A Runoff System Restores the Principle of Minimum Differentiation

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Abstract

We show that the principle of minimum differentiation holds in two-round elections, for any number of candidates, regardless of the presence of entrants, or the distribution of voters' preferences.

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1. Introduction

Hotelling (1929) and Downs (1957) showed that two candidates competing in an election will both choose the position of the median voter as their platform. This result was coined the principle of minimum differentiation (or MD) by Bouder (1966). Later work, however, showed that this result is highly sensitive to the assumptions made. In particular, with more than two candidates, the possibility of entry, or a change in the underlying distribution of voters' preferences, the principle of minimum differentiation no longer holds. In this letter, we show that MD is restored under a runoff system, where only the two most successful candidates in the first round are allowed to run in the second. Our result is robust to changes in the number of candidates, to the possibility of entry, and the distribution of voters' preferences.

In a seminal paper, Hotelling (1929) showed that two firms, in choosing a location on a straight line, will locate exactly in the middle. Both Hotelling (1929) and Downs (1957) suggested that this model can also be used to study political competition. When two candidates participating in an election locate in an ideological space, the model then implies that both will choose a position that is equal to that of the median voter. Eaton and Lipsey (1975) relax several of the assumptions Hotelling makes. They show that MD no longer holds when more than 2 candidates are competing. Notably, with 3 candidates, a Nash equilibrium fails to exist. These results were also proven by Selten (1971). Also, for $n > 2$, the equilibrium depends on the probability distribution of voters' position on the line. Prescott and Visscher (1977) show that the possibility of entry will also induce candidates not to locate in the middle. In the context of firms, d'Aspremont, Gabszewicz and Thisse (1979) note that when firms also set prices, they will locate at the extremes when transportation costs are linear. Quadratic transportation costs restore MD.¹

All these results, when interpreted in a political context, assume that candidates choose a location so as to maximize the share of the vote. Real-world elections, however, often do not work this way. In numerous countries, such as in France, Portugal, and Austria, the winner of, for example, a presidential election is decided in two rounds (see e.g. Lijphart 1994). In the first round, many candidates run. In the second round, or runoff, only the two most successful candidates from the first round are allowed to compete.² Voters choose between these two to decide who ultimately

1 For a survey of more contributions to this literature, see e.g. Martin (1993), chapter 10 for the economic, and Shepsle (1991) for the political interpretation of the model.

2 Osborne and Slivinski (1996) also study a runoff system. But they assume citizen-candidates, i.e. each candidate uses his own preference as a platform in the election, rather than choosing a platform

wins the election. This system is also used in all presidential elections in Latin America (see Cox 1997). Other countries, such as Australia, use an Alternative Vote system to elect the house of representatives. In this system, voters already announce in the first round who they will vote for should their favorite candidate be eliminated in the first round. For more on this system, see, for example, Bogdanor (1983). It can easily be seen that the results we derive in this paper also hold for an Alternative Vote system.

In this paper, we show that MD is restored when elections are held under a runoff system. We show that, regardless of the number of initial candidates, all of them will choose the position of the median voter. Moreover, this result carries through in circumstances when the distribution of voters' preferences are not uniform. Finally, we show that our result also holds when potential entry is taken into account. Section 2 considers the standard case, with n candidates and a uniform distribution of voters' preferences. In section 3 we show that entry does not change our results. Section 4 generalizes to any distribution of voters' preferences.

2. Why a Runoff System Restores MD

Consider the following set-up. We have an election with two rounds. In the first round, n candidates participate. The 2 candidates with the highest share of the vote proceed to the second round. The candidate with the largest share of the vote in that round wins the election.

Preferences are represented by a horizontal line, normalized to $[0, 1]$. Before the first round, every candidate i chooses his position P_i . We assume that this position cannot be changed between rounds, for example since such a shift in position would undermine the candidate's credibility, destroying his chances to win the election. Voters are uniformly distributed on $[0, 1]$, an assumption we will relax in section 4. Voters always vote for the candidate with the position that is closest to theirs. In case of a tie, they decide randomly which candidate to vote for. We can now establish the following result.

Theorem 2.1 *When voters' preferences are uniformly distributed, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter: $P_i = \frac{1}{2} \forall i$.*

that maximizes the chance of winning the election.

PROOF. Since the case $n = 2$ is the standard one-round case already considered by Hotelling (1929), we restrict attention to $n > 2$. First assume that all candidates $i = 1, \dots, n$ set $P_i = \frac{1}{2}$. Now suppose that candidate n considers a defection to some $P'_n \neq \frac{1}{2}$. Without loss of generality, suppose $P'_n < \frac{1}{2}$. The share of the vote of candidate n in the first round of the election then equals $S_n^1 = P'_n + (\frac{1}{2} - P'_n)/2 = \frac{1}{4} + \frac{1}{2}P'_n$. The other $n-1$ candidates each have a share $S_i^1 = \{\frac{1}{2} + (\frac{1}{2} - P'_n)/2\}/(n-1) = (\frac{3}{4} - \frac{P'_n}{2})/(n-1)$. Candidate n will reach the second round with certainty whenever $S_n^1 > S_i^1$, thus if

$$(n-1)(1 + 2P'_n) > 3 - 2P'_n, \quad (1)$$

or

$$P'_n > \frac{4-n}{2n}. \quad (2)$$

With $n = 3$, this condition is satisfied for $P'_n > \frac{1}{6}$. Any defection with $P'_n < \frac{1}{6}$ then implies that candidate n already drops out in the first round. With $n = 4$, it is satisfied for $P'_n > 0$. For larger n , it always holds. However, when candidate n follows such a defection, he will face a run-off with some candidate j who has $P_j = \frac{1}{2}$. He will always lose this run-off: his share of the vote will equal $S_n^2 = \frac{1}{2}P'_n + \frac{1}{4}$, whereas that of his competitor equals $S_i^2 = \frac{3}{4} - \frac{1}{2}P'_n$, which, with $P'_n < \frac{1}{2}$, is always higher. Thus, defecting from an equilibrium with $P_i = \frac{1}{2} \forall i$, is never profitable. The uniqueness of the symmetric Nash equilibrium is trivial. Suppose there is a unique symmetric Nash equilibrium with $P_i \neq \frac{1}{2}$, and equal for all i . Then any candidate j can improve by defecting to $P_j = \frac{1}{2}$. By doing so, he wins both the first and second round. $\square$

The above proof also provides an intuition for this result. By choosing a position different from that of the median voter, a candidate may win the first round. In the second round, however, he will be beaten by one of the candidates that did choose the median voter's position.³

3 Admittedly, the equilibrium described in the theorem is not the unique Nash equilibrium. For example, with $n = 4$, the same argument as in the proof can be used to show that $P_1 = P_2 = \frac{1}{2} - \varepsilon$, and $P_3 = P_4 = \frac{1}{2} + \varepsilon$ is also an equilibrium, for small enough ε . Yet, such an equilibrium does not exist for all n . With n odd, it can be shown that no other Nash equilibrium exists. Also, we believe that the equilibrium in the text is the most natural one. In any non-symmetric equilibrium some coordination mechanism is needed to determine which candidate chooses which position. Such problems do not exist in our equilibrium. Therefore, we believe our equilibrium is focal in the sense of Schelling (1960).

3. The Case of Potential Entry

Prescott and Visscher (1977) argue that MD no longer holds when candidates consider the possibility of entry of third candidates. When two incumbents do choose the median position, they argue, such an entrant can secure almost half of the vote by entering just to the left or just to the right of the median voter, leaving the original incumbents with only $\frac{1}{4}$ of the vote. When incumbents anticipate this possibility, they will not locate at the median voter but, rather, at positions $\frac{1}{4}$ and $\frac{3}{4}$.

In our model, however, this argument does not hold. Suppose that n candidates are located at $\frac{1}{2}$, and a new candidate enters. The best the new candidate can do is locate at $\frac{1}{2}$ as well, by the same argument as in the proof of theorem 1. Entering at any other location will result in losing the election in the second round. Hence, the incumbents do not have an incentive to locate differently. We thus have

Theorem 3.1 *With potential entry, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter.*

4. The Case of a General Distribution Function

So far, we have restricted attention to the case in which the preferences of the voters are uniformly distributed. Eaton and Lipsey (1975) show that, with $n > 2$, the equilibrium depends on the distribution of voters. In this section, we show that, in our set-up, this distribution does not affect the results.

Theorem 4.1 *For any continuous distribution of voters' preferences, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter: $P_i = p_m \forall i$.*

PROOF. The proof is virtually identical to that of theorem 1. Suppose voters' preferences are described by a probability density function $F(p)$, with $F(0) = 0$ and $F(1) = 1$. The median voter p_m is the one for whom $F(p_m) = \frac{1}{2}$. Restrict attention to the case $n > 2$, the case $n = 2$ being the standard one-round case already proven by Eaton and Lipsey (1975). Assume that all candidates $i = 1, \dots, n$ set $P_i = p_m$. Now suppose that candidate n considers a defection to some $P'_n \neq p_m$. Without loss of generality, suppose $P'_n < p_m$. The share of the vote of candidate n in the first round of the election then equals $S_n^1 = F\left(\frac{P'_n + p_m}{2}\right)$. The other $n - 1$ candidates have a share $S_i^1 = \left\{1 - F\left(\frac{P'_n + p_m}{2}\right)\right\} / (n - 1)$. Candidate n will reach the second round

with certainty whenever $S_n^1 > S_i^1$, thus if $F\left(\frac{P'_n + p_m}{2}\right) > \frac{1}{n}$. This is intuitive; it implies that the share of the vote of candidate n is higher than the average share of the vote of all n voters. Suppose n chooses a defection such that this condition is met. Then, in the second round, he will face a run-off with some candidate j with $P_j = p_m$. He will always lose this run-off. $\square$

5. Conclusion

A wealth of literature suggests that the principal of minimum differentiation does not hold with more than two candidates, when the distribution of voters' preferences is not uniform and/or there is potential entry. In this paper we showed that, in all these circumstances, every candidate does choose the position of the median voter when the election is held under a runoff system.

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