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Quenched penguin operators and the $\Delta I = 1/2$ ruleMaarten Golterman^{1,*} and Elisabetta Pallante^{2,†}¹*Department of Physics and Astronomy, San Francisco State University, 1600 Holloway Ave, San Francisco, California 94132, USA*²*Institute for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands*

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The transformation properties of strong penguin operators under the action of the flavor group change when they are considered as operators in (partially) quenched QCD instead of the unquenched theory. As a result, additional operators and new low-energy constants appear in the effective theory describing nonleptonic kaon decay matrix elements in the partially quenched setting. These new low-energy constants do not have a counterpart in the unquenched theory, and should thus be considered as an artifact of the quenched approximation. Here we consider strong penguin operators consisting of products of two left-handed flavor currents, and give a complete one-loop analysis in the effective theory for K^0 to vacuum and K^+ to π^+ matrix elements. We find that the new low-energy constants already appear in these matrix elements at leading order. This implies that (partially) quenched lattice computations of for instance the $\Delta I = 1/2$ rule are affected by ambiguities intrinsic to the use of the quenched approximation at leading order. The only exception is the partially quenched case with three light sea quarks, consistent with general expectations. Our results are also relevant when the charm quark is kept in the theory.

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I. INTRODUCTION

Recently, there has been a renewed interest in lattice computations of weak matrix elements relevant for the understanding of aspects of nonleptonic kaon decays, such as the $\Delta I = 1/2$ rule and ε'/ε , which parametrizes direct CP violation in $K \rightarrow \pi\pi$ decays—for a recent review, see Ref. [1]. However, while this renewed interest is due to the advent of lattice fermions with very good chiral symmetry, to date all such computations have been done in the quenched approximation, in which sea-quark effects are ignored—recent quenched results for both the $\Delta I = 1/2$ rule and ε'/ε can be found in Refs. [2,3].

In the theory where the charm quark has been integrated out penguin operators play an important role. In particular, referring to a commonly used basis, the left-left (LL) penguin operator $\mathcal{O}_2$ plays an important role in the $\Delta I = 1/2$ rule, while the left-right (LR) operator $\mathcal{O}_6$ gives a major contribution to ε'/ε [4].

When one makes the transition from unquenched QCD to partially quenched (PQ) QCD, the theory is changed from the physical theory with three light quarks to a theory with K light valence quarks and N light sea quarks. Fully quenched QCD is the special case with $N = 0$. This implies that the flavor symmetry group changes from the usual $SU(3)_L \times SU(3)_R$ to the graded group $SU(K + N|K)_L \times SU(K + N|K)_R$ [5]. In general, this implies that the classification of weak operators with respect to the flavor symmetry group also changes. In particular, what happens for strong penguins is that the penguin operator which transformed as a component of one irreducible representation (irrep) of $SU(3)_L \times SU(3)_R$ (the octet rep-

resentation) now splits into several parts, each of which transforms in a *different* representation of the PQ symmetry group. One of those is the “natural” generalization of the original penguin operator to the PQ theory, whereas the other transforms in a more complicated way under $SU(K + N|K)_L \times SU(K + N|K)_R$. We will refer to these two parts as the “singlet” and “adjoint” operators, respectively—for reasons that will become clear in Sec. II.

At the level of the effective theory, this means that new low-energy constants (LECs) occur for the adjoint operators, with no counterpart in the unquenched theory, and which thus should be considered an artifact of the use of an approximation where the number of light sea quarks is not equal to three.¹ These new LECs appear in physical matrix elements, unless one decides to drop the corresponding adjoint operators from the PQ theory—meaning that the definition of the penguin operators themselves is changed in the transition to the PQ theory. We observe that also the LECs for the singlet operators do not have to be equal to those of the physical three-flavor theory if $N \neq 3$; even their scale dependence will in general be different.

This problem was considered in a previous paper for the strong LR operators $\mathcal{O}_{5,6}$ [6]. Here we consider the same problem for the LL operators $\mathcal{O}_{1,2}$. While the group theory involved in the LL case is a little more complicated than for the LR case, our results are rather similar. We find that also in the LL case the new adjoint operators do contribute to physical matrix elements already at the leading order (at tree level) in chiral perturbation theory (ChPT).² Here we

¹We will see that for $N = 3$, and when choosing valence- and sea-quark masses equal, physical matrix elements of all new operators vanish.

²A speculation in Ref. [6] that they would only start contributing at order p^4 turns out to be incorrect.

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demonstrate this explicitly with the examples of the K^0 to *vacuum* and K^+ to π^+ matrix elements. We also find that the one-loop corrections for the adjoint operators differ from those of the singlet operators, calculated in Ref. [7]. In other words, the singlet and adjoint LECs do not occur in some fixed, given linear combinations in physical matrix elements beyond tree level.

The outline of this paper is as follows. In Sec. II we show how the standard LL strong penguins break up into terms that transform differently under the enlarged symmetry group of PQ QCD with K valence quarks and N sea quarks. In Sec. III we construct the corresponding weak operators in ChPT, while Sec. IV contains a discussion of some relevant peculiarities that arise for representations of the graded symmetry group $SU(K + N|K)$ in the PQ theory. Section V contains explicit expressions for the K^0 to *vacuum* and K^+ to π^+ matrix elements to one loop in ChPT. Section VI summarizes our results for the fully quenched case, and the final section contains our conclusions. Some group-theoretical details and useful relations amongst weak operators are given in two appendices.

II. LEFT-LEFT PENGUINS IN PARTIALLY QUENCHED QCD

We consider the LL penguin operators

$$\begin{aligned}\mathcal{O}_1 &= (\bar{s}d)_L(\bar{u}u)_L - (\bar{s}u)_L(\bar{u}d)_L \\ &= (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta + \bar{d}_\beta d_\beta + \bar{s}_\beta s_\beta)_L \\ &\quad - (\bar{s}_\alpha d_\beta)_L(\bar{u}_\beta u_\alpha + \bar{d}_\alpha d_\beta + \bar{s}_\beta s_\alpha)_L, \\ \mathcal{O}_2 &= (\bar{s}d)_L(\bar{u}u)_L + (\bar{s}u)_L(\bar{u}d)_L + 2(\bar{s}d)_L(\bar{d}d + \bar{s}s)_L \\ &= (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta + \bar{d}_\beta d_\beta + \bar{s}_\beta s_\beta)_L \\ &\quad + (\bar{s}_\alpha d_\beta)_L(\bar{u}_\beta u_\alpha + \bar{d}_\alpha d_\beta + \bar{s}_\beta s_\alpha)_L,\end{aligned}\quad (2.1)$$

where

$$(\bar{q}_1 q_2)_L \equiv \bar{q}_1 \gamma_\mu P_L q_2, \quad (2.2)$$

with the projection operator on left-handed chirality $P_L = (1 - \gamma_5)/2$. In the second expression for each operator, we have made the color indices α, β explicit. Both operators $\mathcal{O}_{1,2}$ are penguin operators, and each is a linear combination of color unmixed and color mixed terms. Both operators transform in the octet representation of $SU(3)_L$, and, trivially, in the singlet representation of $SU(3)_R$. They are part of a basis of irreducible representations of the chiral group that are *CPS* invariant and with definite isospin $I = 1/2$ and $I = 3/2$ [4]. In Appendix A we clarify how this basis is related to a set of weak operators more frequently used in phenomenological analyses. Our basis is especially convenient for working out group theoretical properties.

As already mentioned in Sec. I, when we consider the LL penguin operators $\mathcal{O}_{1,2}$ of Eq. (2.1) in the partially quenched theory, the representation content of these operators changes. A general realization of PQ QCD contains

K valence quarks, each accompanied by one of K ghost quarks with the same mass in order to suppress the valence-fermion determinant, and N sea quarks—the dynamical quarks—which can all have masses different from those of the valence quarks. The relevant flavor symmetry group enlarges from the physical $SU(3)_L \times SU(3)_R$ to the graded group $SU(K + N|K)_L \times SU(K + N|K)_R$ [5].³

It is clear that the $(\bar{s}d)_L$ factors of both operators in Eq. (2.1) are still a component of the adjoint representation of $SU(K + N|K)$,⁴ while the factors $(\bar{u}u + \bar{d}d + \bar{s}s)_L$ no longer transform as singlets. Instead, the operators can now be written as

$$\begin{aligned}\mathcal{O}_1 &= \frac{K}{N} \mathcal{O}_-^{\text{PQS}} + \mathcal{O}_-^{\text{PQA}}, \quad \mathcal{O}_2 = \frac{K}{N} \mathcal{O}_+^{\text{PQS}} + \mathcal{O}_+^{\text{PQA}}, \\ \mathcal{O}_\pm^{\text{PQS}} &= (\bar{q}_\alpha \Lambda q_\alpha)_L (\bar{q}_\beta q_\beta)_L \pm (\bar{q}_\alpha \Lambda q_\beta)_L (\bar{q}_\beta q_\alpha)_L, \\ \mathcal{O}_\pm^{\text{PQA}} &= (\bar{q}_\alpha \Lambda q_\alpha)_L (\bar{q}_\beta A q_\beta)_L \pm (\bar{q}_\alpha \Lambda q_\beta)_L (\bar{q}_\beta A q_\alpha)_L,\end{aligned}\quad (2.3)$$

where we introduced the spurion fields Λ and A with values

$$\Lambda_i^j = \delta_{i3} \delta^{j2}, \quad A = \text{diag}\left(1 - \frac{K}{N}, \dots, -\frac{K}{N}, \dots\right). \quad (2.4)$$

The first K diagonal elements of A are equal to $(1 - K/N)$ —corresponding to the K valence quarks—and the last $N + K$ diagonal elements are equal to $(-K/N)$ —corresponding to the N sea quarks and the K ghost quarks, both of which do not occur in $\mathcal{O}_{1,2}$. The quark fields are graded vectors in flavor space, with fermionic components given by the valence and sea quarks, and bosonic components by ghost quarks. The indices i and j are graded flavor indices, and run over valence, sea, and ghost flavors.⁵ For the down (strange) quark we have $i = 2$ ($i = 3$). The completely quenched theory, i.e. the theory with $N = 0$, for which the split of Eqs. (2.3) and (2.4), is singular, will be dealt with in Sec. VI.

We note at this point that for $N = K = 3$ we regain the physical three-flavor theory. The adjoint operators $\mathcal{O}_\pm^{\text{PQA}}$ now contain only terms involving either sea or ghost quarks, and it is rather straightforward to see that their contributions to physical matrix elements (i.e. those with only valence quarks on the external lines) vanishes because of cancellation between sea-quark and ghost-quark loops.

³For a detailed analysis of the actual symmetry group in the Euclidean lattice theory, we refer to Ref. [8]. The upshot is that for our purposes, it is appropriate to consider the PQ symmetry group to be $SU(K + N|K)_L \times SU(K + N|K)_R$ [9].

⁴We will often drop the subscript L on the group from now on, since all operators considered in this paper are trivial with respect to the right-handed group.

⁵Wherever explicitly written, we denote flavor indices with $i, j, \dots$, color indices with $\alpha, \beta, \dots$, and Dirac indices with $a, b, \dots$.

For this cancellation to happen, valence masses and sea masses should be chosen equal.

The spurions Λ and A both transform in the adjoint representation of $SU(K + N|K)_L$, as can be seen from the fact that both have a vanishing supertrace (str) [10]. The operators $\mathcal{O}_\pm^{\text{PQS}}$ thus transform in the adjoint representation, while the operators $\mathcal{O}_\pm^{\text{PQA}}$ transform as the product representation of two adjoint irreps, and they are thus reducible. The corresponding decomposition of $\mathcal{O}_\pm^{\text{PQA}}$ is accomplished by (anti-)symmetrization in covariant and contravariant indices, and by “removing” supertraces on pairs of covariant and contravariant indices, much as is done in the case of $SU(N)$ [10]. Here we take the quark fields q_i as covariant, and the antiquark fields $\bar{q}^i$ as contravariant. It turns out that the operators $\mathcal{O}_-^{\text{PQA}}$ and $\mathcal{O}_-^{\text{PQS}}$ ($\mathcal{O}_+^{\text{PQA}}$ and $\mathcal{O}_+^{\text{PQS}}$) are already symmetric (antisymmetric) in both their two covariant and their two contravariant flavor indices—see Appendix B for details.

In the next section, we are going to construct the low-energy bosonized effective Lagrangian for the operators $\mathcal{O}_{1,2}$ in the PQ theory. For this we do not need the details of the decomposition into irreps, and we postpone further discussion of this decomposition to Sec. IV.

III. THE EFFECTIVE LAGRANGIAN

The bosonized low-energy effective Lagrangian is constructed in terms of the nonlinear field

$$\Sigma = \exp(2i\Phi/f), \quad (3.1)$$

where Φ is the $(2K + N) \times (2K + N)$ Hermitian field describing the Goldstone mesons, and f is the pion-decay constant in the chiral limit (normalized such that $f_\pi = 132$ MeV). Out of this field, we construct the necessary building blocks

$$L_\mu = i\Sigma\partial_\mu\Sigma^\dagger, \quad X_+ = 2B_0(\Sigma M^\dagger + M\Sigma^\dagger), \quad (3.2)$$

where M is the $(2K + N) \times (2K + N)$ quark-mass matrix, and $2B_0 = -\langle\bar{u}u\rangle/f^2$ in the chiral limit [11]. These building blocks as well as the spurions Λ and A all transform in the same way under the left-handed group $SU(K + N|K)$, and to lowest order in the chiral expansion, we can construct the effective operators (note that $[\Lambda, A] = 0$ and that⁶ $\text{str}(L_\mu) = 0$)

$$\begin{aligned} \mathcal{L}_1^A &= \text{str}(\Lambda L_\mu) \text{str}(A L_\mu), & \mathcal{L}_2^A &= \text{str}(\Lambda L_\mu A L_\mu), \\ \mathcal{L}_3^A &= \text{str}(\Lambda A L_\mu L_\mu), & \mathcal{L}_4^A &= \text{str}(\Lambda A X_+). \end{aligned} \quad (3.3)$$

In deriving this list, we have also used *CPS* symmetry [12]. For the singlet operators $\mathcal{O}_\pm^{\text{PQS}}$ one replaces $A \rightarrow \mathbf{1}$, and the above operators reduce to

$$\mathcal{L}_1^S = \text{str}(\Lambda L_\mu L_\mu), \quad \mathcal{L}_2^S = \text{str}(\Lambda X_+). \quad (3.4)$$

⁶If we consider the PQ theory with the η' integrated out.

The singlet operators $\mathcal{O}_\pm^{\text{PQS}}$ are represented in chiral perturbation theory by the leading-order Lagrangian [7,12]

$$\mathcal{L}^S = -\alpha_1^{(8,1)} \mathcal{L}_1^S + \alpha_2^{(8,1)} \mathcal{L}_2^S, \quad (3.5)$$

where $\alpha_{1,2}^{(8,1)}$ are two weak LECs.⁷ Note that the LECs for the two different operators $\mathcal{O}_\pm^{\text{PQS}}$ are independent of each other, even though we use the same symbol for both of them.

The operators $\mathcal{O}_\pm^{\text{PQA}}$ correspond to different representations of $SU(K + N|K)$, and are thus represented by different linear combinations of $\mathcal{L}_{1,2,3,4}^A$. A mostly straightforward analysis leads to the bosonization rules⁸

$$\mathcal{O}_\pm^{\text{PQA}} \rightarrow \mathcal{L}_\pm^A = \alpha_{1a}^{A\pm} (\mathcal{L}_1^A \pm \mathcal{L}_2^A) + \alpha_{1b}^{A\pm} \mathcal{L}_3^A + \alpha_2^{A\pm} \mathcal{L}_4^A. \quad (3.6)$$

Here we have explicitly indicated the dependence of the LECs on the operator through the superscripts $\pm$, because they refer to different representations of the PQ flavor group. We conclude that the transition from the unquenched theory to the PQ theory leads to the introduction of three new LECs for each of the two operators $\mathcal{O}_1$ and $\mathcal{O}_2$.

IV. MIXING OF FOUR-QUARK OPERATORS

As stated already in the previous section, the two operators $\mathcal{O}_\pm^{\text{PQA}}$ correspond to two different representations of $SU(K + N|K)$. Both four-quark operators may be written in the form $\bar{q}^i \bar{q}^j T_{ij}^{kl} q_k q_l$, with T symmetric (or antisymmetric) in both $i \leftrightarrow j$ and $k \leftrightarrow l$.⁹ Further decomposition of these representations is generally possible by splitting the tensor T into a part which is supertraceless on any pair of covariant and contravariant indices, and a supertrace part. Note that, because of the value of the spurion Λ , the double supertrace of T on both pairs of indices vanishes in our case.

Taking the supertrace of T on one pair, one obtains a tensor S_i^k , and one may thus construct new four-fermion operators by replacing the tensor T by the new tensor $S_i^k \delta_j^l$, of course after (anti-)symmetrizing this new tensor in correspondence with the symmetry properties of the tensor T . This results in the two new four-quark operators,

⁷The minus sign is there to make the definition of $\alpha_1^{(8,1)}$ conform to that of Ref. [12], in which the effective Lagrangian was defined in Minkowski space. We work in Euclidean space.

⁸Some of the less straightforward aspects will be discussed in the next section. See also Appendix B.

⁹(Anti-)symmetrization here is understood as appropriate for representations of graded groups. “Symmetrization” implies symmetrization in bosonic indices and antisymmetrization in fermionic indices, and vice versa (see Appendix B and Ref. [10]).

$$\begin{aligned}\mathcal{O}_{\pm}^{\text{PQT}} &= (\bar{q}_{\alpha} \Lambda A q_{\alpha})_L (\bar{q}_{\beta} q_{\beta})_L \pm (\bar{q}_{\alpha} \Lambda A q_{\beta})_L (\bar{q}_{\beta} q_{\alpha})_L \\ &= \left(1 - \frac{K}{N}\right) ((\bar{s}_{\alpha} d_{\alpha})_L (\bar{q}_{\beta} q_{\beta})_L \pm (\bar{s}_{\alpha} d_{\beta})_L (\bar{q}_{\beta} q_{\alpha})_L).\end{aligned}\quad (4.1)$$

Our task is now to see whether these new operators are contained in the original four-quark operators $\mathcal{O}_{\pm}^{\text{PQA}}$, i.e. whether the operators $\mathcal{O}_{\pm}^{\text{PQA}}$ and $\mathcal{O}_{\pm}^{\text{PQT}}$ mix. We observe that these operators look just like the singlet operators $\mathcal{O}_{\pm}^{\text{PQS}}$, but for the factor $1 - K/N$, which is the only remnant of the spurion A .

$$\begin{aligned}\mathcal{O}_{\pm}^{\text{PQA}} &\rightarrow (\bar{s}_{\alpha} \gamma_{\mu} P_L d_{\beta}) \left(-\left(1 - \frac{K}{N}\right) (\text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}^s) + \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}^d)) \mp \sum_{q \text{ valence}} \left(1 - \frac{K}{N}\right) \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}^q) \right. \\ &\quad \pm \sum_{q \text{ sea}} \frac{K}{N} \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}^q) \mp \sum_{q \text{ ghost}} \frac{K}{N} \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}^q) \Big) \pm (\bar{s}_{\alpha} \gamma_{\mu} P_L d_{\alpha}) \left(-\left(1 - \frac{K}{N}\right) (\text{tr}(\gamma_{\mu} P_L S_{\beta\beta}^s) + \text{tr}(\gamma_{\mu} P_L S_{\beta\beta}^d)) \right. \\ &\quad \mp \sum_{q \text{ valence}} \left(1 - \frac{K}{N}\right) \text{tr}(\gamma_{\mu} P_L S_{\beta\beta}^q) \pm \sum_{q \text{ sea}} \frac{K}{N} \text{tr}(\gamma_{\mu} P_L S_{\beta\beta}^q) \mp \sum_{q \text{ ghost}} \frac{K}{N} \text{tr}(\gamma_{\mu} P_L S_{\beta\beta}^q) \Big),\end{aligned}\quad (4.2)$$

where $S_{\alpha\beta}^q$ is the quark propagator for flavor q in an arbitrary gluon background—i.e. the quark propagator with an arbitrary number of gluons attached—and we used that

$$\gamma_{\mu} P_L S_{\alpha\beta}^q \gamma_{\mu} P_L = -\gamma_{\mu} P_L \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}^q). \quad (4.3)$$

The latter relation is easily proved by expanding the left-hand side on a basis of 2^4 Euclidean Hermitian Dirac matrices for fixed color indices α and β . In these equations, tr stands for a trace over Dirac indices only.

Quark masses in our theory can be thought of as insertions leading to higher-dimension weak operators, and we may thus consider the same result (4.2) in the chiral limit, in which case the fermion propagator $S^q \rightarrow S$ becomes flavor independent. Equation (4.2) thus simplifies to

$$\begin{aligned}\mathcal{O}_{\pm}^{\text{PQA}} &\rightarrow -2 \left(1 - \frac{K}{N}\right) ((\bar{s}_{\alpha} \gamma_{\mu} P_L d_{\beta}) \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}) \\ &\quad \pm (\bar{s}_{\alpha} \gamma_{\mu} P_L d_{\alpha}) \text{tr}(\gamma_{\mu} P_L S_{\beta\beta})).\end{aligned}\quad (4.4)$$

The above expression vanishes if we take $K = N$, consistent with the fact that the operators $\mathcal{O}_{\pm}^{\text{PQT}}$ vanish in this case (cf. Eq. (4.1)). When $K \neq N$, this result shows that indeed the operators $\mathcal{O}_{\pm}^{\text{PQA}}$ mix with $\mathcal{O}_{\pm}^{\text{PQT}}$, because the gluons attached to the fermion propagator couple to the flavor-singlet bilinear $\bar{q} \gamma_{\mu} P_L q$.

Performing the same contraction on the operators $\mathcal{O}_{\pm}^{\text{PQT}}$, we find

$$\begin{aligned}\mathcal{O}_{\pm}^{\text{PQT}} &\rightarrow \left(1 - \frac{K}{N}\right) (\mp N - 2) ((\bar{s}_{\alpha} \gamma_{\mu} P_L d_{\beta}) \text{tr}(\gamma_{\mu} P_L S_{\alpha\beta}) \\ &\quad \pm (\bar{s}_{\alpha} \gamma_{\mu} P_L d_{\alpha}) \text{tr}(\gamma_{\mu} P_L S_{\beta\beta})).\end{aligned}\quad (4.5)$$

There are three types of diagrams when one considers QCD corrections to all these operators: self-energy corrections on the external lines, vertex corrections, and penguin-like diagrams. The mixing is only possible through the penguinlike diagrams, because the first two types of diagrams do not change the flavor structure of the operator. This means that we have to consider what happens when we contract one quark field with one antiquark field in each of these operators. Such a contraction, in group-theoretical language, corresponds to taking a supertrace on a pair of quark and antiquark indices of the tensor T . Performing the contraction inside the operators $\mathcal{O}_{\pm}^{\text{PQA}}$, we obtain¹⁰

Comparison of Eqs. (4.4) and (4.5) implies that there exist the supertraceless linear combinations

$$\mathcal{O}_{\pm}^{\text{PQA}} + \frac{2}{(\mp N - 2)} \mathcal{O}_{\pm}^{\text{PQT}} \quad (4.6)$$

that do not mix with the operators $\mathcal{O}_{\pm}^{\text{PQT}}$. In group-theoretical language, these linear combinations are irreducible under $SU(K + N|K)$. We thus find that in general, except for $K = N$, our four-quark operators do “contain” the operators $\mathcal{O}_{\pm}^{\text{PQT}}$. In the effective theory, the latter correspond to the operator $\mathcal{L}_3^A$ in Eq. (3.3), justifying the bosonization (3.6).

A closer look at Eq. (4.6) reveals that when the number of sea quarks, N , is equal to two, a singularity arises when one tries to decompose $\mathcal{O}_{\pm}^{\text{PQA}}$ into irreps. Clearly, for $N = 2$ $\mathcal{O}_{\pm}^{\text{PQA}}$ does mix with $\mathcal{O}_{\pm}^{\text{PQT}}$, but it is not possible to define an operator which does *not* mix with $\mathcal{O}_{\pm}^{\text{PQT}}$. In group-theoretical language, the corresponding statement is that the representation in which $\mathcal{O}_{\pm}^{\text{PQA}}$ transforms is not fully reducible. This is indeed what one finds if one analyzes the tensor T_{ij}^{kl} for this operator, and in accordance with the fact that such reducible but not decomposable representations do exist for graded groups [13].¹¹ In any case, we still have to include the operator $\mathcal{L}_3^A$ in the effective theory, and the bosonization rule (3.6) thus stays the same.

¹⁰Wick contraction of graded quark bilinears gives $\overline{q}_{\beta}^j q_{\alpha}^i = (-1)^{g(j)} S_{\alpha\beta}^j \delta_i^j$, with index $g(j)$ defined in Appendix B.

¹¹The operator $\mathcal{O}_{\pm}^{\text{PQA}}$ corresponds to a tensor which is symmetric in both $i \leftrightarrow j$ and $k \leftrightarrow l$, with the appropriate grading (see Appendix B).

V. $K^0 \rightarrow \text{vacuum}$ AND $K^+ \rightarrow \pi^+$ MATRIX ELEMENTS

In this section we give results for the simplest kaon matrix elements of the new weak effective operators, $\mathcal{L}_\pm^A$ (cf. Eq. (3.6)). We first give the tree-level results, and then include also the chiral logarithms which occur at $O(p^4)$. We will not present a detailed analysis of $O(p^4)$ contact terms, because it is unlikely that the matrix elements of $\mathcal{O}_\pm^{\text{PQA}}$ will be numerically computed in the future. The reason is that only the singlet LECs $\alpha_{1,2}^{(8,1)}$ are the interesting ones for physical predictions, and in a PQ setting they can be obtained from the operators $\mathcal{O}_\pm^{\text{PQS}}$, as long as the number of light sea quarks is physical, i.e. $N = 3$ [14]. For a complete $O(p^4)$ analysis of the singlet operators $\mathcal{O}_\pm^{\text{PQS}}$, represented at lowest order by $\mathcal{L}^S$ in Eq. (3.5), we refer to Ref. [7] (see also Ref. [15]). On the other hand, it is relevant to verify whether or not the tree-level and one-loop contributions to physical matrix elements of $\mathcal{L}^A$ have the same form as those arising from the singlet operator $\mathcal{L}^S$.

At tree level, we find that

$$\begin{aligned}\langle 0 | \mathcal{L}_\pm^A | K^0 \rangle &= \frac{4i(M_K^2 - M_\pi^2)}{f} \left(1 - \frac{K}{N}\right) \alpha_2^{A\pm}, \\ \langle \pi^+ | \mathcal{L}_\pm^A | K^+ \rangle &= \frac{4M^2}{f^2} \left(1 - \frac{K}{N}\right) (\mp \alpha_{1a}^{A\pm} - \alpha_{1b}^{A\pm} - \alpha_2^{A\pm}),\end{aligned}\quad (5.1)$$

where in the case of $K^+ \rightarrow \pi^+$ we took the kaon and pion

masses to be equal, $M_K = M_\pi = M$. It is clear that the new LECs $\alpha_{1a,1b,2}^{A\pm}$ already appear at leading order in ChPT, competing with the leading-order contributions coming from the singlet LECs $\alpha_{1,2}^{(8,1)}$.

Next, we give the nonanalytic terms arising at $O(p^4)$. We express our results in terms of bare meson masses, where we take all sea quarks degenerate for simplicity, and also work in the isospin limit, setting $m_u = m_d = m$, i.e.

$$\begin{aligned}M_K^2 &= B_0(m + m_s), \\ M_\pi^2 &= 2B_0m, \\ M_{iS}^2 &= B_0(m_i + m_{\text{sea}}), \\ M_{SS}^2 &= 2B_0m_{\text{sea}}, \\ M_{ij}^2 &= B_0(m_i + m_j),\end{aligned}\quad (5.2)$$

where m_i is the mass of the i th valence quark—we use the labels $i = u, d, s$, respectively $i = 1, 2, 3$ interchangeably to label valence-quark masses. For simplicity, we set $M_K = M_\pi = M$ in the $K^+ \rightarrow \pi^+$ matrix elements, thus working in the degenerate limit. It is further assumed that the η' of the PQ theory is heavy, and therefore has been integrated out [7,9,16,17]. We give $\overline{MS}$ expressions for all one-loop results.

For the nonanalytic terms in $\langle 0 | \mathcal{O}_\pm^{\text{PQA}} | K^0 \rangle$ at one loop we find

$$\begin{aligned}\langle 0 | \mathcal{O}_\pm^{\text{PQA}} | K^0 \rangle_{\text{one-loop}} &= \frac{4i\alpha_{1a}^{A\pm}}{f(4\pi f)^2} \left\{ \left(1 - \frac{K}{N}\right) M_{33}^4 \left(1 - \log \frac{M_{33}^2}{\mu^2}\right) + \frac{1}{N} \sum_{i \text{ valence}} \frac{M_{ii}^2 - M_{SS}^2}{M_{33}^2 - M_{ii}^2} \left[-M_{33}^4 \left(1 - \log \frac{M_{33}^2}{\mu^2}\right) \right. \right. \\ &\quad \left. \left. + M_{ii}^4 \left(1 - \log \frac{M_{ii}^2}{\mu^2}\right) \right] \pm \sum_{i \text{ valence}} M_{3i}^4 \left(1 - \log \frac{M_{3i}^2}{\mu^2}\right) \pm \frac{1}{N} \left(1 - \frac{K}{N}\right) M_{33}^2 (3M_{33}^2 - 2M_{SS}^2) \log \frac{M_{33}^2}{\mu^2} \right. \\ &\quad \left. \mp KM_{3S}^4 \left(1 - \log \frac{M_{3S}^2}{\mu^2}\right) \mp \frac{1}{N} \left(1 - \frac{K}{N}\right) M_{33}^2 (2M_{33}^2 - M_{SS}^2) - (m_3 \leftrightarrow m_2) \right\} \\ &\quad + \frac{4i\alpha_{1b}^{A\pm}}{f(4\pi f)^2} \left(1 - \frac{K}{N}\right) \left\{ \frac{1}{N} M_{33}^2 (3M_{33}^2 - 2M_{SS}^2) \log \frac{M_{33}^2}{\mu^2} + NM_{3S}^4 \left(1 - \log \frac{M_{3S}^2}{\mu^2}\right) - \frac{1}{N} M_{33}^2 (2M_{33}^2 - M_{SS}^2) \right. \\ &\quad \left. - (m_3 \leftrightarrow m_2) \right\} + \frac{2i\alpha_2^{A\pm}}{f(4\pi f)^2} (M_K^2 - M_\pi^2) \left(1 - \frac{K}{N}\right) \left\{ \frac{1}{N} \left[2M_{33}^2 - M_{SS}^2 + 2M_{33}^2 \frac{M_{SS}^2 - M_{33}^2}{M_{22}^2 - M_{33}^2} \right] \log \frac{M_{33}^2}{\mu^2} \right. \\ &\quad \left. + \frac{1}{N} (-3M_{33}^2 + M_{SS}^2) + NM_{3S}^2 \left(1 - \log \frac{M_{3S}^2}{\mu^2}\right) + (m_3 \leftrightarrow m_2) \right\},\end{aligned}\quad (5.3)$$

where $m_3 \leftrightarrow m_2$ stands for the exchange of strange and down valence-quark masses, and we took the exact isospin limit.

For the nonanalytic terms in $\langle \pi^+ | \mathcal{O}_\pm^{\text{PQA}} | K^+ \rangle$ at one loop, with $M_K = M_\pi = M$ and $M_{VS}^2 = (M^2 + M_{SS}^2)/2$, we find

$$\begin{aligned}
\langle \pi^+ | \mathcal{O}_{\pm}^{\text{PQA}} | K^+ \rangle_{\text{one-loop}} &= \frac{4M^2 \alpha_{1a}^{A\pm}}{f^2(4\pi f)^2} \left\{ \left[\frac{2K}{N} M_{SS}^2 + \left(4 - \frac{6K}{N} \right) M^2 \right] \log \frac{M^2}{\mu^2} + \frac{K}{N} M_{SS}^2 - \left(2 - \frac{K}{N} \right) M^2 \pm (K - 2N) M_{VS}^2 \left(1 - \log \frac{M_{VS}^2}{\mu^2} \right) \right. \\
&\quad \left. \pm M^2 \left(K + \frac{4}{N} \left(1 - \frac{K}{N} \right) \right) \pm \frac{2}{N} \left(1 - \frac{K}{N} \right) M_{SS}^2 \pm \left[\frac{6}{N} \left(1 - \frac{K}{N} \right) M_{SS}^2 - M^2 \left(K + \frac{16}{N} \left(1 - \frac{K}{N} \right) \right) \right] \log \frac{M^2}{\mu^2} \right\} \\
&\quad + \frac{4M^2 \alpha_{1b}^{A\pm}}{f^2(4\pi f)^2} \left(1 - \frac{K}{N} \right) \left\{ -N M_{VS}^2 \left(1 - \log \frac{M_{VS}^2}{\mu^2} \right) + \frac{2}{N} (3M_{SS}^2 - 8M^2) \log \frac{M^2}{\mu^2} + \frac{2}{N} M_{SS}^2 + \frac{4}{N} M^2 \right\} \\
&\quad + \frac{4M^2 \alpha_2^{A\pm}}{f^2(4\pi f)^2} \left(1 - \frac{K}{N} \right) \frac{2}{N} \left\{ M_{SS}^2 + (M_{SS}^2 - 4M^2) \log \frac{M^2}{\mu^2} \right\}. \tag{5.4}
\end{aligned}$$

Equations (5.3) and (5.4) represent the partially quenched one-loop contamination which affects K to *vacuum* and K^+ to π^+ amplitudes through the operators $\mathcal{O}_{\pm}^{\text{PQA}}$. By construction, these contributions vanish in the full QCD limit, when $N = K$ and the valence- and sea-quark masses are chosen to be pairwise equal. We note that in Eq. (5.4) M denotes the mass of a meson made out of two valence quarks, i.e. $M = M_{ij}$ for all i, j valence. The contribution of sea quarks can be traced through the index S on M_{VS} , which is the mass of a meson made out of one valence and one sea quark, and M_{SS} , which is the mass of a meson made out of two sea quarks.

VI. THE QUENCHED CASE

In this section we summarize the results for the quenched case, i.e. the case without any sea quarks, for which the symmetry group is $SU(K|K)$. The quenched case needs to be treated separately, because the η' cannot be integrated out [18,19]. Moreover, the phenomenon that not all reducible representations of graded groups are fully reducible already shows up when one tries to construct the adjoint representation: the product of the fundamental irrep and its complex conjugate is not fully reducible [6]. The decomposition (2.3) now takes the form

$$\begin{aligned}
\mathcal{O}_1 &= \frac{1}{1+\gamma} (\gamma \mathcal{O}_{\pm}^{\text{QS}} + \mathcal{O}_{\pm}^{\text{QNS}}), \\
\mathcal{O}_2 &= \frac{1}{1+\gamma} (\gamma \mathcal{O}_{\pm}^{\text{QS}} + \mathcal{O}_{\pm}^{\text{QNS}}), \\
\mathcal{O}_{\pm}^{\text{QS}} &= (\bar{q}_\alpha \Lambda q_\alpha)_L (\bar{q}_\beta q_\beta)_L \pm (\bar{q}_\alpha \Lambda q_\beta)_L (\bar{q}_\beta q_\alpha)_L, \\
\mathcal{O}_{\pm}^{\text{QNS}} &= (\bar{q}_\alpha \Lambda q_\alpha)_L (\bar{q}_\beta \hat{N} q_\beta)_L \pm (\bar{q}_\alpha \Lambda q_\beta)_L (\bar{q}_\beta \hat{N} q_\alpha)_L, \tag{6.1}
\end{aligned}$$

where the spurion field $\hat{N}$ is given by

$$\hat{N} = \text{diag}(1, \dots, -\gamma, \dots). \tag{6.2}$$

The first K diagonal elements of $\hat{N}$ are equal to 1, and the last K diagonal elements are equal to $-\gamma$, with γ arbitrary except $\gamma \neq -1$.¹² Note that $\hat{N}$ has a nonvanishing supertrace, unlike A in the PQ case, while the supertrace of the

unit matrix $\text{diag}(1, \dots, 1)$ does vanish, again unlike the PQ case. The operators $\mathcal{O}_{\pm}^{\text{QS}}$ and $\mathcal{O}_{\pm}^{\text{QNS}}$ each are represented by their own LECs in the effective theory, just as in the case of LR penguins [6]. Naively, it looks like one could avoid those LECs which correspond to $\mathcal{O}_{\pm}^{\text{QS}}$ by choosing $\gamma = 0$. However, the operators $\mathcal{O}_{\pm}^{\text{QNS}}$ mix with $\mathcal{O}_{\pm}^{\text{QS}}$ for any choice of γ , as can be shown in the quark language using arguments similar to those of Sec. IV.

Once again, in order to analyze results from quenched QCD, one has to develop the effective theory for the operators $\mathcal{O}_{\pm}^{\text{QNS}}$. This is straightforward: one exactly follows the construction of Sec. III for the PQ case, by replacing the PQ spurion A everywhere with $\hat{N}$. However, there is one extra operator, because in the quenched case $\text{str}(L_\mu) \neq 0$ —due to the η' , which cannot be integrated out in the quenched case—and the quenched nonsinglet effective Lagrangian replacing Eq. (3.6) reads now (see also Appendix B)

$$\begin{aligned}
\mathcal{L}_{\pm}^{\text{QNS}} &= \alpha_{1a}^{N\pm} (\text{str}(\Lambda L_\mu) \text{str}(\hat{N} L_\mu) \pm \text{str}(\Lambda L_\mu \hat{N} L_\mu)) \\
&\quad + \alpha_{1b}^{N\pm} (\text{str}(\Lambda \hat{N} L_\mu L_\mu) \pm \text{str}(\Lambda \hat{N} L_\mu) \text{str}(L_\mu)) \\
&\quad + \alpha_2^{N\pm} \text{str}(\Lambda \hat{N} X_+). \tag{6.3}
\end{aligned}$$

The tree-level K^0 to *vacuum* and $K^+ \rightarrow \pi^+$ matrix elements are given by

$$\begin{aligned}
\langle 0 | \mathcal{L}_{\pm}^N | K^0 \rangle &= \frac{4i(M_K^2 - M_\pi^2)}{f} \alpha_2^{N\pm}, \\
\langle \pi^+ | \mathcal{L}_{\pm}^N | K^+ \rangle &= \frac{4M^2}{f^2} (\mp \alpha_{1a}^{N\pm} - \alpha_{1b}^{N\pm} - \alpha_2^{N\pm}), \tag{6.4}
\end{aligned}$$

where we labeled the nonsinglet effective operators and LECs with a superscript N instead of a superscript A for the quenched case.

Before presenting quenched one-loop results, we remind the reader that the tree-level propagator of a neutral valence meson made out of quark and antiquark flavor i is given by

$$D_{ij}(p) = \frac{\delta_{ij}}{p^2 + M_{ii}^2} - \frac{1}{3} \frac{m_0^2 + \alpha p^2}{(p^2 + M_{ii}^2)(p^2 + M_{jj}^2)}, \tag{6.5}$$

where m_0^2 is the “double-hairpin” vertex at zero momentum, and α parametrizes its momentum dependence [18]. For the nonanalytic terms at one loop, we find

¹²In Ref. [6] we chose $\gamma = 1$.

$$\begin{aligned}
\langle 0 | \mathcal{O}_{\pm}^{\text{QNS}} | K^0 \rangle_{\text{one-loop}} = & \frac{4i\alpha_{1a}^{N\pm}}{f(4\pi f)^2} \left\{ M_{33}^4 \left(1 - \log \frac{M_{33}^2}{\mu^2} \right) - \frac{1}{3} (1 + \gamma) \sum_{i \text{ valence}} \frac{1}{M_{33}^2 - M_{ii}^2} \left[M_{ii}^4 (m_0^2 - \alpha M_{ii}^2) \left(1 - \log \frac{M_{ii}^2}{\mu^2} \right) \right. \right. \\
& - M_{33}^4 (m_0^2 - \alpha M_{33}^2) \left(1 - \log \frac{M_{33}^2}{\mu^2} \right) \left. \right] \pm \frac{1}{3} M_{33}^2 (m_0^2 - 2\alpha M_{33}^2) \mp M_{33}^2 \left(\frac{2}{3} m_0^2 - \alpha M_{33}^2 \right) \log \frac{M_{33}^2}{\mu^2} \\
& \pm (1 + \gamma) \sum_{i \text{ valence}} M_{3i}^4 \left(1 - \log \frac{M_{3i}^2}{\mu^2} \right) - (m_3 \leftrightarrow m_2) \left. \right\} + \frac{4i\alpha_{1b}^{N\pm}}{f(4\pi f)^2} \left\{ \left(\frac{1}{3} m_0^2 - \frac{2}{3} \alpha M_{33}^2 \right) M_{33}^2 \right. \\
& - \left(\frac{2}{3} m_0^2 - \alpha M_{33}^2 \right) M_{33}^2 \log \frac{M_{33}^2}{\mu^2} \pm M_{33}^4 \left(1 - \log \frac{M_{33}^2}{\mu^2} \right) - (m_3 \leftrightarrow m_2) \left. \right\} \\
& + \frac{2i\alpha_2^{N\pm}}{3f(4\pi f)^2} (M_K^2 - M_\pi^2) \left\{ m_0^2 - 3\alpha M_{33}^2 + \left[-m_0^2 + 2\alpha M_{33}^2 - 2M_{33}^2 \frac{m_0^2 - \alpha M_{33}^2}{M_{33}^2 - M_{22}^2} \right] \log \frac{M_{33}^2}{\mu^2} \right. \\
& \left. \left. + (m_3 \leftrightarrow m_2) \right\}, \tag{6.6}
\end{aligned}$$

for the $\langle 0 | \mathcal{O}_{\pm}^{\text{QNS}} | K^0 \rangle$ matrix element, and

$$\begin{aligned}
\langle \pi^+ | \mathcal{O}_{\pm}^{\text{QNS}} | K^+ \rangle_{\text{one-loop}} = & \frac{4M^2 \alpha_{1a}^{N\pm}}{3f^2(4\pi f)^2} \left\{ 2(6M^2 + K(1 + \gamma)(m_0^2 - 3\alpha M^2)) \log \frac{M^2}{\mu^2} - 6M^2 + K(1 + \gamma)(m_0^2 + \alpha M^2) \right. \\
& \left. \pm (2m_0^2 + (4\alpha + 3K(1 + \gamma))M^2) \pm (6m_0^2 - (16\alpha + 3K(1 + \gamma))M^2) \log \frac{M^2}{\mu^2} \right\} \\
& - \frac{8M^2 \alpha_{1b}^{N\pm}}{3f^2(4\pi f)^2} \left\{ -(3m_0^2 - 8\alpha M^2) \log \frac{M^2}{\mu^2} - m_0^2 - 2\alpha M^2 \pm 3M^2 \left(1 - 2 \log \frac{M^2}{\mu^2} \right) \right\} \\
& + \frac{8M^2 \alpha_2^{N\pm}}{3f^2(4\pi f)^2} \left\{ m_0^2 + (m_0^2 - 4\alpha M^2) \log \frac{M^2}{\mu^2} \right\} \tag{6.7}
\end{aligned}$$

for the $\langle \pi^+ | \mathcal{O}_{\pm}^{\text{QNS}} | K^+ \rangle$ matrix element.

VII. CONCLUSIONS

We add a few comments to the main conclusions already stated in the Introduction.

First, a consistency check on all our explicit tree-level and one-loop results is that they should vanish when the number of valence quarks K is chosen equal to the number of sea quarks N and when the valence- and sea-quark masses are also chosen to be pairwise equal. We noted this already in Sec. II, and it is straightforward to see that indeed all results contained in Sec. V do satisfy this requirement.

We emphasize that the new adjoint operators $\mathcal{O}_{\pm}^{\text{PQA}}$ occurring in the PQ theory are genuinely new operators, and one thus expects that one-loop corrections in ChPT for matrix elements of these operators differ from those of the singlet operators $\mathcal{O}_{\pm}^{\text{PQS}}$. We find that this is indeed the case. If one however only considers the tree-level results, only one particular linear combination of singlet and adjoint LECs appears in all matrix elements (including $K \rightarrow \pi\pi$ matrix elements). This is unlike the case of LR penguins,

where enhancement of the adjoint operators leads to the appearance of chiral logarithms already at leading order in ChPT [6]. Thus, Refs. [2,3], while aiming for the leading-order LECs for $\mathcal{O}_1$ and $\mathcal{O}_2$ in the quenched approximation, have actually computed the linear combinations of LECs that appear at tree level, namely

$$\begin{aligned}
& \frac{1}{1 + \gamma} (\gamma \alpha_2^{(8,1)} + \alpha_2^{N\pm}), \\
& \frac{1}{1 + \gamma} (\gamma \alpha_1^{(8,1)} \mp \alpha_{1a}^{N\pm} - \alpha_{1b}^{N\pm} - (\gamma \alpha_2^{(8,1)} + \alpha_2^{N\pm})). \tag{7.1}
\end{aligned}$$

At first, the appearance of the arbitrary parameter γ looks a bit strange. However, this just reflects the fact that the nonsinglet operators $\mathcal{O}_{\pm}^{\text{QNS}}$ are only defined modulo mixing with the singlet operators $\mathcal{O}_{\pm}^{\text{QS}}$, since they do not by themselves constitute an irrep, as discussed in Sec. VI. In the PQ case no such arbitrariness arises.

Our results show that quenching artifacts do modify, already at the leading chiral order, those $\Delta S = 1$ weak matrix elements that receive contributions from LL pen-

guin operators.¹³ This is especially the case for the $\Delta I = 1/2$ rule where the dominant contributions come from the current-current operators Q_1 and Q_2 of Eq. (A2). In addition, quenching contaminations to LL penguin operators can in principle affect a lattice determination of ε'/ε .¹⁴ While quenching artifacts analyzed in Ref. [6] affect a dominant contribution to ε'/ε coming from the LR penguin operator Q_6 , the quenching ambiguity affecting the operator Q_4 through $\mathcal{O}_1$ and $\mathcal{O}_2$ can be relevant in the presence of a large cancellation of the dominant contributions from Q_6 and the electroweak penguin operator Q_8 [21].

Finally, we note that our results are also relevant for the theory in which the charm quark is kept. In that case, the relevant weak operators are $\mathcal{O}_\pm$, which can be written as

$$\begin{aligned}\mathcal{O}_- &= (\bar{s}d)_L(\bar{u}u)_L - (\bar{s}u)_L(\bar{u}d)_L - (u \rightarrow c) = \mathcal{O}_1 - (u \rightarrow c), \\ \mathcal{O}_+ &= (\bar{s}d)_L(\bar{u}u)_L + (\bar{s}u)_L(\bar{u}d)_L - (u \rightarrow c) \\ &= \frac{1}{5}\mathcal{O}_2 + \frac{2}{15}\mathcal{O}_3 + \frac{2}{3}\mathcal{O}_4 - (u \rightarrow c),\end{aligned}\quad (7.2)$$

where $\mathcal{O}_{3,4}$ transform in the 27-dimensional irrep of $SU(3)_L$ —see also Appendix A. The operators $\mathcal{O}_{1,2}$ appear, and the discussion of this paper thus also applies to the operators $\mathcal{O}_\pm$. The charm quark transforms as a singlet under both $SU(3)$ and $SU(K + N|K)$, and the “ $(u \rightarrow c)$ ” terms in Eq. (7.2) thus transform in the adjoint representa-

tions of both $SU(3)$ and $SU(K + N|K)$. Our observations here do not apply to the $SU(4)$ case of an unphysically light charm quark, because the $SU(4)$ transformation properties of $\mathcal{O}_\pm$ are different.

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APPENDIX A: $\Delta S = 1$ WEAK OPERATORS

In this appendix, we clarify the relation between the operator basis we used and a set of weak operators often used in phenomenological analyses of kaon decays—a comprehensive review can be found in Ref. [22]. Our basis of $\Delta S = 1$ four-quark effective weak operators corresponds to irreducible representations of the chiral group $SU(3)_L \times SU(3)_R$, that are CP invariant [12] (i.e. invariant under the product of CP and the exchange of strange and d quark), and with definite isospin, $I = 1/2$ and $I = 3/2$:

$$\begin{aligned}(8_L, 1_R) \ I = 1/2: \ \mathcal{O}_1 &= (\bar{s}d)_L(\bar{u}u)_L - (\bar{s}u)_L(\bar{u}d)_L, \\ I = 1/2: \ \mathcal{O}_2 &= (\bar{s}d)_L(\bar{u}u)_L + (\bar{s}u)_L(\bar{u}d)_L + 2(\bar{s}d)_L(\bar{d}d + \bar{s}s)_L, \\ (27_L, 1_R) \ I = 1/2: \ \mathcal{O}_3 &= (\bar{s}d)_L[(\bar{u}u)_L + 2(\bar{d}d)_L - 3(\bar{s}s)_L] + (\bar{s}u)_L(\bar{u}d)_L, \\ I = 3/2: \ \mathcal{O}_4 &= (\bar{s}d)_L[(\bar{u}u)_L - (\bar{d}d)_L] + (\bar{s}u)_L(\bar{u}d)_L, \\ (8_L, 1_R) \ I = 1/2: \ \mathcal{O}_5 &= (\bar{s}d)_L[(\bar{u}u)_R + (\bar{d}d)_R + (\bar{s}s)_R], \\ I = 1/2: \ \mathcal{O}_6 &= (\bar{s}_\alpha d_\beta)_L[(\bar{u}_\beta u_\alpha)_R + (\bar{d}_\beta d_\alpha)_R + (\bar{s}_\beta s_\alpha)_R], \\ (8_L, 8_R) \ I = 1/2: \ \mathcal{O}_7 &= (\bar{s}d)_L[(\bar{u}u)_R - (\bar{s}s)_R] - (\bar{s}u)_L(\bar{u}d)_R, \\ I = 1/2: \ \mathcal{O}_8 &= (\bar{s}_\alpha d_\beta)_L[(\bar{u}_\beta u_\alpha)_R - (\bar{s}_\beta s_\alpha)_R] - (\bar{s}_\alpha u_\beta)_L(\bar{u}_\beta d_\alpha)_R, \\ (8_L, 8_R) \ I = 3/2: \ \mathcal{O}_9 &= (\bar{s}d)_L[(\bar{u}u)_R - (\bar{d}d)_R] + (\bar{s}u)_L(\bar{u}d)_R, \\ I = 3/2: \ \mathcal{O}_{10} &= (\bar{s}_\alpha d_\beta)_L[(\bar{u}_\beta u_\alpha)_R - (\bar{d}_\beta d_\alpha)_R] + (\bar{s}_\alpha u_\beta)_L(\bar{u}_\beta d_\alpha)_R.\end{aligned}\quad (A1)$$

The operators $\mathcal{O}_5$ and $\mathcal{O}_6$ are the LR penguin operators considered in Ref. [6]. A set of $\Delta S = 1$ four-quark effective operators frequently used in phenomenological analyses of kaon decays are the Q_i , $i = 1, 10$. They are related to our basis as follows:

¹³We disagree with the conclusion of Ref. [20] that the $\Delta I = 1/2$ rule is not affected by (partial) quenching artifacts at tree level. In particular, we point out that it is not possible to decide unambiguously within the quenched approximation what is the “best” linear combination of singlet and nonsinglet operators to choose. Only an unquenched computation can, in hindsight, decide this issue.

¹⁴We thank the referee for pointing this out.

$$\begin{aligned}
Q_1 &= \frac{1}{2}O_1 + \frac{1}{10}O_2 + \frac{1}{15}O_3 + \frac{1}{3}O_4, \\
Q_2 &= -\frac{1}{2}O_1 + \frac{1}{10}O_2 + \frac{1}{15}O_3 + \frac{1}{3}O_4, \\
Q_3 &= \frac{1}{2}O_1 + \frac{1}{2}O_2, \\
Q_4 &= -\frac{1}{2}O_1 + \frac{1}{2}O_2, \\
Q_5 &= O_5, \\
Q_6 &= O_6, \\
Q_7 &= \frac{1}{2}(O_7 + O_9), \\
Q_8 &= \frac{1}{2}(O_8 + O_{10}), \\
Q_9 &= \frac{1}{2}O_1 - \frac{1}{10}O_2 + \frac{1}{10}O_3 + \frac{1}{2}O_4, \\
Q_{10} &= -\frac{1}{2}O_1 - \frac{1}{10}O_2 + \frac{1}{10}O_3 + \frac{1}{2}O_4,
\end{aligned} \tag{A2}$$

where we have used $Q_4 = Q_2 + Q_3 - Q_1$, $Q_9 = 3/2Q_1 - 1/2Q_3$, and $Q_{10} = 3/2Q_2 - 1/2Q_4 = Q_2 - 1/2Q_3 + 1/2Q_1$. It appears that any modification induced by partial quenching of the LL penguin operators $O_{1,2}$ considered in this paper will affect the operators $Q_{1,2}$, $Q_{3,4}$, and $Q_{9,10}$ according to the decomposition in Eq. (A2). The change of the current-current operators $Q_{1,2}$ is relevant to any (partially) quenched lattice calculation of the $\Delta I = 1/2$ rule. A change of the LR penguin operator Q_6 can drastically affect a (partially) quenched lattice calculation of ε'/ε as has been analyzed in Ref. [6]. The fact that Q_4 is affected as well may also be relevant for the determination of ε'/ε .

APPENDIX B: GRADED GROUPS

In this appendix, we give a few details of graded-group technology as applied to the PQ operators considered in this paper.

First, our definition of the supertrace of a matrix M_i^j is

$$\text{str}(M) = -\sum_i (-1)^{g(i)} M_i^i, \tag{B1}$$

where $g(i) = 1$ if the index i is fermionic (corresponding to a valence or sea quark), and $g(i) = 0$ if the index i is bosonic (corresponding to a ghost quark). The extra overall minus sign is there to make the supertrace reduce to the normal trace in flavor space in the case of unquenched QCD.

Next, we derive in some detail our claim that the operators $O_{\pm}^{\text{PQS}}$ and $O_{\pm}^{\text{PQA}}$ are already symmetric or antisymmetric in the pairs of flavor indices of the quarks (which we refer to as covariant, following Ref. [10]) and antiquarks (which we refer to as contravariant).

Begin with considering the operator

$$(\bar{q}_{\alpha a} \Lambda q'_{\alpha a})(\bar{q}_{\beta b} A q'_{\beta b}) = (\bar{s}_{\alpha a} d'_{\alpha a})(\bar{q}_{\beta b} A q'_{\beta b}) \tag{B2}$$

in which α, β are color indices and a, b are Dirac indices with summation convention for all explicit indices, and we abbreviate

$$q' = \gamma_{\mu} P_L q \tag{B3}$$

for fixed μ . Symmetrizing this in both covariant and contravariant flavor indices leads to

$$\begin{aligned}
&(\bar{q}_{\alpha a} \Lambda q'_{\alpha a})(\bar{q}_{\beta b} A q'_{\beta b}) + (\bar{q}_{\alpha a} \Lambda q'_{\beta b})(\bar{q}_{\beta b} A q'_{\alpha a}) \\
&+ (\bar{q}_{\beta b} \Lambda q'_{\alpha a})(\bar{q}_{\alpha a} A q'_{\beta b}) + (\bar{q}_{\beta b} \Lambda q'_{\beta b})(\bar{q}_{\alpha a} A q'_{\alpha a}) \\
&= 2(\bar{q}_{\alpha a} \Lambda q'_{\alpha a})(\bar{q}_{\beta b} A q'_{\beta b}) + 2(\bar{q}_{\alpha a} \Lambda q'_{\beta b})(\bar{q}_{\beta b} A q'_{\alpha a}).
\end{aligned} \tag{B4}$$

We used that the symmetric product of two quark fields is given by [10]

$$q_{1i} q_{2j} + q_{2i} q_{1j} = q_{1i} q_{2j} + (-1)^{g(i)g(j)} q_{1j} q_{2i}, \tag{B5}$$

and likewise for antiquark fields and antisymmetric products. We now wish to write this in a form in which we can use the shorthand (2.2). We rewrite the second term on the right-hand side of Eq. (B4) as

$$\begin{aligned}
(\bar{q}_{\alpha a} \Lambda q'_{\beta b})(\bar{q}_{\beta b} A q'_{\alpha a}) &= \sum_i (-1)^{g(i)} (\bar{s}_{\alpha} q_{\alpha i})_L (\bar{q}_{\beta}^i d_{\beta})_L A_i^i \\
&= -(\bar{q}_{\alpha} \Lambda q_{\beta})_L (\bar{q}_{\beta} A q_{\alpha})_L,
\end{aligned} \tag{B6}$$

where in the last step we fierzed the operator, taking into account that this involves an extra minus sign if $q_{\alpha i}$ and $\bar{q}_{\beta}^i$ are bosonic (i.e. ghosts). We conclude that symmetrizing the operator of Eq. (B2) in both quark and antiquark flavor indices yields the operator $O_{\pm}^{\text{PQA}}$.

A similar argument shows that antisymmetrizing in both quark and antiquark flavor indices leads to the operator $O_{\pm}^{\text{PQA}}$, and that operators with mixed symmetry (symmetric in quark flavor indices and antisymmetric in antiquark flavor indices, or vice versa) vanish. The operators $O_{\pm}^{\text{PQS}}$ and $O_{\pm}^{\text{PQT}}$ are obtained in the same way. Note that these last two pairs of operators are in principle different, even though for the particular value of the spurion field A relevant for this paper (cf. Eq. (2.4)) we have that $O_{\pm}^{\text{PQT}} = (1 - \frac{K}{N}) O_{\pm}^{\text{PQS}}$.

We may now bosonize these four-quark operators following standard techniques, see, for example, Ref. [4]. The

only difference with respect to the usual, nongraded case is that care has to be taken with extra signs due to the grading of our symmetry group, as discussed briefly above, and in much more detail in Ref. [10]. This leads to the weak Lagrangians given in Eqs. (3.5) and (3.6). In particular, bosonization of $\mathcal{O}_{\pm}^{\text{PQT}}$ leads to $\mathcal{L}_3^A$. In fact, the effective operators corresponding to $\mathcal{O}_{\pm}^{\text{PQT}}$ are

$$\text{str}(\Lambda A L_{\mu} L_{\mu}) \pm \text{str}(\Lambda A L_{\mu}) \text{str}(L_{\mu}), \quad (\text{B7})$$

but we have that $\text{str}(L_{\mu}) = 0$ in the PQ theory with the η' integrated out, leading to $\mathcal{L}_3^A$ in Eq. (3.6). In the quenched theory the spurion A is replaced by $\hat{N}$, and the η' cannot be integrated out, whence $\mathcal{L}_3^A$ of Eq. (3.6) is replaced by the quenched version

$$\text{str}(\Lambda \hat{N} L_{\mu} L_{\mu}) \pm \text{str}(\Lambda \hat{N} L_{\mu}) \text{str}(L_{\mu}). \quad (\text{B8})$$

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