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Stabilization of Nonlinear RLC Circuits: Power Shaping and Passivation

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Abstract—In this paper we prove that for a class of RLC circuits with convex energy function and weak electromagnetic coupling it is possible to “add a differentiation" to the port terminals preserving passivity—with a new storage function that is directly related to the circuit power. The result is of interest in circuits theory, but also has applications in control problems as it suggests the paradigm of power shaping stabilization as an alternative to the well-known method of energy shaping. We show in the paper that, in contrast with energy shaping designs, power shaping is not restricted to systems without pervasive dissipation and naturally allows to add “derivative" actions in the control. These important features, that stymie the applicability of energy shaping control, make power shaping very practically appealing, as illustrated with examples in the paper. To establish our results we exploit the geometric property that voltages and currents in RLC circuits live in orthogonal spaces, i.e., Tellegen’s theorem, and heavily rely on the seminal paper of Brayton and Moser in 1964.

Note: This paper is an abridged version of (Ortega et al., 2003).

I. INTRODUCTION

In this paper we are interested in (possibly nonlinear) RLC circuits consisting of arbitrary interconnections of resistors, inductors, capacitors and voltage and current sources. It is well-known that, if the resistors, inductors and capacitors are passive, i.e., if their energy functions are positive, then the overall interconnected circuit is also passive with port variables the external sources voltages and currents, and storage function the total stored energy (Desoer and Kuh, 1969). This property was exploited by Youla in 1959 (Youla et al., 1959) who proved that terminating the port variables of a passive RLC circuit with a passive resistor would ensure that “finite energy inputs will be mapped into finite energy outputs," what in modern parlance says that adding damping injection to a passive system ensures $L_2$-stability. Passivity can also be used to stabilize a non-zero equilibrium point, but in this case we must modify the storage function to assign a minimum at this point. If the storage function is the total energy we refer to this step as energy shaping, which combined with damping injection constitute the two main stages of passivity-based control (PBC) (Ortega and Spong, 1989). As explained in (Ortega et al., 1998) there are several ways to achieve energy shaping, the most physically appealing being the so-called energy balancing PBC (or control by interconnection) method. With this procedure the storage function assigned to the closed-loop passive map is the difference between the total energy of the system and the energy supplied by the controller, hence the name energy balancing. Unfortunately, energy balancing PBC is stymied by the presence of pervasive dissipation, that is, the existence of resistive elements whose power does not vanish at the desired equilibrium point. Another practical drawback of energy-shaping control is the limited ability to “speed up" the transient response (preserving, of course, a provable stable behavior.) Indeed, as tuning in this kind of controllers is essentially restricted to the damping injection gain, the transients may turn out to be somehow sluggish, and the overall performance level below par.

Our main contribution in this paper is the establishment of a new passivity property for a class of RLC circuits that provides the basis for a novel PBC design methodology that does not suffer from the two aforementioned drawbacks. To define the class, we assume that the energy of the inductors and capacitors are not just positive but actually convex functions, and assume that the electromagnetic coupling between the dynamic elements is weak. Indeed, for the case of RC or RL circuits this condition is conspicuous by its absence—as already reported in (Ortega and Shi, 2002).

The new passivity property, which is by itself of interest in circuits theory, has two key features that makes it attractive for control design as well. First, that the storage function is not the total energy, but a function directly related with the energy) storage functions immediately suggests the paradigm of stabilization via power shaping. To generalize the idea to a broad class of RLC we need some preli-
nary material from the ground breaking paper (Brayton and Moser, 1964), that is introduced in Section IV. Finally, we present the main result in Section V.

II. ENERGY BALANCING PASSIVITY-BASED CONTROL

In (Ortega et al., 2001) a new method is presented to stabilize the following class of nonlinear systems—that includes passive systems.

Definition 1: We say that the \( m \)-port system

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= y(x),
\end{align*}
\]

with state \( x = \col(x_1, \ldots, x_o) \in \mathbb{R}^o \), and power port variables \( u, y \in \mathbb{R}^m \), satisfies the energy balance inequality if, along all trajectories compatible with state \( x \), we have

\[
E[x(t)] - E[x(0)] = \int_0^t u^T(t')y(x(t'))dt' \geq 0,
\]

where \( E : \mathbb{R}^m \rightarrow \mathbb{R} \) is the stored energy function. If \( E(x) \) is positive semidefinite then we say that the system is passive with port variables \( u, y \).

The proposition below, established in (Ortega et al., 2001), constitutes the basis for energy-balancing PBC. (For simplicity, we present only the case of static state feedback, the case of dynamic controllers may be found in (Ortega et al., 2001).)

Proposition 1: Consider \( m \)-port systems that satisfy the energy balance equation (2). If we can find a vector function \( \hat{u}(x) : \mathbb{R}^o \rightarrow \mathbb{R}^m \) such that the partial differential equation

\[
\nabla^T \dot{E}_a(x)[f(x) + g(x)\hat{u}(x)] = -\dot{y}^T(x)\hat{u}(x),
\]

(3)

can be solved for the scalar function \( \dot{E}_a \) \( : \mathbb{R}^m \rightarrow \mathbb{R} \), and the function \( \dot{E}_a(x) := E(x) + E_a(x) \) has an isolated minimum at \( x^* \), then the state-feedback \( u = \hat{u}(x) \) is an energy balancing PBC, i.e., \( x^* \) is a stable equilibrium with the difference between the stored and the supplied energies constituting a Lyapunov function.

This result, although quite general, is of limited interest. First of all, these kind of state models do not reveal the role played by the energy function in the system dynamics. Hence it is difficult to incorporate prior information to select a \( \hat{u}(x) \) to solve the PDE (3). In (Ortega et al., 2002) energy balancing PBC is developed for a more suitable class of models, the so-called port-controlled Hamiltonian systems, that explicitly exhibit the existence of dynamic invariants. Second, and perhaps more importantly, it is shown in (Ortega et al., 1998) that, beyond the realm of mechanical systems, the applicability of energy balancing control is severely stymied by the system's natural dissipation. Indeed, it is easy to see that a necessary condition for the global solvability of the PDE (3) is that \( y^T(x)\dot{u}(x) \) vanishes at all the zeros of \( f(x) + g(x)\hat{u}(x) \), that is, the implication

\[
f(x) + g(x)\hat{u}(x) = 0 \Rightarrow y^T(x)\dot{u}(x) = 0
\]

should hold. Now, \( f(x) + g(x)\hat{u}(x) \) is obviously zero at the equilibrium \( x^* \), hence the power extracted from the controller should also be zero at the equilibrium. This means that energy balancing PBC is applicable only if the system does not have pervasive damping, i.e., if it can be stabilized extracting a finite amount of energy from the controller. This is the case in regulation of mechanical systems where the extracted power is the product of force and velocity and we want to drive the velocity to zero. Unfortunately, it is no longer the case for most electrical or electromechanical systems where power involves the product of voltages and currents and the latter may be nonzero for nonzero equilibria. For instance, a series RC circuit is energy-balancing stabilizable (because in steady state there is no current drained from the source), but not an RL circuit—see the following section.

Remark 1: For linear systems it is, of course, possible to overcome the dissipation obstacle by shifting the equilibrium of the systems equation to zero. As the terms dependent on \( x^* \), \( u^* \) cancel in the incremental model, the original (quadratic) storage function—but expressed now in terms of the incremental variables—qualifies as a storage function for the shifted model. Unfortunately, this simple solution is not applicable for the nonlinear case, as there is no systematic procedure to generate, from the knowledge of \( E(x) \), a storage function for the “input-shifted” system

\[
\dot{x} = f(x) + g(x)u^* + g(x)w, \quad y = y(x),
\]

with \( w := u - u^* \), and \( (w, y) \) the new port variables. As shown in (Maschke et al., 2000) the natural solution of adding to \( E(x) \) a term \(-\int_0^t w^T(t')y(x(t'))dt'\) is also restricted to systems without pervasive damping.

III. TOWARDS POWER SHAPING CONTROL

Let us illustrate with an example how the limitations of energy balancing PBC can be overcome via power balancing. Consider a voltage-controlled nonlinear series RL circuit. The behavior of the inductor is characterized by a function, \( p_L = p_L(i_L) \), relating the flux linkages \( p_L \) and the current \( i_L \), and Faraday's law: \( \dot{p}_L = v_L \), where \( v_L \) is the inductor voltage. The resistor is a static element described by its characteristic function \( v_R = v_R(i_R) \), where \( v_R, i_R \) are the resistors voltage and current, respectively. The dynamics of the circuit is obtained from Kirchhoff's voltage law as

\[
v_L = L(i_L)\frac{di_L}{dt} = -v_R(i_R) + v_S,
\]

(5)

where \( v_S \) is the voltage at the port terminal, which is our control action. Furthermore, we have that \( i_R = i_L \), and the property \( L(i_L) := \nabla_{i_L}p_L(i_L) \). Differentiating the inductor's energy \( \dot{E}_L(p_L) \) we obtain

\[
\dot{E}_L(p_L) = \nabla_{p_L}E_L(p_L)\dot{p}_L (= i_Lv_L) = i_Sv_S - i_Rv_R(i_R),
\]
where, to obtain the last equation, we used the fact that \( i_S \), the port current, is equal to \( i_L \). If we assume that the resistor is passive, that is, that the energy that it dissipates is nonnegative, i.e., \( \int_0^t i_R(t')v_R(t')dt' \geq 0 \), and integrate from 0 to \( t \), we recover the energy balance inequality (2). If we further assume that the inductor is also passive—that is, its stored energy is nonnegative—we verify that the circuit is nonnegative, i.e., \( J \), the energy stored in the circuit, is zero only at zero, it is clear that, at any equilibrium \( i_L^* \neq 0 \), the extracted power \( i_L^* \hat{v}_R(i_L^*) \) is nonzero, hence the circuit is not energy-balancing stabilizable—not even in the linear case!

To overcome this problem let us define the function

\[
F(i_R) := \int_0^t \hat{v}_R(i_R')di'_R,
\]

known in the circuits literature (Millar, 1951) as the resistors content, which has units of power—in particular, for linear resistors it is half the dissipated power. Furthermore, notice that for passive resistors the function is nonnegative. Summarizing, we have the following result.

**Proposition 2:** Consider a series RL circuit. If the inductor is passive and has a twice differentiable convex energy function, that is,

\[
\nabla_{i_L}^2 E_L(i_L) \geq 0,
\]

then, along the trajectories of the system, we have the power balance inequality

\[
F[i_L(t)] - F[i_L(0)] \leq \int_0^t \hat{v}_S(t')d_s(t')dt'.
\]

Furthermore, if the resistor is passive, then the circuit defines a passive system with port variables \( (v_S, i_S) \) and storage function \( E_L(p_L) \).

If the resistance characteristic is exactly known we can take \( F_S(i_L) = -F(i_L) + \frac{R}{2}(i_L - i_L^*)^2 \), with \( R > 0 \) some tuning parameter. But clearly, we only need to “dominate” \( F(i_L) \) to assign the desired minimum, which (together with the fact that \( L(i_L) \) is completely unknown) exhibits the robustness of the design procedure.

Detailed proofs for general RL and RC circuits can be found in (Jeltsema et al., 2003; Ortega and Shi, 2002). An important observation, that will be proved for more general nonlinear RLC circuits in the following section, is that we can express the circuit dynamics (5) in terms of the resistor content as

\[
L(i_L) \frac{di_L}{dt} = -\nabla_{i_L} F(i_L) + v_S.
\]

The identification of a gradient-like description of RLC circuits is the main contribution of the seminal paper (Brayton and Moser, 1964).

**IV. PASSIVITY OF BRAYTON-MOSER CIRCUITS**

The previous developments show that, using the content (resp. co-content in the RC case (Jeltsema et al., 2003; Ortega and Shi, 2002)) as a storage function, we can identify new passivity properties of RL (resp. RC) circuits. In this section we will establish similar properties for RLC circuits. Towards this end, we strongly rely on some fundamental results reported in (Brayton and Moser, 1964). Furthermore, we assume that the current-controlled resistors are contained in \( \Sigma_L \) and the voltage-controlled resistors are contained in \( \Sigma_C \). The class of RLC considered here is then composed by an interconnection of \( \Sigma_L \) and \( \Sigma_C \). For a detailed derivation, see (Ortega et al., 2003).

**A. Brayton and Moser’s Equations**

In the early sixties Brayton and Moser (Brayton and Moser, 1964) have shown that the dynamic behavior of a topologically complete circuit (where we restrict, for simplicity, to circuits having only voltage sources in series with the inductors) is governed by the following differential equation

\[
Q(x)\dot{x} = \nabla_x P(x) - Bv_S
\]

where \( x = \text{col}(i_L, v_C) \), \( B = \text{col}(B_S, 0) \) with \( B_S \in \mathbb{R}^{n_L \times n_S} \), \( Q(x) = \text{diag}(-L(i_L), C(v_C)) \in \mathbb{R}^{n \times n} \), \( n = n_L + n_C \), and \( P : \mathbb{R}^n \to \mathbb{R} \) is called the mixed-potential and is given by

\[
P(x) = i_L^T \Gamma v_C + F(i_L) - G(v_C),
\]

where \( \Gamma \in \mathbb{R}^{n_L \times n_C} \) is a (full rank) matrix that captures the interconnection structure between the inductors and capacitors. The functions \( F(i_L) \) and \( G(v_C) \) are the resistors content \( F(i_L) \) (like in (6)) and co-content \( G(v_C) \) having the form

\[
G(v_R) := \int_0^r i_R(v_R')dv_R',
\]

respectively.
B. Generation of New Storage Function Candidates

Let us next see how the Brayton-Moser equations (9) can be used to generate storage functions for RLC circuits. From (9) we have

$$\dot{P}(x) = x^T Q(x) x + x^T B v_S.$$  \hfill{(12)}

Compare the latter with the right-hand side of (7) of Proposition 2 (notice that $x^T B v_S = \frac{1}{2} x^T v_S$). Unfortunately, even under the reasonable assumption that the inductor and capacitor have convex energy functions, the presence of the negative sign in the first main diagonal block of $Q(x)$ makes the quadratic form sign-indefinite, and not negative (semi-)definite as desired. Hence, we cannot establish a power-balance inequality from (12). Moreover, to obtain the passivity property an additional difficulty stems from the fact that $P(x)$ is also not sign-definite. To overcome these difficulties we borrow inspiration from (Brayton and Moser, 1964) and look for other suitable pairs, say $Q_A(x)$ and $P_A(x)$, which we call admissible, that preserve the form of (9). More precisely, we want to find matrix functions $Q_A(x)$ verifying

$$Q^T_A(x) + Q_A(x) \leq 0,$$  \hfill{(13)}

and scalar functions $P_A : \mathbb{R}^n \to \mathbb{R}$ (if possible, positive semi-definite) that describe the same dynamics as (9). If (13) holds, it is clear that $P_A(x) \leq x^T B v_S$, from which we obtain a power balance equation with the desired port variables. Furthermore, if $P_A(x)$ is positive semi-definite we are able to establish the required passivity property.

A complete characterization of the admissible pairs $(Q_A, P_A)$ has been reported in (Ortega et al., 2002), but it requires the solution of a partial differential equation. A more constructive procedure to generate admissible pairs is given in the following proposition which, for ease of reference, is enunciated in terms of the original RLC circuit data. For ease of notation, we write (9) in the more compact form

$$Q(x) x = \nabla_x \dot{P}(x),$$  \hfill{(14)}

where $\dot{P}(x) = P(x) - x^T B v_S$.

**Proposition 3:** Consider a complete RLC circuit with regulated voltage sources in series with the inductors. Assume that the energy functions of the dynamic elements are strictly convex, i.e., $\nabla^2_{v_S} E_C(v_C), \nabla^2_{i_L} E_L(i_L) > 0$. Then,

(i) **(Sufficiency)** For all $\lambda \in \mathbb{R}$, and symmetric matrix functions $M(1_L, v_C)$, with $M : \mathbb{R}^n \to \mathbb{R}^{n \times n}$, the pair

$$\tilde{P}_A := \lambda \tilde{P}_A + \frac{1}{2} \nabla^2 \tilde{P}_A M \nabla^T \tilde{P}_A$$  \hfill{(15)}

$$Q_A := \left[ \frac{1}{2} \nabla^2 P_A M + \frac{1}{2} \nabla (M \nabla P_A) + \lambda I \right] Q$$  \hfill{(16)}

is admissible, i.e., is such that the time integrals of $Q_A(x) x = \nabla_x \tilde{P}_A(x)$ coincide with the time integrals of (14).

(ii) **(Partial converse)** Assume the circuit (9) admits only isolated equilibrium points. Then, given any admissible pair $(Q_A, P_A)$ there exists $\lambda$, and $M$ such that, almost everywhere, $P_A$ takes the form (15).

**Proof:** See (Ortega et al., 2003).

An important observation regarding Proposition 3 is that, for suitable choices of $\lambda$ and $M$, we can now try to generate a matrix $Q_A(x)$ with the required negativity property (13).

**Remark 2:** Some simple calculations show that a change of (state) coordinates on the dynamical system (14) acts as a similarity transformation on $Q$. Therefore, is of no use for our purposes where we want to change the sign of $Q$ to render the quadratic form sign-definite.

C. Power Balance Inequality

Before we present our main result we first remark that in order to preserve the port variables $(v_S, \frac{d v_S}{d t})$, we must ensure that the transformed dynamics can be expressed in the form (9), which is equivalent to requiring that $\dot{P}_A(x) = P_A(x) - x^T B v_S$. This naturally restricts the freedom in the choices for $\lambda$ and $M$ in Proposition 3.

**Theorem 1:** Consider a (possibly nonlinear) RLC circuit described by (9). Assume:

A.1 The inductors and capacitors are passive and have strictly convex energy functions.

A.2 The voltage-controlled resistors in $\Sigma_C$ are passive, linear and time-invariant. Also, det $(R_C) \neq 0$, and thus $G(v_C) = \frac{1}{2} v_C^T R_C v_C \geq 0$.

A.3 Uniformly in $x$ we have

$$\|C^{\frac{1}{2}} (v_C) R_C T^T L^{-1} (i_L)\| < 1,$$

where $\| \cdot \|$ denotes the spectral norm of a matrix.

Under these conditions, we have the following power balance inequality

$$P_A[x(t)] - P_A[x(0)] \leq \int_0^t v_T^T \frac{d v_S}{d t} d t,$$  \hfill{(17)}

where the transformed mixed-potential function is defined as

$$P_A(x) = F(1_L) + \frac{1}{2} (\Gamma^T i_L \Gamma R_C T^T i_L) + \frac{1}{2} (\Gamma^T i_L - R_C^{-1} v_C)^T R_C (\Gamma^T i_L - R_C^{-1} v_C)$$

If, furthermore

A.4 The current-controlled resistors are passive, then, the circuit defines a passive system with port variables $(v_S, \frac{d v_S}{d t})$ and storage function the transformed mixed-potential $P_A(x)$.

**Proof:** The proof consists in first defining the parameters $\lambda$ and $M$ of Proposition 3 so that, under the conditions A.1-A.4 of the theorem, the resulting $Q_A(x)$ satisfies (13) and $P_A(x)$ is a positive semi-definite function. First, notice

3As shown in the proof, the qualifier "regularization procedure" stands for the existence of possible singular points. These points can be avoided with standard regularization procedures, but is omitted here for brevity.
that under assumption A.2 the co-content is linear and quadratic. To ensure that $P(x)$ is linear in $v_S$, as is required to preserve the desired port variables, we may select $\lambda = 1$ and $M = \text{diag}(0, 2R_C)$.

Now, using (16) we obtain after some straightforward calculations:

$$Q_A(x) = \begin{bmatrix} -L(i_L) & 2\Gamma R_C C(v_C) \\ 0 & -C(v_C) \end{bmatrix}.$$

Assumption A.1 ensures that $L(i_L)$ and $C(v_C)$ are positive definite. Hence, a Schur complement analysis proves that, under assumption A.2, the co-content quadratic. To ensure that some straightforward calculations

$$\text{Suppose that under Assumption A.2 and A.4, the mixed-potential function } P_A(x) \text{ is positive semi-definite for all } x.$$

Remark 3: Assumption A.3 is satisfied if the voltage-controlled resistances in $R_C$ are 'small'. Recalling that these resistors are contained in $\Sigma_C$, this means that the coupling between $\Sigma_L$ and $\Sigma_C$, that is, the coupling between the inductors and capacitors, is weak.

V. STABILIZATION VIA POWER SHAPING

The theorem below proves that complete RLC circuits with strictly convex energy function and linear voltage controlled resistors are stabilizable via power-shaping—without requiring Assumptions A.3 or A.4—but only provided that the number of control signals is "sufficiently large" to shape the mixed potential function and add the damping.

Theorem 2 (Stabilization via power shaping): Consider a complete RLC circuit satisfying Assumptions A.1 and A.2 of Theorem 1, and a desired (admissible) equilibrium $(i_L^*, v_C^*) \in \mathbb{R}^n$. Assume there exists a function $P_a : \mathbb{R}^{n_L} \to \mathbb{R}$ verifying:

A.5 (Realizability) $B_S^T \nabla P_a = 0$, where $B_S^T B_S = 0$.
A.6 (Equilibrium assignment) $\nabla P_a(i_L^*) + \nabla v_c F(i_L^*) + \Gamma R_C \Gamma^T i_L^* = 0$.
A.7 (Damping injection) Uniformly in $i_L$, $\nabla^2 P_a + \nabla^2 v_c F \geq R_n I$, for some sufficiently large $R_n > 0$.

Under these conditions, the circuit is stabilizable via power-shaping. More precisely, the control law

$$v_S = -(B_S^T B_S)^{-1} B_S^T \nabla P_a$$

ensures that all bounded trajectories satisfy

$$\lim_{t \to \infty} (i_L(t), v_C(t)) = (i_L^*, v_C^*).$$

Furthermore, if the characteristic functions of the dynamic elements are such that $(p_L, q_C) = (p_L(i_L), q_C(v_C))$ is a global diffeomorphism then all trajectories are bounded and the equilibrium is globally attractive.

Proof: The circuit dynamics are described by (9) and (10). Now, under Assumption A.5, the control law (18) satisfies $B_S v_S = -\nabla P_a$. This leads to the closed-loop dynamics

$$Q_A \frac{d}{dt} \begin{bmatrix} i_L \\ v_C \end{bmatrix} = \nabla P_a,$$

where $P_a(i_L, v_C) := P + P_a$. From Assumption A.1 we have that $Q$ is full rank and consequently the equilibria are the extrema of $P_a$. Furthermore, from (10) and Assumption A.2 we have that

$$\nabla P_a = \begin{bmatrix} \Gamma v_C + \nabla v_c F + \nabla P_a \\ \Gamma^T i_L - R_C v_C \end{bmatrix}.$$
Assumption A.1 we have that the total energy, $\mathcal{E} = \mathcal{E}_C + \mathcal{E}_L$, is a positive radially unbounded function. Evaluating its time derivative we get

$$
\dot{\mathcal{E}} = -\nabla^T \mathcal{E}_C \dot{\mathcal{R}}_C \nabla \mathcal{E}_C - \nabla^T \mathcal{E}_L \dot{l}_L F(\dot{l}_L(p_L)) + \nabla \mathcal{P}_a(\dot{l}_L(p_L))
$$

(19)

Assumption A.7 states that the function $F(\dot{l}_L) + \mathcal{P}_a(\dot{l}_L)$ is strongly convex. The latter ensures that the lower term in (19) is positive outside some ball $|\dot{l}_L| = b$, and consequently $\dot{\mathcal{E}}$ is negative outside a compact set. This proves global boundedness of the solutions and completes the proof.

Remark 4: Clearly, all assumptions of Theorem 2 are constraints related with the “degree of under-actuation” of the circuit. All conditions are obviated in the extreme case where $B_S = I$ when we can add an arbitrary power function $\mathcal{P}_a$. Also, the rather restrictive Assumption A.3 of Theorem 1 is conspicuous by its absence—this means that we do not assume that the circuit to be controlled is already passive.

VI. CONCLUSION

Our main motivation in this paper was to propose an alternative to the well-known method of energy shaping stabilization of physical systems—which as pointed out in (Ortega et al., 2002; Ortega et al., 2001; Schaft, 2000) is severely stymied by the existence of pervasive damping. In this paper we have, for nonlinear RLC circuits, put forth the paradigm of power shaping and shown that it is not restricted to systems without pervasive dissipation. The starting point for the formulation of the power shaping idea are some new power balancing and passivity properties established for a class of nonlinear RLC circuits with convex energy function and weak electromagnetic coupling. To enlarge the class of circuits that enjoy these properties we have made extensive use of Proposition 3 which provides a procedure to generate alternative circuit topologies that reveal, through the new admissible pairs $(Q_A, P_A)$, properties of the original circuit that we can exploit in our controller design. Future research includes the extension of our results beyond the realm of RLC circuits, e.g., to mechanical or electromechanical systems. A related question is whether we can find Brayton–Moser like models for this class of systems.

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VII. REFERENCES


