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Piaget's neo-Gödelian turn: Between biology and logic, origins of the New Theory

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Abstract

Jean Piaget's research program—which involved the study of child development as a central feature (viz. “stages”), but which can be understood more broadly as advancing a constructive theory of knowledge (i.e., “genetic epistemology”)—is thought by many contemporary developmentalists to have been guided by a coherent, complete, and unchanging meta-theoretical framework: “equilibration.” While this is correct philosophically, it is incorrect historically. Briefly put: the *formal meaning* of equilibration changed over time, and thus so too did the entirety of the theory that relied upon it. To focus in on one specific change of particular importance, this article examines how Piaget appealed to the changing ideas of Kurt Gödel and their interpretations by French-speaking logicians. This historical analysis (a Foucauldian archaeology) thereby excavates a “neo-Gödelian turn” in Piaget's research program. The resulting framework is then sketched in outline: the updated formal meta-theory that made possible “Piaget's new theory.”

Keywords

constructivism, epigenetic epistemology, equilibration, Gödel, hierarchy of levels, Piaget

“Gödel's theorems supply impressive arguments in favor of constructivism.”

Piaget, in *Biology & Knowledge* (1967/1971a, p. 80)

While the early works of Jean Piaget (1896–1980) were being “rediscovered” in the United States, in the late 1950s and early 1960s, he was simultaneously working on a complete revision of the underlying theory using approaches drawn from biology,

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cybernetics, and something he came to call “the epistemology of logic” (see esp. Piaget, 1967a; also 1970a, p. 487; 1970/1972, pp. 63–68). The result of these efforts was the construction of a new meta-theoretical framework formalizing the *processes* driving developmental change (Gallagher, 1972/1977; see also Campbell, 2009). And this then informed more than a decade of new experiments (see Ducret, 2000; Vuyk, 1981).

This later work extends far beyond the limits of the earlier stage theory of child development, popularized especially by Flavell (see Müller, Burman, & Hutchinson, 2013). For this reason, it is referred to in the secondary literature as “Piaget’s new theory” (see esp. Beilin, 1992). The most well-known aspect of this is his unfinished move toward “relevance,” and its formalization in a logic of meanings (Piaget & Garcia, 1987/1991).

Of course, Piaget’s attempts to formalize his theory—and especially his writings on logic—have been the subject of innumerable critiques. Yet most of these have been presented by outsiders who failed to fully understand his intent (see e.g., Piaget, 1963). What follows therefore pursues a quite different goal than that typically sought: even as we recognize the present interests of the discipline, we focus here on providing the view “from within” (following Chapman, 1988, p. 1).

The task is accomplished by tracing the ways in which Piaget himself described the formal aspects of his theory. And we simplify this by focusing on a particular change in his appeals to the works of Kurt Gödel (1906–1978). The resulting history is then of necessity incomplete: there are many more aspects of the New Theory that need to be developed further, and a change in formalism was not the only change that occurred. (Elsewhere, for example, I have discussed some of the similarly timed changes in Piaget’s biology; see Burman, 2013b). Yet what follows does usefully clarify some of what Piaget said about the assumptions that underlie his later works, and so some new translations are also provided where space permits.

To be clear, though: we are not imputing to Piaget any variation of the loose and relativistic crypto-Gödelian interpretations that have subsequently come to be criticized for their abuse of the primary source texts (see e.g., Franzén, 2005). Instead, we trace the emergence of a “neo-Gödelian” view. And while we find this first in Gödel’s own writings, its influence on Piaget came through the efforts of his French-speaking contemporaries. But to get to these, we must begin at the beginning.

Kurt Gödel and his proofs of “incompleteness”

The program that interests us emerged from Gödel’s (1929/1986b) doctoral dissertation. This included a proof of the “completeness” of first-order predicate calculus: it demonstrated the non-contradiction of arithmetical methods and thereby provided a firm foundation for the proof-making activities of mathematicians. (He showed that their tools are trustworthy in a strong sense: *proofs*, if constructed using such methods, are *true*.) Building on this, he then undertook the project that became synonymous with his name: a multi-part proof of the “incompleteness” of all general systems of number theory that are sufficiently complex to allow arithmetical methods to function within them without inconsistencies. That work was presented in 1930, published—in German—in 1931, and accepted as his *Habilitationsschrift* in 1932.¹

The key assumption of Gödel's (1931/1986a) discovery is that *statements about* arithmetic, if formally presented, can themselves be *treated as* arithmetical propositions. Using a technique based on prime factorization that he invented for this purpose (now called "Gödel numbering"), he showed that certain properties of these meta-mathematical statements could be understood to correspond to truth, falsity, provability, etc., in a systematic way. He then showed that these statements could be manipulated using standard arithmetical methods: they could be made to make strong and trustworthy claims, including about themselves. These claims could thus be shown to be either true or false.

The incompleteness project took this one step further. By building on the Liar Paradox—typically presented as, simply, "this statement is false"—Gödel was able to use his numbering technique to set up a self-referential contradiction: "If this statement is true, then it is false."² This corresponds to an arithmetical impossibility (i.e., $1 = 0$).

The discovery that such a claim could be proven afforded a peculiar conclusion: "truth" and "proof" are not formally identical. This was then generalized: not every *statement* can be proved to be true or false, within the system that contains it, despite the completeness of that system's *methods*.

Gödel showed, in other words, that *every formal system* above a certain degree of complexity is afflicted by "undecidable" propositions. (This was demonstrated by his "first" incompleteness theorem.) Furthermore, if such a system *claims*—of itself—that it is complete and consistent, and thus not internally contradictory, then it also suffers from the Liar Paradox and is therefore necessarily *incomplete* and *inconsistent*. (This was demonstrated by his "second" incompleteness theorem.) The details of how this was accomplished, exactly, have been discussed by several well-known secondary sources (most famously by Hofstadter, 1979; but earlier, and most influentially, by Nagel & Newman, 1958). But it's what happened afterward that is of special interest to us.

On the changing implications of incompleteness

The relationship between Piaget and the evolving Gödelian view is a complicated thing to trace. Although Piaget and Gödel almost certainly met at the Institute for Advanced Study, when Piaget was a visiting fellow there in 1954, there is no correspondence between them held in the accessible collections at any of the archives we consulted. (This is not in itself especially meaningful, though: the majority of the collection held by the Piaget Archives is unprocessed and not available for consultation; Burman, 2013a, although things are indeed expected to open up starting in 2018; M. J. Ratcliff, personal communication, January 24, 2016.) As a result, we can only trace the changes in the larger discourse in which they both participated: preparatory spadework, in anticipation of future in-depth archival excavations (e.g., Heinzmann, Trognon, & Tremblay, 2014; Ratcliff & Burman, 2015).

It is toward this end that we pursued a variation on the approach taken by Michel Foucault (1926–1984) in his early works: an "archaeology." We traced Piaget's appeals to Gödel in order to identify the "ruptures" that mark out the sedimentary layers separating different "discourses" (see esp. Foucault, 1969/1972). And this in turn led us to examine the changes in the formal justification for Piaget's theory of "grouping" (*groupement*)—which we can define most usefully here as *that formalism the contents of which*

are “equilibrated”—in order to help the mass of archaeological findings cohere. The result is a narrative that extends our present understanding of how Piaget’s New Theory came to be.

The importance of logic

The value of a formal logical framework is that it enables the valid inference of *truth* or *falsity* about a conclusion drawn from a set of propositions that are themselves known to be either *true* or *false*, but without reference to the content of the claims represented (i.e., if you have trustworthy methods, the rest follows necessarily). And, indeed, this can be put in terms understood by Piaget’s “standard” theory of developmental stages. To explain this, though, we must make explicit an assumption inherited from his earliest works: a “stage” is the product of internally consistent groupings of “structures,” inside of which transformations can be made which do not change the quality of the whole (i.e., they are “equilibrated”). These structures then produce actions that either lead to the desired outcome or don’t (i.e., they are *functionally* true or false).

This assumption, on its own, is supportable by a loose reading of Gödel’s *completeness* theorem: transformations are formalizable as arithmetical statements. In application, it is further supported by the longstanding argument that formal principles can be used to understand the operation of the mind (see Boden, 2006). The result is that Piaget’s “structures” (referred to variously in translation as “schemes,” “schemas,” and “schemata”; see Brown, 2001, pp. 181, 189n2) are theorized as coherent action-producing systems with internal workings having formal characteristics.

Thus, to infer the existence of a particular structure, a Piagetian developmentalist might engage a child in a series of questions related to what is now referred to as a Conservation task. For example: *When it is true that a volume of water is moved between a tall-thin-cup and a short-fat-cup, what must also be true about the volume of water during the transformation?* And then, crucially, in terms of the Piagetian method of differentiating kinds of groupings in the children’s responses: *Why?* (see Bond & Tryphon, 2009).

Such an approach is all well and good for constructing a classification, because—contrary to what one might expect—children at different ages do not all reply in the same way. But things then began to fall apart as a result of how Piaget (1942, 1949, 1952b) formalized this insight further, in an attempt to *explain* his otherwise-acceptable empirical descriptions of child development by positing the existence of different kinds of structures in operation at different ages: “our real problem is to discover the actual operational mechanisms which govern behaviour, and not simply to measure it” (Piaget, 1953, p. xviii).

Briefly put: if the stages of child development are the product of grouped structures, Piaget reasoned the expected outcomes for each stage could be formalized in a kind of behavioral “truth table” listing all of the possible transformations. Different stages would thus have different possible outcomes associated with them, as a consequence of what must follow necessarily given their different operations. (A young child’s response to a problem like volume could then be internally coherent, even though incorrect; the result of their truth table indexing only the level of the water in each glass, rather than coordinating height, width, and depth.) Gödel’s *incompleteness* theorems suggested, however, that—even if such a table could be constructed for each stage—there would still be

functional *truths* of that system which could not be contained within the structural table as *provable* statements: children would be able to produce responses and behaviors that didn't belong to their "appropriate" stage. (There would, in other words, be unevenness in development: *décalages*.) All of the stage-tables, not only the "formal operational" one, would therefore be functionally incomplete. And thus so too would be the theory that produced them. QED.

It would have been devastating for Gödel's discoveries to have become widely understood within developmental psychology before Piaget had prepared a response. The entire formal foundation of Stage Theory had been undermined, using the same language in which it had originally been supported.

For Gödel, however, proving the incompleteness of complex formal systems was only a result: a necessary consequence. And, contrary to what one might now think after reading about it in popular commentaries, he didn't stop working on or with incompleteness once he had proved that Bertrand Russell (1872–1970) and Alfred North Whitehead (1861–1947)—and subsequently David Hilbert (1862–1943) too—had been wrong to insist that logicians could reduce all of mathematical philosophy (perhaps even all human knowledge) to logico-mathematical first principles. (This position is called "logicism," and is traceable ultimately to Gottlob Frege, 1848–1925; see van Heijenoort, 1967; also Smith, 1999.) Thus, we must now ask: *How did Gödel's ideas change, and how did these changes enable the emergence of Piaget's later functional-structuralism?*

From "Gödelian" incompleteness to a "neo-Gödelian" hierarchy

In December of 1933, Gödel delivered a lecture at Cambridge in which he argued for the continuing importance of finding a means to achieve Hilbert's goal of grounding mathematics on firm foundations (published as Gödel, 1995b; see Feferman, 2008). He also argued that, because of incompleteness, a proof of non-contradiction would have to be found by "constructive" means. (In mathematics, a formal system is "constructive" if it produces the object it intends to prove: the proof is in the production, not in the inference.)

Gödel then delivered a similar lecture at Yale, in April of 1941, in which he described how "intuitionistic" logics—which treat mathematics and logic as internally consistent tools for thought, rather than as revelations of the natural order—could be considered "constructive" in this way (published as Gödel, 1995a; see also van Atten, 2014; van Atten & Kennedy, 2003). Still though, as he later explained of the work's reception at the time: "Nobody was interested" (as cited in Wang, 1996, p. 86).

A much longer (and more formal) version of the Yale talk was published, in German, in December of 1958. And it had such an impact that it became known among logicians as the "Functional interpretation" (Gödel, 1958/1990b; see Avigad & Feferman, 1995). That is also the basis for what we are calling the "neo-Gödelian" perspective, as it came to influence Piaget through the writings of Gödel's French-speaking contemporaries.

The Functional interpretation

Gödel's Functional interpretation was published at a time when interest in his early work was on the rise (via esp. Nagel & Newman, 1956, 1958). His purpose, however, was not to preempt the various misunderstandings that would emerge following its popularization.

Instead, he approached a quite different problem: *Where do the insights mathematicians rely upon in constructing new proofs come from?*

His solution is once again too complex to go into here in detail, but he introduced the effort simply enough: “in the proofs we make use of insights... that spring not from the... properties of the sign combinations representing the proofs, but only from their *meaning*” (Gödel, 1958/1990b, p. 241; emphasis as in the original). In this view, proofs do not result from the arrangements of the letters and numbers and symbols which comprise them. Nor are they a function of the contents of a truth table, in which a result could be looked up. (Such a result would be “trivial.”) Instead, the elegance of proof-making is a function of the theorist’s competence in *manipulating* the implications *entailed* by those symbols; by *the theorist’s ability at transforming* the symbols’ signified relations, using accepted operations, to say something *new and non-obvious*. In other words, from this perspective, *success* in the genesis of mathematical knowledge is a function of the mathematician’s *understanding*. It is not a function of the *symbols* themselves.

This is interesting in itself, and Piaget’s work certainly did move in that direction in the first experimental works of the New Theory period (esp. Piaget, 1974/1978). But Gödel’s intent was different: to formalize this fundamental mathematical concern, and thereby make the meaning of “meaning” more rigorously examinable.

To do this, Gödel relied on the notion of “recursion” to distinguish between levels of complexity in mathematical explanation: a proof requiring one operation is simpler than one requiring a transformation of that operation. (Similarly: A statement that relies on recursive self-reference is more complex than one that doesn’t.) “Meaning,” from this perspective, could thus be understood as that *abstraction* which is reflected down through the levels of this many-layered system, from the nuanced understanding of the constructing mathematician to constrain the elegance-in-formalization of the final proof. The result is so, therefore, because it *must be* according to what it *says* and *also* because of what it *means*. Once constructed, the proof then simply *is*: it exists, whether or not there is a formal system to make sense of it.

In contrast to earlier logicist conceptions of the activities involved in proof-making, Gödel’s Functional interpretation implies that the necessity of results is constrained both “bottom-up” (by the denotation of the given symbols) *and* “top-down” (by the understanding, competence, and insight of the proof-making mathematician). In other words: meaning-*signification* is projected upward, while meaning-*implication* is projected downward.³ All proofs thus exist in a middle realm, between the structures which comprise them (having signification) and the functions that they have (having implication).

Piaget’s later works contain elements of all of this. Indeed, it is our contention that the two approaches are intimately related: The “neo-Gödelian” insight of the Functional interpretation—that there are levels of relative incompleteness, and that these exist in a hierarchy—informed Piaget’s reconstruction of his “standard” theory at a new level with greater scope. But without direct evidence of contact between Piaget and Gödel’s Functional interpretation, how can such a claim be supported?

Archaeology and its results

To conduct a Foucauldian archaeology is, in broad strokes, to adopt an agnostic approach to history. Instead of bringing our own interpretive lenses to the events as we perceive

them to have occurred, the following analysis has been guided simply—as it were—“by the fact that words have happened, that these events have left traces behind them” (trans by Gutting, 1989, p. 228). These words have thus been excavated, laid out, explained as clearly as possible, and presented as the product of an evolving “discourse” (Gutting, 1989, pp. 244–245). Of course, because there can’t be a discourse without discussants, we have also traced the influence of specific individuals as well. For reasons of space, however, only the key figures of the neo-Gödelian excavation are introduced (see also Ratcliff, 2016). And, once again, we must begin at the beginning: with the “groupings” that provide the basis for Piaget’s stages.

Back to “grouping”

Piaget’s theory of grouping originated in his earliest studies: in around 1914–1915, when he was in his late teens, he realized—while half-paying attention in a course on logic taught by Arnold Reymond (1874–1958)—that the coherence of biological species as “wholes” could be defined as being a consequence of the relationship between their individual “parts.” He then began to generalize this insight to all systems with similar whole–part relationships. As he explained later in his autobiography:

I suddenly understood that at all levels (*viz.* that of the living cell, organism, species, society, etc., but also with reference to states of conscience [*sic*], to concepts, to logical principles, etc.) one finds the same problem of relationship between the parts and the whole; hence I was convinced that I had found the solution. ... In all fields of life (organic, mental, social) there exist “totalities” qualitatively distinct from their parts and imposing on them an organization. Therefore there exist no isolated “elements”; elementary reality is necessarily dependent on a whole which pervades it. (Piaget, 1952a, pp. 241–242)

This, he felt, could provide the basis for a scientific epistemology: if knowledge claims could be treated as formal wholes, in the same way as could the biological species examined to that point in his early studies,⁴ then strong logical tools could be used in the examination of different conflicting knowledge claims.

Coupled with the training he subsequently received in clinical interviewing at Zurich in 1918, this insight is the source of his interest in grouping children according to their own justifications: *why* the Parisian children he examined in 1919–1921 responded to his test questions in particular ways, rather than *whether their answers were correct*. It also thereby provided the basis for his “standard” theory of stages: different groupings of children, at different ages, justify their knowledge claims in different ways that are nonetheless coherent relative to each other. The later *Traité de Logique* (1949) then attempted to formalize these results more completely. That is also where Gödel is mentioned in Piaget’s writings for the first time.

A misleading appeal

In the *Traité*, a “grouping” (*groupement*) is defined formally as the theoretical intermediary between a “mathematical group” (*groupe*) and a “mathematical lattice” (*réseau*). It was therefore intended as a way of recognizing and preserving the relationship between a whole and its parts (Piaget, 1949, pp. 91–103). This combination enables several parts

to be grouped as wholes at different levels, each of which is then coherent and separate from the other, but with lower levels subsumed by the functional “operations” of the higher levels.

This move is similar to Russell’s (1903, 1908) replacement of Frege’s “general set” with a hierarchy of set-types, as a way to avoid self-referential contradictions like the Liar Paradox. That was then carried into the *Principia Mathematica* (Russell & Whitehead, 1910–1913). Through this it had widespread influence, although it’s also worth mentioning that Russell (1922) made a similar yet differently influential move in his introduction to the English translation of Wittgenstein’s *Tractatus*. And that also had important consequences. As he put it there: “Even if this very difficult hypothesis should prove tenable, it would leave untouched a very large part of Mr. Wittgenstein’s theory, though possibly not the part upon which he himself would wish to lay most stress” (p. 19). But a simpler way to explain the result is to refer back to Piaget’s early background and training in biology.⁵

Just as a *species* is composed of individual organisms, collected together in a group, so too is the next higher level up (genera) composed of individual species. And so it goes: up from species and genera to families, orders, classes, phyla, and kingdoms (extending up still further to *life* and even to *existence*). This is then logically coherent, relative to the definition of a grouping, because each rank in the taxonomic hierarchy can be formalized in ways that are consistent with Gödel’s original *completeness* theorem: a species is complete, for example, because its members form an inter-breeding (intra-translatable) functional whole. Higher taxonomic levels are also complete in the same way because they are relatable by reference to their evolutionary history (as an inter-generational lineage of structural transformations), while at the same time remaining qualitatively (functionally) distinct in the present: by definition, couplings between members of different species produce no fertile offspring (cf. “vicariance” in Note 10). Piaget’s approach in the *Traité* (1949) thus attempted to provide the formal means to describe complex nested wholes within a single interconnected *structure d’ensemble des parties* (typically translated as “structure-of-the-whole,” or “structural whole,” but more appropriately rendered as “power set”; Campbell in Piaget, 1977/2001, p. 122n).

The philosophical challenge posed by this formulation is in determining its relationship to Piaget’s later constructivism, especially given what he came to say about Gödel (in the epigraph). Although Gödel is indeed mentioned in the *Traité de Logique* (1949; and referred to again in a similar way in the three-volume *Introduction à l’épistémologie génétique*; Piaget, 1950), it seems clear from those texts that Piaget had not yet digested the implications of Gödel’s *incompleteness* for groupings: early on, Gödel is for Piaget simply a footnote in the history of logic. As a result, there is no formal support within this conception of grouping for species-change—or indeed stage-change—except by forbidden Lamarckian mechanisms, which Piaget had indicated at the time as being a source of discomfort (1952a, p. 241).

The problem with this early version of the theory, then, is this: although the children’s different groupings of knowledge justification could be shown to exist empirically, at different age ranges, there was no biological explanation for developmental change between the groupings except by the old Haeckelian doctrine of “recapitulation” (as Piaget was criticized by, e.g., Gould, 1977, pp. 144–147). From this perspective, earlier

stages of evolution are complete wholes through which development simply proceeds; they are given, a priori. And while this could indeed move children through the required evolutionary-developmental lineage, in the sense of recapitulating different “species of mind” at different “stages of development,” it did not have associated with it an acceptable constructive cause (aside from “re-equilibration,” which remained inadequately defined until after the neo-Gödelian turn).⁶

That said, however, there was indeed a possible non-constructive cause: “maturation.” And this explains the initial impression by several Americans, during the “rediscovery” period, that Piaget’s works were “neo-maturationist” (noted by Piaget in Evans, 1973, p. 39; also in Voyat, 1982, p. xiii). Yet that is only really a problem if species, and thus also stages, are *real* in a strong sense. While this seems to be required by Piaget’s logic, that interpretation is undermined by his biology.

As Piaget (1952a) had noted in his autobiography, he was a “nominalist” when it came to species definitions. (A species is what one *calls* a grouping of intra-translatable organisms, not what that grouping *is*: “The ‘species’ has no reality in itself and is distinguished from the simple ‘varieties’ merely by a greater stability,” p. 241.) Yet this perspective conflicts with the fundamental assumption of the logical model to which he appealed: hierarchical “types” are separate and distinct. They are formally *real*: they exist independently from our thinking about them, like Platonic objects. And that’s how Gödel conceived of them (see Davis, 2005; van Atten & Kennedy, 2003). But that’s not how Piaget used them. Instead, for Piaget, even the a priori categories of experience are constructed. This is a fundamental assumption of his research program (see esp. Piaget, 1925, 1950, 1968/1970c).

In short, it seems that it was Piaget’s *appeal* to logic in subsuming both his original biological interests and the results of his subsequent psychological experimentation to a larger epistemological structure—and specifically the reference to “groups” and “lattices” in defining *groupement*—that was misleading: as meta-theory it departed from his avowed “nominalism,”⁷ and introduced hard separations where he needed smooth gradations. This is because it is only through the continuity of species that it would be possible to “move” from one grouping to another on the basis of assimilation and accommodation alone. Therefore, from this inconsistency we see that Piaget’s early logic was incompatible with his intent. Indeed, as he recognized explicitly much later, his formalisms needed to be “clean[ed] up” (as cited in Piaget & Garcia, 1987/1991, p. 157; see also Piaget, 1971d).

Beth and the changing formal discourse

We can now consider dismissing claims that Piaget’s early attempts at formalism were anything more than a tool for thinking with. As he put it later: philosophizing in this way is a source of “wisdom,” to be sure, but also one of “illusions” (Piaget, 1965/1971c). Reflection must be tempered by engagement with reality, as well as with the facts of historicity and development. Hence, his life-long insistence—referring to, among others, James Mark Baldwin (1861–1934)—on the need for a “genetic [constructive] logic” (e.g., Piaget, 1928/1977/1995a, p. 184; 1970/1972, p. 15). This, however, was not a concern shared by the logicians who read his work.

Evert Beth (1908–1964), for example, lampooned him: the big book that formalized these early proposals—*Traité de logique* (Piaget, 1949)—was called “mediocre” and “negligent.” Worse, Beth described it as redolent with “failures hidden by pretenses of technicality capable of impressing only a reader naïve in logic” (Beth, 1950, p. 258, author’s translation). Indeed, this was a common reaction among professionals. Willard van Orman Quine (1908–2000) even referred, in correspondence with Beth, to “Piaget’s persistent and evidently incorrigible stupidity over matters of logic” (Quine, 1960; see also 1940a, 1940b).

Still, Piaget’s response was conciliatory: recognizing the failure, he invited Beth to collaborate on a project aligning their perspectives (Piaget, 1951, p. 244). This led to a sustained correspondence (Heinzmann et al., 2014). And that in turn led Beth—and ultimately Quine too—to join the International Center for Genetic Epistemology as a member of Piaget’s advisory board (Burman, 2012, p. 284).

Beth participated in symposia at the Center in 1956, 1959, and 1960 (Beth & Piaget, 1961/1966, p. 5). They also undertook a series of writing projects together. These then informed an important book: *Mathematical Epistemology and Psychology* (Beth & Piaget, 1961/1966).

In this book, Piaget’s appeals to Gödel followed Beth’s. Indeed, Beth wrote his entire first half of the book before Piaget wrote his own half in response (p. xxi). They then commented on each other’s work, revised their respective halves, and wrote the general conclusions together. In other words, after the failure of the *Traité* to achieve his aims, Piaget followed Beth’s lead in matters of logic.

In the final published text, Beth discussed the results of Gödel’s 1931 paper on incompleteness in great detail: he introduced the proof historically, albeit using more complex and rigorous terms than we have used here, and developed it following in the style of Gödel’s application of recursive self-referential meta-mathematical statements (Beth & Piaget, 1961/1966, pp. 54–55, 70, 120–122). He then generalized the results to all formal systems, showing that even a new system brought in to replace an earlier one (proven to be incomplete) will itself be incomplete in a different way.

This is not yet “neo-Gödelian” in the way we intend to mean the phrase, but it comes close: Beth cited Gödel’s (1944/1990a) first extended philosophical statement, which expanded upon his earlier critique of Russell and introduced into print the notions of “construction” that would later be formalized in the Functional interpretation (Beth & Piaget, 1961/1966, p. 112). He then pointed to work showing that unprovable truths could become provable—with their supervening system “rendered more adequate” (Beth & Piaget, 1961/1966, p. 122)—through the addition of new axioms. He also indicated that the resulting system would be “more powerful” (Beth & Piaget, 1961/1966, p. 59). Yet he was clear: one can never predict, in advance, what the innovations will be. Otherwise, they would already exist; a new proof would have been constructed. (Or, in biological terms, a new species introduced: It would exist, whether or not there is a taxonomic system to make sense of it; cf. Note 10.)

Ladrière and levels of relative incompleteness

In December of 1960, less than a year before the publication of Beth’s book with Piaget and almost exactly two years after the publication of Gödel’s Functional interpretation, a

French-speaking Belgian logician—Jean Ladrière (1921–2007)—published in the same journal a commentary in French examining the same sorts of problems discussed by Beth. And he began, as did Beth, by explaining Gödel’s result of 1931. But it is where Ladrière took the discussion afterward that seems to have pushed Piaget toward the neo-Gödelian perspective that came to characterize the New Theory.

There are two ways to read Gödel, explained Ladrière in 1960, referring only to “recent research” (p. 287).⁸ The first is to disabuse oneself of the notion that one fully understands the implications of the operations used in constructing any individual proposition. (This, as it happens, is consistent not only with Beth’s review but also with Piagetian methods; see Bond & Tryphon, 2009.) The second is to posit an open system with indefinite extension:

If this second way is taken, one is brought to envision an infinite, and even transfinite, hierarchy of systems. . . . On the first level, one can formalize a given domain; then, while grounding oneself on this first level, one can then formalize a larger domain, and so on. One can thus also go as far as one wants, but without ever arriving at an end. (Ladrière, 1960, p. 299, author’s translation)

This second response to the necessity of incompleteness, for Ladrière (and for Piaget’s subsequent understanding of Gödel—in the epigraph), was therefore the necessity of constructivism itself. Systems shown to be lacking at one level are simply reconstructed at a higher one. Ever larger in scope; ever broader in reach.

Piaget’s referrals to Ladrière

While we have not yet found any reference by Piaget to Gödel’s later works (there is only the rupture in Piaget’s use of Gödelian ideas), the earliest citation that we have found by Piaget to the Ladrière commentary is from a subsequent article that was published in the year of Beth’s death. Indeed, while arguing for the use of the limits of formalism as a means to bridge the gap between logic and psychology—and thereby also provide the means to “reintroduce an operator constructivism which refers . . . to the subject’s activities” (the original intent of the *Traité de Logique*)—Piaget referred specifically to “the great work by Ladrière” (Piaget, 1964/1971b, p. 135).

Piaget expanded on these initial comments in a large volume that he edited in response to what Beth had earlier called “the Gödelian crisis” (Beth & Piaget, 1961/1966, p. 53; see Piaget, 1967b, p. 8). There, he highlighted Ladrière’s suggestion that “formal systems are the abstracted objects [*objectivation*] of mental activity” (Ladrière, 1960, p. 321, author’s translation; cited by Piaget, 1967a, p. 378). And, more importantly for our purposes here, Piaget pointed explicitly to Ladrière—not Beth, although he is mentioned—as the source of the insight that *levels of relative incompleteness* must exist in a *hierarchy* (Piaget, 1967a, p. 383; citing Ladrière, 1960, 1967). He also repeated Beth’s comment about power, but in slightly different terms: In Piaget’s interpretation, each higher level is “stronger” than the last (e.g., Piaget, 1964/1971b, p. 146; 1967/1971a, p. 319; 1970/1972, pp. 67–68, 70, 90; 1977/1986, p. 307).

That said, however, the most significant discussion of Ladrière’s contributions is made in *Structuralism* (Piaget, 1968).⁹ Those passages are so revealing of the reasoning

informing Piaget's later works that I will provide an extended retranslation of the most relevant section. Then we will build, from there, toward a conclusion.

New translations from structuralism

Piaget began by blending the insights of logic with the facts of biology, integrating Ladrière's neo-Gödelian perspective into his earlier framework:

The first point of interest of such observations is that they introduce, into structures, the notion of greater or lesser strength and weakness (relative to the domains in which they are comparable). The hierarchy thus introduced therefore suggests the idea of construction, just as in biology the hierarchy of characters suggested evolution. Indeed, it seems reasonable that a weak structure uses more elementary means and that more powerful forces correspond to instruments whose development is more complex. (Piaget, 1968, p. 30, author's translation; cf. Piaget, 1968/1971e, pp. 33–34)

He continued, updating his earlier descriptions of how knowledge evolves:

The second fundamental lesson of Gödel's discoveries is ... that to complete a theory in the sense of demonstrating its non-contradiction, it is no longer sufficient to analyze its presuppositions. It has also become necessary to construct its replacement. Before this, one could justify believing in a lineage of theories as a kind of beautiful pyramid, resting upon a foundation of self-sufficiency; the lowest stage the most solid, since it had been formed of the simplest instruments. But if this simplicity itself becomes a sign of weakness, and reinforcing a stage requires the construction of its replacement, then the overall consistency of the pyramid as a whole is in reality suspended from its peak. With this height itself unfinished (and having to be unendingly high), the image of the pyramid must then be reversed and—more precisely—replaced by that of an upwardly broadening spiral. (Piaget, 1968, pp. 30–31, author's translation; cf. Piaget, 1968/1971e, p. 34)

In other words, the requirement for a firm (Hilbertian) foundation is here replaced. There need only be an initial spark—an inclination—followed by construction driven by exploratory behavior (see esp. Piaget, 1976/1979).

Piaget finished his exposition in *Structuralism* by abridging—and then finally citing—Ladrière's 1960 article. Here, again, the levels are made explicit:

The idea of the structure as system-of-transformations thus becomes interdependent with a constructivism of continuous formation. However, despite its general significance, the reason for this is simple: one can draw from Gödel's results important considerations regarding the limits of formalization. One can show, for example, the existence of levels—in addition to the formal levels—that are distinct levels of semi-formal and semi-intuitive knowledge ... that are awaiting, so to speak, their turn at formalization. The frontiers of formalization are thus mobile, or "vicariant,"^[10] and are not closed once and for all like a wall marking the limits of an empire. J. Ladrière proposed the ingenious interpretation: "we cannot survey, at one glance, all the operations possible of thought." (Piaget, 1968, p. 31, author's translation; citing Ladrière, 1960, p. 321; cf. Piaget, 1968/1971e, pp. 34–35)

From this perspective, Piaget's view of children is not unlike Gödel's view of mathematicians: novelty emerges from the projection of meaning, and the results take on a life of their own according to the logic extant at the levels they pass through along the way. Note, too: we can't always predict the outcome, even if we can see all the inputs.

Implications for the standard theory

In the sequel to *Structuralism*—a short volume published in the same series, and translated as *The Principles of Genetic Epistemology* (Piaget, 1970/1972)—we begin to see the emergence of the New Theory's version of the well-known stages of child development. Here, though, the hierarchy of levels acquires traits associated with both “form” and “content”:

Sensorimotor structures are forms in relation to the simple movements they coordinate, but content in relation to the actions that are internalized and conceptualized at the next higher level. Similarly: “concrete” operational structures are forms in relation to sensorimotor actions, but content in relation to the formal operational level of 11–15 years. And these are in turn just content with respect to operations acting on them from still-higher levels. Likewise, in the example given by Gödel, elementary arithmetic is a form that subsumes as content the logic of classes and relations ... and it is itself content ... from the perspective of transfinite arithmetic. (Piaget, 1970b, p. 84, author's translation; cf. Piaget, 1970/1972, pp. 67–68)

Concrete operational reasoning—thinking about objects that can be felt and acted upon as parts and wholes, like the water in a glass—is therefore made possible by the sensorimotor and pre-operational structures that come before it: reaching, grasping, manipulating, etc. Similarly, formal operational reasoning about “imaginary objects” (like those involved in mathematics) cannot be achieved without first inventing the notion of “an object” that it relies upon and refers to.

Piaget expanded on similar themes in his discussion of a talk delivered by Ladrière at the International Center for Genetic Epistemology in 1970 (Ladrière, 1973). There, he explained, Ladrière had mentioned two other things that were of particular interest. The first relates to this notion of possibility *made possible* by past constructions:

in the deductive branches of knowledge, one always finds “opacities” to illuminate, whether as a result of paradoxes or—more generally—because, once a system is constructed and has become internally consistent, it remains to find the reason for its existence and for its global [higher-level] properties. (Piaget, 1973, pp. 215–216)

Indeed, for Piaget, the discovery of such “opacities” is an invitation for enlightenment. More importantly, though, it was also after this that the pursuit of such possibilities came to serve in partnership with the dictates of necessity as a key driver of exploration-without-end (Piaget, 1977/1986; citing Ladrière, 1973, pp. 55–56).

In the second highlight, we see this insight extended to explain how new knowledge—nominally, “an explanation” (but treated functionally, in the abstract, as a *reflection producing desired outcomes*)—is actually constructed:

Ladrière's response is doubly instructive with regard to what we call reflecting abstraction (that which proceeds from operational coordination and not from objects) and the reciprocal assimilation between superior [higher-level] and inferior [lower-level] structures ... Explanation consists in deriving from the preceding structure that which is reorganized on a higher plane, while also enriching it by disengaging what it contained implicitly and then reassembling those functions in a new structure. (Piaget, 1973, p. 216, author's translation)

This insight was then developed further in a book on exactly this topic: *Studies in Reflecting Abstraction* (Piaget, 1977/2001).

We end our discussion of what Piaget learned from Ladrière's neo-Gödelian interpretation of incompleteness by referring to how he used these ideas to make the connection back to biology and psychology:

The relationship between two levels is neither one of reduction from ulterior [farther] to anterior [closer] nor of simple subordination from one to the other [lower to higher], but is rather one of reciprocal assimilation. ... The explanatory relationship between the superior [higher-level] system and the inferior [lower-level] system is a reciprocal assimilation in the sense, not—of course—of identification, but of mutual dependence, and thus of a sort of integration according to the biological or psychological significance of the result. (Piaget, 1973, p. 216, author's translation)

In short: the formal processes are the same, for mind and body, but they have their action each at their own level. These levels also complete each other, and make each other possible, such that the level of biology is as necessary to that of the mind as the mind is to that of knowledge (see esp. Piaget, 1967/1971a; Piaget & Garcia, 1983/1989).

Conclusion

Before the publication of the Functional interpretation, and the subsequent commentaries by Beth and Ladrière, Gödel was to Piaget simply a footnote in the history of logic. (Gödel's results on *completeness* supported Piaget's use of truth tables, but were no more interesting than that.) Afterward, however, changes in the meaning of *incompleteness* provided the means to reconstruct the Genevan research program at a new level. And it was outward from there that the New Theory was built.

That said, of course, it is important to note that Piaget delighted in his role as "one of the chief 'revisionists of Piaget'" (Piaget, 1968/1970c, p. 703n). We are, in other words, discussing change within a lineage. There is therefore no hard or transcendent *philosophical* rupture here, despite what is implied by Beilin's (1992) use of the term "new" in referring to the New Theory. Yet there is indeed an *historical* rupture, in the sense meant by Foucault (1969/1972). And that is why it was examinable using an "archaeological" method: tracing evidence through historical layers, identifying the boundaries, and seeking influences that can be named.

Curiously, though, the result of identifying the neo-Gödelian turn is that we now see that the New Theory is not really a theory of "stages" at all. Rather, it is a theory of constructive processes in which stages play a role as part of the hierarchy of levels but—being potentially infinite in scope and extension—lose their interest as a fundamental

feature of the theory. Indeed, from this perspective, the New Theory is not at all about what children can't do at different stages of their development. It is rather about what happens before, during, and afterward.

Piaget explained this in an interview, but assuming a background that has until now remained implicit:

Too many people take the theory of stages to be simply a series of limitations. That is a disastrous view. The positive aspect is that as soon as each stage is reached, it offers new possibilities to the child. There are no "static" stages as such. Each is the fulfillment of something begun in the preceding one, and the beginning of something that will lead on to the next. (Piaget & Duckworth, 1973, p. 25)

Of course, this is likely to be something of a surprise. But it can be made consistent with what we know by referring to the results of our archaeology.

Reflecting on Piaget's nominalism, we can now suggest that "stages" might always have been little more than a name for something much more dynamic and complex than the name itself implied. This is of course a philosophical claim, rather than an historical one; akin to Piaget's own musings about the meaning of "species" (see Vidal, 1992). Still, it follows that a constructive mechanism was always underneath the descriptive label. And the neo-Gödelian turn seems to provide such a mechanism, at least formally. From this, though, it also follows that it's the *structures* that count. And so the insights of the neo-Gödelian turn apply not only to cognitive functions like *memory* (Piaget & Inhelder, 1968/1973), but also to *affect* (Piaget, 1970/1976, 1954/1981) and *the will* (Piaget, 1962). What, then, is the result?

We are left with a relatively simple theoretical statement: change is driven by the assimilation of the world to structures, followed by the accommodation of those structures when they are found to be incomplete. This is a minor variation of the old assimilation–accommodation–equilibration story that is presently reflected in textbooks. But then the New Theory adds a new layer of complexity: change occurs within and across levels of decreasing relative incompleteness and increasing scope, resilience, or power (see esp. Piaget, 1980).

That said, however, children—or, more generally, "developing knowledge users"—don't *experience* this as a kind of Popperian conjecture and refutation. This is because they *assume* their structures are complete (i.e., they are "egocentric"; see Kesselring & Müller, 2011). Proofs of incompleteness must therefore be delivered in a "relevant" way; they need to be "meaningful," according to how this has already been defined in the extant context. And that can now be characterized, formally, according to how found incompleteness relates to the extant levels from which structures derive their significations and their implications (see esp. Piaget & Garcia, 1983/1989, 1987/1991; Piaget & Henriques, 1978; Piaget et al., 1990/1992).

This has important repercussions. Among other things, we come to see "the social" in a different guise: it's just another level (Burman, 2013b). And while even mentioning this may sound like it extends our discussion beyond the limits of what Piaget actually said, that is not the case (Burman, 2015). Indeed, as Piaget put it explicitly during the neo-Gödelian turn: "society is the supreme unit, and the individual can only achieve his

intentions and intellectual constructions insofar as he is the seat of collective interactions that are naturally dependent, in level and value, on society as a whole” (Piaget, 1967/1971a, p. 368; also Piaget, 1977/1995b). The turn toward “logics of meaning” that is considered to characterize the New Theory can thus now be reinterpreted as a way of modelling this necessary context dependence. Yet these are, in large part, new issues; further research is clearly required.

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Notes

1. For additional biographical details, see esp. *Logical Dilemmas: The Life and Work of Kurt Gödel* (Dawson, 1997). Each of the articles cited from Gödel’s *Collected Works* also have introductory notes, which often provide more useful specifics about any individual piece than the glosses provided elsewhere.
2. Gödel explained this directly to Hao Wang, who reproduced the explanation in *A Logical Journey* (Wang, 1996, pp. 82–83).
3. We are indebted to Tyson Gofton for highlighting this crucial distinction, and to Jagdish Hattiangadi for helping us to understand it.
4. For more on Piaget’s early life, see Vidal (1994), Perret-Clermont and Barrelet (1996/2008), and Ratcliff (2010). On his intellectual influences, see esp. Ducret (1984).
5. The most comprehensive source on Piaget’s biology is Messerly (1996, 2009).
6. Piaget (1975/1985) noted that *Equilibration of Cognitive Structures* represents “a complete reworking” (p. xvii) of the models presented in an earlier book: *Logique et équilibre* (Apostel, Mandelbrot, & Piaget, 1957). The discourse examined here is therefore part of how that earlier material came to be describable as “inadequate.”

7. Beth (1960) later reminded Piaget of the importance of this nominalism in a letter that discusses, among other things, both Gödel and constructivism (p. 2; citing Chapter 16 of Beth, 1959).
8. Ladrière (1960) doesn't cite Gödel's Functional interpretation directly (he cites nothing directly in that piece and includes no references), but it seems likely—since his own article was also published in *Dialectica*—that this paper is included in the “recent research” to which he did refer (p. 287). The reference was then made explicitly later (see Ladrière, 1981, pp. 298–299).
9. This book also contains the only reference I have found by Piaget to Nagel and Newman's (1958) popularization of Gödel's theorem (p. 33n). Yet that reference was added by the translator. And because the change in Piaget's appeals to Gödel precedes this usage, it would be inappropriate to suggest that it was they who caused it.
10. “Vicariant” is erroneously translated as “vicarious” in the English edition. This is a regrettable error, and one repeated in another New Theory-period work that would also otherwise be extremely important (Piaget, 1977, pp. 353–356). But we can make the correction: it is properly to be understood as an allusion to the evolutionary process in which a new species first emerges from an earlier grouping: “vicariance,” such that—prior to the moment of speciation—members from either proto-grouping can be substituted for each other without functional impact (see also Piaget, Henriques, & Ascher, 1990/1992, pp. 45n, 54–57, 91–109, 224–226).

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