“Living in Sin” and Marriage: a Matching Model[‡]

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Abstract

This paper develops a two sided matching model of premarital cohabitation and marriage in which premarital cohabitation serves as a period of learning. We solve for the optimal policy to be followed by individuals by treating the model as a three stage dynamic programming problem. We find that couples are more discriminating when forming marital unions than when forming cohabiting unions. Cohabitation unions arise among members of the same “class” and there is overlap between the classes formed by marital unions and cohabiting unions. This implies that some cohabiting unions progress to marriage while others do not, a finding borne out by empirical studies.

Keywords: cohabitation, marriage, matching, assortative mating.

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1. Introduction

The now old fashioned term “living in sin” has been used to describe unions in which couples live together without first having a formal marriage. This type of living arrangement is more commonly referred to as cohabitation or as a common law union. The incidence of cohabitation has increased dramatically over the last three decades in North America and Europe. For example, between the mid 1970s and the late 1980s the percentage of women aged 20-24 who were in cohabiting unions rose from 11% to 49% in France (Kiernan 1996). In Canada, only 1 in 16 Canadian couples in 1975 reported to have ever been in a cohabiting union while this number had risen to 1 in 10 in 1995 (Wu and Balakrishnan 1995).

Cohabiting unions however, tend to be short-lived. Evidence from the United States indicates that 40% of all cohabiting couples either marry or stop living together within a year and only a third of cohabiting couples are still cohabiting after two years (Bumpass and Sweet 1989; Thornton 1988). Therefore, although more couples live together without marriage than in the past, this appears to be a transitory stage in their lives. Empirical studies find that 60% of those who marry after living in a cohabiting union, marry their cohabiting partners (Bumpass and Sweet 1989). In addition, the most frequently cited reason for unmarried individuals living together is to assess compatibility before marriage (Bumpass et al. 1991). More recent evidence based on data collected in 1992 from the British Household Panel Study also indicates that more than half of first cohabitations turn into marriage (Ermisch and Francesconi 1999).

Since so many of today’s marriages are preceded by cohabitation, it is natural to ask whether this affects future marital success. Some empirical studies have found that couples who have lived together before marriage, face a *higher* probability of subsequent marital dissolution (Balakrishnan et al. 1987; Bumpass and Sweet 1989; Teachman et al. 1991). Following these studies, research has investigated issues of selectivity; that is, whether couples who are more likely to cohabit are also less averse to divorce. Findings suggest that prior cohabitation has no effect on subsequent marital dissolution once selectivity corrections have been made (Lillard et al. 1995). One explanation offered for this finding is that it is the events that occur after marriage rather than before that determine marital stability (Waite 1995). Of course, cohabitation need not be viewed only as a prelude to marriage but also as an alternative to marriage, or even as an alternative to singlehood.

Since many cohabitation unions are short-lived and usually end in marriage, we develop a model in which at least the initial motivation for premarital cohabitation is to learn about the quality of a potential marriage partner. Therefore, it serves as an information collecting period during which potential partners can assess one another and decide if they should embark upon a formal marriage. At the end of this period of cohabitation, some unions progress to marriage while others are terminated. We find that individuals are more discriminating when forming marital unions than when forming cohabiting unions. That is, they sometimes form cohabiting unions

with individuals whom they eventually reject as marriage partners. Individuals are shown to partition themselves within classes when forming cohabiting unions and that cohabiting unions occur only among members of the same class. We establish that there is overlap between the classes formed by cohabiting unions and marital unions suggesting that some of the unions that began as cohabiting unions end in marriage. The modelling framework used is a steady state matching model with nontransferable utility and heterogeneous agents as developed in Burdett and Coles (1997) hereafter simply Burdett and Coles.

A model of a decentralized marriage market in which positive assortative mating¹ arises as an equilibrium outcome is developed by Burdett and Coles. In their model, an individual's attractiveness as a marriage partner is captured by a single number called "pizazz" which is instantly observed. The assortative mating found in Burdett and Coles is in terms of the pizazz measure. That is, high pizazz men and women marry one another and low pizazz men and women marry one another. The utility an individual derives from a marriage is equal to the pizazz of his or her spouse. They demonstrate that the optimal policy for a single individual to follow is to marry the first single encountered of the opposite sex whose pizazz is above a certain reservation level. Equilibrium is characterized by a class partition: singles partition themselves into classes according to their pizazz and marriages occur only between couples who belong to the same class. The steady state is characterized by a constant distribution of pizazz among the pool of single individuals and a balanced flow of market exits and entries.

In this paper, we depart from the Burdett and Coles framework and assume that singles can observe the true worth of a potential partner only imperfectly. This provides a motivation for cohabiting unions which are initiated by single men and women in an attempt to learn each other's true worth as marriage partners. All couples are assumed to cohabit prior to marriage. Of course, cohabitation is just one interpretation of this information collecting period; it can very well be called dating or a "getting acquainted" period. We show that positive assortative mating also arises among cohabiting unions.

The paper is organized as follows. Section 2 explains the working of the marriage market and calculates the optimal policy to be followed by market participants. Demonstrating the existence of a class partition is the subject of Section 3. Section 4 concludes the paper and details of some of the proofs in the paper are provided in the Appendix.

¹This describes the propensity of individuals to choose marriage partners who are similar to themselves in attributes such as height, intelligence and education level. Empirical evidence on positive assortative mating may be found in Becker (1973, 1974). Assortative mating has been researched extensively in the sociology and biosocial literature.

2. Dynamics of the Marriage Market

The marriage market is assumed to comprise a large and equal number of single men and women who are seeking potential mates. Retaining the terminology in Burdett and Coles, we assume that each person's attractiveness as a marriage partner can be quantified by a single number called pizazz. If a single man decides to marry, his utility from the marriage equals the pizazz of his wife and vice versa. Having high pizazz allows one to attract members of the opposite sex who also have high pizazz and therefore increases one's utility from a marriage.

It is assumed that there is a constant inflow of new singles into the market and that the distribution of pizazz among new singles remains constant over time. There exists a flow-in distribution of pizazz, that is, the distribution of pizazz among new men and new women entering the market. The distribution of pizazz among men and women need not be the same.

Clearly, as men and women either cohabit or marry and leave the singles market, the distribution of pizazz will change over time. However, we assume as in Burdett and Coles that singles are only partially rational, and believe that the environment is stationary. This implies that they believe that the distribution of pizazz in the singles market for both men and women is constant over time. They are then assumed to adopt utility maximizing strategies given the behaviour of other singles in the market. Pairs of market distributions of pizazz for men and women which are consistent with flow-in and flow-out distributions that are equal, make up the set of steady state equilibria. In this paper, we do not focus on the calculation of the steady state equilibrium but concentrate instead on the individual's decision problem and on establishing the existence of a partition when cohabiting unions are formed. In our model, the calculation of the steady state is complicated by the fact that in addition to inflows and outflows of singles due to birth and death, cohabiting unions and marital unions may dissolve resulting in individuals re-entering the singles market.

Participants in the marriage market seek to maximize the expected discounted lifetime utility by searching for the best possible matches, discounted at a rate r to the present. We assume that singles use stationary strategies; there is a certain set of opposite sex singles to whom they will propose if they meet. The instantaneous utility of being single is zero. Individual lifetimes follow an exponential distribution, and the probability that an individual dies in time interval h is δh .

For simplicity of exposition, we consider the decision maker to be a single woman searching for a potential marriage partner. Unlike Burdett and Coles, who assume that pizazz x is instantly observed, we assume that when a woman encounters a man, she only observes his pizazz imperfectly. She observes y , where y and x are correlated. Symmetrically, the man also obtains a noisy observation y on the woman's pizazz x . We assume that singles believe that the distribution of observed pizazz is time-invariant. We refer to x as true pizazz and y as observed pizazz.

Upon observing y , the woman forms an expectation of the true pizazz of the man,

$E(X | Y = y)$ denoted $m(y)$ or simply m . She then faces two choices: either enter into a cohabiting union with him or remain single in the hope of drawing a better match next period from the same distribution. It is assumed that m is increasing in y . This appears reasonable as one would expect that high values of observed pizazz would result in the woman's perception of true pizazz to be high (see the Appendix for further details on distributional assumptions that ensure that this condition holds).

We demonstrate that the woman will choose to cohabit with the man whose observed pizazz she observes to be y , if m exceeds a certain threshold level, denoted m_r . If she enters a cohabiting union with a man with observed pizazz y , she will receive utility equal to $m = m(y)$ during that period. If not, she will remain single and the process repeats itself in the next period. We assume, for simplicity, that cohabitation lasts only one period (this may be generalized as in Jovanovich 1979 in which a case of gradual learning in a labour market context is considered). Men and women are assumed to continue to search for partners during cohabitation; an assumption made in the tradition of the "on-the-job search" assumption in labour economics. Therefore, cohabiting individuals may enter into cohabiting relationships with new partners without an intervening period of singlehood. At the end of the cohabitation period, the true pizazz of each partner is revealed to the other. At this point, each party must decide whether to maintain or quit the current match.

If they choose to maintain the match, they are assumed to establish a formal marriage and each partner's utility from the marriage equals the other's true pizazz. If the cohabiting couple do not to marry, they are assumed to separate. Marital unions may dissolve, and in this case it is assumed that individuals return to the pool of singles before forming other partnerships (Burdett and Coles assume that couples who marry never return to the singles market). Figure 1 illustrates the inflows and outflows from the singles market.

Figure 1 about here.

Therefore, the matching process can be viewed as occurring in three stages. In the first stage, which can be regarded as a "pre-draw" stage, a single individual awaits a potential partner in the marriage market. In this stage, she only knows the distribution of expected true pizazz conditional on observed pizazz, $F(m)$. In the second stage, the woman draws x but observes y . In the third stage, she discovers the true pizazz of the man she has been living with and based on this, she decides whether or not to establish a formal marriage with him.

In the next three sections, we derive the optimal policy to be followed at each stage. Following standard dynamic programming techniques, we begin with the last stage, stage 3. In the exposition that follows, we maintain the convention that variables and functions corresponding to the woman (the decision maker) are subscripted by i while variables and functions corresponding to her (potential) partner are subscripted by j .

2.1. Decision Making at Stage 3

At stage 3, the true pizazz x , of the man she has been living with is revealed to the woman. She is to then make a choice between two alternatives: either separate from him and return to the singles market or establish a formal marriage with him. If she marries him, she will acquire utility x in the current period. In all future periods, she will face two possibilities: of either staying married to him or divorcing to return to the singles market. Let U denote the woman's expected discounted lifetime utility of being single in the current period and following an optimal policy from the next period onward. The expected discounted lifetime utility of being married in the current period and following an optimal policy next period onward is denoted Ψ . Marital dissolutions are assumed to be exogenous and follow a Poisson process with parameter γ (see Weiss and Willis 1997 for an empirical investigation of the determinants of marital dissolution in another type of matching context). The woman's choice is then based on the following equation:

$$J(x) = \max\{\Psi(x), U\}, \quad (1)$$

where

$$\Psi(x) = \frac{(1 - \delta h)}{(1 + rh)} \{xh + (1 - \gamma h)\Psi(x) + \gamma h U\}. \quad (2)$$

Simplifying and letting $h \rightarrow 0$, yields

$$\Psi(x) = \frac{x + \gamma U}{c_1}, \quad (3)$$

where $c_1 = r + \delta + \gamma$. In the calculation of the optimal policy in stage 1, we will show that U does not depend on the offered value of x . From eq(3), it is clear that Ψ is increasing in x . That is, the greater the true pizazz of the husband, the greater the woman's expected discounted lifetime utility of the woman from the marriage. Therefore, the optimal policy is a reservation match policy and takes the form:

$$J(x) = \begin{cases} U & \text{if } x < x_r \\ (x + \gamma U)/c_1 & \text{if } x \geq x_r. \end{cases} \quad (4)$$

The woman's optimal policy is determined by her reservation level of the man's true pizazz, x_r . She will marry him if his true pizazz is greater than x_r and separate from him otherwise. The reservation level of pizazz can be written as

$$x_r = \Psi^{-1}(U) = c_2 U, \quad (5)$$

where $c_2 = r + \delta$. The argument is identical when we treat the man as the optimizer.

It should be pointed out that in keeping notation simple, we have omitted certain subscripts which capture the relationship between some of the random variables and functions in the model. For instance, a more precise way of expressing U is $U_i(m_i)$. That is, for woman i , the expected discounted lifetime utility of being single depends

on her expected true pizazz conditional on observed pizazz m_i (as observed by potential partners). Also, $\Psi(x)$ can be more precisely expressed as $\Psi_i(x_j, m_i)$ which denotes the expected discounted lifetime utility obtained by woman i with own expected true pizazz conditional on observed pizazz equal to m_i upon marrying a man with true pizazz x_j . The term $\Psi_i(x_j, m_i)$ is an increasing function of x_j while m_i is treated as a parameter (it is a constant in (x_j, m_j) space). Similarly, the function $J(x)$ may be written as $J_i(x_j, m_i)$.

2.2. Decision Making at Stage 2

At this stage, the woman encounters a man and based on her perception of him which is his observed pizazz, she proceeds to estimate his true pizazz. That is, she calculates $E(X | Y = y)$ denoted $m(y)$. The woman is confronted with two alternatives: continue to be single or enter into a cohabiting union with the man she has encountered.

Let Φ denote the woman's expected discounted lifetime utility of being in a cohabiting union in the current period and following an optimal policy from the next period onward. In this case,

$$V(m) = \max\{\Phi(m), U\}, \quad (6)$$

where

$$\Phi(m) = \frac{(1 - \delta h)}{(1 + rh)} (mh + \lambda h E J + \xi h U + [1 - (\lambda + \xi)h] \Phi(m)). \quad (7)$$

In the above, the first term m represents the instantaneous utility obtained from a cohabiting union in the current period. The second term represents the expected discounted utility of progressing to a formal marriage. The third and fourth terms represent the expected discounted utility of returning to the singles market, and of finding a new cohabiting partner respectively. It is assumed that the probability that a cohabiting union evolves into a marital union in time interval h is λh , and this is governed by a Poisson process. The probability that a cohabitation union fails in time interval h and the participant returns to the pool of singles is assumed to be ξh , also following a Poisson process. Therefore, $1 - (\lambda + \xi)h$ can be interpreted as the probability of finding a new cohabiting partner while living with one during time interval h . By the same procedure used in obtaining eq(3) we have

$$\Phi(m) = \frac{m + \lambda E J + \xi U}{c_3}, \quad (8)$$

where $c_3 = r + \delta + \lambda + \xi$. A reservation policy is optimal in this stage as well. The validity of this policy depends on $\Phi(m)$ being increasing in m and U being constant in m . It is shown in proposition 1 that $\Phi(m)$ is increasing in m , while in the calculations in stage 1 it is demonstrated that U is constant in m . Therefore, the woman's optimal policy at this stage, is to reject men whose $m(y)$ is lower than

a threshold value denoted m_r , where

$$m_r = \Phi^{-1}(U). \quad (9)$$

This is equivalent to a policy in which the woman rejects men whose y is less than $y_r = y(\Phi^{-1}(U))$ and accepts others. Therefore,

$$V(m) = \begin{cases} U & \text{if } m < m_r. \\ \frac{m + \lambda EJ + \xi U}{c_3} & \text{if } m \geq m_r. \end{cases} \quad (10)$$

Again, we have chosen to keep the exposition simple by omitting certain subscripts. More precisely, the function $\Phi(m)$ should be expressed as $\Phi_i(m_j, m_i)$. That is, the expected discounted lifetime utility of being in cohabiting union for woman i depends on the expected true pizazz conditional on observed pizazz of partner j denoted m_j and on her own expected true pizazz conditional on observed pizazz denoted m_i . Similar to the calculations in stage 3, the term $\Phi_i(m_j, m_i)$ is a function of m_j while m_i is treated as a parameter, constant in (x_j, m_j) space. In addition, it is reasonable to assume that the probability that an individual's cohabiting union fails is a function of the attractiveness of that individual. Therefore, ξ may be written as $\xi_i(m_i)$ and $\xi'_i(\cdot) < 0$. The term EJ is more complex and is written more precisely as $E_{X_j|Y_j} J_i(x_j, m_i)$ which is given by:

$$E_{X_j|Y_j} J_i(x_j, m_i) = \int_{S_{X_j}} J_i(x_j, m_i) dQ_j(x_j, m_j \mid m_i). \quad (11)$$

In the above expression, $Q_j(x_j, m_j \mid m_i)$ is used to denote the distribution of true pizazz x_j conditional on observed pizazz $y_j = y(m_j)$ among singles j who are willing to cohabit with an optimizer i with conditional expected pizazz m_i . A detailed specification of the distribution Q can be found in the Appendix. Therefore, a more detailed version of eq(8) is:

$$\Phi_i(m_j, m_i) = \frac{m_j + \lambda E_{X_j|Y_j} J_i(x_j, m_i) + \xi_i(m_i) U_i(m_i)}{c_2 + \lambda + \xi_i(m_i)}, \quad (12)$$

and a more detailed version of eq(6) can be written as:

$$V_i(m_j, m_i) = \max\{\Phi_i(m_j, m_i), U_i(m_i)\}. \quad (13)$$

2.3. Decision Making at Stage 1

This is the “pre-draw” stage, when the woman observes neither x nor y . She only knows the distribution of M , the distribution of expected true pizazz conditional on observed pizazz. Standard dynamic programming arguments yield,

$$U = \frac{(1 - \delta h)}{(1 + rh)} [0h + \alpha h EV + (1 - \alpha h)U], \quad (14)$$

as the instantaneous utility from being single is assumed to be 0. The arrival rate of offers of cohabitation, that is, offers from single men who would like to cohabit with the woman is denoted α . Similar calculations to those in stages 2 and 3 yield,

$$U = \left(\frac{\alpha}{r + \delta + \alpha} \right) EV \equiv \left(\frac{\alpha}{c_2 + \alpha} \right) EV. \quad (15)$$

Again, introducing subscripts to make the above expression more precise yields,

$$U_i(m_i) = \left(\frac{\alpha_i(m_i)}{c_2 + \alpha_i(m_i)} \right) E_{M_j} V_i(m_j, m_i), \quad (16)$$

where

$$E_{M_j} V_i(m_j, m_i) = \int_{S_{M_j}} V_i(m_j, m_i) dF_j(m_j | m_i). \quad (17)$$

Here, $\alpha_i(m_i)$ stands for the arrival rate of proposals faced by single i who possesses conditional expected pizazz m_i . The individuals who appear more attractive will have more singles of the opposite sex interested in them, and therefore we assume that $\alpha'_i(\cdot) > 0$. In the above expression, the distribution $F_j(m_j | m_i)$ denotes the distribution of expected conditional pizazz m_j among potential partners, faced by an individual with own expected conditional pizazz m_i . We can see from equations (16) and (17) that U does not depend on the value of the partner's pizazz, x . Next, we show that the expected discounted utility received by a woman is increasing in the expected pizazz of her potential husband.

Proposition 1: $\Phi'(m) > 0$.

Proof: See Appendix.

Proposition 2: If the optimizer is indexed by i , and j indicates his or her potential partner and the optimizer i 's reservation utility is

$$U_i \equiv U_i(m_i) = \frac{\alpha_i(m_i)}{c_2 [c'_2 + \xi_i(m_i)]} \times \int_{\Phi_i^{-1}(U_i)}^{\bar{m}_j} [1 - F_j(m_j | m_i)] \left[1 - \frac{\lambda}{c_1} \int_{\Psi_i^{-1}(U_i)}^{\bar{x}_j} \frac{\partial Q_j(x_j, m_j | m_i)}{\partial m_j} \right] dm_j \quad (18)$$

where $c'_2 = c_2 + \lambda$.

Proof: See Appendix.

Having calculated the optimal strategies to be followed by market participants, we now turn attention to the issue of class partitions under incomplete information.

3. The Class Partition of Cohabiting and Married Couples

This section is devoted to proving the following results: (1) couples set a higher reservation value when forming cohabiting unions than when forming marital unions;

(2) cohabiting unions occur only between couples in the same class; (3) the classes corresponding to the partition formed by cohabiting unions partially overlap with the classes from the partition formed by marital unions.

We begin first by showing that couples are more choosy when forming marital unions than when forming cohabiting unions.

Proposition 3: $m_r < x_r$.

Proof: Recall that if a woman encounters a man whose expected true pizazz is m_r , she is indifferent between forming a cohabiting union with him or remaining single. Therefore, from eq(10) we have,

$$\frac{m_r + \lambda EJ|_{m=m_r} + \xi U}{c_3} = U. \quad (19)$$

Using $x_r = c_2 U$ from eq(5) and rearranging terms yields,

$$\lambda [EJ|_{m=m_r} - U] = x_r - m_r.$$

Denoting the woman's own expected true pizazz conditional on her observed pizazz as m_i and using eq(38) from the Appendix we have,

$$x_r - m_r = \left(\frac{\lambda}{c_1} \right) \int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m_r | m_i) > 0. \quad (20)$$

The above equation can be interpreted as a comparison of the costs and benefits of search. The left hand side represents the opportunity cost of searching one more time with a marriage proposal of x_r at hand. The right hand side represents the prospective benefit of this search which has the expected discounted utility associated with a future draw of $x > x_r$. Although we have not explicitly modelled the cost of dissolving a cohabitation union vs. the cost of dissolving a marital union, this result is consistent with the generally held belief that dissolving a marriage is more costly than dissolving a cohabiting union (See Bougheas and Georgellis 1999 for a study in which divorce costs are shown to affect both the probability of divorce and the probability of marriage).

During cohabitation, the couple's knowledge of each other is incomplete. However, true pizazz is revealed to both partners at the end of the cohabitation period making couples more discriminating at that point. To ensure the reliability of a prospective marriage and reduce its risk, the reservation level of pizazz for marriage must be raised beyond its counterpart in cohabitation, i.e. $x_r > m_r$.

Proposition 4: The reservation values associated with cohabitation unions are consistent with the reservation values associated with marital unions. That is, consider two women indexed i , $i = k$ and $k + 1$. Assume that woman i will cohabit with men whose expected true pizazz conditional on observed pizazz exceeds m_r^i and marry men whose true pizazz exceeds x_r^i . Then,

$$m_r^k > m_r^{k+1} \Rightarrow x_r^k > x_r^{k+1}. \quad (21)$$

Proof: From equations (5) and (9), we have $x_r = c_2 U$ and $\Phi(m_r) = U$. Therefore, we can write $x_r = c_2 \Phi(m_r)$. From proposition 1, we know that $\Phi'(m) > 0$ and it follows that $dx_r/dm_r = c_2 \Phi'(m_r) > 0$. Since $m'(y) > 0$, this implies that if woman k 's reservation demand for her partner's observed pizazz is higher in cohabitation than woman $k + 1$'s, then it must be that woman k 's reservation demand for the true pizazz of her partner in marriage is higher than woman $k + 1$'s. Therefore,

$$y_r^k > y_r^{k+1} \Rightarrow x_r^k > x_r^{k+1}. \quad (22)$$

Notice that since $x_r = c_2 U$, this implies that

$$U^k > U^{k+1}. \quad (23)$$

That is, if a woman indexed k has higher cut-off levels for observed and true pizazz than another woman indexed $k + 1$, then it must be that woman k 's reservation utility must be higher than that of woman $k + 1$.

Proposition 5: Cohabitation unions occur only among men and women of the same class.

Proof: Consider the situation of the woman who appears most attractive, that is the woman indexed i , whose conditional expected pizazz achieves the supremum of the distribution denoted $\bar{m}_i$. All single men indexed j will propose to her if they contact her. That is, $\alpha_i(\bar{m}_i) = \alpha_0$, where α_0 is the total arrival rate of singles of the opposite sex. In addition, if she wants to enter into a cohabiting union, no man would reject her and therefore, $\xi_i(\bar{m}_i) = 0$. She also faces the distribution of expected conditional pizazz of all single men which implies that $F_j(m_j | \bar{m}_i) = F_j(m_j)$. The distribution of true pizazz conditional on observed pizazz faced by the most attractive woman is $Q_j(x_j, m_j | \bar{m}_i) = Q_j(x_j, m_j)$. The optimal policy for the most attractive woman is to accept proposals from men with conditional expected pizazz $m_j \in (\Phi_i^{-1}(\bar{U}_i), \bar{m}_j] \equiv (m_{j1}, m_{j0}]$ where $\bar{U}_i$ is defined by the following equation:

$$\bar{U}_i \equiv U_i(\bar{m}_i) = \frac{\alpha_0}{c_2 c'_2} \int_{\Phi_i^{-1}(\bar{U}_i)}^{\bar{m}_j} [1 - F_j(m_j)] \left[1 - \frac{\lambda}{c_1} \int_{\Psi_i^{-1}(\bar{U}_i)}^{\bar{x}_j} \frac{\partial Q_j(x_j, m_j)}{\partial m_j} dx_j \right] dm_j \quad (24)$$

The above equation, which follows from Proposition 1, yields a fixed point $\bar{U}_i$ as its solution, which is ensured by Blackwell's Theorem (See Sargent 1987). As it is not guaranteed that the woman who appears most attractive, will indeed meet the man who appears most attractive (that is, the man whose expected conditional pizazz achieves the supremum of the distribution of men $\bar{m}_j$), she also accepts offers of cohabitation from men with conditional expected pizazz in the interval $(\Phi_i^{-1}(\bar{U}_i), \bar{m}_j)$. Symmetrically, for the most desirable man with conditional expected pizazz $\bar{m}_j$, his

reservation utility takes the form of

$$\overline{U}_j \equiv U_j(\overline{m}_j) = \frac{\alpha_0}{c_2 c_2'} \int_{\Phi_j^{-1}(\overline{U}_j)}^{\overline{m}_i} [1 - F_i(m_i)] \left[1 - \frac{\lambda}{c_1} \int_{\Psi_j^{-1}(\overline{U}_j)}^{\overline{x}_i} \frac{\partial Q_i(x_i, m_i)}{\partial m_i} dx_i \right] dm_i. \quad (25)$$

His optimal policy is similar to the one adopted by the most attractive woman and he will accept proposals from women with $m_i \in (\Phi_j^{-1}(\overline{U}_j), \overline{m}_j] \equiv (m_{i1}, m_{i0}]$.

Consider other women with $m_i \in (m_{i1}, m_{i0})$ whom we shall term “top class” women. Since the most desirable man is willing to propose to any top class woman, it must be that all men will woo such a woman if they contact her. Therefore, for any woman with $m_i \in (m_{i1}, m_{i0})$, we have

$$\begin{aligned} \alpha(m_i) &= \alpha, \\ \xi_i(m_i) &= 0, \\ F_j(m_j | m_i) &= F_j(m_j), \\ \text{and } Q_j(x_j, m_j | m_i) &= Q_j(x_j, m_j). \end{aligned}$$

This yields,

$$U_i(m_i) = U_i(\overline{m}_i) \equiv \overline{U}_i \equiv U_{i0} \text{ for all } m_i \in (m_{i1}, m_{i0}). \quad (26)$$

Therefore,

$$U_i(m_i) = \overline{U}_i \equiv U_{i0} \text{ for all } m_i \in (m_{i1}, m_{i0}). \quad (27)$$

Symmetry implies the same for all top class men,

$$U_j(m_j) = \overline{U}_j \equiv U_{j0} \text{ for all } m_j \in (m_{j1}, m_{j0}). \quad (28)$$

Therefore, men and women in the top class cohabit with one another.

Next, we show that second class women will accept proposals of cohabitation from men in the second class and vice versa. Consider a woman i not in the top class, but she is the best of her kind, i.e. her $m_i = m_{i1}$. In what follows, we refer to her as the second best woman. She will be rejected by top class men in accordance with their reservation match strategies mentioned above, and so the only contacts of interest to her are with men not in the top class. This implies that for this woman, $\alpha_i(m_{i1}) = \alpha_0 \Pr(M_j \leq m_{j1}) = \alpha_0 F_j(m_{j1})$ and $\xi_i(m_{i1}) = \alpha_0 [1 - F_j(m_{j1})]$. Also,

$$F_j(m_j | m_{i1}) = \Pr(M_j \leq m_j | M_j \leq m_{j1}) = F_j(m_j)/F_j(m_{j1}) \text{ for } m_j \leq m_{j1}.$$

Similarly, $Q_j(x_j, m_j | m_{i1}) = Q_j(x_j, m_j)/Q_j(x_{j1}, m_j)$ for $m_j \leq m_{j1}$ and $x_{j1} = x(m_{j1})$. In the above expressions, the optimizer i 's conditioning variable m_{i1} has been converted into the partner j 's pizzazz m_{j1} or x_{j1} . Dropping j for simplicity, the new distributions $F(m)/F(m_1) \equiv F_1(m)$ and $Q(x, m)/Q(x_1, m) \equiv Q_1(x, m)$ have their own supports $[\underline{m}, m_1]$ and $[\underline{x}, x_1]$ respectively. It is then legitimate to use eq(18) from Proposition 2 to write down the reservation utility of the second best woman

denoted $U_{i1} \equiv U_i(m_{i1})$, based on these supports. Substituting these four expressions into eq(18) yields

$$U_{i1} = \frac{\alpha_0}{c_2 c_3'} \int_{\Phi_i^{-1}(U_{i1})}^{m_{j1}} [F_j(m_{j1}) - F_j(m_j)] \left[1 - \frac{\lambda}{c_1} \int_{\Psi_i^{-1}(\bar{U}_{i1})}^{\bar{x}_{j1}} \frac{\partial}{\partial m_j} \left[\frac{Q_j(x_j, m_j)}{Q_j(x_{j1}, m_j)} \right] dx_j \right] dm_j \quad (29)$$

where $c_3' = c_2' + \xi_i(m_{i1})$. Since we can write $x_{j1} = x(m_{j1})$ we find that m_{i1} is suppressed in eq(29) and we are left with an implicit relationship $U_{i1} \equiv U_i(m_{j1})$. As in eq(24), a unique value of U_{i1} for m_{j1} exists as the fixed point of eq(29). Therefore, the optimal policy for the best woman in the second class is to accept all men with $m_j \in (\Phi_i^{-1}(U_{i1}), m_{j1}] \equiv (m_{j2}, m_{j1}]$. Symmetry renders it optimal for the best man who is not in the top class to propose to the women with $m_i \in (\Phi_j^{-1}(U_{j1}), m_{i1}] \equiv (m_{i2}, m_{i1}]$. Notice that because all women in a particular class receive the same offers, they all use the same reservation level of conditional expected pizzazz to determine which partners to cohabit with. This is true as well for all men in a particular class.

It is interesting to note that the distribution faced by members of the top class lie entirely in their domains below the distribution faced by members of the second class, that is, $F(m) \leq F_1(m)$, and $Q(x, m) \leq Q_1(x, m)$. This implies that $F(m)$ and $Q(m)$ first order stochastically dominate $F_1(m)$ and $Q_1(x)$. This phenomenon originates in differences in the perceived attractiveness of individuals.

As for the other second-class women with $m_i \in (m_{i2}, m_{i1})$, the argument is the same as the one used in determining the optimal policy for top class women other than the best of the top class women. Therefore, for any $m_i \in (m_{i2}, m_{i1})$, $\alpha(m_i) = \alpha_i(m_{i1})$ and $\xi_i(m_i) = \xi_i(m_{i1})$. Also, $F_j(m_j | m_i) = F_j(m_j | m_{i1})$ and $Q_j(x_j, m_j | m_i) = Q_j(x_j, m_j | m_{i1})$. Therefore, $U_i(m_i) = U_i(m_{i1}) \equiv U_{i1}$ for all $m_i \in (m_{i2}, m_{i1}]$. By symmetry, for men we have $U_j(m_j) = U_j(m_{j1}) \equiv U_{j1}$ for all $m_j \in (m_{j2}, m_{j1}]$.

The argument can be extended to the third-class, fourth-class etc. Since the following holds for both men and women, we omit the i, j subscripts to simplify notation. In general, we can write,

$U(m) = U_k$ for all $m \in (m_k, m_{k+1}]$, for $k = 0, 1, 2 \dots n-1$ and (from proposition 4) it follows that,

$$\begin{aligned} m_n &< m_{n-1} < \dots < m_1 < m_0 = \bar{m}, \\ x_n &< x_{n-1} < \dots < x_1 < x_0 = \bar{x}, \\ U_n &< U_{n-1} < \dots < U_1 < U_0 = \bar{U}. \end{aligned} \quad (30)$$

Proposition 6: The number of classes for both cohabitation and marital unions, n , is finite.

Proof: A contradiction argument can be used to prove this proposition. Even if the upper bound of the support, $\bar{m}$ is $+\infty$, then the lower bound of the integral, $m_1 = \Phi^{-1}(\bar{U})$ has to be finite to ensure that $\bar{U}$ is meaningful. Since $m_k > \underline{m} > 0$ for all k , it must be that $[\underline{m}, m_1]$ must be an interval of finite length. Now suppose

that there do exist infinitely many disjoint intervals subintervals $(m_k, m_{k+1}]$. Then it must be that many of them are degenerate in the sense that $(m_k - m_{k+1}) \rightarrow 0$ as $k \rightarrow \infty$. Under this condition, the integrand and length of the integral interval in the reservation equations approach zero and hence U_k tends to zero as well. This results in degenerate intervals in the class partition which are meaningless, and therefore n must be finite.

We have so far established that singles partition themselves into classes when cohabiting unions are formed and that the number of classes is finite. It is also the case that once true pizazz is revealed to both partners, that marital unions occur only within classes. We omit the proof of this result here as it is similar to the proof given in Burdett and Coles.

Proposition 7: Classes formed by the partition for cohabitation and the classes formed by the partition for marital unions partially overlap. That is,

$$m_{k+1} < x_{k+1} < m_k < x_k.$$

Proof: Suppose that the above expression did not hold and that $x_{k+1} \geq m_k$. This implies that $U(m_{k+1}) > U(m_k)$ which contradicts eq(30) from Proposition 4.

Proposition 7 implies that some cohabitation unions fail because the woman eventually rejects some of the men she cohabits while she is herself rejected by some of the men she cohabits with. As all marriages are preceded by cohabitation in this model, marriages occur within a narrower class of couples than cohabiting unions. One outcome of this is that in the case of the pizazz distributions being the same for both sexes, cohabitation unions reduce the differences in pizazz measures of men and women in the same class.

4. Conclusion

This paper develops a theoretical model of premarital cohabitation in which some unions progress to marriage. We extend the framework developed in Burdett and Coles (1997) to incorporate incomplete information and learning. We solve for the optimal policy to be followed by market participants by treating the model as a three stage dynamic programming problem. We find that couples set higher standards when forming marital unions than when forming marital unions. Cohabitation unions arise among members in the same class. There is overlap between the classes formed by marital unions and cohabiting unions, implying that some cohabiting unions progress to marriage while others do not.

In the simple setting of our model, we have not considered other issues that could lead to interesting extensions. For example, some couples may decide to marry without an initial period of cohabiting. In what respect do these individuals differ from the individuals who first cohabit? There still exist traditional societies in which there is stigma attached to cohabiting before marriage. While this stigma is a cost borne by either one or both partners, it may be partly offset by the gain from cohabiting which reduces the chances of marrying an unsuitable partner. Assessing

the true worth of a partner can be related to the correlation between the true pizzazz and observed pizzazz distributions. The more noisy the signal about a partner, the more worthwhile it becomes to first cohabit and then marry.

A related extension is to consider a model in which individuals are not only characterized by a single measure that captures their attractiveness as a spouse but also a “commitment measure” which measures how loyal an individual is to a relationship, be it cohabitation or marriage. Assuming for simplicity that there are two types, a high and low commitment type of individual, suppose that high commitment types are less likely to dissolve a union while low commitment types are more likely to dissolve a union. If these types are unobserved, it may be possible to identify high and low commitment types from observables such as durations of marriages and cohabiting unions. We leave these extensions as topics for further research.

Appendix

A note on the joint distribution of observed and true pizazz.

We assume that true pizazz, X , and observed pizazz, Y , are jointly distributed. Suppose that their joint density is denoted $f_{X,Y}(x, y)$. The distribution of true pizazz conditional on observed pizazz is:

$$Q_{X|Y}(x | y) = \Pr(X \leq x | y) = \frac{\int_{\underline{x}}^x f_{X,Y}(x, y) dx}{\int_{\underline{x}}^{\bar{x}} f_{X,Y}(x, y) dx}. \quad (31)$$

The expectation of true pizazz conditional on observed pizazz denoted $m(y)$ can be written as

$$m(y) = E(X | Y = y) = \int_{\underline{x}}^{\bar{x}} x q_{X|Y}(x | y) dx = \frac{\int_{\underline{x}}^{\bar{x}} x f_{X,Y}(x, y) dx}{\int_{\underline{x}}^{\bar{x}} f_{X,Y}(x, y) dx}. \quad (32)$$

We assume that m is increasing in y . This appears to be a reasonable assumption to make, since it means that a high value of observed pizazz would result in a high estimate of true pizazz. Since m is increasing in y , the support of m is $[\underline{m}, \bar{m}] = [m(\underline{y}), m(\bar{y})]$. It is then legitimate to invert the function m , denoted $y(m)$. The distribution of the random variable $M = E(X | Y)$ is given by

$$F(m) = \Pr(M \leq m) = \Pr[Y(m) \leq y(m)] = \int_{\underline{y}}^{y(m)} \int_{\underline{x}}^{\bar{x}} f_{X,Y}(x, y) dx dy \quad (33)$$

Therefore,

$$Q_{X|Y}(x | y) = Q_{X|Y}(x | y(m)) \equiv Q(x, m), \quad (34)$$

with domain $S_X \times S_M$ where $S_X = [\underline{x}, \bar{x}]$ and $S_M = [\underline{m}, \bar{m}]$.

Assumption 1: $\partial Q(x, m)/\partial m < 0$

We assume that $\partial Q(x, m)/\partial m < 0$ which ensures that a high value of observed pizazz will lead to a high expectation of true pizazz. This implies that for any given $m_1 < m_2$ we have $Q(x, m_1) > Q(x, m_2)$ for all $x \in S_X$. That is, $Q(x, m_2)$ strictly first order dominates $Q(x, m_1)$. This assumption of stochastic dominance rising with m is important in establishing the positive assortative mating that occurs. This property holds, for instance, for the normal distribution. Suppose that $X \sim N(\mu, \sigma^2)$, $Y = X + \epsilon$, and $\epsilon \sim N(0, 1)$. Then, $\partial Q(x, m)/\partial m = -\left[\frac{1}{\sigma_1}\right] \phi((x - m)/\sigma_1) < 0$ where $\sigma_1 = \sigma^2/(\sigma^2 + 1)$ and ϕ denotes the standard normal density function.

In the exposition in the paper, the distribution $Q_j(x_j, m_j | m_i)$ is used to denote the distribution of true pizazz x_j conditional on observed pizazz $y_j = y(m_j)$ among singles j who are willing to cohabit with an optimizer i with conditional expected pizazz m_i . Alternatively, it is the distribution of true pizazz conditional on observed pizazz among singles who are potential partners for an optimizer indexed i . Therefore, assumption 1 is more precisely written as $\partial Q(x_j, m_j | m_i)/\partial m_j < 0$.

Lemma 1: $\partial EJ/\partial m > 0$ if Assumption 1 holds.

Proof: In what follows, the decision maker is a woman with expected true pizazz conditional on observed pizazz denoted m_i . The true pizazz of the potential husband is denoted x and his expected true pizazz conditional on his observed pizazz y is denoted m .

From the calculations in stage 3, recall that

$$EJ = E_{X|Y} \max [\Psi(x), U]. \quad (35)$$

Therefore,

$$EJ = \Pr(\Psi(x) \geq U) E[\Psi(x) | \Psi(x) \geq U] + \Pr(\Psi(x) < U)U. \quad (36)$$

From the calculations in stage 3, we know that $\Pr(\Psi(x) \geq U) = \Pr(x \geq x_r)$ and $\Pr(\Psi(x) < U) = \Pr(x < x_r)$. Also, we know from the calculations in stage 1, that U does not depend on x . Using equations (3) and (5), we obtain,

$$EJ = \left(\frac{1}{c_1}\right) \Pr(x \geq x_r | y) [E(x | x \geq x_r) + \gamma U] + \Pr(x < x_r | y)U \quad (37)$$

It should be pointed out that in the above expression, $\Pr(x \geq x_r | y)$ is in fact conditional on the woman's own expected pizazz m_i . From eq(31) and eq(34), we can write $\int_{x_r}^{\bar{x}} dQ(x, m | m_i)$ in place of $\Pr(x \geq x_r | y)$. Using $\gamma = c_1 - c_2$ and $x_r = c_2 U$, eq(37) may be written as,

$$\begin{aligned} EJ &= \left(\frac{1}{c_1}\right) \int_{x_r}^{\bar{x}} x dQ(x, m | m_i) + U - \left(\frac{x_r}{c_1}\right) \int_{x_r}^{\bar{x}} dQ(x, m | m_i) \\ &= \left(\frac{1}{c_1}\right) \int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m | m_i) + U. \end{aligned} \quad (38)$$

The term $\int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m | m_i)$ can be integrated by parts (this is done using a change of variable technique; denoting $Q(x, m | m_i)$ by $G(x)$, and using $dG(x) = -d[1 - G(x)]$). This yields

$$EJ = \left(\frac{1}{c_1}\right) \int_{x_r}^{\bar{x}} [1 - Q(x, m | m_i)] dx + U. \quad (39)$$

Therefore,

$$\frac{\partial EJ}{\partial m} = \left(\frac{1}{c_1}\right) \int_{x_r}^{\bar{x}} \left(-\frac{\partial Q(x, m | m_i)}{\partial m}\right) dx > 0. \quad (40)$$

since we have assumed that $\partial Q(x, m | m_i)/\partial m < 0$, and U does not depend on m .

Proposition 1: $\Phi'(m) > 0$.

Proof: We denote expected true pizazz conditional on observed pizazz of the woman by m_i . The true pizazz of the potential husband is denoted x and his expected true pizazz conditional on his observed pizazz y is denoted m . From eq(8) and eq(39) we have

$$\Phi(m) = \frac{1}{c_3} \left[m + \left(\frac{\lambda}{c_1} \right) \int_{x_r}^{\bar{x}} (1 - Q(x, m | m_i)) dx + (\lambda + \xi) U \right]. \quad (41)$$

Using lemma 1,

$$\Phi'(m) = \frac{1}{c_3} \left[1 - \left(\frac{\lambda}{c_1} \right) \int_{x_r}^{\bar{x}} \frac{\partial Q(x, m | m_i)}{\partial m} dx \right] > 0. \quad (42)$$

Proposition 2: If the optimizer is indexed by i , and j indicates his or her potential partner and the optimizer i 's reservation utility is

$$U_i = \frac{\alpha_i(m_i)}{c_2 [c'_2 + \xi_i(m_i)]} \int_{\Phi_i^{-1}(U_i)}^{\bar{m}_j} [1 - F_j(m_j | m_i)] \left[1 - \frac{\lambda}{c_1} \int_{\Psi_i^{-1}(U_i)}^{\bar{x}_j} \frac{\partial Q_j(x_j, m_j | m_i)}{\partial m_j} dx_j \right] dm_j \quad (43)$$

where $c'_2 = c_2 + \lambda$ and $U_i = U_i(m_i)$.

Proof: From equations (6) and (15) we have,

$$\begin{aligned} (\alpha + c_2) U &= \alpha E_M \max \{ \Phi(m), U \} \\ &= \alpha P(\Phi(m) \geq U) E[\Phi(m) | \Phi(m) \geq U] + \alpha P(\Phi(m) < U) U. \end{aligned}$$

Therefore,

$$c_2 U = \alpha P(\Phi(m) \geq U) (E[\Phi(m) | \Phi(m) \geq U] - U).$$

Using $\Phi(m_r) = U$, the above equation can be re-written:

$$\begin{aligned} U &= \frac{\alpha}{c_2} \int_{m_r}^{\bar{m}} [\Phi(m) - \Phi(m_r)] dF(m) \\ &= -\frac{\alpha}{c_2} \int_{m_r}^{\bar{m}} [\Phi(m) - \Phi(m_r)] d(1 - F(m)). \end{aligned}$$

Integrating by parts and noting that $F(\bar{m}) = 1$ yields

$$U = \frac{\alpha}{c_2} \int_{m_r}^{\bar{m}} [1 - F(m)] \Phi'(m) dm. \quad (44)$$

Substituting eq(42) in the above yields

$$U = \frac{\alpha}{c_2} \int_{m_r}^{\bar{m}} [1 - F(m)] \frac{1}{c_3} \left[1 - \left(\frac{\lambda}{c_1} \right) \int_{x_r}^{\bar{x}} \frac{\partial Q(x, m | m_i)}{\partial m} dx \right] dm. \quad (45)$$

Using equations (5) and (9), introducing the appropriate subscripts and noting that $c_3 = r + \delta + \lambda + \xi$ yields,

$$U = \frac{\alpha_i(m_i)}{c_2 [c'_2 + \xi_i(m_i)]} \times \int_{\Phi_i^{-1}(U_i)}^{\bar{m}_j} [1 - F_j(m_j | m_i)] \left[1 - \left(\frac{\lambda}{c_1} \right) \int_{\Psi_i^{-1}(U_i)}^{\bar{x}_j} \frac{\partial Q_j(x_j, m_j | m_i)}{\partial m_j} dx_j \right] dm_j. \quad (46)$$

It is clear from the above expression that U_i does not depend on x_j and m_j .

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Figure 1: The dynamics of the marriage market