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Load to capacitance transfer using different spring elements in capacitive transducers

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Abstract

Many physical sensors in which a displacement is the result of a change in the variable to be measured rely on the principle of capacitive transduction to transfer this displacement into a suitable electric signal. Commonly, the linearity of this transduction is one of the design criteria. The total transduction of physical load to capacitance change can be subdivided into the transfer from load to displacement and from displacement to capacitance change. The latter is non-linear due to its well-known hyperbolic behaviour. The first one — that of load to displacement — depends on the nature of the spring elements between the two plates of the capacitor. When, e.g., a rubber elastic spring is applied, the load to displacement transfer is also not linear. It is the aim of this paper to show that the two non-linear transfer functions of the mentioned subsystems result into a remarkably increased linearity for the transfer of the total capacitive transducer. The thus obtained theoretical relation is experimentally verified for the most favourable situation, using rubber elastic springs. The results are in good agreement with the theory. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Capacitive transducer; Rubber elasticity; Linearity

1. Introduction

Capacitive transduction is used in many physical sensors, e.g., in pressure, force and acceleration sensors. A range of criteria, such as the allowed creep and hysteresis, the technology used, the operational bandwidth and the sensitivity determine the final design. One of the criteria that should be taken into account is the linearity of the total transduction. Although digital signal processing can, in principle, linearize any transfer function, a more or less linear relation between measurand and sensor output remains favourable to guarantee uniform precision and resolution over the whole measurement range.

Basically, the operational principle of any capacitive transducer can be subdivided into the transfer of physical load to displacement, and of displacement to change in capacitance. From the well-known hyperbolic behaviour of the latter transfer function, it is clear that a linear transfer of a capacitive transducer is far from obvious. Nevertheless, when the physical load to displacement transfer is

also not linear, but in such a way that the non-linearity of this transfer partly counteracts the first-mentioned non-linearity, the transfer of the total transducer can become more linear.

In this paper, four spring elements are treated. The first two are based on plan-parallel movement of the two plates of an capacitor, but with different springs in between: an ideal spring, behaving according to Hooke's law, and a rubber elastic spring [1,3]. The second two springs are part of one of the two capacitor plates itself: bending of a clamped plate and of a membrane, respectively [2]. In addition to the theoretical treatment, the most favourable of the four configurations is experimentally verified and the results are presented.

2. Theory

In this paper, two basically different spring systems are discussed. On the one hand, the system where the spacers between two plan-parallel moving rigid plates form the spring elements, as depicted in Fig. 1a, and, on the other hand, the situation where the thin plate itself forms the spring element and bends as a result of an applied pressure, P , as depicted in Fig. 1b.

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Fig. 1. (a) Plan-parallel moving rigid plates, separated by spring elements, and (b) bending membrane. The relevant variables are shown and defined in Table 1.

In the latter case, the air gap, $t(r)$, is a function of the position, r , on the plate. A general expression for the change in capacitance as a result of pressure, P , on a circular plate is

$$\frac{dC(r)}{dP} = \frac{\partial C(r)}{\partial t(r)} \frac{\partial t(r)}{\partial P} \quad (1)$$

For the situation as shown in Fig. 1a, there is no dependence on the radius, r , whereas the curvature of the

plate, $w(r)$, shown in Fig. 1b, requires the integration over the total surface of the plate. In this cylinder symmetrical case, only the r dependence must be elaborated.

Eq. 1 comprises the relation of change in air gap to change in capacitance as well as the relation of change in pressure to change in air gap. As these relations are not always available in an explicit form to be substituted into Eq. 1, it was decided to present the relevant equations, sometimes in an implicit form, and solve them numerically.

The four different spring elements, the resulting curvature of the electrode, $w(r)$, determining the air gap, $t(r)$, the air gap to pressure relation and the capacitance to air gap relation are all summarized in Table 1.

The equations in Table 1 describing the bending plate and membrane, both their curvature and their elastic behaviour, can be found in Ref. [2]. The equations describing the elastic behaviour of rubber is from Ref. [1] and is briefly treated here.

Consider a rubber cube with unit volume ($V_{\text{cube}} = 1$) that is transformed under strain into a rectangular block, now having unequal edge lengths λ_1 , λ_2 and λ_3 , but the conditions for constancy of volume during deformation holds:

$$\lambda_1 \lambda_2 \lambda_3 = 1 \quad (2)$$

Table 1

Four different spring elements in capacitive transducers with resulting relevant relations

t , air gap thickness; t_0 , air gap at zero pressure; w_0 , center deflection; $w(r)$, curvature; P , pressure; R , radius of the circular plate; A_R , area of the rubber spring; G , shear modulus of rubber; E , Young modulus; h , thickness of diaphragm; σ , initial stress in diaphragm; ε_r , ε_0 , relative and absolute dielectric constant.

	Air gap, curvature	Air gap–pressure relation	Capacitance–air gap relation
Free plate, ideal Hooke's spring	t , plan-parallel	$P(t) = \frac{\text{Constant}}{\pi R^2} (t - t_0)$	$C(t) = \frac{\varepsilon_0 \varepsilon_r A}{t}$
Free plate, rubber elastic spring	t , plan-parallel	$P(t) = \frac{A_R G}{\pi R^2} \left\{ \left(1 + \frac{t - t_0}{t_0} \right) - \frac{1}{\left(1 + \frac{t - t_0}{t_0} \right)^2} \right\}$	$C(t) = \frac{\varepsilon_0 \varepsilon_r A}{t}$
Stress free, thin plate, clamped edges	$t(r) = t_0 - w(r)$,	$P(w_0) = \frac{E h^3}{0.171 R^4}$	$dC(t) = \frac{\varepsilon_0 \varepsilon_r r dr d\varphi}{t(r)}$,
Large stress, membrane, clamped edges	$w(r) = w_0 \left(1 - \left(\frac{r}{R} \right)^2 \right)^2$	$\times \left(w_0 + 0.488 \frac{w_0^3}{h^2} \right)$	$C(t) = \int_{\varphi=0}^{2\pi} \int_{r=0}^R dC(t)$
	$t(r) = t_0 - w(r)$,	$P(w_0) = \frac{4\sigma h}{R^2} w_0$	$dC(t) = \frac{\varepsilon_0 \varepsilon_r r dr d\varphi}{t(r)}$,
	$w(r) = w_0 \left(1 - \left(\frac{r}{R} \right)^2 \right)$		$C(t) = \int_{\varphi=0}^{2\pi} \int_{r=0}^R dC(t)$

From the total entropy of deformation, ΔS , the work of deformation per unit volume, W , can be derived, using $W = -T\Delta S$:

$$W = \frac{1}{2} G (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) \quad (3)$$

G being the elastic constant or shear modulus of the rubber. Uniaxial compression of the unit cube, resulting in a compressed edge length λ_1 ($\lambda_1 < 1$), and using Eq. 2 yields:

$$\lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}} \quad (4)$$

Substitution of Eq. 4 into Eq. 3 results in

$$W = \frac{1}{2} G \left(\lambda_1^2 + \frac{2}{\lambda_1} - 3 \right) \quad (5)$$

Now the stress–strain relation can be found using Eq. 5 and $P = dW/d\lambda_1$:

$$P = \frac{dW}{d\lambda_1} = G \left(\lambda_1 - \frac{1}{\lambda_1^2} \right) \quad (6)$$

It can be shown that this equation (Eq. 6) is also valid for rubber springs, other than cubic-shaped under the assumption that the area of the rubber in contact with the top and bottom electrode (A_R in Table 1) does not change during deformation. Using $\lambda_1 = 1 + (t - t_0)/t_0$, where t is the air gap thickness and t_0 is the air gap at zero pressure, and keeping in mind that the pressure on the top electrode with area πR^2 results in a force exerted on the area A_R of the rubber spring only, results in the equation included in Table 1:

$$P(t) = \frac{A_R G}{\pi R^2} \left\{ \left(1 + \frac{t - t_0}{t_0} \right) - \frac{1}{\left(1 + \frac{t - t_0}{t_0} \right)^2} \right\} \quad (7)$$

Relevant for this paper is the resulting equivalent spring constant, in case two springs are connected in series. Spring constant k_{total} for two springs with constants k_1 and k_2 in series is

$$\frac{1}{k_{\text{total}}} = \frac{1}{k_1} + \frac{1}{k_2} \quad (8)$$

The resulting displacement for the series configuration of the springs when a force is applied, is simply the sum of the displacements of the separate spring systems for that force.

3. Protocol

In order to be able to compare the different results, relevant design parameters were fixed and material con-

Table 2
Parameters used throughout this paper

Plate radius	$R = 1000$
Plate thickness	$h = 1$
Zero-pressure air gap	$t_0 = 10$
Minimum air gap	$t_{\min} = t(r=0)_{\min} = 8$
Maximum pressure (at min. air gap)	$P = 2$
Nominal capacitance	$C(P=0) = 0.3$

stants were adapted in such a way that the (maximum) change in the air gap was in all cases reached at one and the same maximum applied pressure. The parameters and variables are all expressed in arbitrary units and listed in Table 2.

In case two springs were connected in series, the parameters were chosen in such a way, that the maximum total decrease in the air gap, $(t_0 - t(r=0))_{\max, \text{total}} = 2$, was equally divided over spring 1 and spring 2 (1 + 1).

The calculations were carried out using Mathcad 6 (by MathSoft).

4. Spring materials

It may be worthwhile to discuss some of the materials that lead to the spring behaviour, as described here. A prerequisite is compatibility with the usual processing steps in micromachined transducers.

An important and well-documented mechanical material in this respect is silicon. A silicon diaphragm of a few up to 10 μm , obtained by wet chemical etching of a Si wafer, using, e.g., an electrochemical etch-stop, yields a structure that complies with the spring, described by a stress-free, thin plate with clamped edges, as mentioned in Table 1 (with $E_{\text{Si}} = 1.3 \times 10^{11}$ Pa).

Thin silicon beams, which suspend a proof mass (e.g., for accelerometers), will behave as ideal springs as long as the displacements of the mass are small with respect to the length (and thickness) of the beams.

A good candidate for rubber elastic springs is Poly-DiMethylSiloxane (PDMS). PDMS is a commercially available physically and chemically stable silicone rubber, well suited for application in micromachined sensors, because of its cleanroom processability [3–6]. PDMS can be spin-coated on wafer-scale and is photocurable and thus can be shaped by photolithography, in case a proper photo-initiator is added. The flexibility of PDMS is high, expressed by its low shear modulus (typically $G = 250$ kPa). In addition, PDMS can be used in mechanical structures up to relatively high frequencies without noticeable loss, due to its low loss tangent ($\tan \delta < 0.001$). Film thicknesses between 2 and 40 μm can be obtained in one spin-coating step by appropriate choice of spin rate and time, as indicated in Fig. 2.

Fig. 2. Measured thickness of PDMS PS851 at different spin rates and spin times.

PDMS has been successfully applied as spring elements in a triaxial accelerometer [3].

5. Experimental

In order to verify the most favourable of the theoretical results, a load to change in capacitive transducer has been manufactured in which the plan-parallel moving top plate was separated from the lower plate by rubber elastic springs, representing the second system of Table 1: free plate, rubber elastic spring.

5.1. Design and fabrication

Both the lower and the upper plate consist of a quarter of a 3-in. Si wafer, 10–15 m Ω cm. After deposition of a primer (TMSM, Aldrich), PDMS (RMS033, ABCR) with photo-initiator (1 wt.% DMAP, Janssen Chimica) was spun on the front side of the bottom wafer, resulting in a film thickness of about 10 μ m. By photolithography, three spring elements were obtained near the ‘corners’ of the quarter of the wafer. Thus, the three springs consist of rubber pillars with height $t_0 \approx 10$ μ m and area of about 0.12 mm² each, resulting in a total spring area of $A_R \approx 0.36$ mm². After a bond-wire was glued with silver paint on the reverse side of the Si top plate, it was carefully deposited with its front side down on top of the PDMS spring elements. The lower Si plate was glued with silver paint on a metallic carrier for easy handling. The bond-wire of the top electrode was bonded to a connector.

5.2. Measurement set up and protocol

The transducer was connected to a digital capacitance meter. In order to prevent noticeable bending of the upper plate due to the application of force, loads with a large flat area of 2.8 cm² were carefully placed on top of the upper plate. The loads had a mass of 3.00 g each. The quarter Si wafer itself weighs approximately 1.0 g. The capacitance of the bond and connection wires turned out to be circ. 13 pF.

6. Results and discussion

The non-linearity of the total transfer from pressure to capacitance is not only caused by the hyperbolic relation between air gap and capacitance, but also by the possibly non-linear spring elements and the curvature of the diaphragm, when pressure is applied. It is therefore hardly possible to predict or optimize the total transfer, when only one of the three above-mentioned factors is known.

As an intermediate result, the last factor of Eq. 1, describing the change in air gap as a result of applied pressure, is plotted in Fig. 3 for both the free-moving plate with rubber elastic springs (curve I) and the stress-free thin plate with clamped edges, wherein the latter case the center deflection is shown (curve II). Also, the linear P -to- t transfer of the free-moving plate with an ideal spring is plotted (curve III).

From Fig. 3 and knowing the hyperbolic relation between capacitance and air gap, it can be reasoned, that the two non-linear transfers, shown in Fig. 3, curves I and II, will counteract this hyperbolic relation, and thus possibly linearize the total P -to- C transfer.

This total transfer from applied pressure to change in capacitance is plotted in Fig. 4 for all four configurations of Table 1, using the data of Table 2.

Some observations can be made: the plan-parallel moving plates (curves A and B) show an overall superior dC/dP over the curved plates (curves C and D). In addition, the non linear transfer of the rubber elastic springs (Fig. 3, curve I) remarkably linearizes the total transfer (curve B) with respect to curve A, where the non-linearity is solely caused by the hyperbolic air gap to C transfer, as the transfer of the ideal spring is linear, shown in curve III of Fig. 4. This phenomenon was expected from Fig. 3.

The non-linear transfer of the clamped plate (Fig. 3, curve II), however, dramatically overcompensates the hyperbolic effect, resulting in a very non-linear and, incidentally, quite insensitive total transfer (Fig. 4, curve C).

Fig. 3. The changing air gap under the influence of applied pressure; (curve I) rubber spring, (curve II) thin plate, (curve III) ideal spring.

Fig. 4. The capacitance as a function of applied pressure for the spring elements listed in Table 1; (curve A) ideal spring, (curve B) rubber spring, (curve C) bending plate, (curve D) bending membrane.

Another option is to apply a bending diaphragm on top of one of the spring elements, resulting in two springs in series. The result of the practically possible membrane on top of rubber spring elements (curve E) and of the not so practically realizable bending plate with clamped edges on top of ideal spring elements (curve F) is shown in Fig. 5.

As to be expected, the sensitivity is within the group of plan-parallel moving plates on the one hand (curves A and B of Fig. 4) and the bending only diaphragms on the other hand (curves C and D of Fig. 4). Curve F of Fig. 5 shows a virtually ideal behaviour with respect to its linearity.

The results of all six cases are summarized in Table 3, using the data, as mentioned, of Table 2. The sensitivity is the linearized sensitivity over the given pressure range (0 to 2).

From the value of the regression coefficient R , indicating how well the curve fits to a linear fit, it can be concluded that it is certainly possible to linearize the intrinsically non-linear capacitive transfer by using non-linear spring elements, e.g., either rubber elastic springs (curve B) or a combination of a bending plate and ideal spring elements (curve F).

Fig. 5. The capacitance as a function of applied pressure for two spring elements in series; (curve E) membrane and rubber, (curve F) plate and ideal spring.

Table 3

Summarized results showing both the sensitivity and the linear regression, R , of all discussed cases

Spring element	Curve	Sensitivity	R
Ideal spring	A	0.037	0.9983
Rubber spring	B	0.038	0.9999
Bending plate	C	0.013	0.9716
Bending membrane	D	0.017	0.9992
Membrane + rubber	E	0.026	0.9990
Plate + ideal spring	F	0.023	0.9999

In order to experimentally verify one of the most favourable theoretical results, that of rigid plates with rubber spring elements in between, tests were carried out with a load to change in capacitance transducer with the device as described in the experimental section. The results of the measured capacitance under load, corrected for the capacitance of the bond and connection wires (though only 1% of the measured capacitance), and with the load of the quarter Si wafer itself taken into account, is shown in Fig. 6.

Due to some lateral movement of the upper plate at the moment the first load was applied, the capacitance reading for zero load is meaningless and is not included in the curve of Fig. 6. The capacitance of the transducer without load is circ. 1.2 nF, whereas 1.01 nF is theoretically expected. Apparently, the thickness of the spring elements is $t_0 \cong 8.4 \mu\text{m}$. Under a load of 0.157 N the measured capacitance of 1.835 nF corresponds to $t \cong 5.5 \mu\text{m}$. Using Eq. 7 with $A_R = 0.36 \text{ mm}^2$, the elastic constant or shear modulus, G , can be calculated: $G = 260 \text{ kPa}$, which is close to the reported value of 250 kPa for PDMS [3,6]. It can be observed from Fig. 6 that within the experimental error, the theoretical, nearly linear behaviour as shown in curve B of Fig. 4 is experimentally found. For comparison, the theoretical curve in case ideal springs would have been used (curve A of Fig. 4), now fitted within the scale of Fig. 6, is included in that figure. The increased linearity is obvious.

Fig. 6. The measured capacitance as a function of the applied load for the configuration of rigid plates with rubber springs; solid line: experimental results. For comparison, the dashed curve shows the non-linear theoretical relation for a configuration with ideal springs.

7. Conclusions

It is shown that the basically non-linear transfer from load to change in capacitance can be linearized by making use of the combination of both the non-linear transfer from displacement to change in capacitance and that from load to displacement.

One of the theoretically most favourable combinations, that of rigid plates with rubber elastic springs in between, has been experimentally verified. The results are in agreement with the theory and clearly show the increase in linearity of the total transfer from load to change in capacitance.

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