SUMMARY.

In the present thesis the diffraction theory for arbitrary aberrations of a symmetrical optical system is developed for the case that the amount of aberration is small.

We introduce a spherical surface \( U \), passing through the centre of the exit-pupil and with its centre at the Gaussian image point (which is situated at the radial distance \( a \) from the optical axis). The Gaussian image is chosen as the origin of a rectangular system of axes in the image space (Ch. I, § 1). The position of a point on \( U \) is specified by the plane polar coordinates \( r \) and \( \varphi \), \( r \) measuring the radial distance to the principal ray divided by the radius \( a \) of the exit-pupil, so that \( r \) varies from 0 to 1. For a system without aberrations \( U \) is a wave surface for a pencil converging to the Gaussian image.

The deviation of the actual wave front from \( U \) is then represented by the aberration function \( V(\sigma, r, \varphi) \). Whereas \( V \) or the closely allied characteristic function of Hamilton is usually expanded in a series of ascending powers of the three rotational invariants \( \sigma^2, r^2 \) and \( \sigma r \cos \varphi \), we have followed a different way by expanding \( V \) as a series of the so-called circle polynomials \( R_n^m(r) \cos m\varphi \), which were introduced by Prof. Zernike in a problem closely related to the one treated here. The circle polynomials form a set of functions orthogonal for the interior of the unit circle, \( R_n^m(r) \) is a polynomial in \( r \) involving the terms \( r^n, r^{n+1}, \ldots, r^m \) (\( n \) and \( m \) are integers \( \geq 0 \), while \( n > m \) and \( n - m \) is even). In Ch. II they are investigated in some detail.

Consequently, whereas the general term in the customary expansion, defining a single aberration, is given by

\[
 b_{lmn} \sigma^{l+m} r^n \cos^m \varphi
\]  

(1)
a single aberration is defined now by the aberration function

\[
 b_{lmn} \sigma^{l+m} R_n^m(r) \cos m\varphi
\]  

(2)

where \( l \) is an integer \( \geq 0 \).

For the diffraction theory of aberrations we find the new definition to have many advantages. So, for instance, the problem of the counter-balancing of aberrations of various orders can be solved...
now completely, for, owing to the orthogonality of the circle polynomials, the “definition” (Definitionshelligkeit) in the presence of aberrations is found to be smaller than the ideal value 1 by a sum of positive terms, each corresponding to a single aberration as defined by (2) (Ch. I, § 4). Thus, the addition of a low order aberration to a high order one always decreases the definition.

In Ch. III, § 1 we investigate the geometrical aberration figure connected with a single aberration, defining the latter by an expression of the form

\[ b_{lnm}\rho^{2l+m}r^n \cos m\varphi \]  

which contains the factor \( \cos m\varphi \) instead of \( \cos m\varphi \) in the usual definition. This modification is found to lead to a considerable simplification. The introduction of the polynomials \( R_{n}^{m}(r) \), on the contrary, seems to be useful only for the diffraction treatment. Hence, our geometrical considerations are based on the definition (3).

In Ch. III, § 2 the classification of aberrations is discussed on the basis of the definitions (2) or (3) for a single aberration. The value of \( m \) determines the general type to which a particular image error belongs (spherical aberration, when \( m = 0 \); coma, when \( m = 1 \); astigmatism, when \( m = 2 \) etc.), while the aberrations belonging to the same type can be distinguished by the value of \( n \). The index \( l \) governs the dependence on the position in the field, but does not affect the appearance either of the geometrical aberration figure or of the diffraction pattern.

In Ch. IV we examine the diffraction pattern connected with an arbitrary single aberration (2). The application of Kirchhoff’s principle leads to a diffraction integral which is expanded in a series of ascending powers of the aberration coefficient \( b_{lnm} \). From the general expansion (4, 18) the properties of symmetry of the diffraction pattern are deduced. Due to the remarkable relation (2, 20), viz.

\[ \int_{0}^{1} R_{n}^{m}(r)J_{l}(qr)rdr = (-1)^{\frac{n-m}{2}} q^{-l}J_{n+l}(q) \]

the intensity distribution in the diffraction pattern may be numerically determined without much labour, provided the amount of aberration is small. For a receiving plane through the origin of co-ordinates the result reduces to the relatively simple expansion (4, 20).

In Ch. V some interesting cases are considered in more detail.
Orthogonality of the circle (Schellingkeit) in the presence of the ideal value 1 by a non-linear growth to a single aberration as an addition of a low order decreases the definition. Geometrical aberration figure defining the latter by an addition of a low order decreases the definition.

\[ \cos^n \varphi \] (3)

Presented is the usual lead to a considerable polynomials \( R_m(r) \), on the basis of the diffraction treatment. Based on the definition (3), aberrations is discussed on a single aberration. The pattern to which a particular image \( m = 0 \); coma, when \( m = 1 \); the aberrations belonging to the value of \( n \). The index \( l \) in the field, but does not geometrical aberration figure or pattern connected with an application of Kirchhoff's formulae which is expanded in a aberration coefficient \( b_{\text{aberr}} \). From properties of symmetry of the to the remarkable relation

\[ \sum_{m=0}^{\infty} q^{-m} J_{m+1}(q) \]

The pattern may be numerically provided the amount of line through the origin of relatively simple expansion considered in more detail.

The results of the numerical computations are given in diagrams showing the lines of equal intensity. The following cases have been dealt with:

1°. Astigmatism, represented by \( b_{\text{ast}} \sigma^{m+3} R_{\varphi}^2(r) \cos 2\varphi \), in the mid-astigmatic plane for 2 values of the aberration coefficient (fig. 6, p. 54 and fig. 7, p. 55).

2°. Astigmatism in the plane through the horizontal focal line (fig. 8, p. 60).

3°. Coma, represented by \( b_{\text{comp}} \sigma^{m+3} R_{\varphi}^2(r) \cos \varphi \) in the Gaussian image plane (fig. 9, p. 62).

On comparison with the results of Steward, which were obtained on the basis of the customary expansion (1), it would seem that neither case numerically evaluated by him is correct. This may be a consequence of the intricacy of his formulae which hardly permit calculation of the intensity in a sufficient number of points.