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An integrated transient model for simulating the operation of natural gas transport systems

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ABSTRACT

This paper presents an integrated transient hydraulic model that describes the dynamic behavior of natural gas transport systems (GTS). The model includes sub models of the most important facilities comprising a GTS, such as pipelines, compressor stations, pressure reduction stations, underground gas storage facilities and LNG Terminals. The submodels are combined to an integrated network model and the algorithm for solving the resulting system of equations is detailed. The accuracy of the model is confirmed by benchmarking the model against results from the scientific literature and the commercial software SIMONE. Finally, the ability of the model to simulate the normal operation of a real world gas system and the operation in case of a supply disruption from a major entry point is demonstrated.

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1. Introduction

Natural gas plays a vital role in the energy portfolio of the European Union. In 2013 it accounted for almost one quarter of the primary energy consumption in the EU-28 (EUROGAS, 2014). It is mainly used for power generation, heating, transportation and as a feed stock for industrial production. The location of natural gas deposits is usually many kilometers away from where it is demanded, therefore, natural gas is typically transported to market areas through a designated pipeline transport system. In cases where a direct pipeline connection between production and demand sites is inefficient, or even infeasible, natural gas is liquefied and then shipped as liquefied natural gas (LNG) in large vessels. At ports of destination, it is stored and finally regasified and injected into the pipeline system.

The driving force for the transport of natural gas in pipelines is the gas pressure. To balance pipeline capacity and cost, the system operates at elevated pressure. When gas is flowing, the pressure decreases (by approx. 0.1 [bar/km]) mainly due to friction between the gas and the inner surface of the pipeline. These losses are

compensated for by compressor stations typically installed at 150–200 [km] intervals along the pipeline system. Compressor stations increase the upstream gas pressure to a higher downstream pressure in order to ensure a continuous transport and delivery of natural gas to its customers at the contracted nominations and delivery pressures. Natural gas arriving from the transport pipelines to consumption regions is either directly served to large customers connected to the transport pipeline, such as gas fired power plants and large industrial customers, or delivered to the local gas distribution system, where it is distributed to smaller customers (households, commercial customers, public services, etc.). In both cases, the gas pressure and gas quantity exiting the transport system are metered and controlled by pressure reduction and metering stations, which consist of a number of valves, regulators, metering devices and other components.

Natural gas may also be injected into underground gas storage (UGS) for later withdrawal. UGS may be for seasonal storage - store gas at long periods of gas surplus, usually during summer, and withdrawal at periods of increased or even peak demand in winter. Furthermore, the function of a UGS (e.g. salt cavern storage) can also be to facilitate market arbitrage - accommodating rapid cycles of storage and withdrawal, to let parties benefit from dynamic gas prices. This function to some extent can also be performed using line pack - the storage volume available in the pipeline. UGS facilities increase the flexibility of the transport system to react to load

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Nomenclature

EU	European Union	p_c	critical pressure
CBE	cross border export	p_{in}	inlet pressure
CBI	cross border import	p_{out}	outlet pressure
CBP	cross border point	P_{set}	pressure set point
CGS	city gate station	p_m	mean pressure
CV	control volume	p_n	standard pressure
FTE	fast transient equation	POW_d	driver power
GPP	gas fired power plant	Q	element flow rate
GTS	gas transport system	Q_{vol}	volumetric flow rate
IND	large industry	Q_{set}	flow rate set point
LNG	liquefied natural gas	Q_{wdr}	withdrawal rate
MOP	max. operating pressure	Q_{inj}	injection rate
NGTS	national gas transport system	R	gas constant
PDE	partial differential equation	r	pipe roughness
PRO	production field	Re	Reynold's number
SCADA	supervisory control and data acquisition	R_f	pipe resistance factor
STE	slow transient equation	R_i	pipe inertia factor
TSO	transmission system operator	$SUML$	load balance
UGS	underground gas storage	$SUMLP$	total line pack
$\mathbf{A}$	incidence matrix	t	time
A	cross-sectional area	Δt	time step
c	speed of sound	T	temperature
c_v	isochoric heat capacity	T_c	critical temperature
D	inner diameter	T_n	standard temperature
d	relative density	v	gas velocity
e	Euler's number	V	pipe volume
g	gravitational acceleration	V_i	nodal volume
H	elevation	x	pipeline coordinate
I	storage inventory	Δx	pipe segment length
J	number of pipe segments	z	height coordinate
k	iteration number	Z	compressibility factor
$k_{c,max}$	max. constraint iterations	α	inclination
k_{max}	max. iterations	$\in$	residual tolerance
K_v	valve sizing factor	ζ	resistance factor
L	nodal load (vector)	η	dynamic viscosity
L_{set}	load set point	η_{ad}	adiabatic efficiency
l	pipe length	η_e	pipe efficiency
l_e	equivalent pipe length	η_{mech}	driver efficiency
LP	line pack	κ	isentropic exponent
m	number of elements	λ	Darcy friction factor
n	number of nodes	λ_e	effective friction factor
P	square pressure (vector)	Π	pressure ratio
ΔP	square pressure drop	ρ	gas density
p	gas pressure (vector)	τ	shear stress
Δp	pressure drop	$\dot{Q}$	heat exchange rate

fluctuations and supply disruptions.

The operation of natural gas transport systems is restricted by physical, technical and contractual constraints imposed by the different facilities and stakeholders involved in the gas supply chain. Pipelines, for instance, have a maximum operating pressure (MOP), which if exceeded can cause great damage to the transport system. Compressor stations, on the other hand, have a limited compression power and usually require a minimum gas inflow for surge prevention, while gas fired power plants can only operate above a certain delivery pressure threshold (Walloopilai and Laud, 2003). In order to keep the network within its operational envelope and to meet the contractual nominations, transmission system operators (TSO) are usually equipped with a designated SCADA

(Supervisory Control and Data Acquisition) - System together with software applications to monitor and control the facilities and components in their transport system. Controlling the system involves finding the most economical configurations and control set points for the facilities without violating technical and legal constraints.

While the TSO may have a good picture of the current and projected state of its network, it may however, not be fully aware of potential threats to security of gas supply originating from external sources. For instance, there may be a supply disruption from an important transit pipeline due to a failure in a facility located in an upstream transmission system, or an interrupted supply from a major LNG import terminal due to a geopolitical crises. Moreover,

the growing dependency of the EU-28 upon gas imports from non-EU countries – mainly Russia, Norway and Algeria (EUROGAS, 2014) – stresses the importance to examine the resilience and vulnerabilities of gas transport systems from an independent multinational perspective. By doing this, we can develop strategies to mitigate the potential threats to security of gas supply. The examination of multinational gas transport systems in terms of security of gas supply requires the use of numerical models that are able to reflect the behavior of gas transport systems and the reaction of the system to supply disruptions in adequate and accurate manner. The following three model requirements can be distinguished.

1.) System dynamic behavior

The model should be able to capture accurately the reaction of (multi-) national transport systems to load variations (i.e. daily and seasonal changes of gas demands at offtake points) and disruption events (e.g. loss of supply from an entry point, failure in a compressor station etc.) with reasonable computation cost, taking into account the physical laws governing the dynamic behavior of gas transport systems.

2.) System components

The model should include reasonable sub models of all important facilities comprising a gas transport system, such as pipelines, compressor stations, production fields, cross-border entry and exit stations, city gate stations, stations of direct served customers, LNG terminals and UGS facilities.

3.) System constraints

The model should implement appropriately the constraints imposed by major facilities and stakeholders (e.g. maximum operating pipeline pressures (MOP), maximum available compression power, maximum withdrawal rate from storages and production fields, minimum delivery pressures at offtake points, etc.).

The goal of this paper is to present an integrated transient hydraulic model that fulfills the criteria listed above. The scope of this paper will be on the first criteria, namely, the development of the system equations for the integrated gas network system and the algorithm for solving these equations. Criteria two and three will be addressed briefly by presenting generic sub models of the relevant non-pipe facilities.

To achieve this goal the paper follows the following pattern. In the first part, we present the state of the art of hydraulic models dealing with the simulation of gas transport systems and highlight the main contribution of this paper. Secondly, we derive the equations describing the gas flow in pipelines and make use of commonly used assumptions in the state of the art to simplify and adapt the equations to the prevailing conditions in gas transport systems. In addition, we present a linearisation for the pipe equations which has been used in the context of steady state computations for pipelines without inclination but not to the transient simulation of non-horizontal pipelines. The derivation and linearisation of the pipe equations is followed by a mathematical description of the network system and the basic non-pipe components included in the system, such as compressors, regulators, valves, resistors and nodes. Furthermore, we apply the integral form of the continuity equation to derive the equation system describing the integrated network model which includes generic sub models of all relevant facilities. The derivation is followed by a description of the algorithm for the solution of the system equations based on an implicit time integration with an iterative linearisation of the pipe equation for each simulation time step.

In the last part of the paper, the accuracy of the model is demonstrated by benchmarking it against results from the

literature and the commercial software SIMONE. Finally, the model is applied to perform a dynamic simulation on a real-world network, namely, the Bulgarian and Greek bi-national gas transmission system.

2. State of the art

The hydraulic gas models in the literature can be divided into two groups, namely, steady state models and transient models. Steady state models are characterized by a balance between total gas inflow and outflow, since pressures and flows are assumed constant in time. Transient models in contrast, consider the time evolution of pressures and flows and the changes in line pack. Steady state models are primarily used for optimization and design purposes, mainly, due to their simplicity compared to transient models. While the simulation of transients involves the solution of a set of non-linear hyperbolic partial differential equations (PDEs), a steady state problem requires only the solution of a set of non-linear scalar equations, which is typically solved with a Newton–Raphson approach. There are a number of references addressing both groups of models in the scientific literature, which are summarized in the following.

References mainly dealing with steady state models are (Szoplik, 2012; Woldeyohannes and Abd Majid, 2011; Mohring et al., 2003; Schroeder, August 2001; van der Hoeven, April 2004). SZOPLIK (Szoplik, 2012) applies a steady state model to perform a succession of steady state simulations to capture the behavior of a low pressure distribution network for different air temperatures. The size of the investigated network amounts to 316 nodes, 319 branches. WOLDEYOHANNES and ABD MAJID (Woldeyohannes and Abd Majid, 2011) use a steady state approach with a detailed non-linear compressor model to simulate and analyze different configurations of an existing gas transmission network. The size and complexity of the simulated network is relatively low (1 source, 1 compressor station, 11 pipelines, 5 off take points, 1 loop). MOHRING et al. (Mohring et al., 2003) present a method to reduce the complexity of gas pipeline networks. The method is based on finding a bottleneck model using symbolic simplification, i.e. identifying and extracting dominant network components by combining computer algebra and numerical analysis. Finally, the method is applied on the transmission network of Belgium by comparing steady state simulations of the full network and the reduced network. Van der Hoeven (van der Hoeven, 2004) presents an approach for linearizing the pressure drop equation for horizontal pipelines to perform steady state analysis on gas transport systems. The linearisation presented makes use of the positive slope of the pressure drop curve due to the positive sign of the pipe resistance. He presents three linearisation approaches which differ in the way the linear approximation of the pressure drop curve is obtained from the result of a previous iteration.

The majority of the references dealing with transient gas models either focus on the numerical schemes for solving the partial differential equations describing the gas flow through pipelines like for instance in (Herrán-González et al., 2009; Ebrahimzadeh et al., 2012; Reddy et al., 2006; Hai et al., 2011; Herty and Mohring, 2009; Gato and Henriques, 2005; Behbahani-Nejad and Bagheri, 2010; Behbahani-Nejad and Shekari, 2010; Dorao and Fernandino, 2011; Alamian et al., 2012; Lewandowski, 1995; Grundel et al., 2013) or on the comparison of different transient models obtained by simplifying the flow equations as it is done in (Herrán-González et al., 2009; Osiadacz, 1996; Chaczykowski, 2010, 2009; Osiadacz and Chaczykowski, 2001). The numerical approaches adopted range

from implicit finite difference methods to the method of characteristics, finite element and finite volume methods. HERRÁN-GONZÁLEZ et al. (Herrán-González et al., 2009) implemented the implicit Crank-Nicholson method and the method of characteristics to develop a Matlab-Simulink library which can be used to simulate the transient behavior of a gas network. Moreover, they stress the importance of including the gravitational term in the momentum equation by comparing results for different pipe inclinations. The simulations they performed were applied to a single pipeline and a triangular network with three pipelines. KE and TI (Ke and Ti, 2000) present a more unconventional method for solving transient flow in pipelines which is based on an analogy between the electrical grid and the gas network. The basic idea of this approach is the transformation of the gas model to an analogous electrical model with voltage, resistance, capacitance and inductance. Results computed with this method are benchmarked against those computed for the triangular network in other papers. REDDY et al. (Reddy et al., 2006) developed a dynamic simulator based on a transfer function model to obtain an accurate on-line state estimation from measurements and to estimate the demands. The model is applied on an example pipeline network containing 9 pipelines, one source node and two off-take points. VAN DER HOEVEN (van der Hoeven, 2007) discusses the dynamics in gas transport pipeline based on a concept of distributing the dominant properties of a pipeline, namely, line pack and resistance by segmenting the pipeline into a succession of volumes and resistance. Using this approach he shows how the dynamic in a gas transport systems can be described by a first order differential equation, which if solved yields an exponential function. Moreover he presents the concept of a time constant which is an indicator for how long a pipeline may need to recover from a sudden change of demand at its outlet.

Furthermore, the suitability of steady state and transient methods for different applications has been discussed by a number of authors. OSIADACZ (Osładacz, 1996) discusses the use of steady state and transient models in the industry. According to OSIADACZ (Osładacz, 1996) steady state models are traditionally used for design purposes or in low pressure distribution networks where the dynamics are very rapid and steady state conditions are reached very fast, whereas in high pressure transmission networks, where a very large amount of gas is stored in the pipeline, the dynamics are much slower and therefore transient models are more appropriate. MODISSETTE and MODISSETTE (Modissette and Modissette, 2001) discuss the suitability of transient and succession of steady state models for different purposes like for instance product tracking, line balance, line pack distribution, pressure monitoring or deliverability. The suitability of each model depends on the application. Product tracking for instance can be adequately performed with succession of steady state due to large time step information points, while deliverability, pressure monitoring and line pack distribution cannot be captured correctly by steady state models due to the amplitudes in pressure and flows caused by short term changes in the network (e.g. use of line pack at delivery points till demand is balanced by the supply source). BACHMANN and GOODREAU (Bachman and Goodreau, 2003) discuss different sources of error causing steady state simulations to be unrealistic. One of these sources pointed out by BACHMANN and GOODREAU (Bachman and Goodreau, 2003) is the use of steady state models for solving problems of dynamic nature, like for instance, the use of line pack as short term gas storage to balance a ramp up of demand at industrial delivery points.

The state of the art of hydraulic gas models shows there are a number of hydraulic models addressing the steady state and transient simulation of gas network systems, however, the issue of security of gas supply has not been addressed in any of the

Fig. 1. Forces acting on a control volume in a general gas pipeline.

hydraulic models in the literature. Since most of the models were not developed from a security of gas supply perspective they do not consider all relevant facilities which may play an important role in case of a gas supply disruption. UGS facilities and LNG terminals, for instance, have not been addressed in any of the hydraulic models in the literature. Moreover, the transient models in the literature are primarily applied to relatively small networks, without considering the technical and contractual constraints inherently present in a gas transport system. Thus, there is a gap in the literature which we would like to fill with the work in this paper. The main contribution of this paper is the development of an integrated transient hydraulic model for gas transport systems, that includes all relevant facilities and their constraints and moreover enables the simulation of gas supply disruptions at a regional (multinational) level.

3. Methodology

In this section we develop the model to perform transient simulation in gas transmission networks. Firstly, we set the fundamental equations of gas flow and the corresponding simplifying assumptions for transport pipelines, including linearisation of equations. Next, we elaborate the system of equations for the integrated network system and present a generic model for the key non-pipe facilities in the gas transport system. Finally, we present an algorithm for solving the developed model.

3.1. Pipeline model

The gas flow through pipelines is generally governed by the constitutional laws of fluid dynamics and thermodynamics, namely, the law of conservation of mass, Newton's second law of motion (conservation of momentum), the first law of thermodynamics (conservation of energy) and the real gas law. Applying these laws on an infinitesimal control volume (CV) of a general pipeline with a constant cross-sectional area A and an infinitesimal length dx (see Fig. 1) and moreover, assuming the parameters describing the gas flow along the pipe coordinate x are averaged over A , yields the following set of partial differential equations (PDEs)³:

³ The assumption of averaging the flow parameters over the cross-sectional area can be justified as long as the pipe length L is much greater than the pipe diameter D which is the case in transmission networks where D/L is of order $O(10^{-5})$ or lower.

Law of Conservation of Mass – Continuity Equation⁴:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \quad (1)$$

Newton's Second Law of Motion – Momentum Equation:

$$\underbrace{\frac{\partial(\rho v)}{\partial t}}_{\text{inertia}} + \underbrace{\frac{\partial(\rho v^2)}{\partial x}}_{\text{convective term}} + \underbrace{\frac{\partial p}{\partial x}}_{\text{pressure force}} + \underbrace{\frac{\lambda \rho v |v|}{2D}}_{\text{shear force}} + \underbrace{\rho g \sin \alpha}_{\text{force of gravity}} = 0 \quad (2)$$

First Law of Thermodynamics – Energy Equation:

$$\frac{\partial}{\partial t} \left[\left(c_v T + \frac{1}{2} v^2 \right) \rho A \right] + \frac{\partial}{\partial x} \left[\left(c_v T + \frac{p}{\rho} + \frac{1}{2} v^2 \right) \rho v A \right] + \rho v A g \sin \alpha = \dot{Q} \quad (3)$$

Real Gas Law – State Equation:

$$\frac{p}{\rho} = Z R T \quad (4)$$

The shear force term in the momentum equation is derived from the Darcy–Weisbach relation, which relates the frictional shear stress τ to the dynamic pressure $\rho v^2/2$ as follows:

$$\tau = \lambda \frac{D}{4} \frac{\rho v |v|}{2} \quad (5)$$

For turbulent flow (typical flow condition in transport pipelines) the friction factor λ is generally described by the empirical Colebrook–White correlation:

$$\frac{1}{\sqrt{\lambda}} = -2 \log_{10} \left(\frac{2.51}{Re \sqrt{\lambda}} + \frac{r}{3.71D} \right) \quad (6)$$

where k is the integrated pipeline roughness and Re the Reynolds number, which is the ratio of inertia and frictional forces. Re is defined as follows:

$$Re = \frac{\rho v D}{\eta} \quad (7)$$

where η is the kinematic viscosity of the flowing gas. The Colebrook–White equation is implicit in λ , thus, it can only be solved iteratively. There are a number of explicit approximations for the Colebrook–White equation which are applicable for a specific range of Reynolds numbers. One of these equations is the approximation by HOFFER (Hofer, 1973), which is valid for the turbulent flow regime in transport pipelines:

$$\lambda = \left[2 \log_{10} \left(\frac{4.518}{Re} \log_{10} \left(\frac{Re}{7} \right) + \frac{r}{3.71D} \right) \right]^{-2} \quad (8)$$

The friction factor λ is referred to a straight pipe without curvature. To account for the curvature and the form of the pipeline an efficiency factor η_e is typically introduced as follows:

$$\sqrt{\frac{1}{\lambda_e}} = \eta_e \sqrt{\frac{1}{\lambda}} \quad (9)$$

where λ_e is the effective friction factor. Furthermore, the compressibility factor Z in the state equation is a correction factor for real gases, which accounts for the deviation from ideal gas behavior. Z depends on the gas pressure and temperature and can be approximated by the following equation derived by PAPAY (Papay, 1968), which is valid for natural gas up to a pressure of 150 [bar]:

$$Z = 1 - 3.52 \left(\frac{p}{p_c} \right) \exp \left[-2.260 \left(\frac{T}{T_c} \right) \right] + 0.274 \left(\frac{p}{p_c} \right)^2 \exp \left[-1.878 \left(\frac{T}{T_c} \right) \right] \quad (10)$$

where p_c and T_c are the critical pressure and temperature, respectively.

3.2. Simplifying assumptions

As can be seen from the above derivations, the set of equations describing the gas flow in pipelines is very complex and can only be solved numerically. For large pipeline systems with hundreds or even thousands of interconnected pipeline elements the solution of the integrated PDE system may be very costly regarding computing time and storage. Therefore, it is reasonable to adapt the equations to the prevailing conditions in gas transport systems by means of neglecting some of the terms in the PDE system, but at the same time maintaining an appropriate level of accuracy. The process of simplifying the PDE system has been thoroughly discussed in the available literature, thus, we make use of some of the most common assumptions applied by a number of authors (Herrán-González et al., 2009; Osiadacz, 1996, 1987). The following assumptions are generally accepted as reasonable approximations for the prevailing conditions in gas transport systems.

1.) Isothermal flow

The changes in gas temperature are negligible, therefore we can assume isothermal flow, i.e. the gas temperature is constant in time and space and equal to the ground temperature.

2.) Creeping motion

The influence of the convective term is negligible compared to the other terms in the momentum equation, due to the typically small flow velocities in transport pipelines ($v \leq 15$ [m/s]).

3.) Slow changes in boundary conditions

The inertia term can be neglected if the boundary conditions in terms of pressures and flows do not change rapidly, which is the case in a normal operation of gas transport systems, where load changes typically occur in a time scale of an hour.

Applying the first two assumptions to eqs. (1)–(4) yields the following reduced set of hyperbolic PDEs, which we will denote fast transient equation (FTE) in the following⁵:

$$\frac{\partial p}{\partial t} = -\frac{\rho_n c^2}{A} \frac{\partial Q}{\partial x} \quad (11)$$

⁴ See nomenclature for variable definitions.

⁵ See Appendix A for derivation of Eq. (12).

$$\frac{\partial p}{\partial x} = -\frac{\rho_n}{A} \frac{\partial Q}{\partial t} - \frac{\lambda \rho_n^2 c^2}{2\eta_e^2 DA^2 p} \left| Q \right| Q - \frac{g \sin \alpha}{c^2} p \quad (12)$$

where c is the isothermal speed of sound, ρ_n and Q the gas density and flow rate at standard conditions, respectively. From these equations, we can conclude the following. A pipeline section has volume (the ability to store a certain amount of gas) which is proportional to the gas pressure. Pressure and volume change in time whenever there is an imbalance between inflow and outflow at the boundary of the pipeline section (continuity equation). At steady state (i.e. time derivatives are equal to zero) there is a balance between incoming and outgoing flow and the flow rate is constant along the pipeline. The second equation explains the pressure drop along the pipeline as a result of inertia (force acting opposite to the direction of flow acceleration), resistance (frictional force acting opposite to the flow direction) and gravity (gravitational force due to pipeline inclination). Moreover, if we consider the third assumption, Eq. (12) can be further simplified to the following ordinary differential equation with respect to the square pressure $P = p^2$, which we will refer to as slow transient equation (STE) in the following:

$$\frac{dP}{dx} + \frac{2g \sin \alpha}{c^2} P = -\frac{\lambda \rho_n^2 c^2}{\eta_e^2 DA^2} \left| Q \right| Q \quad (13)$$

The STE can be solved analytically if we assume the flow rate Q the compressibility factor Z and the friction factor λ are averaged over the pipe section with length $\Delta x = l$. The solution yields the following pressure drop equation:

$$\Delta P = R_f \cdot \left| Q \right| Q \quad (14)$$

with

$$\Delta P = P_1 - P_2 e^s, \quad s = \frac{2g(H_2 - H_1)}{c^2}, \quad R_f = \frac{16\lambda \rho_n^2 c^2 l_e}{\pi^2 \eta_e^2 D^5}, \quad l_e = \begin{cases} l, & H_1 = H_2 \\ \frac{e^s - 1}{s} l, & H_1 \neq H_2 \end{cases}$$

where ΔP is the (corrected) square pressure drop, R_f the pipe resistance coefficient, e Euler's number and H_1 and H_2 the elevation at the pipe inlet and outlet, respectively. The pressure drop equation (Eq. (14)) is parabolic and has a positive resistance R_f , i.e. the slope of the parabolic curve is always positive, which is a beneficial characteristic in terms of solving the set of equations for the entire network.

The assumption of slow transients may not be appropriate if the boundary conditions change rapidly. The most rapid change one can think of is the opening or closure of a valve, for safety or maintenance reasons, which may occur in a time scale of minutes. Thus, for the simulation of fast transients in the transport system (e.g. disruption in a compressor station, valve closure for safety reasons etc.) the inertia term is considered. In contrast to the slow transient case, the momentum equation for fast transients contains a time derivative which complicates the solution of the differential equation. Thus, we make an intermediate step to approximate the inertia term by performing an implicit time integration of the FTE in $\Delta t = t_{n+1} - t_n$. This approach yields the following finite difference equation.

$$\frac{dP^{n+1}}{dx} + \frac{2g \sin \alpha}{c^2} P^{n+1} = -\frac{2\rho_n P^{n+1}}{\Delta t A} (Q^{n+1} - Q^n) - \frac{\lambda \rho_n^2 c^2}{\eta_e^2 DA^2} \left| Q^{n+1} \right| Q^{n+1} \quad (15)$$

Similarly to the slow transient case, if we average the coefficients of the flow rate Q^{n+1} on the right hand side over the pipe segment $\Delta x = l$ we can obtain a pressure drop equation which includes an additional term reflecting the influence of inertia for rapid changes in the transport system:

$$\Delta P^{n+1} = R_f \cdot \left| Q^{n+1} \right| Q^{n+1} + R_i \cdot (Q^{n+1} - Q^n) \quad (16)$$

with

$$\Delta P^{n+1} = P_1^{n+1} - P_2^{n+1} e^s, \quad R_i = \frac{2\rho_n L_e p_m}{\Delta t A}, \quad p_m = \frac{2}{3} \frac{p_1^2 + p_1 p_2 + p_2^2}{p_1 + p_2}$$

As can be seen from the above equation, the pressure drop equation for fast transients turns into the equation for slow transient if the changes in flow rate between two time steps are marginal.

3.3. Linearisation of the pipe equation

Both derived pressure drop equations are non-linear, thus, in order to solve the set of equations for a pipeline network the equations have to be linearized and then solved iteratively for each time step t_{n+1} . We make use of the methods applied by VAN DER HOEVEN (van der Hoeven, 2004) for the linearisation of the steady state pressure drop equation for horizontal pipelines and extend his approach to the transient case taking into account the gravitational term. Linearisation basically means approximating the non-linear pressure drop curve with a tangent line, that touches the curve at a specific point. The linearized pressure drop equation for a pipe (i, j) around a point $(\Delta P_{ij}^{k,n+1}, Q_{ij}^{k,n+1})$, can be expressed as follows:

$$\Delta P_{ij}^{k+1,n+1} - \Delta P_{ij}^{k,n+1} = \frac{d\Delta P_{ij}^{k,n+1}}{dQ_{ij}^{k,n+1}} \left(Q_{ij}^{k+1,n+1} - Q_{ij}^{k,n+1} \right) \quad (17)$$

The FTE yields the following linear equation:

$$\Delta P_{ij}^{k+1,n+1} - \left(2R_f \left| Q_{ij}^{k,n+1} \right| + R_i \right) Q_{ij}^{k+1,n+1} = -R_f \left| Q_{ij}^{k,n+1} \right| Q_{ij}^{k,n+1} - R_i Q_{ij}^{k,n} \quad (18)$$

For the STE this expression yields:

$$\Delta P_{ij}^{k+1,n+1} - 2R_f \left| Q_{ij}^k \right| Q_{ij}^{k+1,n+1} = -\Delta P_{ij}^{k,n+1} \quad (19)$$

The square pressure drop term ΔP may be converted to normal pressure drop Δp by dividing the pressure drop equation by $(p_i + p_j e^{s/2})$.

In the following subsection, we firstly, give an overview of the different elements and node types comprising a gas network system, secondly, we present a general graph theoretical representation of the network topology and finally, we use the integral form of the continuity equation to derive the system of equations describing the dynamics in the integrated gas system.

Table 1
Basic elements comprising gas transport networks.

Element	Function	Equations (Osiaadacz, 1987; Pflüger, 2014)
Pipe	Transport of natural gas from point a to b, short term gas storage	$\Delta P^{n+1} = R_f \cdot Q^{n+1} Q^{n+1} + R_i \cdot (Q^{n+1} - Q^n)$
Compressor	Compensates the pressure and head losses due to friction and heat transfer by increasing the gas pressure	$POW_d = \frac{\kappa Z_1 T_1 R_{pn}}{(\kappa-1) \eta_{ad} \eta_{mech}} Q \left[\frac{p_2}{p_1} \frac{\kappa-1}{\kappa} - 1 \right]$
Regulator	Reduces the upstream pressure to a lower downstream pressure and/or regulates the gas flow rate	$Q = 514 K_v \sqrt{\frac{(p_1 - p_2) p_2}{\rho_n T_1}}$
Valve	Interrupts the gas flow and shuts-off sections of the gas network for maintenance or safety reasons	$p_1 = p_2 \quad \vee \quad Q = 0$
Resistor	Passive devices that cause a local pressure drop (e.g. meters, inlet piping, coolers, heaters, scrubbers etc.)	$p_1 - p_2 = \zeta \frac{\rho}{2} v v $

3.4. System description

The components in a gas transport system can be described using basic elements of graph theory, namely, nodes and directed edges. A directed edge represents an element with an inlet, an outlet and a flow direction. The following elements listed in Table 1 are described by directed edges, namely, pipelines, compressors, regulators, valves and resistors. Pipelines and resistors are passive elements, since their behavior is fully described by the physical equation, while compressors, regulators and valves are considered active elements, since their states can be controlled externally. Furthermore, for each element in Table 1 a description of the function and the basic equation describing their operation are listed. The operation of a compressor station, for instance, is generally expressed by an equation describing the required compression power from the driver, which can be either a gas turbine or an electric driver.

The interconnection points between the individual network elements in Table 1 are referred to as nodes. Each element typically has at least one common inlet or outlet node with another element in the network. Moreover, nodes are the only locations in the network where gas can be injected or extracted. Similar to elements, we can distinguish the following type of nodes listed in Table 2, namely, demand, supply, storage and junction nodes. Demand nodes are points in the transport system, where gas is extracted from the network, such as city gate stations (CGS, connection to the local, low pressure distribution network), cross border export stations (CBE, transport of natural gas to neighboring network systems) gas power plants (GPP) and large industries (IND), which are directly served from the transport system. Supply nodes, in contrast, are entry points to the gas system from production fields (PRO), cross border import stations (CBI, gas import from neighboring countries through pipelines) and LNG terminals,⁶ while storage nodes are locations of underground gas storage (UGS) facilities,⁷ where gas can either enter or exit the network. The topology of the entire network is described by the following node-branch incidence matrix:

$$\mathbf{A} = [a_{ij}]^{n \times m} \quad a_{ij} = \begin{cases} +1, & \text{node } i \text{ is outlet of element } j \\ -1, & \text{node } i \text{ is inlet of element } j \\ 0, & \text{node } i \text{ and element } j \text{ are not connected} \end{cases} \quad (20)$$

where n is the number of nodes, m the number of elements and a_{ij}

the elements of matrix $\mathbf{A}$. Matrix $\mathbf{A}$ can be decomposed in a node-pipe incidence matrix $\mathbf{A}_P$ describing only pipe connections and a node-non-pipe incidence matrix $\mathbf{A}_N$ describing non-pipe connections.

$$\mathbf{A} = [\mathbf{A}_P | \mathbf{A}_N] \quad (21)$$

Each node in the network is characterized by its nodal pressure p_i and nodal load L_i , while each branch is characterized by its gas flow rate Q_j . The set of nodal pressures, nodal loads and element flow rates can be described by their corresponding vectors as follows:

$$\mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} p_1^2 \\ p_2^2 \\ \vdots \\ p_n^2 \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{pmatrix} \quad (22)$$

where

$$p_i \geq 0, \quad L_i : \begin{cases} > 0, & \text{demand} \\ < 0, & \text{supply} \end{cases}, \quad Q_j : \begin{cases} > 0, & \text{flow direction is inlet to outlet} \\ < 0, & \text{flow direction is from outlet to inlet} \end{cases} \quad (23)$$

Similarly to the incidence matrix $\mathbf{A}$, the flow vector $\mathbf{Q}$ can be decomposed in a pipe and non-pipe component.

$$\mathbf{Q} = [\mathbf{Q}_P | \mathbf{Q}_N]^T \quad (24)$$

3.5. System equations

The dynamic behavior of a gas transport system is primarily determined by the pipeline elements. As discussed in the previous section, gas pipelines have four basic properties, namely, volume, resistance, inertia and gravity. Volume and resistance are the most dominant properties, while gravity and inertia play a secondary role. A pipeline network can be segmented into a number of pipeline sections, by distributing the aforementioned properties to the corresponding pipe segments. Fig. 2 demonstrates how this can be done for a section of a pipe network. In this example, the volumes of the pipelines are equally distributed and assigned to the inlet and outlet nodes of each pipeline. The quantity of gas Q_{ij} transferred between two nodal volumes depends on the pressure difference p_i and p_j , the resistance, the inertia and the inclination of the pipeline segment between the two nodal volumes. This relation is described by the pressure drop equation derived in the previous section. According to the continuity equation, the pressure in the

⁶ See Appendix C for more details on LNG model.

⁷ See Appendix B for more details on UGS model.

Table 2
Classification and characteristics of nodes in the network.

Node type	Description	Facilities	Control mode	Constraints
Demand	Point, where gas is extracted from the network	CGS, CBE, GPP, IND	Flow control: $L = L_{set}$ Pressure control: $p = P_{set}$ Inactive: $L = 0$	Min. load $L \geq L_{min}$ min. pressure $p \geq P_{min}$
Supply	Point, where gas is injected into the network	PRO, CBI, LNG	Flow control: $L = L_{set}$ Pressure control: $p = P_{set}$ Inactive: $L = 0$	Max. load $ L \leq L_{max}$ max. pressure $p \leq P_{max}$
Storage	Point, where gas is injected or extracted from the network	UGS	Flow Control: $L = Q_{wdr}^{set}$ or $L = Q_{inj}^{set}$ Pressure Control: $p = P_{set}$ Inactive: $L = 0$	Max. withdrawal $ L \leq Q_{wdr}^{max}(I_w)$ max. injection $L \leq Q_{inj}^{max}(I_w)$
Junction	Point, where a topological change or a change in pipe properties occurs (e.g. diameter, inclination)	—	—	—

Fig. 2. Law of mass conservation applied to a nodal control volume at a pipeline intersection.

nodal volume may change in time if there is an imbalance between inflows and outflows to the nodal volume V_i . This can be expressed by the integral form of the continuity equation applied to node i as follows:

$$\frac{V_i}{\rho_n c_{ij}^2} \frac{dp_i}{dt} = \sum_{j=1}^k a_{ij} Q_{ij} - L_i \quad (25)$$

with

$$V_i = \frac{\pi}{8} \sum_{j=1}^k D_{ij}^2 \Delta x_{ij} \quad (26)$$

where L_i is the external load extracted/injected into node i with volume V_i and D_{ij} the diameter of pipe segment (ij) . The continuity equation can be expressed for each node in the network, thus, we

will obtain a set of equations equal to the number of nodes defined for the network. If we perform an implicit time integration of the set of integral continuity equations for a time step $\Delta t = t_{n+1} - t_n$ and order the equation in terms of known variables and boundary conditions at time t_n and t_{n+1} (right hand side) and unknown variables at time t_{n+1} (left hand side) we obtain the following finite difference matrix equation for the integrated network:

$$\Phi p^{n+1} - A Q^{n+1} = \Phi p^n - 0.5(L^n + L^{n+1}) \quad (27)$$

where A is the incidence matrix described in Eq. (20), Q and L the vectors of element flows and nodal loads described in Eq. (22) and Φ the following diagonal matrix describing the pressure coefficients ϕ_i :

$$\Phi = \text{diag}\{\phi_1, \phi_2, \dots, \phi_n\}, \quad \phi_i = \frac{V_i}{\rho_n c_{ij}^2 \Delta t} \quad (28)$$

As can be seen from the right hand side of Eq. (27), to obtain the state of the network for a time in the future (t_{n+1}) we need information on the state in the past (t_n) and the boundary conditions in the future (L^{n+1}), thus, a transient simulation requires an initial state and boundary conditions. Typically, a steady state solution of the network is chosen as an initial condition to start the transient calculation. The equations for the steady state can be obtained if we set $p^n = p^{n+1}$ and $L^n = L^{n+1}$ or the nodal volume V_i to zero, which will result in an equation describing Kirchhoffs' first law, which implies the sum of incoming and outgoing flows Q_j in a node i to be equal to the nodal load L_i .

$$A Q = L \quad (29)$$

3.6. Boundary conditions

The matrix equation derived in the previous section requires additional linear independent equations in order to close the entire problem, since the number of unknowns is greater than the number of equations. These equations are provided by the linearized pressure drop equations derived in the previous section (Eq. (17)) and additional equations describing the control mode of the non-pipe elements (e.g. compressors and regulators), and the control of the facilities at entry and exit points of the network. The operation of compressor stations and regulator stations are typically controlled by a designated automated control system, which

ensures the desired operating set point (e.g. outlet pressure, flow rate, inlet pressure etc.) is maintained and constraints are not violated. At each gas entry and exit point in the transport system there is typically a regulator station that controls the flow rate or pressure at a desired set point. Thus, we can reflect this control by assigning to each demand, supply and storage node the corresponding control of the connected facility. In the following the missing equations describing the control modes are elaborated.

3.6.1. Non-pipe equations

The general equation for non-pipe elements are provided by the following linear equation:

$$C_{p,j,i}p_i^{n+1} + C_{p,j,k}p_k^{n+1} + C_{Q,j}Q_j^{n+1} = E_j \quad (30)$$

where $C_{p,j,i}$, $C_{p,j,k}$ and $C_{Q,j}$ are scalar coefficients of the inlet node i , the outlet node $k \neq i$ and the flow rate of the non pipe element j , respectively. In addition E_j is a constant on the right hand side. The different control modes for each non-pipe element are summarized in Table 3. The set of control modes for compressor stations are similar to that for regulator stations, while for valves only the control mode bypass and inactive is possible. Moreover, compressor and regulator elements may have constraints, which will be checked during the time integration process. For the scope of this paper the following generic constraints listed in Table 3 are considered.

3.6.2. Nodal equations

For each node either the nodal pressure p_i or the nodal load L_i has to be known for a future time step t_{n+1} . If the node is pressure controlled the pressure p_i is set to a desired set point P_{set} , while the corresponding load L_i is calculated for the equivalent time step. In contrast, if the node is load controlled, the load L_i is assigned a desired set point L_{set} and the resulting nodal pressure p_i is calculated. For junction nodes the control mode is always inactive since these nodes are characterized by a zero nodal load, thus, for these nodes a resulting pressure is calculated. The control mode for each node i can be generally described by the following linear scalar equation.

$$K_{p,i} \cdot p_i^{n+1} + K_{L,i} \cdot L_i^{n+1} = S_i \quad (31)$$

where $K_{p,i}$ and $K_{L,i}$ are scalar coefficients of the nodal pressure p_i and nodal load L_i , respectively, and S_i is the value of the desired load or pressure set point. Moreover, for each demand, supply and storage node constraints on load and pressure can be defined, which are listed in the last column of Table 2. For instance, for storage nodes the maximum load (withdrawal or injection rate) depends on the working inventory in the storage and may change during a transient simulation.⁸

3.7. Algorithm

The equations elaborated for the nodes, non-pipe and pipe elements can be combined with Eq. (27) to the following linear matrix equation for computing the state of the network for a future time step t_{n+1} based on an initial time step t_n and prescribed boundary conditions at nodes and non-pipe elements:

$$\begin{pmatrix} \Phi & -A_p & -A_N & I \\ A_{DP} & -R & 0 & 0 \\ C_p & 0 & C_N & 0 \\ K_p & 0 & 0 & K_L \end{pmatrix} \begin{pmatrix} p^{n+1} \\ Q_p^{n+1} \\ Q_N^{n+1} \\ L^{n+1} \end{pmatrix} = \begin{pmatrix} \Phi p^n \\ B \\ E \\ S \end{pmatrix} \quad (32)$$

The first row of the matrix equation describes the set of nodal continuity equations (s. Eq. (27)), while the second row describes the linearized pipe equation (s. Eqs. (19) and (18)), where matrix R is a diagonal matrix representing the slope of the linearized pressure drop equations, B the intersection with the y-axis and A_{DP} the pipe-node incidence matrix describing the pressure drop term DP . Finally, the third and fourth row describes the control mode of non-pipe elements and node facilities, respectively.

Fig. 3 shows a flow chart of the algorithm for the transient simulation. After an initial steady state solution is obtained, the network is discretized in space (Δx) and time (Δt). In addition, the initial steady state solution is projected on the generated simulation grid, i.e. pipe flows and pressure distribution of the original network are assigned to the corresponding grid pipes and grid nodes. After the simulation grid is generated the matrix equation Eq. (32) is solved iteratively for each time step t_{n+1} . First an initial approximation is made for the pressure drop equation using the linearisation described in Eq. (17) then the matrices and vectors in Eq. (32) are assembled in order to solve the linearized matrix equation for the iteration step k . The obtained solution is then used to calculate the residual vector for the non-linear pressure drop equation.

$$Res = \Delta P^{n+1} - [R_f \cdot |Q^{n+1}| Q^{n+1} + R_i \cdot (Q^{n+1} - Q^n)] \quad (33)$$

The euclidean norm of the residual vector

$$\|Res\| = \sqrt{Res_1^2 + Res_2^2 + \dots + Res_{mp}^2}, \quad \|Res\| : \begin{cases} \leq \varepsilon, & \text{converged solution} \\ > \varepsilon, & \text{solution did not converge} \end{cases} \quad (34)$$

is then compared against a tolerance $\varepsilon = [10^{-3}, \dots, 10^{-8}]$. The iteration continues if the residual is above the tolerance and if a maximum number of iterations (k_{max}) has not been exceeded. The residual will typically converge quadratically if the solution for an iteration step is relatively close to the final solution, which is the case if the boundary conditions between two time steps do not change rapidly. Depending on the magnitude of the changes between two time steps and the prescribed residual tolerance a converged solution is usually obtained within 1–5 iterations. In case a converge solution is not obtained for a time step one can adjust the residual tolerance ε and the maximum number of iterations k_{max} and then reiterate.

After a converged solution is obtained for a time step the solution is checked for constraints violation. If violations are found the iteration for the last time step will be repeated adapting the boundary conditions to the violated constraints. The algorithm continues with the next time step if a converged and feasible solution is found, otherwise the simulation is terminated. The entire simulation process ends successfully if for each time step a converged and feasible solution is obtained.

The algorithm was implemented in a numerical code using the programming language Visual Basic.NET.

4. Model benchmarking

In this section, we benchmark the integrated model against results obtained from other models in the literature for a well-

⁸ See Appendix B & C for more details on UGS and LNG model.

Table 3
Control modes and generic constraints of active elements.

Active element	Control mode	Constraints
Compressor	Outlet pressure control: $p_{out} = PO_{set}$ Inlet pressure control: $p_{in} = PI_{set}$ Flow control: $Q = Q_{set}$ Pressure ratio control: $\frac{p_{out}}{p_{in}} = \Pi_{set}$ Bypass mode: $p_{out} = p_{in}$ Inactive: $Q = 0$ See compressor	Max. outlet pressure PO_{max} Min. inlet pressure PI_{min} Max. volumetric flow rate $Q_{vol,max}$ Max. pressure ratio Π_{max} Max. driver power $POW_{d,max}$
Regulator	See compressor	Max. outlet pressure PO_{max} Min. inlet pressure PI_{min} Max. flow rate Q_{max}
Valve	Opened: $p_{out} = p_{in}$ Closed: $Q = 0$	

known triangular network, in order to verify the accuracy of the proposed model. Moreover, we make a specific study about the accuracy concerning the gravitational term in the momentum equation, and its importance in the solution for relatively small average slopes. Proper validation is almost impossible given the scarcity of experimental data available for real networks.

4.1. Simulation of a triangular network

The model is applied to solve the gas network depicted in Fig. 4 using the STE. The example network has been used in a number of references (Herrán-González et al., 2009; Hai et al., 2011; Behbahani-Nejad and Bagheri, 2010; Behbahani-Nejad and Shekari, 2010; Alamian et al., 2012; Ke and Ti, 2000; Osiadacz, 1987) to benchmark the results with different transient methods. The results obtained with the model will be benchmarked against two of these references, namely, OSIADACZ (Osiadacz, 1987) and KE and TI (Ke and Ti, 2000) and the commercial software SIMONE. The network in Fig. 4 consists of three pipelines and two demand nodes (node 2 and node 3) that are supplied with gas by a single supply node (node 1). The network topology and pipe properties are listed in Table 4 while the simulation properties and gas properties are shown in Table 5. The properties are chosen according to those from the references in order to compare the results. To show the efficiency and the accuracy of the proposed model for low space resolutions the pipes are discretized by only one segment per pipeline ($\Delta x = l$).⁹ Unlike the pipe properties in the references (Ke and Ti, 2000; Osiadacz, 1987)¹⁰ the pipe roughness is taken into account for the computation of the friction factor. Thus, for each pipeline a roughness value typical for transport pipelines is assigned ($r = 0.012$ [mm]). Moreover, for the transient simulation a time step of 180 [s] and a total simulation time of one day is chosen. The boundary conditions for the supply and demand nodes are shown in Fig. 5, where in a) the prescribed pressure profile of the supply node and in b) the load profile of the demand nodes are depicted. The supply node is pressure controlled with a constant pressure of 50 [bar] while the demand nodes are flow controlled with a fluctuating load profile according to Fig. 5 b). The required initial condition for the transient simulation is obtained through an initial steady state computation using the boundary conditions of the initial time step $t_0 = 0$ [s] ($p_1 = 50$ [bar], $L_2 = 20$ [sm³/s]¹¹ and $L_3 = 40$ [sm³/s]). For both steady state and transient simulation a computation time of less than 1 [s] was needed to obtain a

converged solution. The computations were run on an 2.4 GHz Intel Core i7 CPU with 8 GB RAM.

Fig. 6 compares the computed pressure profile at the demand nodes with results from OSIADACZ (Osiadacz, 1987), KE and TI (Ke and Ti, 2000) and the SIMONE software. In general, the pressure at the demand nodes decreases with increasing nodal loads and increases as the nodal loads are decreasing. The results obtained with the integrated model is very similar to the results from the SIMONE software. In contrast, there are deviations between the model and the results from OSIADACZ (Osiadacz, 1987) and KE and TI (Ke and Ti, 2000), which could be caused by the different treatment of the friction and compressibility factor. However, the deviations are marginal (below 1%).

Fig. 7 shows the evolution of a) the average pipeline flow and b) the total line pack and Fig. 8 a) the resulting load profile at supply node 1 and b) the total load balance of the network. Since for these parameters no results are provided by OSIADACZ (Osiadacz, 1987) and KE and TI (Ke and Ti, 2000) the results obtained from the integrated model are only compared to SIMONE results.

The evolution of the gas flow for pipe 1 and 2 fluctuates very similar to the corresponding nodal loads, while for pipe 3 the flow remains constant (s. Fig. 7). Latter, is due to the fact that the load difference between the inlet and outlet of pipe 3 (i.e. between node 2 and 3) is constant throughout the simulation even though both loads fluctuate. The results obtained for the pipe flows from the integrated model is again very similar to the SIMONE results.

The correlation between the total line pack of the network and the flow balance can be seen if we compare Fig. 7b) to Fig. 8b). The load balance is equal to the time derivative of the line pack, i.e. the slope of the line pack curve. Whenever the load balance is zero the line pack is at its local minimum/maximum and whenever the load balance is negative/positive the line pack decreases/increases, respectively. The line pack can be viewed as a buffer to balance short term load fluctuations until the supply node is able to react to the changes. The results obtained for the line pack and the flow balance from the integrated model and from SIMONE are similar. The deviation of the line pack is less than 0.5% and originates from the initial steady state calculation, then remains constant throughout the transient simulation. The deviations of the load balance is less than 1% compared to the calculated load profile at the supply node shown in Fig. 8a). These deviations observed for the load balance is actually the difference between the two curves in Fig. 8a).

4.2. Influence of the inclination term

The majority of the hydraulic gas models in the literature do not consider the gravitational term in the momentum equation. In this section, we demonstrate the influence of the gravitational term for even small pipe inclinations and compare the results to SIMONE

⁹ In (Osiadacz, 1987) the pipelines are discretized in 10 segments while in (Ke and Ti, 2000) the pipelines are discretized in 5 segments.

¹⁰ In (Osiadacz, 1987) the friction factor is assumed to be constant with a value of $\lambda = 0.012$ while in (Ke and Ti, 2000) the friction factor is computed without considering the pipe roughness.

¹¹ Standard cubic meters per second.

Fig. 3. Flow chart for transient simulation.

computation in order to verify the accuracy of the model. The computations are conducted on the three pipeline network from the previous section using the same data as in Tables 4 and 5 and Fig. 5. The elevation H_1 of the supply node 1 is varied between $(-1000 [m], \dots, 1000 [m])$ in $500 [m]$ intervals which corresponds to the following inclinations α_j for each pipe j in the network (see Table 6). The inclination angle of pipe 3 remains zero since it is not connected to the supply node. The results are shown in Fig. 9, where the resulting pressure profile for node 2 and 3 are plotted for different elevations of the supply node H_1 . The results show

pressure shifts of approx. 2 [bar] per every 500 [m] of elevation compared to the horizontal case. The pipe outlet pressure decreases if the pipe is ascending ($\alpha > 0$) and increases if the pipeline is descending ($\alpha < 0$). Moreover, a remarkable observation can be made if the inlet and outlet pressures for the descending pipes are compared. For the chosen inclination for this pipelines the pressure increases in flow direction, i.e. the outlet pressure is greater than the inlet pressure, since the potential energy of the gas at the inlet is transferred to static pressure. The elevation change does not influence the shape of the pressure profile since for each elevation the

Fig. 4. Topology of the 3 pipeline network.

Table 4

Pipe data for the 3 pipeline network.

Pipe	Inlet node	Outlet node	Diameter [mm]	Length [km]	Roughness [mm]
1	1	3	600	80	0.012
2	1	2	600	90	0.012
3	2	3	600	100	0.012

Table 5

Input parameters for the transient simulation of the 3 pipeline network.

Parameter	Symbol	Value	Unit
Grid segments per pipe	J	1	[-]
Time step	Δt	180	[s]
Total simulation time	t_{max}	24	[h]
Residual tolerance	ϵ	10^{-4}	[-]
Gas temperature	T	278	[K]
Dynamic viscosity	η	10^{-5}	[kg/m s]
Standard pressure	p_n	1.01325	[bar]
Standard temperature	T_n	273.15	[K]
Relative density	d	0.6	[-]

different inclinations indicate the importance of this term, which should not be neglected in large areas of the EU transmission networks, where these average slopes are exceeded easily.

5. Model application: Bulgarian and Greek national gas transport system (NGTS)

The integrated model is applied to perform a transient simulation on a simplified model of the Bulgarian and Greek NGTS depicted in Fig. 10. The model comprises of 210 pipe elements (total pipe length of approx. 3600 [km] and total geometric pipe volume of approx. 1.6 [Mm³]¹²), 11 compressor stations (10 located in Bulgaria and 1 in Greece) and 217 nodes (67 exit stations to the local distribution system (CGS) and to direct served customers (GPP, IND), two Cross Border Export Stations (CBE), 2 Cross Border Import Stations (CBI), one LNG Terminal and one Underground Gas Storage Facility (UGS)). The Bulgarian-Greek NGTS is basically structured into two national transmission systems and a transit pipeline transporting a large quantity of gas from CBI Negru Voda to the CBEs at the border to Turkey (Malcoclar), FYRO Macedonia (Zidilova) and Greece (Sidirokastron). Apart from the CBI in Negru Voda, there are three additional entry points to the transport system, namely, UGS Chiren in Bulgaria (supply from storage during winter and injection during summer), the LNG Terminal Revythoussa in Greece, and CBI Kipi at the Greek-Turkish border. Fig. 10 shows the steady state pressure and load distribution and the flow direction for the network model computed with the integrated model. The input data for the loads are based on peak winter consumption in 2011. Moreover, each supply node in the model is pressure controlled while each compressor station (except the compressor station at UGS Chiren, which is typically used to increase the gas pressure for storage injection) is pressure ratio controlled with a pressure ratio set point ranging between 1.02–1.2. The diameter of the circles representing demand and supply nodes correspond to

Fig. 5. Initial conditions and boundary conditions for the transient simulation of the 3 pipeline network a) pressure condition at supply node 1 and b) Load profile at the demand nodes 2 and 3.

shape of the pressure profile remains unchanged and is just shifted by a certain amount along the vertical axis. The results obtained from the integrated model are very similar to those obtained from SIMONE, which confirms the accuracy of the integrated model.

Moreover, the magnitude of the differences in pressure for the

the magnitude of the steady state loads in logarithmic scale, as can be seen from the legend in the bottom left corner. Moreover, the

¹² Million cubic meter.

Fig. 6. Computed pressure profiles at node 2 and node 3 compared to results from the literature and SIMONE.

Fig. 7. a) Average gas flow rate in pipelines. b) Total line pack in network.

colors of the pipe elements correspond to the pressure levels indicated in the color bar on top.

In the following, we use the steady state solution of the network depicted in Fig. 10 as an initial state to simulate and compare two different transient scenarios. In the first scenario (case 1) we simulate the normal operation of the Bulgarian-Greek NGTS by assigning the characteristic (relative) load profile depicted in Fig. 11 to all exit stations to the local distribution system (CGS), while for all other exit stations we assume a constant load profile corresponding to the steady state load. The absolute values of the load profile for the CGS nodes is obtained by multiplying the steady state load with the relative values in Fig. 11. Moreover, we change the control of CBI Negru Voda and CBI Kipi to flow control with a flow rate set point corresponding to the steady state load, while the pressure set point for UGS Chiren and LNG Revythoussa as well as the pressure ratio set points for the compressor stations remain unchanged. Furthermore, we set a minimum pressure constraint for both CBE nodes to 30 [bar-g]¹³ and a minimum delivery pressure for all CGS nodes to 20 [bar-g]. In addition, we consider

the constraints on the maximum withdrawal rate from UGS Chiren (175 [ksm³/h]¹⁴) and the maximum regasification rate from LNG Terminal Revythoussa (570 [ksm³/h]).

In the second scenario (case 2) we simulate a supply shortage of 25% at the largest entry point to the NGTS, namely, CBI Negru Voda by successively reducing the steady state load from the initial time to 18:00 and then maintaining the value till the end of the simulation, as shown in Fig. 13. All other settings remain the same as in case 1. Both scenarios are computed for two gas days using the fast transient equation (FTE) derived in Eq. (18). Table 7 lists additional settings for the simulation parameters.

The simulation grid is generated with a time resolution of 300 [s] and a space discretization based on the following criterion proposed by KRALIK (Králík, 1988):

$$J = \text{ceiling} \left\{ \max \left\{ \frac{|H_2 - H_1|}{200[m]}, \frac{l}{30000 \cdot D} \right\} \right\} \quad (35)$$

which basically means each pipe element is segmented in such a

¹³ Gauge pressure = absolute pressure minus atmospheric pressure.

¹⁴ Thousand standard cubic meters per hour.

Fig. 8. a) Resulting load profile at supply node 1 b) load balance (sum of inflow minus sum of outflow) for the entire network.

Table 6
Inclination angles for different elevations of node 1.

Elevation H_1 [m]	α_1 [°]	α_2 [°]	α_3 [°]
–1000	0.716	0.637	0
–500	0.358	0.318	0
500	–0.358	–0.318	0
1000	–0.716	–0.637	0

way that the elevation change between the inlet and outlet node is not greater than 200 [m] and the ratio between the pipe length l and pipe diameter D is not greater than 30,000. Applying this criterion to the network model yields a total grid size of 345 pipe segments and 352 nodes. The computation for each case took approximately 83 s for 576 time steps. Simulation results are shown in Figs. 12–18 and are discussed in the following. Furthermore, an animation video of the results for each case can be viewed in the electronic version of the paper (Supplementary data related to this article can be found online at <http://dx.doi.org/10.1016/j.jngse.2015.11.036>).

The results show how the supply reduction from the main entry point in CBI Negru Voda propagates through the network and affects the pressures and loads at downstream exit and entry points. Fig. 12 illustrates the time evolution of the load balance (i.e. sum of inflow minus sum of outflow) and the total line pack in the combined NGTS for the examined cases. In both cases, the initial line pack in the network decreases, however, the decrease in line pack is much greater for case 2 than for case 1, which can be explained by the difference in load balance for both cases. In case 2, the amount of gas extracted from the network is significantly higher than the amount injected due to the supply reduction in CBI Negru Voda, thus, the line pack in the pipelines is used as a buffer to satisfy demands until the system catches up. The observation for the total line pack are in line with the pressure profiles at the downstream entry and exit points. The nodal pressures for case 2 are either lower or equal to those for case 1 (see pressure profiles in Figs. 13–18).

In addition, the results show the capability of the model to consider constraints imposed on the network. For example, the load set point at CBE Malcoclar (Fig. 14) is not attained in case 2 due

Fig. 9. Computed pressure profiles at node 2 and node 3 for different elevations H_1 for node 1 compared to results from SIMONE.

Fig. 10. Steady state pressure and load distribution for the Bulgarian and Greek NGTS.

to the delivery pressure constraint (30 [bar–g]) imposed on the CBE stations. After a simulation time of approx. 24 [h] the pressure reaches minimum delivery pressure. In order, to maintain the minimum pressure the load is reduced continuously. Another example of constraint consideration can be observed in Fig. 15, where the load and pressure evolution for UGS Chiren is depicted. Due to the supply reduction at CBI Negru Voda (case 2) the storage withdrawal increases rapidly in order to provide gas to customers in the consumption area around Sofia. Around 12:00 on simulation day 2 the withdrawal rate reaches its maximum value (175 [ksm³/h]), thus, the pressure set point cannot be maintained and reduces slightly, while the withdrawal rate is kept at its maximum value.

Fig. 11. Load profile assigned to city gate stations (interface to the local gas distribution system).

Table 7
Input parameters for transient simulation of the Bulgarian-Greek network model.

Parameter	Symbol	Value	Unit
Time step	Δt	300	[s]
Total simulation time	t_{max}	48	[h]
Residual tolerance	ϵ	10^{-4}	[-]
Gas temperature	T	288.15	[K]
Dynamic viscosity	η	10^{-5}	[kg/m s]
Standard pressure	p_n	1.01325	[bar]
Standard temperature	T_n	273.15	[K]
Relative density	d	0.6	[-]

Moreover, looking at the pressure and load evolutions for the exit and entry points downstream CBI Negru Voda, we can notice that the farther an entry or exit point is located from CBI - Negru Voda the later its loads and pressures are affected by the supply reduction. For instance, the resulting load profile for the LNG Terminal in Revythoussa is indifferent for both examined cases up until 12 h after the beginning of the supply reduction at CBI Negru Voda, while for UGS Chiren and CBE Malcoclac an impact can be noticed a few hours after the start of the supply reduction. The magnitude of the reaction time roughly corresponds to the gas travel time, i.e. the ratio of total pipe length to mean velocity from point A to B which corresponds to the amount of time a gas particle would need to travel between both points.

In conclusion, the results demonstrate the capability of the developed model to simulate the dynamic behavior of a real gas transport system and the reaction of the system to supply disruptions.

Fig. 12. Load balance (sum of inflow minus sum of outflow) & line pack evolution for the entire network for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

Fig. 13. Load & pressure evolution at CBI Negru Voda for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

Fig. 14. Load & pressure evolution at CBE Malcoclac for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

Fig. 15. Load & pressure evolution at UGS Chiren for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

6. Conclusion

In this paper, we have developed a transient hydraulic gas model for simulating the operation of real world natural gas transport systems in the context of security of gas supply. The model considers the physical laws governing the gas flow in pipelines, the most important facilities (e.g. compressor stations, entry and exit stations, storage facilities and LNG Terminals) and their technical constraints. The set of equations describing the dynamics

of the transport system were obtained by simplifying and adapting the full equations to the prevailing conditions in gas transport systems. Next, the equation system was discretized and linearised using an implicit time integration and a linearisation technique for the non-linear pipe equations. Furthermore, the model was benchmarked against results from the literature and the commercial software SIMONE. The comparison of the results confirmed the capability of the model to reflect the dynamic behavior of gas transport systems in an adequate and accurate manner. In addition,

Fig. 16. Load & pressure evolution at CBE Zidilova for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

Fig. 17. Load & pressure evolution at CBI Kipi for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

Fig. 18. Load & pressure evolution at LNG Terminal Revythoussa for case 1 (normal operation) and case 2 (supply reduction at entry point Negru Voda).

the model was applied to a real world instance, namely, the operation of the Bulgarian-Greek national gas transport system under normal condition and in the case of a supply reduction from a main entry point. Again, the results obtained demonstrate the ability of the model to capture appropriately the propagation of load and pressure fluctuations at entry and exit points and the flexibility to adapt the boundary conditions of the network in case of constraints violation.

In the near future, the model will be expanded to include more features, such as an enhanced constraint handling algorithm and a graphical user interface (GUI) for analysing the impact of gas supply disruptions and the effectivity of strategies and policies to mitigate the consequences of supply disruptions.

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Appendix A. Simplification of the PDEs

The most common simplification is to assume isothermal flow, which means the gas temperature is constant along the pipeline at

all time. Since pipelines are typically installed approx. 2 [m] underneath the ground the gas temperature is assumed to be equal to the prevailing ground temperature. In reality the gas temperature may change along the pipeline, especially in submarine pipelines and in downstream pipelines of compressor stations (increase in gas temperature) and regulator stations (decrease in gas temperature), though these stations typically mitigate large temperature changes by cooling or preheating the gas. Changes in gas temperature along the pipeline are primarily caused by the Joule-Thompson-Effect and heat exchange between the pipeline and its surroundings. To capture this process adequately one would require a good knowledge of the thermal resistance of the ground and the distribution of ground temperature, which is typically difficult to estimate. Moreover, due to the slow dynamics in transport pipelines ($v < 15$ [m/s]) the flowing gas typically has sufficient time to exchange heat with the ground and adapt its temperature to ground temperature. Thus, it is reasonable to neglect the temperature changes and assume a constant temperature equal to the ground temperature as it is done by many authors in the literature (Herrán-González et al., 2009; Osiadacz, 1996, 1987). If the isothermal condition is applied to the set of PDEs the energy equation becomes redundant and can, therefore, be neglected. Furthermore, the following relation for the isothermal speed of sound c can be obtained from the state equation:

$$\frac{p}{\rho} = c^2 = ZRT \quad (\text{A.1})$$

The PDEs can be further simplified by comparing the different terms in the momentum equation for typical conditions in transport pipelines. The convective term and the pressure term, for instance, are both partial derivatives of the space coordinate x , thus, it is reasonable to compare the magnitude of both terms for typical values for the flow velocity ($v = 10$ [m/s]) and the speed of sound ($c = 300$ [m/s]) as shown in the following:

$$\begin{aligned} \frac{\partial}{\partial x} [\rho v^2 + p] &= \frac{\partial}{\partial x} \left[p \left(1 + \frac{v^2}{c^2} \right) \right] = \frac{\partial}{\partial x} \left[p \left(1 + \frac{10^2}{300^2} \right) \right] \\ &= \frac{\partial}{\partial x} [p (1 + 10^{-3})] \approx \frac{\partial p}{\partial x} \end{aligned} \quad (\text{A.2})$$

The convective term is negligible compared to the pressure term for typical flow conditions in gas pipelines, therefore, this term can be neglected. Applying the above assumptions and the following relation for the flow rate Q and density ρ_n at standard conditions and the mass flow rate M ,

$$M = \rho_n Q = \rho v A \quad (\text{A.3})$$

the PDEs are reduced to the following form referred to as fast transient equations (FTE) in this paper:

$$\frac{\partial p}{\partial t} = - \frac{\rho_n c^2}{A} \frac{\partial Q}{\partial x} \quad (\text{A.4})$$

$$\frac{\partial p}{\partial x} = - \frac{\rho_n}{A} \frac{\partial Q}{\partial t} - \frac{\lambda \rho_n^2 c^2}{2 \eta_e^2 D A^2 p} \left| Q \right| Q - \frac{g \sin \alpha}{c^2} p \quad (\text{A.5})$$

Appendix B. Modeling of underground storages

Underground gas storage facilities (UGS) play an important role in terms of security of gas supply in gas transport systems. They increase the ability and flexibility to react to unexpected disruptions in the transport system. UGS are basically characterized by two processes, namely, the withdrawal and the injection process. The withdrawal process uses the available pressure in the storage reservoir to inject gas into the pipeline, while the injection process utilizes the gas pressure in the pipeline and additional pressure provided by compressor stations to inject gas into the reservoir. The dynamic of both processes differ mainly due to the compressibility of natural gas, which causes the reservoir pressure to decrease as working gas (the quantity of gas in the storage reservoir available for withdrawal) is withdrawn and to increase as working gas is injected into the reservoir. In general, three types of UGS facilities are distinguished, namely, depleted gas fields, aquifer and salt cavern storage. Depleted gas fields can usually store a very large quantity of gas in their reservoirs, but have relatively low withdrawal rates. They are used for long-term seasonal storage. Salt caverns, in contrast, have relatively high withdrawal rates, but smaller working gas inventory compared to depleted gas fields and are used for short term storage. Aquifer, in turn, have characteristics of both depleted gas fields and salt cavern. They can be used as long-term as well as short term storage.

The operating region of a storage facility is typically reflected by its storage envelope, which depicts the technically possible operating points of the facility in terms of withdrawal rate (Q_{wdr}), injection rate (Q_{inj}) and working gas inventory (I_w). The shape of the storage envelope depends on a number of properties, such as, the storage type, the maximum and minimum reservoir pressure,

minimum gas inventory (cushion gas), the available compression power for injection and withdrawal etc.. Fig. B.19 shows typical storage envelopes for the withdrawal and injection process of the three storage types.

A storage facility can be modeled as a single node, which is either a supply node, during the withdrawal process or a demand node during the injection process. During withdrawal, the load of the node corresponds to the withdrawal rate, while during injection, the load is equal to the injection rate of the storage facility.

The actual operation of the storage facility will be considered by monitoring the operating point (withdrawal/injection rate vs. working gas inventory) in each time step and ensuring this point lies within the storage envelope. The working gas inventory in the storage reservoir at each time step can be obtained as follows:

$$I_w(t) = I_w(t_0) + \int_{t_0}^t Q_{inj}(t) - Q_{wdr}(t) dt \quad (\text{B.6})$$

where t_0 is the initial time point. Knowing the working gas inventory and the withdrawal or injection rate, the operating point can be determined and checked for each time step. If the operating point is outside the storage envelop, the time step will be recalculated using the boundary values. Moreover, similar to demand and supply nodes a storage node can be either flow or pressure controlled. If flow controlled a flow rate set point below injection or withdrawal capacity is set, while if pressure controlled a desired pressure set point will be set for the storage node, respectively.

Appendix C. Modeling of LNG Terminals

LNG terminals play an important role in many national transport systems in Europe. They increase the storage capacity of the transport system and diversify the sources of natural gas imports. Similar to UGS facilities, LNG terminals can be regarded as supply nodes with an operating region limited by the LNG working inventory (the amount of LNG in the tanks available for regasification) and regasification capacity Q_{reg}^{max} . The working gas inventory decreases during withdrawal from storage and increases when LNG is discharged from an arriving LNG vessel and injected into storage, while the regasification capacity, remains constant, and is independent of the fluctuations in storage level. The storage inventory in the tanks has to be monitored each time step in order to capture the operating point of the station correctly. Therefore, the LNG in storage will be transformed to an equivalent quantity of gaseous natural gas at standard conditions using the following relation for the relative density d_{lng} :

$$d_{lng} = \frac{\rho_{n,lng}}{\rho_{n,gas}} \quad (\text{C.7})$$

which is the ratio between the density of LNG ($\rho_{n,lng} \cong 421$ kg/m³) and gaseous natural gas $\rho_{n,gas}$ at standard condition. Using this relation the equivalent standard volume of working gas inventory in the LNG tank at any instant of time $I_w(t)$ can be obtained as follows:

$$I_w(t) = d_{lng} \cdot I_{LNG}(t_0) + \int_{t_0}^t d_{lng} \cdot Q_{dis}(t) - Q_{reg}(t) dt \quad [sm^3] \quad (\text{C.8})$$

where $I_{LNG}(t_0)$ is the LNG in storage at the initial time point t_0 , $Q_{dis}(t)$ the vessel discharge rate and $Q_{reg}(t)$ the regasification rate. If I_w exceeds the equivalent maximum working gas capacity of the LNG terminal the load of the supply node will be set to zero.

Figure B.19. Typical UGS envelopes for depleted gas fields, aquifer and salt cavern storage for the withdrawal process (top) and the injection process (bottom) (Lochner and Dieckhoener, 2010; E.ON GAS STORAGE GmbH, 2015).

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