tullock (1980). It has many applications, including litigation contests, lobbying by firms for a monopoly position, research and development patent races between firms, and sport competitions. Nitzan (1994), Lockard and Tullock (2001), Congleton, Hillman, and Konrad (2008), and Konrad (2009) give overviews of the rent-seeking literature. We investigate the case where each player has the option to hire a delegate who competes on his behalf. We show that delegation can arise in equilibrium if the delegates have better information about the value of the prize than the players. This offers a new explanation for the occurrence of delegation in rent-seeking contests. Examples of delegation abound in practice: parties hire lawyers to win lawsuits, firms hire lobbyists to acquire monopoly rents from the government, etc. It is thus interesting to explain why delegation can happen endogenously.

In the standard contest, the value of the prize is given and known. However, we examine the case where it is uncertain. We investigate a two-player contest for a prize of common value, which is either high or low with given probabilities. The

players are called uninformed, since they only know the prior distribution of the value of the prize. We assume that each (risk-neutral) player can hire a (risk-neutral) delegate who will act on his behalf in the contest. If the player does not hire a delegate, he competes himself. The delegates are informed, that is, if a delegate is hired, she privately learns the realized value of the prize before the start of the actual contest. She can use this information while choosing her effort in the contest. There is moral hazard, since the player does not observe the effort exerted by his delegate, he only observes the contest outcome. We further assume that a delegate is protected by limited liability, she does not have to transfer money to the player in case she does not win the contest. In this setup, the optimal (incentive) contract offered by the player to his delegate is a contingent fee contract, cf. Wärneryd (2000) and Baik (2007, 2008). According to the contract, the delegate receives a deliberately chosen fraction of the realized prize if she is successful, and nothing otherwise. Under the optimal contract, the fractions offered may differ for the two values of the prize. It turns out that we cannot find a closed form solution of our game if players offer the optimal contracts. Therefore, in order to obtain analytical results, we focus on the more simple case where the fractions must be identical for both values of the prize. We call this a uniform-fraction contingent fee contract. Notice that under this more restrictive type of contract, the absolute amount of money received by a successful

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ABBREVIATION

FOCs: First-Order Conditions

delegate still depends on the value of the realized prize. As a result, the effort exerted by a delegate differs for both values of the prize.

A player's decision whether or not to delegate depends on the intricate interplay of a number of effects. First, it depends on the delegation decision of the opponent. Second, if the player competes himself, his *ex ante* payoff is given by his expected revenues minus his nonrefundable cost of effort. He factors in the realization of both values of the prize while determining his optimal effort. He further takes into account that he may keep the entire prize if he wins the contest. Third, if a player delegates, his *ex ante* payoff is given by his expected revenues under delegation. He has no cost of effort now, since the effort will be exerted by the delegate. The delegate can tailor her effort to the realized value of the prize since she is informed about it. The delegate factors in that she will only receive a fraction of the prize herself in the event of winning the contest. This will make her less aggressive in the contest than the player would be (if he would be informed and compete himself). The player also takes into account that he has to share the prize with his delegate in case she is successful.

Our main result characterizes the conditions under which the game has either a unique (pure-strategy subgame-perfect) equilibrium where no player delegates, or two asymmetric equilibria wherein only one of the players delegates, or a unique equilibrium where both players delegate. It turns out that the game has equilibria with endogenous delegation if and only if both the probability of the high prize is small enough and the ratio of the high and low value of the prize is large enough. We identify the effects of delegation which are responsible for this result. We briefly examine the case with the optimal contracts numerically. We find that the equilibrium results for that case are very similar to those derived for the uniform-fraction case.

Our analysis applies to a court case where two parties make a claim on a complicated legacy of which they do not know the exact value. Each party might then decide to hire a legacy lawyer who is able to determine the true value of the legacy and can act on the party's behalf in court. As another example, consider two firms lobbying a local government for a license to be awarded to one of them. The license gives the winning firm a monopoly position on a given market, say for local bus transport. It then might happen that the firms do not know the exact value of the monopoly position since they are uncertain if the

government will introduce new regulation (e.g., regarding the number of routes or timetable) affecting this value. Each firm might then decide to hire a professional lobbyist who is able to assess whether the government will introduce the new regulation (the lobbyist might have direct access to important decision makers within the government and receive private information from them) and can lobby on the firm's behalf. In each case we thus assume that the players involved know that the relevant delegates are able to learn the true value of the prize, given their specific expertise, access to better sources of information, etc.

Our paper is organized as follows. Section II discusses related literature. Section III presents our game. Section IV gives preliminary results and Section V the equilibrium results. We conclude in Section VI. Proofs and technical details are in the Appendix.

II. RELATED LITERATURE

A number of papers have studied delegation in two-player rent-seeking contests, assuming limited liability for the delegates and moral hazard due to unobservable efforts of the delegates. However, in those studies we have complete information about the value of the prize. In a pioneering paper, Baik and Kim (1997) analyze the case with independent valuations of the prize (i.e., each player might attach a different value to winning the prize), where a delegate has a larger ability than the player himself (i.e., the delegate's effort has a larger effect on the probability of winning the prize than the same effort would have if exerted by the player himself). Baik and Kim endogenize the decision to hire a delegate. However, they assume that the payment scheme offered to the delegates is exogenously given. In particular, a delegate receives an exogenous contingent fee, plus a given fixed fee (depending on the delegate's ability) that is paid regardless of the outcome of the contest. Baik (2007, 2008) investigates the case where delegation is compulsory, and players and delegates have identical abilities. He studies in detail the properties of the optimal contracts offered to the delegates.¹ Similar to us, Wärneryd (2000) examines the case where the prize has a common value for both players, and players and delegates have the same abilities. Assuming that delegates are

1. Baik and Lee (2013) study compulsory delegation and endogenous timing in a rent-seeking contest.

offered optimal contracts, he shows that no delegation is a dominant strategy for each player, hence delegation is *not* an equilibrium. In fact, the situation is akin to a prisoners' dilemma, because each player prefers the outcome where both delegate over no delegation at all. The reason is that if delegates are hired, then the delegates' efforts are smaller than the efforts exerted by the players if they would compete themselves. Recall that delegates have a weaker incentive to exert effort than the players, since they only receive a fraction of the prize (rather than the full prize) if they are successful. Focusing on a common-value contest and optimal contracts, Schoonbeek (2002) shows that one-sided endogenous delegation occurs if the two players have different risk-attitudes (in equilibrium, a risk-averse player might decide to hire a risk-neutral delegate), while Schoonbeek (2007) demonstrates that one-sided or two-sided endogenous delegation can arise if delegates have two instruments at their disposal while the players can use only one instrument. Our paper offers an alternative interesting explanation of endogenous delegation.²

A number of papers have studied rent-seeking contests with random valuations of the prize and private information. However, those studies do not include the option of delegation. Hurley and Shogren (1998a) take a two-player contest with independent valuations and analyze the case where one player has private information about his own valuation. Hurley and Shogren (1998b) numerically analyze a similar case with two-sided private information. Malueg and Yates (2004) study a two-player contest with independent valuations and two-sided private information analytically, assuming that the valuations are high or low, while imposing a simple structure on the probabilities associated with these valuations. Similar to us, Wärneryd (2003) considers a two-player common-value contest. The distribution of the value of the prize is continuous. Wärneryd investigates the case where

one (uninformed) player only knows the prior distribution of the value of the prize, while the other (informed) player knows its true value. He demonstrates that in equilibrium the *uninformed* player has a strictly larger ex ante probability of winning the contest than the informed player. On the other hand, the expected payoff is strictly largest for the informed player.³ Wärneryd also examines the cases with symmetric information, where both players are either informed or uninformed about the value of the prize. He shows that the aggregate equilibrium effort is strictly lower under asymmetric information than under symmetric information. Einy et al. (2013) find similar results for a contest where the value of the prize is obtained from a discrete distribution with a finite number of possible values.⁴ Finally, Denter, Morgan, and Sisak (2014) study a two-player rent-seeking contest without the option of delegation, and with independent valuations of the prize. Each valuation can be high or low with given probabilities. The valuation of the first player is common knowledge, the second player is privately informed about his own valuation. The authors investigate the incentives of the first (second) player to strategically acquire (disclose) information about the second player's valuation.

III. THE GAME

We consider a Tullock contest where risk-neutral players 1 and 2 (principals) can compete by exerting nonrefundable efforts for a single prize of common value that will be awarded to one of them. It is common knowledge that the value of the prize is V_H (high) with probability q and V_L (low) with probability $1 - q$, with $V_H > V_L > 0$ and $0 < q < 1$ given. The game has three stages. In stage 1, player $i = 1, 2$ can decide to hire a risk-neutral delegate i who will compete on his behalf in the contest (for simplicity, we

2. There is also a literature on more general delegation problems. For example, Alonso and Matouschek (2008) examine a model where a principal is unable to commit to contingent monetary payments to his delegate, who has private information about the state of the world. In that case, the principal's optimal decision rule specifies what decisions the delegate should and should not be allowed to make. Prendergast (2002) studies the trade-off between risk and incentives in a model where a delegate has private information about the performance of a firm. He shows that the principal rewards the delegate based on (monitored) assigned tasks in certain economic environments. When the situation is more uncertain, the principal delegates responsibility to the delegate and offers output-based contracts.

3. Wärneryd (2013) considers a related two-player common-value all-pay auction (or perfectly discriminatory contest), where the value of the prize is distributed continuously. He shows that in that case, the uninformed and informed player win with equal ex ante probability in equilibrium, while the expected payoff of the informed player is strictly largest. Einy et al. (2016) study a two-player common-value all-pay auction, where the value of the prize is drawn from a distribution with a finite number of possible values and no player has a general information advantage over the opponent. They show that then players do not always have the same ex ante probability of winning.

4. Other related studies are Schoonbeek and Winkel (2006), Fey (2008), Ryvkin (2010), Wärneryd (2012), Wasser (2013), and Einy, Moreno, and Shitovitz (2014).

do not take into account that if player i hires a delegate, he saves time to do something else). In stage 2, Nature draws the true value of the prize from the prior distribution. At the end of stage 2, each delegate (if hired) is privately informed about the realized value, players 1 and 2 do not receive this information (they only know the prior distribution). We therefore call the delegates informed and players uninformed. We say that delegate i has type t if the prize has value V_t ($t = H, L$). In stage 3, the actual contest takes place. We have a contest between players 1 and 2 if neither of them has hired a delegate. If player i has delegated, he will be replaced by delegate i . At the end of stage 3, the player who has won the contest, or whose delegate has won the contest, learns the true and verifiable value of the prize and payoffs are realized.⁵

Next, consider the contract offered to the delegates in stage 1. We suppose that player i does not observe the effort put forward by delegate i (if hired) in stage 3, he only learns whether the contest is won or lost by her ($i = 1, 2$). Further, delegate i will operate under limited liability, that is, she does not have to compensate player i if she loses the contest. The payoff of the delegate is zero if she rejects the contract. The contract offered by player i specifies the payments received by delegate i if she, respectively, wins or loses the contest. Using Baik (2007, 2008), we see that the *optimal* contract is a contingent fee contract such that delegate i of type t will receive a fraction w_{it} ($0 \leq w_{it} \leq 1$) of the prize V_t if she wins the contest and nothing else ($i = 1, 2$; $t = H, L$).⁶ Player i determines the size of w_{it} . The contract offered to delegate i can be observed by her opponent in the contest. As mentioned in Section I, we are not able to find the equilibrium of our game in closed form if players use the

optimal contracts. Therefore, in order to obtain analytical results, we focus on the more simple case where the fractions of the prize offered to delegate i must be the same regardless of her type, that is, we impose $w_{it} = w_i$ ($i = 1, 2$; $t = H, L$). We call this a *uniform-fraction contingent fee* contract. Notice that, although the fraction w_i is the same for both values of the prize, the absolute amount of money, $w_i V_t$, received by a successful delegate i depends on her type. We examine the case where players can offer the optimal contracts numerically. It turns out that the equilibrium results under the optimal and uniform-fraction contracts are very similar.

The contest winner in stage 3 is determined by using the Tullock success function. If players 1 and 2 compete themselves, player i wins with probability $e_i/(e_i + e_j)$, where $e_i \geq 0$ denotes player i 's effort ($i, j = 1, 2$; $i \neq j$). If player i hires a delegate but player j does not delegate, delegate i of type t wins the contest with probability $y_{it}/(y_{it} + e_j)$, where $y_{it} \geq 0$ is the delegate's effort. If both players delegate, delegate i of type t wins the contest with probability $y_{it}/(y_{it} + y_{jt})$ ($i, j = 1, 2$; $i \neq j$; $t = H, L$).⁷

We apply backward induction in order to find the (pure-strategy subgame-perfect) equilibria. It is convenient to write $V_H = \mu V_L$, with $\mu > 1$ the ratio of the high and low value of the prize, and to analyze equilibrium values as a function of μ , given $q \in (0, 1)$.

IV. PRELIMINARY RESULTS

Consider the game with uniform-fraction contingent fee contracts. Take stage 3, given that players 1 and 2 have decided to compete themselves in stage 1. Player i solves

$$(1) \quad \max_{e_i} = \left(\frac{e_i}{e_i + e_j} \right) \mathbb{E} [V(\mu)] - e_i,$$

where $\mathbb{E} [V(\mu)] \equiv (q\mu + 1 - q) V_L$ and $i, j = 1, 2$ ($i \neq j$). Denoting equilibrium values (given the players' delegation decisions of stage 1) with a hat, the first-order conditions (FOCs) give $\hat{e}_i^{nn}(\mu) = \mathbb{E} [V(\mu)] / 4$, where index "nn" indicates that both players do not delegate.⁸ So, rent dissipation (the ex ante aggregate effort) equals

7. In all situations, if both participants in the contest exert zero effort, each one will win the prize with probability one half. From now on we disregard these cases, since they will not happen in equilibrium.

8. The second-order conditions are fulfilled in all cases discussed in the paper.

5. In the first example discussed in Section I, this player thus learns the actual value of the legacy after the court's decision in stage 3. In the second example, the relevant firm then knows with certainty whether the government introduces new bus transport regulation.

6. In fact, Baik (2007, 2008) investigates a contest with complete information about the value of the prize. Given limited liability, he shows that the optimal payment is zero if the delegate loses the contest (cf. Wärneryd 2000). By doing so, the player most strongly motivates his delegate to win the contest. It easily follows that this result applies to our game as well. Baik further points out that the participation constraint of a delegate is nonbinding if her reservation payoff is zero, or positive and small enough. The same holds in our game. For simplicity, we assume that the reservation payoff is zero. Finally, we notice that Baik (2007, 2008) also analyzes the case where the delegate's reservation payoff is so high, that her participation constraint becomes binding. We disregard that case here for simplicity.

$\hat{R}^{nn}(\mu) \equiv \mathbb{E}[V(\mu)]/2$. Player i wins the contest with probability $\hat{p}_i^{nn} = \frac{1}{2}$, for each value of the prize. The ex ante payoff of player $i = 1, 2$ is

$$(2) \quad \hat{\Pi}_i^{nn}(\mu) = \frac{\mathbb{E}[V(\mu)]}{4}.$$

Next, examine stage 3 in case both players have hired a delegate in stage 1. Given $w_i > 0$ and $w_j > 0$, delegate i of type t considers

$$(3) \quad \max_{y_{it}} \left(\frac{y_{it}}{y_{it} + y_{jt}} \right) w_i V_t - y_{it},$$

with $i, j = 1, 2$ ($i \neq j$) and $t = H, L$. We have $y_{it}^{dd} = w_j w_i^2 V_t / (w_i + w_j)^2$ in the equilibrium of stage 3, where index “ dd ” denotes that both players delegate. The probability that delegate i wins the contest is $p_i^{dd} = w_i / (w_i + w_j)$, regardless of her type. Using this, we turn to stage 1, where player $i = 1, 2$ solves

$$(4) \quad \max_{w_i} \left(\frac{w_i}{w_i + w_j} \right) (1 - w_i) \mathbb{E}[V(\mu)].$$

There are two opposing effects if player i increases w_i , given $w_j > 0$. First, the probability that delegate i of either type wins the contest increases (since she increases her effort), which benefits player i . On the other hand, if the contest is won by delegate i , the share of the prize accruing to player i will decrease, which hurts this player. At the optimal w_i , these two effects are in balance. Using the FOCs, we find that $\hat{w}_i^{dd} = \frac{1}{3}$ ($i = 1, 2$). Each delegate of either type wins with probability $\hat{p}_i^{dd} = \frac{1}{2}$ in equilibrium. The ex ante payoff of a delegate is positive, and the ex ante payoff of player $i = 1, 2$ is

$$(5) \quad \hat{\Pi}_i^{dd}(\mu) = \frac{\mathbb{E}[V(\mu)]}{3}.$$

Notice that $\hat{\Pi}_i^{dd}(\mu) > \hat{\Pi}_i^{nn}(\mu)$ and $\hat{R}^{dd}(\mu) < \hat{R}^{nn}(\mu)$ for $i = 1, 2$ and $\mu \in (1, \infty)$, where $\hat{R}^{dd}(\mu) \equiv \mathbb{E}[V(\mu)]/6$ is rent dissipation if both players delegate. Hence, each player benefits if both players delegate rather than compete themselves and rent dissipation is smaller in that case. This corresponds to the related result of Wärneryd (2000) for the contest with complete information (see Section II).

Next, we investigate stage 3 in case only one player delegates. Without loss of generality, we suppose that player 1 has hired a delegate, while

player 2 competes himself. Given $w_1 > 0$, delegate 1 of type $t = H, L$ considers

$$(6) \quad \max_{y_{1t}} \left(\frac{y_{1t}}{y_{1t} + e_2} \right) w_1 V_t - y_{1t},$$

and player 2 solves

$$(7) \quad \max_{e_2} q \left(\frac{e_2 \mu V_L}{y_{1H} + e_2} - e_2 \right) + (1 - q) \left(\frac{e_2 V_L}{y_{1L} + e_2} - e_2 \right).$$

Similar to Wärneryd (2003), we have to distinguish two cases for the corresponding equilibrium: case (a) where both types of delegate 1 are active, that is, exert a positive effort, and case (b) where the high-type delegate 1 is active but the low-type is inactive.

In case (a), the equilibrium efforts of stage 3 and the probability that delegate 1 of type t wins the contest are

$$(8) \quad y_{1t}^a(\mu) = \frac{w_1 \mathbb{E}[\sqrt{V(\mu)}]}{(1 + w_1)^2} \times \left((1 + w_1) \sqrt{V_t} - \mathbb{E}[\sqrt{V(\mu)}] \right),$$

$$(9) \quad e_2^a(\mu) = w_1 \left(\frac{\mathbb{E}[\sqrt{V(\mu)}]}{1 + w_1} \right)^2,$$

$$(10) \quad p_{1t}^a(\mu) = 1 - \frac{\mathbb{E}[\sqrt{V(\mu)}]}{(1 + w_1) \sqrt{V_t}},$$

where $\mathbb{E}[\sqrt{V(\mu)}] \equiv (q\sqrt{\mu} + 1 - q)\sqrt{V_L}$, and $t = H, L$. In turn, player 1 considers the following problem in stage 1:

$$(11) \quad \max_{w_1} q \times p_{1H}^a(\mu) \times (1 - w_1) \times \mu V_L + (1 - q) \times p_{1L}^a(\mu) \times (1 - w_1) \times V_L.$$

Player 1 faces again a trade-off in setting w_1 . An increase of w_1 increases the probability that delegate 1 of type t wins the contest, but decreases the share of the prize received by player 1 if his delegate is successful. Using the FOC, we find the equilibrium value

$$(12) \quad \hat{w}_1^a(\mu) = \frac{\sqrt{2} \mathbb{E}[\sqrt{V(\mu)}]}{\sqrt{\mathbb{E}[V(\mu)]}} - 1.$$

Substituting $\hat{w}_1^a(\mu)$, we obtain (with $t = H, L$)

$$(13) \quad \hat{y}_{1t}^a(\mu) = \left(\sqrt{\frac{\mathbb{E}[V(\mu)] V_t}{2}} - \frac{\mathbb{E}[V(\mu)]}{2} \right) \hat{w}_1^a(\mu),$$

$$(14) \quad \hat{e}_2^a(\mu) = \frac{\mathbb{E}[V(\mu)]}{2} \times \hat{w}_1^a(\mu),$$

$$(15) \quad \hat{p}_{1t}^a(\mu) = 1 - \sqrt{\frac{\mathbb{E}[V(\mu)]}{2V_t}}.$$

The best-response function associated with Equation (6) for $t = L$ yields that case (a) arises if and only if the equilibrium effort of player 2 is smaller than the reward of the low-type delegate 1 if she is successful, that is, $\hat{e}_2^a(\mu) < \hat{w}_1^a(\mu) V_L$. Using Equation (14), we see that case (a) is relevant if and only if $\mu \in (1, \bar{\mu}(q))$, where $\bar{\mu}(q) \equiv 1 + (1/q)$, given $q \in (0, 1)$. Hence, both types of delegate 1 are active if and only if the ratio of the high and low value of the prize is small enough, given the probability of the high prize. We have $\bar{\mu}(q) > 2$ for all $q \in (0, 1)$. Notice that $\bar{\mu}(q)$ is decreasing in q . In order to understand this, consider a pair q and $\mu = \bar{\mu}(q)$ (i.e., where the low-type delegate 1 is just inactive), and next (marginally) increase q . Then player 2's effort (relative to $\hat{w}_1^a(\mu) V_L$) will increase. Intuitively, player 2 will compete more fiercely in that case since he attaches a larger probability to the occurrence of the high prize. As a result, the low-type delegate 1 now becomes just inactive for a smaller value of μ (i.e., a smaller size of the high prize relative to the low prize), because that makes player 2 less aggressive again (since he takes into account the realization of both values of the prize). Hence, $\bar{\mu}(q)$ decreases.

We observe that $\hat{w}_1^a(\mu)$ is positive and decreasing in $\mu \in (1, \bar{\mu}(q))$. The latter can be understood as follows. First, $e_2^a(\mu)$ is increasing in μ , that is, player 2 competes more fiercely if the high prize relative to the low prize increases. Next, delegate 1 of type t is able to tailor her effort to the realized prize but also takes into account that the effort, $e_2^a(\mu)$, of player 2 is the same for both values of the prize. As a result, $p_{1H}^a(\mu)$ is increasing in μ , while $p_{1L}^a(\mu)$ is decreasing in μ . In order to compensate for the decrease of $p_{1L}^a(\mu)$, player 1 increases the share of the prize kept by himself if his delegate is successful, that is, w_1

decreases. Note also that $\hat{p}_{1H}^a(\mu)$ and $\hat{p}_{1L}^a(\mu)$ are, respectively, increasing and decreasing in μ .

The equilibrium efforts of the two types of delegate 1 differ increasingly if μ becomes larger, that is, the ratio $\hat{y}_{1H}^a(\mu)/\hat{y}_{1L}^a(\mu)$ is increasing in $\mu \in (1, \bar{\mu}(q))$. The ex ante probability that delegate 1 wins the contest is $\hat{p}_1^a(\mu) \equiv q \times \hat{p}_{1H}^a(\mu) + (1 - q) \times \hat{p}_{1L}^a(\mu) < \frac{1}{2}$. Hence, delegate 1 has a smaller winning probability than player 2. This corresponds to the finding of Wärneryd (2003), who shows that, in a contest without the option of delegation, but with one informed player and one uninformed player, the informed player has a smaller ex ante probability to win the contest than the uninformed player (see Section II).

Delegate 1's ex ante payoff is positive. The ex ante payoffs of players 1 and 2 are

$$(16) \quad \hat{\Pi}_1^a(\mu) = \left(\sqrt{2\mathbb{E}[V(\mu)]} - \mathbb{E}[\sqrt{V(\mu)}] \right)^2,$$

and

$$(17) \quad \hat{\Pi}_2^a(\mu) = \frac{\mathbb{E}[V(\mu)]}{2}.$$

Next, we turn to case (b), where the equilibrium effort of player 2 is so large that the low-type delegate 1 decides to remain inactive. Case (b) occurs if and only if $\mu \in [\bar{\mu}(q), \infty)$, given $q \in (0, 1)$. The efforts in stage 3 are $y_{1H}^b(\mu) = q\mu V_L w_1^2 / (q + w_1)^2$, $y_{1L}^b = 0$, and $e_2^b(\mu) = w_1 \mu V_L q^2 / (q + w_1)^2$. The high-type delegate 1 wins the contest with probability $p_{1H}^b = w_1 / (q + w_1)$, while the low-type is successful with probability $p_{1L}^b = 0$. In turn, player 1 solves

$$(18) \quad \max_{w_1} q \times p_{1H}^b \times (1 - w_1) \times \mu V_L.$$

Since p_{1H}^b is increasing in w_1 , player 1 faces again a trade-off in setting w_1 . We find

$$(19) \quad \hat{w}_1^b = -q + \sqrt{q(q+1)},$$

which is independent of μ . The reason for the latter is that an increase of μ does not affect p_{1H}^b and p_{1L}^b , and player 1 thus does not have to adjust his contingent fee in that case, contrary to case (a). Substituting $\hat{w}_1^b$, we find

$$(20) \quad \hat{y}_{1H}^b(\mu) = \frac{(\hat{w}_1^b)^2 \mu V_L}{q + 1},$$

$$(21) \quad \hat{e}_2^b(\mu) = \frac{\hat{w}_1^b q \mu V_L}{q+1},$$

$$(22) \quad \hat{p}_{1H}^b = 1 - \sqrt{\frac{q}{q+1}}.$$

Clearly, $\hat{y}_{1L}^b = y_{1L}^b = 0$ and $\hat{p}_{1L}^b = p_{1L}^b = 0$. Both $\hat{p}_{1H}^b$ and $\hat{p}_{1L}^b$ are independent of μ . Similar to case (a), delegate 1 has a smaller ex ante winning probability than player 2 in case (b), that is, $\hat{p}_1^b(\mu) \equiv q \times \hat{p}_{1H}^b(\mu) + (1-q) \times \hat{p}_{1L}^b < \frac{1}{2}$. The ex ante payoff of delegate 1 is positive. The ex ante payoffs of players 1 and 2 are

$$(23) \quad \hat{\Pi}_1^b(\mu) = \left(\sqrt{q(q+1)} - q \right)^2 \mu V_L,$$

and

$$(24) \quad \hat{\Pi}_2^b(\mu) = \left(\frac{q^2 \mu}{q+1} + 1 - q \right) V_L.$$

Combining cases (a) and (b), we define $\hat{\Pi}_1^{dn}(\mu) = \hat{\Pi}_1^a(\mu)$ if $\mu \in (1, \bar{\mu}(q))$, and $\hat{\Pi}_1^{dn}(\mu) = \hat{\Pi}_1^b(\mu)$ if $\mu \in [\bar{\mu}(q), \infty)$. Here “dn” means that player 1 delegates and player 2 does not delegate. Similarly, we define $\hat{\Pi}_2^{dn}(\mu)$, $\hat{w}_{1t}^{dn}(\mu)$, $\hat{y}_{1t}^{dn}(\mu)$, $\hat{e}_2^{dn}(\mu)$, and $\hat{p}_{1t}^{dn}(\mu)$ ($t=H, L$). Taking the case where only player 2 delegates, we immediately see that the ex ante payoffs of the two players are $\hat{\Pi}_1^{nd}(\mu) = \hat{\Pi}_2^{dn}(\mu)$ and $\hat{\Pi}_2^{nd}(\mu) = \hat{\Pi}_1^{dn}(\mu)$, where “nd” has the obvious meaning and $\mu \in (1, \infty)$. We have $\hat{e}_1^{nd}(\mu) = \hat{e}_2^{dn}(\mu)$ and $\hat{p}_{1t}^{nd}(\mu) = 1 - \hat{p}_{1t}^{dn}(\mu)$. All equilibrium values are continuous in $q \in (0, 1)$ and $\mu \in (1, \infty)$.

In the next section, we investigate under which conditions we have delegation in the equilibrium of the full game. Table 1 gives the relevant payoff matrix. We assume that a player hires a delegate if and only if his ex ante payoff is strictly larger in that case. For later use, it is useful to elaborate on a player’s incentive to delegate, given the delegation decision of his opponent. Without loss of generality, we focus on the incentive to delegate of player 1. We call $\widehat{VN}_1(\mu) \equiv \hat{\Pi}_1^{dn}(\mu) - \hat{\Pi}_1^{nn}(\mu)$ player 1’s value of delegation given that player 2 does not delegate for $\mu \in (1, \infty)$, given $q \in (0, 1)$. We decompose the value as

$$(25) \quad \widehat{VN}_1(\mu) = q \times \widehat{RN}_{1H}(\mu) + (1-q) \times \widehat{RN}_{1L}(\mu) + \hat{e}_1^{nn}(\mu),$$

TABLE 1
Payoff Matrix

		Player 2	
		No delegation	Delegation
Player 1	No delegation	$\hat{\Pi}_1^{nn}(\mu), \hat{\Pi}_2^{nn}(\mu)$	$\hat{\Pi}_1^{nd}(\mu), \hat{\Pi}_2^{nd}(\mu)$
	Delegation	$\hat{\Pi}_1^{dn}(\mu), \hat{\Pi}_2^{dn}(\mu)$	$\hat{\Pi}_1^{dd}(\mu), \hat{\Pi}_2^{dd}(\mu)$

Note: $\hat{\Pi}_i^{nn}(\mu)$ and $\hat{\Pi}_i^{dd}(\mu)$ denote player i ’s payoff if both players do not delegate and delegate, respectively; $\hat{\Pi}_i^{dn}(\mu)$ denotes player i ’s payoff if player 1 delegates and player 2 does not delegate; $\hat{\Pi}_i^{nd}(\mu)$ denotes player i ’s payoff if player 1 does not delegate and player 2 delegates ($i=1,2$).

where

$$(26) \quad \widehat{RN}_{1t}(\mu) = (\hat{p}_{1t}^{dn}(\mu) \times (1 - \hat{w}_1^{dn}(\mu)) \times V_t) - (\hat{p}_1^{nn} \times V_t),$$

is the corresponding revenue effect of delegation conditional on the realization of V_t ($t=H, L$), and $\hat{e}_1^{nn}(\mu)$ the cost of effort effect of delegation. For brevity, we call $\widehat{RN}_{1t}(\mu)$ the high-prize revenue effect if $t=H$ and low-prize revenue effect if $t=L$. The high-prize revenue effect measures the difference between the (expected) revenue of player 1 if he delegates and competes himself, respectively, given that the prize is high. The effect depends in turn on the probability that, respectively, the high-type delegate 1 and player 1 wins the contest in these cases, and the share of the prize player 1 then receives (player 1 receives only a share of the prize if he delegates, but the entire prize if he competes himself). We call the difference between $\hat{p}_{1H}^{dn}(\mu)$ and $\hat{p}_1^{nn}$ the probability of winning effect of delegation, and the difference between the share of the prize kept by player 1 in case of winning under delegation or no delegation the prize-share effect. The latter effect is negative. The high-prize revenue effect is positive if and only if the probability of winning effect is positive and offsets the negative prize-share effect. Notice that the low-prize revenue effect can be discussed similarly. The cost of effort effect of delegation measures the effort expenditures saved by player 1 if he delegates rather than competes himself. This effect is positive. We conclude that the value of delegation $\widehat{VN}_1(\mu)$ is positive if and only if the ex ante aggregate of the high-prize and low-prize revenue effects plus the cost of effort effect is positive.

In a similar way, player 1’s value of delegation given that player 2 delegates, $\widehat{VD}_1(\mu) \equiv$

$\hat{\Pi}_1^{dd}(\mu) - \hat{\Pi}_1^{nd}(\mu)$ for $\mu \in (1, \infty)$ given $q \in (0, 1)$, can be decomposed as

$$(27) \quad \widehat{VD}_1(\mu) = q \times \widehat{RD}_{1H}(\mu) + (1 - q) \times \widehat{RD}_{1L}(\mu) + \widehat{\mathcal{E}}_1^{nd}(\mu),$$

where

$$(28) \quad \widehat{RD}_{1t}(\mu) = (\hat{p}_1^{dd} \times (1 - \hat{w}_1^{dd}) \times V_t) - (\hat{p}_1^{nd}(\mu) \times V_t)$$

is the corresponding high-prize and low-prize revenue effect if $t=H$ and $t=L$, respectively, and $\widehat{\mathcal{E}}_1^{nd}(\mu)$ the cost of effort effect of delegation. Each revenue effect is affected by a corresponding probability of winning effect and (negative) prize-share effect.

V. DELEGATION IN EQUILIBRIUM

We now present our main result.

PROPOSITION 1. *Consider the game where players 1 and 2 have the option to hire a delegate using uniform-fraction contingent fee contracts, with $q \in (0, 1)$ and $\mu \in (1, \infty)$. We have the following in equilibrium:*

i. Take $q \in (0, \frac{1}{2})$. Then the game has a unique equilibrium, where both players do not delegate, if $\mu \in (1, \mu_1(q))$; the game has two equilibria, in which either only player 1 or only player 2 delegates, if $\mu \in (\mu_1(q), \mu_2(q))$; the game has a unique equilibrium, where both players delegate, if $\mu \in (\mu_2(q), \infty)$. Here $\mu_1(q)$ and $\mu_2(q)$ are uniquely defined, with $1 < \mu_1(q) < \bar{\mu}(q) < \mu_2(q)$ if $q \in (0, \frac{1}{2}(\sqrt{2} - 1))$, and $\bar{\mu}(q) \leq \mu_1(q) < \mu_2(q)$ if $q \in [\frac{1}{2}(\sqrt{2} - 1), \frac{1}{2})$.

ii. Take $q \in [\frac{1}{2}, \frac{9}{16})$. Then the game has a unique equilibrium, in which both players do not delegate, if $\mu \in (1, \mu_1(q))$; the game has two equilibria, in which either only player 1 or only player 2 delegates, if $\mu \in (\mu_1(q), \infty)$. Here $\mu_1(q) \geq \bar{\mu}(q)$ is uniquely defined.

iii. Take $q \in [\frac{9}{16}, 1)$. Then the game has a unique equilibrium, in which both players do not delegate.

Part (i) of Proposition 1 shows that if we take q small and consider increasing values of μ , then first no player hires a delegate, next only one

FIGURE 1

The case with uniform-fraction contingent fee contracts, $q \in (0, 1)$, $\mu \in (1, 30]$. In equilibrium, either player 1 or player 2 delegates in the region above the line $\mu_1(q)$ and below or on the line $\mu_2(q)$, both players delegate in the region above the line $\mu_2(q)$, and no player delegates otherwise. Further, $\bar{\mu}(q) = 1 + (1/q)$, and the dotted line $\hat{\mu}(q)$ is defined in the Appendix, part A.2.

player hires a delegate, and finally both players hire a delegate in equilibrium. Part (ii) gives a similar result for a small range of intermediate values of q . However, now the equilibrium where both players delegate no longer emerges. Part (iii) shows that for large values of q , none of the players delegates in equilibrium, regardless of the size of μ . Figure 1 illustrates the equilibrium thresholds $\mu_1(q)$ and $\mu_2(q)$ of Proposition 1. It also shows the line $\bar{\mu}(q)$. Recall that if only one player delegates, then in stage 3 of the game both types of the delegate are active for parameter combinations below that line, while only the high-type delegate is active otherwise.⁹

The Appendix presents two lemmas characterizing the players' incentives to delegate. Focusing again on the incentive to delegate of player 1, given the decision of player 2, the lemmas show that $\widehat{VN}_1(\mu) > 0$ if and only if $q \in (0, \frac{9}{16})$ and $\mu \in (\mu_1(q), \infty)$, while $\widehat{VD}_1(\mu) > 0$ if and only if $q \in (0, \frac{1}{2})$ and $\mu \in (\mu_2(q), \infty)$ ($\mu_1(q)$ and $\mu_2(q)$

9. As an example, take $q = \frac{1}{4}$. Then $\mu_1(\frac{1}{4}) = 5.69$, $\mu_2(\frac{1}{4}) = 15$, and $\bar{\mu}(\frac{1}{4}) = 5$. In equilibrium no player delegates if $\mu \in (1, 5.69]$, only one player delegates if $\mu \in (5.69, 15]$ (and the relevant high-type delegate is active while the low-type delegate is inactive), and both players delegate if $\mu \in (15, \infty)$.

of the lemmas are the same as in Proposition 1). Using this, Proposition 1 follows easily.

In order to understand the incentives to delegate intuitively, first take the boundary situation with $q = 1$ and $\mu \in (1, \infty)$. This situation coincides with the complete information case considered by Wärneryd (2000), with a prize equal to V_H . Applying decompositions Equations (25) and (27), we then see that the probability of winning effect of delegation is negative for player 1, given either decision of player 2. The reason is that the delegate of player 1 competes less fiercely in the contest than player 2 since she receives only a (relatively small) fraction of the prize if she is successful, whereas player 2 receives the full prize if he wins. Recalling that the prize-share effect is always negative as well, it follows that the revenue effect of delegation is negative, regardless of the decision of player 2. Moreover, in either case, the negative revenue effect dominates the positive cost of effort effect, hence the value of delegation is negative.¹⁰ This explains that no delegation is a dominant strategy for player 1 if $q = 1$ and $\mu \in (1, \infty)$, cf. Wärneryd (2000). We obtain identical conclusions in the boundary situation with $q \in (0, 1)$ and $\mu = 1$. From this and continuity of the ex ante payoffs as functions of q and μ , it is clear that in our game with private information we have $\widehat{VN}_1(\mu) < 0$ and $\widehat{VD}_1(\mu) < 0$ for parameter combinations sufficiently close to the vertical line $q = 1$ or the horizontal line $\mu = 1$ in Figure 1. In equilibrium no delegation takes place in these cases. The corresponding high-prize and low-prize revenue effects, and the probability of winning effects pertaining to them, are negative as well.

Next, take $q \in (0, 1)$ and $\mu \in (1, \infty)$.¹¹ The points with $\widehat{VN}_1(\mu) > 0$ are located in the area

10. Take $q = 1$ and $V_H = \mu \in (1, \infty)$ (i.e., normalize V_L to 1). Suppose that player 2 does not delegate. Then $\widehat{p}_1^{dn} = 1 - (1/\sqrt{2}) \approx 0.293 < \widehat{p}_1^{nn} = \frac{1}{2}$ and $\widehat{w}_1^{dn} = \sqrt{2} - 1$. The probability of winning and prize-share effects are negative. Further, $\widehat{RN}_1(\mu) = \left((\sqrt{2} - 1)^2 - \frac{1}{2} \right) \mu \approx -0.328\mu < 0$, while $\widehat{e}_1^{nn}(\mu) = \frac{1}{4}\mu$, so $\widehat{VN}_1(\mu) \approx -0.078\mu < 0$. Next, suppose that player 2 delegates. Then $\widehat{p}_1^{dd} = \frac{1}{2} < \widehat{p}_1^{nd} \approx 0.707$ and $\widehat{w}_1^{dd} = \frac{1}{3}$. Again the probability of winning and prize-share effects are negative. Further, $\widehat{RD}_1(\mu) = \left(\frac{1}{3} - (1/\sqrt{2}) \right) \mu \approx -0.374\mu < 0$ and $\widehat{e}_1^{nd}(\mu) = \frac{1}{6}\mu$, thus $\widehat{VD}_1(\mu) \approx -0.207\mu < 0$. We have omitted the index denoting the value of the prize in all variables, since it is not relevant now.

11. The Appendix, part A.2, presents technical details related to this and the next paragraph.

above the line $\mu_1(q)$ in Figure 1. Interestingly, the corresponding probability of winning effect associated with the high prize is positive for $q < \frac{1}{3}$ and μ large enough. Recall that a positive probability of winning effect does not arise in the game with complete information. The positive probability of winning effect, that is, $\widehat{p}_{1H}^{dn}(\mu) > \widehat{p}_1^{nn} (= \frac{1}{2})$, emerges in our game with private information for the following reason. If player 1 delegates, then the high-type delegate 1 is informed that the high prize has been realized and thus competes fiercely, whereas the effort of the uninformed player 2 is small, since he attaches a small probability to the occurrence of the high prize ($q < \frac{1}{3}$). As a result, $\widehat{p}_{1H}^{dn}(\mu)$ is relatively large in this case. On the other hand, if player 1 does not delegate, then both uninformed players compete in the contest and each one has probability one half of winning it. Note that by delegating, player 1 deliberately creates a situation with asymmetric information between the contestants in stage 3, where his delegate has better information than player 2. This cannot happen in the game with complete information. The positive probability of winning effect acts against the negative prize-share effect in determining the high-prize revenue effect. In fact, $\widehat{RN}_{1H}(\mu) > 0$ for points left of the line $q = \frac{1}{8}$ and above the dotted line $\widehat{\mu}(q)$ (which is defined in the Appendix, part A.2) in Figure 1. The positive probability of winning effect then offsets the negative prize-share effect. It can be verified that $\widehat{RN}_{1L}(\mu) < 0$ always.¹² Concluding, if $q < \frac{1}{8}$ and μ is large enough, then the occurrence of $\widehat{VN}_1(\mu) > 0$ is driven by both a positive high-prize revenue effect and positive cost of effort effect, which dominate the negative low-prize revenue effect. If $q \geq \frac{1}{8}$, then $\widehat{VN}_1(\mu) > 0$ is driven by a positive cost of effort effect only (the high-prize revenue effect is negative but small in absolute value in that case).

The occurrence of $\widehat{VD}_1(\mu) > 0$ can be understood similarly. For brevity we only mention here the main conclusions. We have

12. The probability of winning effect associated with $\widehat{RN}_{1L}(\mu)$ is negative, that is, $\widehat{p}_{1L}^{dn}(\mu) < \widehat{p}_1^{nn}$ always. Intuitively, if player 1 delegates, the effort of the low-type delegate 1 is small, since she knows that the low prize has been realized while player 2 factors in the occurrence of both prizes and competes more fiercely (the low-type delegate 1 is even inactive for $\mu \in [\widehat{\mu}(q), \infty)$). If player 1 does not delegate, both uninformed players compete and each one has probability one half of winning the contest.

$\widehat{VD}_1(\mu) > 0$ for points above the line $\mu_2(q)$ in Figure 1. A positive probability of winning effect associated with the high prize, that is, $\widehat{p}_1^{dd} \left(= \frac{1}{2} \right) > \widehat{p}_{1H}^{nd}(\mu) (= 1 - \widehat{p}_{1H}^{dn}(\mu))$, arises if $q < \frac{1}{3}$ and μ is large enough. The reason is that if player 1 delegates and the high prize is realized, then the high-type delegates 1 and 2 compete in the contest and each one has probability one half of winning it. On the other hand, if player 1 does not delegate, then his effort in the contest is small since he takes into account that the high prize will occur with a small probability ($q < \frac{1}{3}$), while the high-type delegate 2 is informed about the realization of the high prize and competes more fiercely. As a result, $\widehat{p}_{1H}^{nd}(\mu)$ is relatively small in this case. Notice that by delegating, player 1 now avoids a situation with asymmetric information in stage 3, where he has to compete against a better informed delegate of player 2. If $q < \frac{1}{8}$, then $\widehat{VD}_1(\mu) > 0$ is driven by both a positive high-prize revenue effect and positive cost of effort effect which dominate the negative low-prize revenue effect, otherwise it is driven by a positive cost of effort effect only.¹³

In sum, given the delegation decision of his opponent, a player has an incentive to delegate if and only if both q is small enough and μ is large enough. The high-prize revenue effect is then either positive, or negative but small in absolute value. As a consequence, the ex ante aggregate of the high-prize revenue effect, low-prize revenue effect and cost of effort effect of delegation is positive. Combining our observations, we obtain an intuitive understanding of Proposition 1 and Figure 1.^{14,15}

Proceeding, we briefly investigate the situation where the players are allowed to employ the optimal contracts. Recall that then the fraction of the prize received by a successful delegate may

13. The probability of winning effect associated with $\widehat{RD}_{1L}(\mu)$ is negative, that is, $\widehat{p}_{1L}^{dd} < \widehat{p}_{1L}^{nd}(\mu)$ always. If player 1 delegates, both low-type delegates compete and each one has probability one half of winning the contest. If player 1 does not delegate, he wins the contest with certainty since the low-type delegate 2 is inactive in this case.

14. Notice that $\widehat{e}_1^m(\mu) > \widehat{e}_1^n(\mu)$, that is, the cost of effort effect associated with $\widehat{VN}_1(\mu)$ is largest. Numerical simulations show that this largely explains that $\mu_1(q)$ is located below $\mu_2(q)$ in Figure 1.

15. Notice that the low-type delegate is usually inactive if delegation happens. Intuitively, when the prize spread (μ) is large, winning the low prize is not really interesting, and trying to activate the low-type delegate would just distort the contingent fee offered to the delegates away from its optimal value.

vary with the size of the prize. If both players do not delegate, or if both players delegate, then the players' ex ante payoffs under the optimal contracts are identical to those of Table 1 (if both players delegate under the optimal contracts, then $\widehat{w}_{it}^{dd} = \frac{1}{3}$, $i = 1, 2$, $t = H, L$). We are not able to find closed form solutions of the players' ex ante payoffs under the optimal contracts in case only one player delegates (in particular, when both types of the relevant delegate are active in that case). Therefore, we have derived these payoffs numerically. Using them, Figure 2 illustrates the equilibrium outcomes under the optimal contracts: both players do not delegate in the white region, either only player 1 or only player 2 delegates in the light gray region, and both players delegate in the dark gray region. The dotted line in Figure 2 is the counterpart of the line $\bar{\mu}(q)$ of Figure 1 (this line is also reproduced in Figure 2). If only one player delegates under the optimal contracts, then the corresponding high-type and low-type delegates are active in stage 3 for points below the dotted line, otherwise only the high-type delegate is active. We see in Figure 2 that in points where the low-type delegate is inactive under the optimal contracts, she is inactive under uniform-fraction contingent fee contracts as well. As a result, in these points the fraction of the prize received by a successful high-type delegate must be identical under both types of contracts. This implies that the lower boundary of the dark gray region in Figure 2 coincides with the line $\mu_2(q)$ in Figure 1. Furthermore, if $q \geq q'$, then the lower boundary of the light gray region in Figure 2 coincides with the line $\mu_1(q)$ in Figure 1. The dashed line in Figure 2 reproduces the line $\mu_1(q)$ of Figure 1 for $q < q'$. Combining, we see that the equilibrium delegation decisions under the optimal contracts are almost identical to those under the uniform-fraction contingent fee contracts. The only difference is that in the very small area of the light gray region below the dashed line in Figure 2, either player 1 or player 2 delegates in the equilibrium under the optimal contracts, whereas no player delegates under uniform-fraction contingent fee contracts. In all other cases the players' decisions whether or not to delegate are the same under both types of contracts.

Concluding this section, we present two remarks.¹⁶

Remark 1: We discuss three benchmarks for our game. First, suppose that both players are also

16. Proofs of the results mentioned in Remarks 1 and 2 are available upon request.

FIGURE 2

The case with optimal contracts, $q \in (0, 1)$, $\mu \in (1, 30]$. In equilibrium, either player 1 or player 2 delegates in the light gray region, both players delegate in the dark gray region, and no player delegates otherwise. If only one player delegates in the case with optimal contracts, then the corresponding high-type and low-type delegates are active in stage 3 for points below the dotted line, else only the high-type delegate is active (for $q \geq q''$ the dotted line falls together with the line $\bar{\mu}(q)$). The dashed line represents $\mu_1(q)$ of Figure 1 for $q < q'$. Further, $\bar{\mu}(q) = 1 + (1/q)$.

informed about the true value of the prize at the end of stage 2, while the rest of the game remains unchanged. Solving this game, while assuming that players offer optimal contracts to the delegates, we obtain that no delegation is a dominant strategy for each player. Hence, this benchmark game, where both players and delegates are informed, has a unique equilibrium, where both players do not delegate. Second, suppose that the delegates do not learn the true value of the prize at the end of stage 2, that is, both players and delegates are uninformed, while the rest of the game is unaltered. Using optimal contracts again, we find that no delegation is a dominant strategy for each player, so this game also has a unique equilibrium, with no delegation. Finally, we investigate the game where each player can only hire a delegate under a fixed-value contingent fee contract. In that case, delegate $i = 1, 2$ receives a given absolute amount of money, $f_i \geq 0$, if she wins the contest regardless of the realized prize, and nothing else. The rest of the game remains the same. Then again no delegation is a dominant strategy for each player, and there is a unique equilibrium without delegation. Note that a delegate exerts

the same effort level for both values of the prize under a fixed-value contingent fee contract. On the contrary, the efforts of the two types of a delegate differ under a uniform-fraction contingent fee contract, since the absolute amount of money received by a successful delegate depends on the value of the prize in that case. Summarizing, we conclude that delegation cannot arise in equilibrium if the players and delegates have the same amount of information, or if the incentives for the delegates are the same under either value of the prize.

Remark 2: As an alternative to our game, take the case where players 1 and 2 are not able to hire a delegate, but instead have the option to learn the realized value of the prize themselves by incurring a given cost $C > 0$. For example, C might be the fixed cost of buying a specialist's report determining the true value of the prize. Take our game and replace the option of delegation by this option, while the rest of the game remains the same. One can verify that (i) the game has a unique equilibrium, where both players buy a report, if C is "small"; (ii) the game has two asymmetric equilibria, in which either only player 1 or only player 2 buys a report, if C is of "intermediate" size; and (iii) the game has a unique equilibrium, where both players do not buy a report, if C is "large."

VI. CONCLUSION

This paper has investigated delegation in a two-player common-value rent-seeking contest with an uncertain prize. The players only know that the prize is high or low with given probabilities. They have the option to hire delegates who receive private information about the true value of the prize. We focused on the situation where players can offer uniform-fraction contingent fee contracts to the delegates. Our main result identified equilibria with endogenous delegation. In order to understand why these equilibria arise, we have decomposed the change in a player's ex ante payoff resulting from hiring a delegate in terms of three effects (given the delegation decision of his opponent): a high-prize revenue effect (which can be positive or negative), a negative low-prize revenue effect, and a positive cost of effort effect. We find that the player has an incentive to delegate if and only if both the probability of the high prize is small enough and the ratio of the high and low value of the prize is large enough. In that case, the high-prize revenue effect is either positive, or negative but small in absolute value.

As a result, the ex ante aggregate of the three aforementioned effects of delegation is positive. The positive high-prize revenue effect is caused by a relatively large increase in the probability of winning the contest if the player delegates rather competes himself. In turn, this follows from the better information possessed by the high-type delegates as compared to the uninformed players. Numerical calculations demonstrate that very similar equilibrium results hold if the players can use optimal contracts, where the fraction of the prize received by a successful delegate might vary with the size of the prize. Our findings contrast with the well-known result for the related game with complete information about the value of the prize. In that case delegation does not arise in equilibrium, since the negative revenue effect always dominates the positive cost of effort effect of delegation. In future work, it is interesting to extend our analysis to all-pay auctions.

APPENDIX: PROOFS AND TECHNICAL DETAILS

A.1. TWO USEFUL LEMMAS AND THE PROOF OF PROPOSITION 1

In order to establish Proposition 1, we first present two lemmas.

Lemma A.1. Let $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. We have the following:

- i. Take $q \in (0, \frac{1}{2}(\sqrt{2} - 1))$. Then there is a unique $\mu_1(q) \in (1, \bar{\mu}(q))$ such that $\widehat{VN}_1(\mu) \leq 0$ if $\mu \in (1, \mu_1(q))$, whereas $\widehat{VN}_1(\mu) > 0$ if $\mu \in (\mu_1(q), \bar{\mu}(q))$.
- ii. Take $q \in [\frac{1}{2}(\sqrt{2} - 1), 1)$. Then $\widehat{VN}_1(\mu) < 0$.
- iii. We have $\widehat{VD}_1(\mu) < 0$.

Proof

Let $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. We have $\widehat{\Pi}_1^{dn}(\mu) = \widehat{\Pi}_1^a(\mu) > \widehat{\Pi}_1^{mn}(\mu)$ if and only if $(\sqrt{2} - f_0(\mu))^2 > \frac{1}{4}$, where $f_0(\mu) \equiv (q\sqrt{\mu} + 1 - q) / \sqrt{q\mu + 1 - q}$ (for $\mu \in \mathbb{R}^+$). We see that $f_0(\mu) < \sqrt{2}$ if $f_1(s) > 0$ for $s (= \sqrt{\mu}) > 1$, where $f_1(s) \equiv (2 - q)qs^2 - 2q(1 - q)s + 1 - q^2$ (for $s \in \mathbb{R}^+$). Note that $f_1(s)$ is an upward opening parabola attaining a minimum value at s_0 , with $s_0 \in (0, 1)$, while $f_1(1) > 0$. Hence, $f_1(s) > 0$ for $s > 1$. It follows that $\widehat{VN}_1(\mu) > 0$ if and only if $f_0(\mu) < \sqrt{2} - \frac{1}{2}$. Note that $f_0(\mu)$ is decreasing in $\mu > 1$, while $f_0(1) > \sqrt{2} - \frac{1}{2}$. Further, $f_0(\bar{\mu}(q)) < \sqrt{2} - \frac{1}{2}$ if and only if $1 - \frac{\sqrt{2}}{2} > \sqrt{q(1 + q)} - q$, or equivalently $q < \frac{1}{2}(\sqrt{2} - 1)$. If $q \in (0, \frac{1}{2}(\sqrt{2} - 1))$, there is a unique $\mu_1(q) \in (1, \bar{\mu}(q))$ such that $f_0(\mu_1(q)) = \sqrt{2} - \frac{1}{2}$. This establishes parts (i) and (ii) of the lemma. Recalling that $\widehat{\Pi}_1^{nd}(\mu) = \widehat{\Pi}_2^a(\mu)$, part (iii) follows directly from Equations (5) and (17).

Lemma A.2. Let $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. We have the following:

- i. Take $q \in (0, \frac{1}{2}(\sqrt{2} - 1))$. Then $\widehat{VN}_1(\mu) > 0$.
- ii. Take $q \in [\frac{1}{2}(\sqrt{2} - 1), \frac{9}{16})$. Then there is a unique $\mu_1(q) \in [\bar{\mu}(q), \infty)$ such that $\widehat{VN}_1(\mu) \leq 0$ if $\mu \in [\bar{\mu}(q), \mu_1(q)]$, whereas $\widehat{VN}_1(\mu) > 0$ if $\mu \in (\mu_1(q), \infty)$.
- iii. Take $q \in [\frac{9}{16}, 1)$. Then $\widehat{VN}_1(\mu) < 0$.
- iv. Take $q \in (0, \frac{1}{2})$. Then there is a unique $\mu_2(q) \in (\bar{\mu}(q), \infty)$ such that $\widehat{VD}_1(\mu) \leq 0$ if $\mu \in [\bar{\mu}(q), \mu_2(q)]$, whereas $\widehat{VD}_1(\mu) > 0$ if $\mu \in (\mu_2(q), \infty)$. Further, if $q \in [\frac{1}{2}(\sqrt{2} - 1), \frac{1}{2})$, then $\mu_2(q) > \mu_1(q)$, where $\mu_1(q)$ is defined in part (ii).
- v. Take $q \in [\frac{1}{2}, 1)$. Then $\widehat{VD}_1(\mu) < 0$.

Proof

Let $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. We have $\widehat{\Pi}_1^{dn}(\mu) = \widehat{\Pi}_1^b(\mu) > \widehat{\Pi}_1^{mn}(\mu)$ if and only if $g_0(\mu) < (\sqrt{q(q+1)} - q)^2 - \frac{q}{4}$, where $g_0(\mu) \equiv (1 - q)/(4\mu)$ (for $\mu > 1$). Note that $g_0(\mu)$ is decreasing in μ , and $\lim_{\mu \rightarrow \infty} g_0(\mu) = 0$. Further, $(\sqrt{q(q+1)} - q)^2 - \frac{q}{4} > 0$ if and only if $q < \frac{9}{16}$, and $g_0(\bar{\mu}(q)) < (\sqrt{q(q+1)} - q)^2 - \frac{q}{4}$ if and only if $q < \frac{1}{2}(\sqrt{2} - 1)$. This proves part (i) and part (iii) of the lemma. If $q \in [\frac{1}{2}(\sqrt{2} - 1), \frac{9}{16})$, there is a unique $\mu_1(q) \in [\bar{\mu}(q), \infty)$ such that $g_0(\mu_1(q)) = (\sqrt{q(q+1)} - q)^2 - \frac{q}{4}$, which implies part (ii). Note that $\mu_1(\frac{1}{2}(\sqrt{2} - 1)) = \bar{\mu}(\frac{1}{2}(\sqrt{2} - 1))$ and $\mu_1(q) > \bar{\mu}(q)$ if $q \in (\frac{1}{2}(\sqrt{2} - 1), \frac{9}{16})$.

Next, we have $\widehat{\Pi}_1^{dd}(\mu) \leq \widehat{\Pi}_1^{nd}(\mu) = \widehat{\Pi}_1^b(\mu)$ if and only if $q(1 - 2q)\mu \leq 2(1 - q^2)$. Part (v) follows directly. If $q \in (0, \frac{1}{2})$, we take $\mu_2(q) \equiv 2(1 - q^2)/(q(1 - 2q))$, and note that $\mu_2(q) \in (\bar{\mu}(q), \infty)$. This gives the first statement of part (iv). If $q \in [\frac{1}{2}(\sqrt{2} - 1), \frac{1}{2})$, then $\mu_1(q) < \mu_2(q)$ since $g_0(\mu_2(q)) < (\sqrt{q(q+1)} - q)^2 - \frac{q}{4}$ if and only if $(1 + q - \sqrt{q(1 + q)})^2 > \frac{3}{8}$, or equivalently $q < 0.67$, which is true in that case. This gives the second statement of part (iv).

Proof of Proposition 1

Owing to the symmetry of the game, it is sufficient to examine the signs of $\widehat{VN}_1(\mu)$ and $\widehat{VD}_1(\mu)$. Given $q \in (0, 1)$, we first consider the cases $\mu \in (1, \bar{\mu}(q))$ and $\mu \in [\bar{\mu}(q), \infty)$ separately.

Let $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. Using part (iii) of Lemma A.1, we know that $\widehat{VD}_1(\mu) < 0$. Take $q \in (0, \frac{1}{2}(\sqrt{2} - 1))$. Part (i) of Lemma A.1 then implies that the game has a unique equilibrium, where both players do not delegate, if $\mu \in (1, \mu_1(q))$. The game has two equilibria, in which either only player 1 or only player 2 delegates, if $\mu \in (\mu_1(q), \bar{\mu}(q))$. Next, take $q \in [\frac{1}{2}(\sqrt{2} - 1), 1)$. Part (ii) of Lemma A.1 then implies that the game has a unique equilibrium, where both players do not delegate.

Next, let $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. First, take $q \in (0, \frac{1}{2}(\sqrt{2} - 1))$. Parts (i) and (iv) of Lemma A.2 then imply that the game has two equilibria, in which either only player 1 or only player 2 delegates, if $\mu \in [\bar{\mu}(q), \mu_2(q)]$. The game

has a unique equilibrium, where both players delegate, if $\mu \in (\mu_2(q), \infty)$. Second, take $q \in [\frac{1}{2}(\sqrt{2}-1), \frac{1}{2}]$. Parts (ii) and (iv) of Lemma A.2 then yield that (a) the game has a unique equilibrium, where both players do not delegate, if $\mu \in [\bar{\mu}(q), \mu_1(q)]$; (b) the game has two equilibria, in which either only player 1 or only player 2 delegates, if $\mu \in (\mu_1(q), \mu_2(q))$; (c) the game has a unique equilibrium, where both players delegate, if $\mu \in (\mu_2(q), \infty)$. Third, take $q \in [\frac{1}{2}, \frac{9}{16}]$. Parts (ii) and (v) of Lemma A.2 then imply that the game has a unique equilibrium, where both players do not delegate, if $\mu \in [\bar{\mu}(q), \mu_1(q)]$, while the game has two equilibria, in which either only player 1 or only player 2 delegates, if $\mu \in (\mu_1(q), \infty)$. Fourth, take $q \in [\frac{9}{16}, 1)$. Parts (iii) and (v) of Lemma A.2 then give that the game has a unique equilibrium, where both players do not delegate.

The proof is established by combining these results, and defining $\mu_1(q)$ according to Lemma A.1 for $q \in (0, \frac{1}{2}(\sqrt{2}-1))$ and according to Lemma A.2 for $q \in [\frac{1}{2}(\sqrt{2}-1), \frac{9}{16}]$, and defining $\mu_2(q)$ according to Lemma A.2 for $q \in (0, \frac{1}{2})$.

A.2. DETAILS ASSOCIATED WITH EQUATIONS (26) AND (28)¹⁷

Here we present details regarding the signs of the high-prize and low-prize revenue effects Equations (26) and (28). Considering Equation (26), we also derive the condition such that $\hat{p}_{1H}^{dn}(\mu) > \hat{p}_1^{nn}$. In that case the probability of winning effect of delegation is positive if the high prize is realized. We further show that the probability of winning effect is always negative if the low prize is realized, that is, $\hat{p}_{1L}^{dn}(\mu) < \hat{p}_1^{nn}$. We do the same with regard to Equation (28).

Consider Equation (26). First, take $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. We then have

$$(A1) \quad \widehat{RN}_{1H}(\mu) = (\hat{p}_{1H}^a(\mu) \times (1 - \hat{w}_1^a(\mu)) - \frac{1}{2}) \mu V_L.$$

We have two cases: (i) Let $q \in (0, \frac{1}{8})$. Then there is a unique $\hat{\mu}(q) \in (1, \bar{\mu}(q))$ (indicated in Figure 1) such that $\widehat{RN}_{1H}(\mu) < 0$ if $\mu \in (1, \hat{\mu}(q))$, $\widehat{RN}_{1H}(\hat{\mu}(q)) = 0$, and $\widehat{RN}_{1H}(\mu) > 0$ if $\mu \in (\hat{\mu}(q), \bar{\mu}(q))$; (ii) Let $q \in [\frac{1}{8}, 1)$. Then $\widehat{RN}_{1H}(\mu) < 0$ for $\mu \in (1, \bar{\mu}(q))$. Note that $\hat{p}_{1H}^{dn}(\mu) = \hat{p}_{1H}^a(\mu) > \hat{p}_1^{nn}$ if and only if $q \in (0, \frac{1}{3})$ and $\mu \in (\frac{2(1-q)}{1-2q}, \bar{\mu}(q))$. Further, we have

$$(A2) \quad \widehat{RN}_{1L}(\mu) = (\hat{p}_{1L}^a(\mu) \times (1 - \hat{w}_1^a(\mu)) - \frac{1}{2}) V_L < 0$$

for $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. Note that $\hat{p}_{1L}^{dn}(\mu) = \hat{p}_{1L}^a(\mu) < \hat{p}_1^{nn}$ in this case.

Second, take $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. Then

$$(A3) \quad \widehat{RN}_{1H}(\mu) = \left((1+q) \left(1 - \sqrt{\frac{q}{q+1}} \right)^2 - \frac{1}{2} \right) \mu V_L.$$

Thus, $\widehat{RN}_{1H}(\mu) > 0$ if $q \in (0, \frac{1}{8})$, $\widehat{RN}_{1H}(\mu) = 0$ if $q = \frac{1}{8}$, and $\widehat{RN}_{1H}(\mu) < 0$ if $q \in (\frac{1}{8}, 1)$. Note that $\hat{p}_{1H}^{dn}(\mu) = \hat{p}_{1H}^b >$

$\hat{p}_1^{nn}$ if and only if $q \in (0, \frac{1}{3})$. Further, $\widehat{RN}_{1L}(\mu) = -\frac{1}{2} V_L < 0$ for $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. Note that $\hat{p}_{1L}^{dn}(\mu) = \hat{p}_{1L}^b < \hat{p}_1^{nn}$ in this case.

Proceeding, examine Equation (28). First, take $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. We then have $\widehat{RD}_{1H}(\mu) = (-\frac{2}{3} + \hat{p}_{1H}^a(\mu)) \mu V_L$. We have two cases: (i) Let $q \in (0, \frac{1}{8})$. Then $\widehat{RD}_{1H}(\mu) < 0$ if $\mu \in (1, \bar{\mu}(q))$, $\widehat{RD}_{1H}(\bar{\mu}(q)) = 0$, and $\widehat{RD}_{1H}(\mu) > 0$ if $\mu \in (\bar{\mu}(q), \bar{\mu}(q))$, where $\bar{\mu}(q) \equiv \frac{9(1-q)}{2-9q} \in (1, \bar{\mu}(q))$; (ii) Let $q \in [\frac{1}{8}, 1)$. Then $\widehat{RD}_{1H}(\mu) < 0$ for $\mu \in (1, \bar{\mu}(q))$. Note that $\hat{p}_1^{dd} > \hat{p}_{1H}^{nd}(\mu) = 1 - \hat{p}_{1H}^a(\mu)$ if and only if $q \in (0, \frac{1}{3})$ and $\mu \in (\frac{2(1-q)}{1-2q}, \bar{\mu}(q))$. Further, $\widehat{RD}_{1L}(\mu) = (-\frac{2}{3} + \hat{p}_{1L}^a(\mu)) V_L < 0$ for $q \in (0, 1)$ and $\mu \in (1, \bar{\mu}(q))$. Note that $\hat{p}_1^{dd} < \hat{p}_{1L}^{nd}(\mu) = 1 - \hat{p}_{1L}^a(\mu)$ in this case.

Second, take $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. Then

$$(A4) \quad \widehat{RD}_{1H}(\mu) = \left(\frac{1}{3} - \sqrt{\frac{q}{q+1}} \right) \mu V_L.$$

Thus $\widehat{RD}_{1H}(\mu) > 0$ if $q \in (0, \frac{1}{8})$, $\widehat{RD}_{1H}(\mu) = 0$ if $q = \frac{1}{8}$, and $\widehat{RD}_{1H}(\mu) < 0$ if $q \in (\frac{1}{8}, 1)$. Note that $\hat{p}_1^{dd} > \hat{p}_{1H}^{nd}(\mu) = 1 - \hat{p}_{1H}^b$ if and only if $q \in (0, \frac{1}{3})$. Next, $\widehat{RD}_{1L}(\mu) = -\frac{2}{3} V_L < 0$ for $q \in (0, 1)$ and $\mu \in [\bar{\mu}(q), \infty)$. Note that $\hat{p}_1^{dd} < \hat{p}_{1L}^{nd}(\mu) = 1 - \hat{p}_{1L}^b$ in this case.

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