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Bergshoeff, E.; Roo, M. de

Published in:
Physics Letters B

DOI:
[10.1016/0370-2693\(96\)00523-0](https://doi.org/10.1016/0370-2693(96)00523-0)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1996

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Bergshoeff, E., & Roo, M. D. (1996). D-branes and T-duality. *Physics Letters B*, 380(3-4), 265 - 272.
[https://doi.org/10.1016/0370-2693\(96\)00523-0](https://doi.org/10.1016/0370-2693(96)00523-0)

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11 July 1996

PHYSICS LETTERS B

Physics Letters B 380 (1996) 265–272

D-branes and *T*-duality

E. Bergshoeff, M. de Roo

Institute for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

Received 23 March 1996

Editor: P.V. Landshoff

Abstract

We show how the *T*-duality between *D*-branes is realized (i) on *p*-brane solutions ($p = 0, \dots, 9$) of IIA/IIB supergravity and (ii) on the *D*-brane actions ($p = 0, \dots, 3$) that act as source terms for the *p*-brane solutions. We point out that the presence of a cosmological constant in the IIA theory leads, by the requirement of gauge invariance, to a topological mass term for the worldvolume gauge field in the 2-brane case.

1. Introduction

Recent developments in string theory have shown that *p*-brane solutions and duality symmetries play an important role in understanding the nonperturbative behaviour of the theory. An important example of a duality symmetry is the *T*-duality [1] which states that a string compactified on a torus with radius *R* is equivalent to a string compactified on a torus with radius α'/R where α' is the inverse string tension.

It turns out that the *p*-brane solutions whose charge are carried by a RR (Ramond/Ramond) gauge field of the type II supergravity theories have a natural place within open string theory as *D*-branes [2]. The relation is established via the requirement that the end-points of the open string are constrained to live on the $p + 1$ -dimensional worldvolume of the Dirichlet *p*-brane. Such a (ten-dimensional) open string state is described by Dirichlet boundary conditions for the $9 - p$ transverse directions and Neumann boundary conditions for the $p + 1$ worldvolume directions. Since under *T*-duality Dirichlet and Neumann boundary conditions are interchanged it follows that all Dirichlet

p-branes ($p = 0, \dots, 9$) are *T*-dual versions of each other. A discussion of how this *T*-duality between *D*-branes arises in string theory can be found in the recent review article [3].

Since all *D*-branes are *T*-dual to each other it is natural to expect that this *T*-duality is also realized on the underlying *p*-brane solutions of the IIA/IIB supergravity theories. Furthermore, the *T*-duality should also be realized on the Dirichlet *p*-brane actions which act as source terms of the *p*-brane solutions. It is the purpose of this letter to give the details of this *T*-duality between Dirichlet *p*-brane solutions and their source terms and to point out a few subtleties that occur in establishing *T*-duality.

2. *D*-brane solutions

We first consider the *T*-duality between the Dirichlet *p*-brane solutions. In [4] it was pointed out that the *T*-duality between the RR *p*-brane solutions of $D = 10$ IIA/IIB supergravity, with $0 \leq p \leq 9$, is an almost immediate consequence of the *T*-duality rules of [5] which are a generalization to curved background of

the type II T -duality rules of [6,7]. The relationship between the 7-brane and 8-brane solutions turns out to be more subtle and it involves the introduction of modified, so-called *massive* T -duality rules [4].

We will be concerned with a class of solutions whose metric and dilaton (σ) for all values of p ($0 \leq p \leq 9$) is given by

$$ds^2 = H^{-\frac{1}{2}} d^2 s_{p+1} + H^{\frac{1}{2}} ds_{9-p}^2, \\ e^{-2\sigma} = H^{\frac{1}{2}(p-3)}, \quad (1)$$

where $d^2 s_{p+1}$ is the Minkowski $(p+1)$ -metric on the worldvolume and ds_{9-p}^2 is the Minkowski $(9-p)$ -metric on the transverse space. The function H only depends on the transverse coordinates and is harmonic with respect to these variables. The dilaton σ is equal to the dilaton ϕ (φ) of IIA (IIB) supergravity depending on whether p is even or odd:

$$\sigma = \phi \text{ (IIA dilaton) } \quad p \text{ even,} \\ \sigma = \varphi \text{ (IIB dilaton) } \quad p \text{ odd.} \quad (2)$$

We use here the same notation and conventions as [4,5]. The fact that the RR p -brane solutions for $0 \leq p \leq 6$ are of the form (1) was shown in [8]. Each of the above RR p -brane solutions has a nontrivial RR (electric or magnetic) charge carried by one of the RR gauge fields. We will first discuss the duality of the metric and dilaton and after that the behaviour under duality of the corresponding RR gauge fields.

It is not too difficult to see that the class of metrics and dilatons given in (1) are transformed into each other under T -duality. Although the T -duality rules are complicated for general backgrounds they simplify for the class of solutions (1) which have a diagonal metric and a vanishing NS/NS 2-form gauge field. The relevant rules of the metric and dilaton are in this case given by¹

$$j_{\mu\nu} = g_{\mu\nu}, \\ j_{xx} = 1/g_{xx}, \\ e^{-2\varphi} = e^{-2\phi}|g_{xx}|. \quad (3)$$

¹ We only give here the rules that lead from a given IIA solution to a dual IIB solution. The inverse rules can be used to construct a dual IIA solution out of a given IIB solution.

Here g (j) is the IIA (IIB) metric. The coordinate x refers to the direction over which the duality is performed and the μ index labels the nine remaining directions.

Starting with the 0-brane, or particle solution, one first dualizes over one of the transverse directions, say x^1 , thereby assuming that the function H of the 0-brane solution is independent of x^1 and is harmonic over the remaining eight transverse directions $x^2, \dots, x^9$. Applying the duality rules (3) with respect to the x^1 direction leads to the 1-brane solution given in (1). Next, one assumes that the harmonic function corresponding to the 1-brane solution is independent of one of its transverse directions, say x^2 . Dualizing over x^2 then leads to the 2-brane solution. This process is repeated until one reaches the 8-brane solution. At this point the transverse space has become one-dimensional so that the harmonic function H only depends on one variable, say x^9 . In the last step one assumes that the dependence on x^9 disappears so that we end up with a 9-brane solution that is given by flat 10-dimensional Minkowski spacetime.

Next, we consider the behaviour of the RR gauge fields under T -duality. The RR gauge fields of the type IIA theory are given by $\{A_\mu^{(1)}, C_{\mu\nu\rho}\}$ while those of the type IIB theory are given by $\{D_{\mu\nu\rho\sigma}, \mathcal{B}_{\mu\nu}^{(2)}, \ell\}$. Again, the duality rules for a general background are complicated but simplify for the class of solutions we are considering. They are given by:

$$D_{\mu\nu\rho x} = \frac{3}{8} C_{\mu\nu\rho}, \\ \mathcal{B}_{\mu\nu}^{(2)} = \frac{3}{2} C_{\mu\nu x}, \\ \mathcal{B}_{x\mu}^{(2)} = -A_\mu^{(1)}, \\ \ell = A_x^{(1)} + mx. \quad (4)$$

Here m is the mass parameter of massive IIA supergravity [9].

Two remarks are in order. First of all, we have only given the duality rule of $D_{\mu\nu\rho x}$. The duality rule for $D_{\mu\nu\rho\sigma}$ follows from the self-duality condition

$$F(D)_{\mu_1 \dots \mu_5} = \frac{1}{120} \frac{1}{\sqrt{-j}} j_{\mu_1 \nu_1} \dots j_{\mu_5 \nu_5} \\ \times \epsilon^{\nu_1 \dots \nu_5 \rho_1 \dots \rho_5} F(D)_{\rho_1 \dots \rho_5}, \quad (5)$$

where $F(D) = dD$ is the curvature of D for vanishing NS/NS 2-form gauge fields. From this it follows that

we cannot determine the duality rule for $D_{\mu\nu\rho\sigma}$ but only of its curvature $F(D)_{\mu_1\cdots\mu_5}$. Secondly, the mass parameter in the last rule of (4) will only play a role in establishing the duality between the 7-brane and 8-brane solutions, as has been explained in [4] and will be discussed further below.

Using the duality rules (4) for the RR gauge fields, it is straightforward to get the expressions of the RR gauge fields (or curvatures) of *all* p -brane solutions, starting from the particle solution. We find the following expressions:

$$\begin{aligned}
 p = 0: & \quad A_0^{(1)} = H^{-1}, \\
 p = 1: & \quad \mathcal{B}_{01}^{(2)} = H^{-1} \\
 p = 2: & \quad C_{012} = \frac{2}{3}H^{-1} \\
 p = 3: & \quad D_{0123} = \frac{1}{4}H^{-1} \quad \text{or} \\
 & \quad F(D)_{ijklm} = \frac{1}{20}\epsilon^{ijklmn}H^2\partial_nH^{-1}, \\
 p = 4: & \quad G_{ijkl} = \frac{1}{6}\epsilon^{ijklm}H^2\partial_mH^{-1} \\
 p = 5: & \quad \mathcal{H}_{ijk}^{(2)} = -\frac{1}{3}\epsilon^{ijkl}H^2\partial_lH^{-1}, \\
 p = 6: & \quad F_{ij}^{(1)} = -\epsilon^{ijk}H^2\partial_kH^{-1}, \\
 p = 7: & \quad \partial_i\ell = \epsilon^{ij}H^2\partial_jH^{-1}, \\
 p = 8: & \quad m = H', \\
 p = 9: & \quad H = 1.
 \end{aligned} \tag{6}$$

Here the $0, 1, 2, \dots$ indices refer to the $(p+1)$ -dimensional worldvolume directions and i denotes the $(9-p)$ transverse directions. Furthermore, $F(D) = dD$, $G = dC$, $\mathcal{H}^{(2)} = d\mathcal{B}^{(2)}$ and $F^{(1)} = 2dA^{(1)}$ are the expressions for the curvatures in the absence of the NS/NS 2-form gauge field.

We should point out a few subtleties in obtaining the expressions (6). Starting from the particle, a straightforward application of the duality rules (4) leads one to the 3-brane. Note that all p -brane solutions with $0 \leq p \leq 2$ are electrically charged. In going from the 2-brane to the 3-brane we dualize over the x^3 direction and apply the first rule of (4) to obtain the expression for D_{0123} . The expression for $F(D)_{ijklm}$ is obtained by using the selfduality relation (5). Note that the 3-brane solution is dyonic: it carries both electric and magnetic charge. Next, in going from the 3-brane to the 4-brane solution we cannot apply the first rule of (4) the reason being that the duality is now performed over x^4 . Instead, we first write $C_{ijk} = 8/3D_{ijk4}$ and take the curl with respect to x^l which gives $G_{ijkl} = 10/3F(D)_{ijkl4}$. We next substitute for $F(D)_{ijkl4}$ the expression corresponding to the 3-brane solution. We

thus obtain the expression for G_{ijkl} given in (6). Note that this 4-brane is magnetically charged. The expressions for the magnetically charged 5-brane and 6-brane solutions are obtained by taking curls of the duality relations (4). The same applies to the transition from the 6-brane to 7-brane solution where we dualize over the x^7 direction, but note that by taking the curl of ℓ the m -dependent term in the duality rule drops out since we take the derivative with respect to x^i ($i = 8, 9$) of the equation $\ell = A_7^{(1)} + mx^7$. Having arrived at the 7-brane one cannot repeat the process again. The reason for this is that the independence of the harmonic function of the 7-brane solution of one of the two transverse directions, say x^8 , does *not* imply that the RR scalar ℓ is independent of x^8 . In fact, in this case the expression for ℓ is given by [4]

$$\ell = (\partial_9 H)x^8, \tag{7}$$

where x^9 refers to the second transverse direction. To go to the 8-brane we now perform duality over the x^8 direction and use the fact that in the massive T -duality rules given in (4) the RR scalar ℓ is allowed to depend linearly on the duality direction x^8 . This leads to the 8-brane solution of [10]. Finally, in the last step, we obtain 10-dimensional Minkowski spacetime as the 9-brane solution.

3. D -brane actions

Having established the T -duality between the Dirichlet p -brane solutions of IIA/IIB supergravity, we next turn our attention to the corresponding D -brane actions. Since the D -brane actions should provide for the source terms of the Dirichlet p -brane solutions we must be able to establish a duality between the D -brane actions as well.

In the recent literature [11–15] the structure of the (bosonic part of the) D -brane actions for $0 \leq p \leq 3$ has been established. It turns out that the coupling to the NS/NS background fields (i.e. the metric, dilaton and 2-form gauge field) is described by a kinetic term of the Born-Infeld (BI) type [16,17]. The action contains besides the usual embedding coordinates X^μ , an additional worldvolume gauge field V_i ($i = 1, \dots, p+1$). For $p = 1$ and flat background, such an action was proposed some time ago in an attempt to give a spacetime scale-invariant formulation of the

superstring [18]. In such a formulation the (dimensionful) string coupling constant arises as an integration constant in solving the equation of motion of the worldvolume gauge field. The D -brane actions contain also a Wess-Zumino (WZ) term that describes the coupling of the D -brane to the RR gauge fields as well as further couplings to the NS/NS 2-form gauge field.

We will show that the structure of both the BI kinetic term and the WZ term is determined by: (1) gauge invariance² and (2) T -duality. In fact, as we will see below, these requirements lead to extra terms in the type IIA D -brane action in case the IIA supergravity background contains a nonvanishing target space cosmological constant.

Since gauge symmetries play an important role in the present discussion, we first summarize the transformation rules of the different RR gauge fields and of the worldvolume gauge field. The (target space) transformations of the IIA gauge fields are given by

$$\begin{aligned}\delta A^{(1)} &= d\Lambda^{(1)} - \frac{m}{2}\eta^{(1)}, \\ \delta B^{(1)} &= d\eta^{(1)}, \\ \delta C &= d\chi + 2B^{(1)}d\Lambda^{(1)} - m\eta^{(1)}B^{(1)},\end{aligned}\quad (8)$$

while those of the IIB gauge fields have the form

$$\begin{aligned}\delta B^{(i)} &= d\Sigma^{(i)}, \quad i = 1, 2, \\ \delta D &= d\rho - \frac{3}{4}d\Sigma^{(1)}\mathcal{B}^{(2)} + \frac{3}{4}d\Sigma^{(2)}\mathcal{B}^{(1)}.\end{aligned}\quad (9)$$

The (target space) transformation of the worldvolume gauge field is given by

$$\begin{aligned}\delta V &= \frac{1}{2}\eta^{(1)} \quad (\text{IIA}, p \text{ even}), \\ \delta V &= \frac{1}{2}\Sigma^{(1)} \quad (\text{IIB}, p \text{ odd}),\end{aligned}\quad (10)$$

where the first rule is an abbreviation for $\delta V_i = 1/2\partial_i X^\mu \eta_\mu^{(1)}$.

As mentioned above, the D -brane action consists of two terms, a kinetic and a WZ term:

$$S^{(p)} = \int d^{p+1}\xi \left(\mathcal{L}_{\text{kin}}^{(p)} + \mathcal{L}_{\text{WZ}}^{(p)} \right). \quad (11)$$

² One can even show that the RR gauge fields cannot be coupled in a gauge-invariant way to the D -brane without the worldvolume gauge field. This provides another explanation for the presence of this gauge field.

For all p ($0 \leq p \leq 9$) the kinetic term is of the following BI-type:

$$\mathcal{L}_{\text{kin}}^{(p)} = e^{-\sigma} \sqrt{|\det(g_{ij} + \mathcal{F}_{ij})|}, \quad (12)$$

where g_{ij} is the embedding metric:

$$\begin{aligned}g_{ij} &= \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} \quad (\text{IIA}, p \text{ even}), \\ g_{ij} &= \partial_i X^\mu \partial_j X^\nu j_{\mu\nu} \quad (\text{IIB}, p \text{ odd}),\end{aligned}\quad (13)$$

and where σ for p even/odd is defined in Eqs. (2). The (gauge-invariant) curvature $\mathcal{F}$ of the worldvolume gauge field is given by

$$\begin{aligned}\mathcal{F} &= 2dV - B^{(1)} \quad (\text{IIA}, p \text{ even}), \\ \mathcal{F} &= 2dV - \mathcal{B}^{(1)} \quad (\text{IIB}, p \text{ odd}),\end{aligned}\quad (14)$$

where the first line is short-hand for $\mathcal{F}_{ij} = \partial_i V_j - \partial_j V_i - \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}^{(1)}$. The explicit dilaton coupling in front of the kinetic term (12) corresponds to the fact that the mass per unit p -volume of the D -brane scales as [19]

$$\mathcal{M}_p \sim \frac{1}{g}, \quad (15)$$

where $g \sim e^\sigma$ is the string coupling constant.

Before giving the WZ terms we first discuss the T -duality of the kinetic terms. We first show, as an example, how the duality works between the 0-brane and 1-brane action and next prove the duality for general p^3 . The kinetic terms are, respectively, given by⁴:

$$\mathcal{L}_{\text{kin}}^{(0)} = e^{-\hat{\phi}} \sqrt{\hat{X}^{\hat{\mu}} \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}}, \quad (16)$$

$$\mathcal{L}_{\text{kin}}^{(1)} = e^{-\hat{\phi}} \sqrt{|\det(\hat{g}_{ij} + \hat{\mathcal{F}}_{ij})|}, \quad (17)$$

where the embedding metric g_{ij} is defined by the second equation of (13).

We first consider the reduction of the 0-brane action to nine dimensions. Like in the previous section, we

³ For flat backgrounds, the T -duality between the kinetic terms has been considered in [20]. We thank P. Townsend for bringing this reference to our attention. For curved backgrounds the discussion below overlaps with the one given in [22], see Note Added.

⁴ Since we will show the duality by reduction to nine dimensions, it is necessary at this stage to distinguish between nine- and ten-dimensional objects. In the discussion below we will indicate the ten-dimensional indices with a hat.

dualize over the $x^{\hat{1}}$ -direction. Since, from the 0-brane point of view, this is a so-called direct dimensional reduction, we obtain after reduction a 0-brane action in nine dimensions where the $x^{\hat{1}}$ coordinate has become an extra worldline scalar S , i.e.⁵

$$X^{\hat{1}} = S. \quad (18)$$

Using the reduction formulae given in [4,5] the kinetic term for the nine-dimensional 0-brane action, coupled to the background fields of $D = 9$ supergravity $\{g_{\mu\nu}, C_{\mu\nu\rho}, B_{\mu\nu}^i, A_{\mu}^i, B_{\mu}, \phi, k, \ell\}$ ($i = 1, 2$)⁶, is given by

$$\begin{aligned} \mathcal{L}_{\text{kin}}^{(0)} &= e^{-\phi} k^{-\frac{1}{2}} \\ &\times \sqrt{\dot{X}^{\mu} \dot{X}^{\nu} (g_{\mu\nu} - k^2 A_{\mu}^{(2)} A_{\nu}^{(2)}) - k^2 \dot{S}^2 - 2k^2 \dot{S} \dot{X}^{\mu} A_{\mu}^{(2)}}. \end{aligned} \quad (19)$$

We next consider the reduction to nine dimensions of the 1-brane action. Again we reduce over the $\hat{1}$ -direction but in this case we call the corresponding embedding coordinate of the 1-brane $Y^{\hat{1}}$ to distinguish it from the 0-brane coordinate $X^{\hat{1}}$ used above. Since in this case we are performing a double dimensional reduction we set

$$Y^{\hat{1}} = \sigma, \quad (20)$$

where σ is the spacelike direction of the 1-brane worldvolume. It turns out that, in order to obtain *the same* nine-dimensional 0-brane action (19) we must compactify the worldvolume gauge field as follows:

$$\hat{V}_{\sigma} = S. \quad (21)$$

⁵ This is similar to the reduction of the eleven-dimensional membrane to ten dimensions [11]. A difference is that in the reduction of the eleven-dimensional membrane the extra scalar becomes, after dualization on the 3-dimensional worldvolume, a worldvolume gauge field. In the particle case the scalar remains a scalar. The role of this extra scalar in nine dimensions is however similar to the role of the worldvolume vector in ten dimensions. In the same way as the worldvolume vector is used to give a scale-invariant description of the string where the string tension arises as an integration constant, the extra scalar in nine dimensions can be used to give a scale-invariant formulation of the massive particle where the mass arises as an integration constant. The massive particle in nine dimensions can then be interpreted as a massless particle in ten dimensions. For more details, see [18].

⁶ The $D = 9$ NS/NS fields are $g, B^{(1)}, A^{(2)}, B, \phi$ and k .

The time component of V drops out of the action after double dimensional reduction.

To work out the dimensional reduction of the 1-brane action it is useful to first use the fact that for a 2×2 matrix we have the identity:

$$\det(g_{ij} + \mathcal{F}_{ij}) = \det g_{ij} - \det \mathcal{F}_{ij}. \quad (22)$$

We thus obtain that $\mathcal{L}_{\text{kin}}^{(1)}$ reduces to the following expression after dimensional reduction:

$$\begin{aligned} e^{-\phi} k^{\frac{1}{2}} \left[\det \begin{pmatrix} \dot{X}^{\mu} \dot{X}^{\nu} (g_{\mu\nu} - k^{-2} B_{\mu} B_{\nu}) & -k^{-2} \dot{X}^{\mu} B_{\mu} \\ -k^{-2} B_{\mu} & -k^{-2} \end{pmatrix} \right. \\ \left. + \hat{\mathcal{F}}_{01}^2 \right]^{\frac{1}{2}}. \end{aligned} \quad (23)$$

All B -terms in the determinant cancel. Using the fact that

$$\hat{\mathcal{F}}_{01} = \dot{S} + \dot{X}^{\mu} A_{\mu}^{(2)}, \quad (24)$$

it is then easy to show that indeed the 1-brane action reduces to the same nine-dimensional 0-brane action (19), thereby establishing the T -duality between the 0-brane and 1-brane action.

It is interesting to see that in order to establish T -duality the worldvolume gauge field component $\hat{V}_{\sigma}$ of the 1-brane gets identified with the 0-brane embedding coordinate $X^{\hat{1}}$ leading to the T -duality relation

$$X^{\hat{1}} = \hat{V}_{\sigma}. \quad (25)$$

Another noteworthy feature is that the T -duality only works in the presence of the dilaton coupling $e^{-\sigma}$ in front of the kinetic term. Thus, T -duality requires that the mass per unit p volume of the D -brane is proportional to the inverse string coupling constant.

Let us now prove T -duality between the kinetic terms for general p . We first consider p even, so that we establish the duality between a IIA p -brane and a IIB $(p+1)$ -brane. We establish duality by reducing both kinetic terms to $d = 9$, by direct and double dimensional reduction respectively, and by then showing that the resulting kinetic terms are equal.

As for the 0-brane, the transverse coordinate $X^{(p+1)}$ of the IIA p -brane becomes a worldvolume scalar: $X^{(p+1)} = S$, while the worldvolume vector reduces as $\hat{V}_i = V_i$. The kinetic term for the p -brane is then

determined by the following $(p+1) \times (p+1)$ -matrix $(i, j = 0, 1, \dots, p)$ in nine dimensions:

$$\mathcal{A}_{ij} = g_{ij}^A + \mathcal{F}_{ij}^A, \quad (26)$$

with

$$g_{ij}^A = G_{ij} - k^2 (A_i^{(2)} + S_i) (A_j^{(2)} + S_j), \quad (27)$$

$$\begin{aligned} \mathcal{F}_{ij}^A &= F(V)_{ij} - B_{ij} + B_i (S_j + A_j^{(2)}) \\ &\quad - B_j (S_i + A_i^{(2)}), \end{aligned} \quad (28)$$

where we have used the notation:

$$\begin{aligned} G_{ij} &= \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}, \\ B_{ij} &= \partial_i X^\mu \partial_j X^\nu (B_{\mu\nu}^{(1)} - A_{[\mu}^{(2)} B_{\nu]}), \\ A_i^{(2)} &= \partial_i X^\mu A_\mu^{(2)}, \\ B_i &= \partial_i X^\mu B_\mu, \\ S_i &= \partial_i S. \end{aligned} \quad (29)$$

For the IIB $(p+1)$ -brane we perform a double dimensional reduction. One of the worldvolume scalars is identified with a spacelike worldvolume coordinate: $X^{(p+1)} = \rho$, while the $(p+1)$ -component of V becomes a worldvolume scalar: $\hat{V}_{p+1} = S$, $\hat{V}_i = V_i$. The IIB $p+1$ -brane then gives in nine dimensions the following $(p+2) \times (p+2)$ -matrix:

$$\mathcal{B} = g^B + \mathcal{F}^B, \quad (30)$$

with

$$\begin{aligned} g^B &= \begin{pmatrix} G_{ij} - k^{-2} B_i B_j & -k^{-2} B_i \\ -k^2 B_j & -k^{-2} \end{pmatrix}, \\ \mathcal{F}^B &= \begin{pmatrix} F(V)_{ij} - B_{ij} & S_i + B_i \\ -(S_j + B_j) & 0 \end{pmatrix}. \end{aligned} \quad (31)$$

The definitions

$$\begin{aligned} U_i^\pm &\equiv S_i + A_i^{(2)} \pm k^{-2} B_i, \\ H_{ij} &\equiv G_{ij} + F(V)_{ij} - B_{ij} - k^{-2} B_i B_j, \end{aligned} \quad (32)$$

lead to the following form of $\mathcal{A}$ and $\mathcal{B}$:

$$\begin{aligned} \mathcal{A}_{ij} &= H_{ij} - k^2 U_i^- U_j^+, \\ \mathcal{B} &= \begin{pmatrix} H_{ij} & U_i^- \\ -U_j^+ & -k^{-2} \end{pmatrix}. \end{aligned} \quad (33)$$

The determinant of $\mathcal{B}$ can now be easily obtained. We rewrite

$$\mathcal{B} = \begin{pmatrix} H_{ij} & 0 \\ 0 & -k^{-2} \end{pmatrix} \begin{pmatrix} \delta_{kj} & H_{kl}^{-1} U_l^- \\ k^2 U_j^+ & 1 \end{pmatrix}, \quad (34)$$

and show by induction that

$$\det \mathcal{B} = -k^{-2} (1 - k^2 U_k^+ H_{kl}^{-1} U_l^-) \det H. \quad (35)$$

The matrix $\mathcal{A}$ we rewrite as

$$\mathcal{A}_{ij} = H_{ik} (\delta_{kj} - k^2 H_{kl}^{-1} U_l^- U_j^+). \quad (36)$$

The second factor in (36) has p eigenvalues equal to one (for the p vectors orthogonal to U^+), and one eigenvalue equal to $1 - k^2 H_{kl}^{-1} U_l^- U_k^+$, for the eigenvector $H_{ij}^{-1} U_j^-$. The calculation of the IIA,B determinants assumes that the inverse of H exists, and that $H_{ij}^{-1} U_j^-$ is not orthogonal to U_j^+ . This is clearly correct for generic field configurations.

The calculation of the determinants establishes the equality

$$\det \mathcal{B} = -k^{-2} \det \mathcal{A}. \quad (37)$$

The factor k^{-2} is cancelled by the dilaton prefactor in the D -brane action. So indeed we obtain by reduction to $d=9$ the same kinetic terms from the IIA p -brane and the IIB $(p+1)$ -brane.

In case p is odd, the roles of the IIA and IIB fields are interchanged. The same argument as above, with the interchange of the $d=9$ fields $A^{(2)} \leftrightarrow B$ and $k \leftrightarrow k^{-1}$ leads to the proof of duality for p odd.

Having established the duality for the kinetic terms, we now turn our attention to the Wess-Zumino terms. These terms have been studied in [11–15]. Most of the contributions to the Wess-Zumino term are fixed by the gauge symmetries given in (8), (9) and (10). We obtain the following result for $0 \leq p \leq 3$ (omitting hats and using form notation):

$$\begin{aligned} \mathcal{L}_{\text{WZ}}^{(0)} &= A^{(1)} + m V_t, \\ \mathcal{L}_{\text{WZ}}^{(1)} &= \frac{1}{2} \mathcal{B}^{(2)} + \frac{1}{2} \ell \mathcal{F}, \\ \mathcal{L}_{\text{WZ}}^{(2)} &= \frac{1}{4} C + \frac{1}{2} A^{(1)} \mathcal{F} + \frac{m}{2} v dV, \\ \mathcal{L}_{\text{WZ}}^{(3)} &= \frac{1}{6} D + \frac{1}{4} \mathcal{B}^{(2)} \mathcal{F} + \frac{1}{8} \mathcal{B}^{(1)} \mathcal{B}^{(2)} + \frac{1}{16} \ell \mathcal{F}^2. \end{aligned} \quad (38)$$

Note that since the target spacetime gauge transformations contain m -dependent terms, gauge invariance requires, for even $p=0, 2$, the presence of m -dependent

terms in the Wess-Zumino term. For $p = 0$ the equation of motion of the worldline gauge field V_i (which is absent in the $p = 0$ kinetic term) leads to $m = 0$. Finally, we note that the 3-brane action contains an explicit $\mathcal{B}^{(1)}\mathcal{B}^{(2)}$ term which arises as a higher order term in string theory (see footnote 10 of [14]).

We have explicitly verified that the above WZ terms are not only gauge-invariant but also T -dual to each other. The requirement of duality fixes the coefficients of the $\ell\mathcal{F}^{(p+1)/2}$ terms in the 1-brane and 3-brane action which are gauge-invariant by themselves. As an illustration we show how the duality between the $p = 0$ and $p = 1$ WZ terms is established. Introducing the hat notation and taking

$$X^{\hat{1}} = S, \quad \hat{V}_i = V_i, \quad (39)$$

the $p = 0$ WZ term reduces to

$$\dot{X}^\mu (A_\mu^{(1)} + \ell A_\mu^{(2)}) + \ell \dot{S} + m V_i. \quad (40)$$

Similarly, taking

$$Y^{\hat{1}} = \sigma, \quad \hat{V}_\sigma = S, \quad \hat{V}_i = V_i, \quad (41)$$

the $p = 1$ WZ term first reduces to⁷

$$\dot{X}^\mu (A_\mu^{(1)} - m\sigma A_\mu^{(2)}) + (\ell + m\sigma)(\dot{S} - \partial_\sigma V_i + \dot{X}^\mu A_\mu^{(2)}). \quad (42)$$

After partial differentiation the σ -dependent terms cancel and one is left with the same expression (40). This establishes the T -duality. The cancellation of the σ -dependent terms is related to the fact that we are performing a Scherk-Schwarz dimensional reduction [21] that makes use of a global $U(1)$ subgroup of $SL(2, R)_{\text{IIB}}$, as is explained in [4]. Note that the duality establishes a relation between the gauge-invariant $\ell\mathcal{F}$ term in the $p = 1$ WZ term with the mV_i topological term in the $p = 0$ WZ term whose coefficient is determined by gauge invariance. It is in this way that we have used T -duality to determine the coefficients of the gauge-invariant $\ell\mathcal{F}^p$ terms in the $p = 1$ and $p = 3$ WZ terms.

⁷ At first sight the term $\partial_\sigma V_i$ might seem odd since V_i is independent of σ and hence this term would seem to vanish. However, our rule of dimensional reduction is that one should first, by partial differentiation, eliminate the σ -dependence from the action. This same ambiguity arises in the dimensional reduction of IIB supergravity discussed in [4].

Finally, we note that the same $p = 1$ WZ term also plays a role in establishing a duality between the $p = 1$ and $p = 2$ WZ terms. However, in this case one should reduce the $p = 1$ WZ term via a direct as opposed to a double dimensional reduction. Assuming that we reduce over the $\hat{2}$ -direction, the $p = 1$ WZ term is reduced according to

$$Y^{\hat{2}} = S, \quad \hat{V}_i = V_i, \quad i = 0, 1. \quad (43)$$

On the other hand, the $p = 2$ WZ term is reduced by double dimensional reduction as follows:

$$X^{\hat{2}} = \rho, \quad \hat{V}_\rho = S, \quad \hat{V}_i = V_i, \quad i = 0, 1, \quad (44)$$

where ρ is a spacelike direction on the $p = 2$ world-volume. The same identifications were used in establishing the duality between the kinetic terms.

4. Comments

In this letter we have shown how T -duality is realized on (i) the Dirichlet p -brane solutions of IIA/IIB supergravity for $0 \leq p \leq 9$, (ii) the kinetic terms of the D -brane actions for $0 \leq p \leq 9$ and (iii) the WZ terms of the D -brane actions for $0 \leq p \leq 3$. To establish T -duality between the WZ terms for $3 \leq p \leq 9$ we first need to know their form. Note that for these cases there is no leading order RR gauge field in the formulation of IIA/IIB supergravity we are using here that naturally couples to the D -brane. Such higher-order RR gauge fields can be obtained from the existing ones by dualization but so far this seems to lead to rather complicated expressions.

We find that the $p = 0, 2$ WZ terms contain m -dependent topological terms for the worldvolume gauge field. For $p = 0$ the field equation of the world-volume gauge field leads to $m = 0$. Note that only for $m = 0$ the 0-branes can be interpreted as KK states of $D = 11$ supergravity [8]. Concerning the $p = 2$ WZ term we note that only for $m = 0$ the 2-brane action can be obtained by direct dimensional reduction of the eleven-dimensional supermembrane [11]. This 11-dimensional interpretation is not possible for $m \neq 0$ since in that case the worldvolume gauge field V cannot be dualized into a scalar due to the topological mass term. This is related to the fact that

massive IIA supergravity has no eleven-dimensional interpretation, at least not that we know of.

Finally, our hope is that the results of the present work will be useful to establish a kappa-symmetric extension of the bosonic D -brane actions. Such a kappa-symmetric extension has been constructed for the 0-brane and for the 2-brane with $m = 0$ [11]. The T -duality relations given here should be helpful in constructing kappa-symmetric extensions of the other D -brane actions. A particularly interesting case is the 9-brane action which in the “physical gauge” becomes equal to a supersymmetric BI action. We hope to report on progress in this direction in a future publication.

Acknowledgements

We thank G. Papadopoulos, T. Ortín and P. Townsend for useful discussions. The work of E.B. has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences (KNAW).

Note added

Upon completing this work we received a preprint [22] which also discusses the T -duality between D -brane actions.

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