

University of Groningen

Lumped Approximation of a Transmission Line with an Alternative Geometric Discretization

Lopezlena, Ricardo; Scherpen, Jacquélien M.A.

Published in:

Proceedings of the 2nd IFAC Symposium on System, Structure and Control

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2004

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Lopezlena, R., & Scherpen, J. M. A. (2004). Lumped Approximation of a Transmission Line with an Alternative Geometric Discretization. In *Proceedings of the 2nd IFAC Symposium on System, Structure and Control* University of Groningen, Research Institute of Technology and Management.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

**LUMPED APPROXIMATION OF A
TRANSMISSION LINE WITH AN
ALTERNATIVE GEOMETRIC
DISCRETIZATION**

Ricardo Lopezlena * Jacquélien M. A. Scherpen **

** Instituto Mexicano del Petróleo (IMP), Eje Central
Lázaro Cárdenas 152, Col. San Bartolo Atepehuacan CP
07730, México D.F., México, Apdo. Postal 14-805.*

Tel. +52-9175-7623, Fax +52-9175-7079. rlopezle@imp.mx

*** Delft Center of Systems and Control, Delft University of
Technology, Mekelweg 2, 2628 CD Delft, The Netherlands.*

Tel. +31-15-278 5489, Fax +31-15-278 6679.

J.M.A.Scherpen@dcsc.TUdelft.nl

Abstract: An electromagnetic one-dimensional transmission line represented in a distributed port-Hamiltonian form is lumped into a chain of subsystems which preserve the port-Hamiltonian structure with inputs and outputs in collocated form. The procedure is essentially an adaptation of the procedure for discretization of Stokes-Dirac structures presented in (Clemente-Gallardo *et al.*, 2002), that does not preserve the port-Hamiltonian structure after discretization. With some modifications essentially inspired on the finite difference paradigm, the procedure now results in a system that preserves the collocated port-Hamiltonian structure along with some other desirable conditions for interconnection. The simulation results are compared with those presented previously in (Golo *et al.*, 2002).

Copyright 2004 ©IFAC

Keywords: Distributed parameter systems, partial differential equations, electrical circuits.

1. INTRODUCTION

The symbiosis of systems and control theory and classical mechanics has resulted in a fruitful field of research that has provided highly structured and systematic tools for a diversified class of physical systems. In particular, the class of port-Hamiltonian systems has interesting properties useful for modelling and control purposes. Since the introduction of distributed port-Hamiltonian systems in (Maschke and van der Schaft, 2000) and lately in (van der Schaft and Maschke, 2001), several important applications could be envisioned

with the use of the so called Stokes-Dirac structures based on the success of the applications that have resulted from its finite dimensional counterpart, the Dirac structures.

At the heart of the interconnection of such infinite-dimensional models of conservation laws is precisely the Stokes-Dirac structure. For an applications point of view the adequate spatial discretization of this structure is fundamental for the preservation of such conservation and interconnection relations. Therefore, some efforts have been devoted to the systematic discretization of such structure (Golo *et al.*, 2003; Clemente-Gallardo

et al., 2002). In order to deal with arbitrary dimensions a common framework using differential geometry concepts is used. Despite having such common ground, each procedure approaches the problem of approximation of the *geometric objects* in a different form.

On the one hand, in (Golo *et al.*, 2003) their geometric procedure is based, roughly speaking, on the approximation of the differentiable n-forms into finite element objects. For instance 0-forms and 1-forms as *linear splines*, etc. With such tools at hand the Stokes-Dirac structure of the port-Hamiltonian model of a one-dimensional transmission line is discretized, simulated and compared with the *exact solution* of the model. This approach allows them to deal with uniform and non-uniform grids (for further details see (Golo *et al.*, 2003)).

On the other hand, in our approach (Clemente-Gallardo *et al.*, 2002), such differentiable n-forms lie in a spatial domain which has associated a grid of points or nodes from where a finite-difference provides a way to approximately express each n-form. In this way a 0-form (a function) lies at the node, a 1-form is defined to lie on the midpoints between two collinear nodes, 2-forms lie at the center of the square, etc. The procedure was tested on the non-homentropic model of a one-dimensional pipeline, but no further analysis on the precision of the discretization was presented. In this paper the precision of the methods is tested for an improved version of the discretization presented in (Clemente-Gallardo *et al.*, 2002), which preserves the port-Hamiltonian structure, for the (simpler) model of the one-dimensional transmission line. The availability of an exact solution for a state variable at the boundary in (Golo *et al.*, 2003) provides us a trustable benchmark on the precision of the method.

The paper is organized as follows. After briefly presenting in Section 2 some necessary adaptations of the procedure shown in (Clemente-Gallardo *et al.*, 2002) in the particular case of the Stokes-Dirac structure of the model of the one-dimensional transmission line, in Section 3 we present some structures in the collocated port-Hamiltonian form along with some comments on the properties of such structures under interconnection. In Section 4 the simulation results are shown along with some preliminary conclusions drawn from the comparison of the results with the exact solution presented in (Golo *et al.*, 2003), to finish with some general conclusions.

2. THE STOKES-DIRAC MODIFIED DISCRETIZATION

Consider a 1-dimensional Riemannian manifold \mathcal{M} with metric g and 0-dimensional boundary $\partial\mathcal{M}$, consider two spaces \mathcal{F} and \mathcal{E} defined by

$$\mathcal{F} = \mathcal{F}_q \times \mathcal{F}_\varphi \times \mathcal{F}_b = \bigwedge^1(\mathcal{M}) \times \bigwedge^1(\mathcal{M}) \times \bigwedge^0(\partial\mathcal{M}),$$

and

$$\mathcal{E} = \mathcal{E}_q \times \mathcal{E}_\varphi \times \mathcal{E}_b = \bigwedge^0(\mathcal{M}) \times \bigwedge^0(\mathcal{M}) \times \bigwedge^0(\partial\mathcal{M}),$$

where the subindex b stands for boundary variables. Throughout the section we denote

$$\begin{cases} f^i = (f_q^i, f_\varphi^i, f_b^i) \in \mathcal{F} \\ e^i = (e_q^i, e_\varphi^i, e_b^i) \in \mathcal{E} \end{cases}$$

along with a product $\langle\langle(f^1, e^1), (f^2, e^2)\rangle\rangle$ in the space $\mathcal{F} \times \mathcal{E}$ defined as

$$\int_{\mathcal{M}} \{e_q^1 \wedge f_q^2 + e_\varphi^1 \wedge f_\varphi^2 + e_q^2 \wedge f_q^1 + e_\varphi^2 \wedge f_\varphi^1\} + \int_{\partial\mathcal{M}} (e_b^1 \wedge f_b^2 + e_b^2 \wedge f_b^1). \quad (1)$$

As described by (van der Schaft and Maschke, 2001) the subspace D associated to the one-dimensional transmission line defined by

$$\begin{bmatrix} f_q \\ f_\varphi \\ e_b \\ f_b \end{bmatrix} = \begin{bmatrix} 0 & -d & 0 & 0 \\ -d & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_q \\ e_\varphi \\ e_q|_{\partial\mathcal{M}} \\ e_\varphi|_{\partial\mathcal{M}} \end{bmatrix}, \quad (2)$$

defines a Stokes-Dirac structure on $\mathcal{F} \times \mathcal{E}$, where q and φ stand for charge and flux densities (one-forms) respectively.

As shown in (van der Schaft and Maschke, 2001), the *distributed* port-Hamiltonian representation of the one-dimensional transmission line is given by

$$\begin{bmatrix} \partial_t q \\ \partial_t \varphi \\ \varphi \\ q \end{bmatrix} = \begin{bmatrix} 0 & -d & 0 & 0 \\ -d & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_\varphi H \\ \delta_q H|_{\partial Z} \\ \delta_\varphi H|_{\partial Z} \end{bmatrix} \quad (3)$$

with energy stored in the Hamiltonian expressed by the functional

$$\mathcal{H} = \int_{\Omega} \frac{1}{2} \left\{ \frac{q^2}{C} + \frac{\varphi^2}{L} \right\} d\Omega. \quad (4)$$

where Ω denotes the boundary. Based on the procedure of discretization of Stokes-Dirac structures presented in (Clemente-Gallardo *et al.*, 2002), it is necessary first to identify the geometric objects involved in the particular structure used to model the transmission line, eq. (2), namely the one-forms and the exterior differential operator d .

The Hamiltonian function H is defined by a sequence of nodes in a uniform grid. Given a Hamiltonian functional (4) and a finite grid defining n -finite sections, a finite dimensional Hamiltonian

$$H_i = \int_{\mathbf{v}_i} \frac{1}{2} \left\{ \frac{q^2}{C} + \frac{\varphi^2}{L} \right\} d\mathbf{V}_i, \quad (5)$$

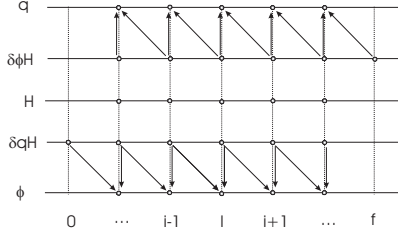


Fig. 1. A schematic grid of nodes interconnection for the transmission line.

can be obtained for each i -volume \mathcal{V}_i , resulting in a total Hamiltonian

$$H = \sum_i^n H_i. \quad (6)$$

Since the integration takes place assuming that the energy is uniformly distributed in each grid-volume, a *lumped Hamiltonian* can be defined for each volume. For instance, for the one-dimensional case define the following lumped Hamiltonian 'per unit length',

$$H = \frac{1}{\Delta(x_i)} H(X), \quad (7)$$

where the length $\Delta(x_i)$ depends on the different block element used. Since both q and φ are one-forms, such forms can be placed at the middle of two adjacent nodes. Finally the approximation of the partial derivative $\partial_x H$ can be performed in several forms namely backward $\partial_x H \approx (H_i - H_{i-1})/\Delta x$, forward $\partial_x H \approx (H_{i+1} - H_i)/\Delta x$ or central differentiation $\partial_x H \approx (H_{i+1} - H_{i-1})/2\Delta x$. Different selections of approximated differentiations, result in different structures which may fail to be skew symmetric.

3. LUMPING THE TRANSMISSION LINE

The structure of the blocks is defined with port-Hamiltonian systems in a particular structure called **collocated port-Hamiltonian systems** (Lopezlena *et al.*, 2003) (see figure 2) the essential characteristic of this arrangement is that the inputs and outputs are paired at each boundary such that they determine the energy transfer, i.e., at each boundary there exist a transfer or energy without a predefined assumption on the direction of transfer. For a given set of power defining variables at the boundary, some variables are defined by the system at one two-port and the

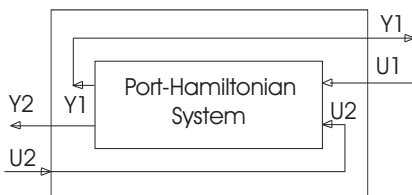


Fig. 2. A PHS with collocated inputs and outputs

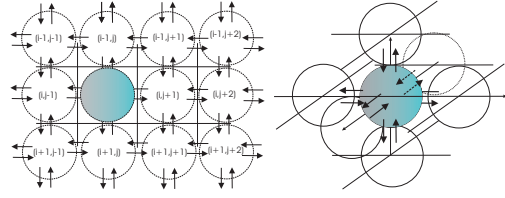


Fig. 3. A schematic of interconnection of 2-D and 3-D PHS blocks.

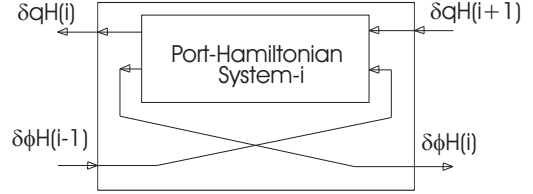


Fig. 4. A single block in A form.

complementary variables are defined at the opposite two-port (for further details see (Lopezlena *et al.*, 2003)). This does not modify at all the structure of the Port-Hamiltonian paradigm. The difference remains in the way of looking at interconnection of these structures: they can be chained in 1-D problems, grided in 2-D and assembled into polyhedral structures for 3-D cases, see figure 3. For more general structures including dissipation and feedforward terms see (Lopezlena and Scherpen, 2004)). In particular, assume that $J(x) = -J^T(x)$. The collocated representation can be written as

$$\begin{bmatrix} \dot{x}_1 \\ -y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} J_1^1 & J_1^2 & J_1^3 \\ J_2^1 & 0 & 0 \\ J_3^1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_1}{\partial x_1} \\ u_1 \\ u_2 \end{bmatrix} \quad (8)$$

With the purpose of series interconnection consider a second system with $G(x) = -G^T(x)$ in the form

$$\begin{bmatrix} \dot{x}_2 \\ -z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} G_1^1 & G_1^2 & G_1^3 \\ G_2^1 & 0 & 0 \\ G_3^1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_2}{\partial x_2} \\ v_1 \\ v_2 \end{bmatrix} \quad (9)$$

Assuming that the interconnection is compatible, i.e. $y_2 = v_1$ and $u_2 = z_1$, then, as can be seen after some simple algebra, the interconnected system has the form :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ -y_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} J_1^1 & -J_1^3 G_2^1 & J_1^2 & 0 \\ G_1^2 J_3^1 & G_1^1 & 0 & G_1^3 \\ J_2^1 & 0 & 0 & 0 \\ 0 & G_3^1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_1}{\partial x_1} \\ \frac{\partial H_2}{\partial x_2} \\ u_1 \\ v_2 \end{bmatrix}$$

and it can be verified that the resulting structure is again skew symmetric.

Proposition 3.1. The following system (block) associated to the lumped Hamiltonian in (7),

$$(A) \left\{ \begin{bmatrix} \dot{q}_i \\ \dot{\varphi}_i \\ -\partial_\varphi H_i \\ \partial_q H_i \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H_i \\ \partial_\varphi H_i \\ -\partial_q H_{i+1} \\ \partial_\varphi H_{i-1} \end{bmatrix} \right.$$

provides a space-discretization of Telegrapher's equations in distributed port-Hamiltonian form and is a finite dimensional port-Hamiltonian system with collocated inputs and outputs. Furthermore, the PHS (A) satisfies the energy conservation equation

$$\frac{dH_i}{dt} = y_i^T u_i \quad (10)$$

and for a chain of n-blocks PHS, the total energy satisfies the energy balance

$$\frac{dH}{dt} = \sum_{i=1}^n \frac{\partial H_i}{\partial t} = y^T u \quad (11)$$

Proof. Consider the following discrete approximation $d(\delta_\varphi H) \approx \Delta^{-1}(\delta_\varphi H_{i+1} - \delta_\varphi H_i)$ and $d(\delta_q H) \approx \Delta^{-1}(\delta_q H_i - \delta_q H_{i-1})$ in the structure (3). The first statement can be proved straightforwardly. The second statement is proved as follows: Consider the Hamiltonian function $H_i = \frac{1}{2}(q_i^2/C_i + \varphi_i^2/L_i)$, which is such that $\nabla^T H_i = (\partial_q H_i, \partial_\varphi H_i)$. Adequate definitions of $J_1^1 = -(J_1^1)^T$, and simple substitutions yield

$$\begin{aligned} \frac{dH_i}{dt} &= \frac{\partial^T H_i}{\partial x} [J_1^1 \frac{\partial H_i}{\partial x} + J_1^2 u_i] = \frac{\partial^T H_i}{\partial x} J_1^2 u_i \\ &= \partial_q H_i \partial_\varphi H_{i-1} - \partial_\varphi H_i \partial_q H_{i+1} = y_i^T u_i \end{aligned}$$

Since $\partial_q H_i = V_i$ and $\partial_\varphi H_i = I_i$, the product defines power which after integration reproduces the required supplied energy or delivered energy. In order to prove the last statement consider the following: By induction, for $i = 1$ was already proved in the previous proposition. Consider the interconnection of two systems. In such case

$$\begin{aligned} \frac{dH}{dt} &= \frac{dH_1}{dt} + \frac{dH_2}{dt} \\ &= \partial_q H_1 \partial_\varphi H_0 - \partial_\varphi H_1 \partial_q H_2 + \partial_q H_1 \partial_\varphi H_0 \\ &\quad - \partial_\varphi H_1 \partial_q H_2 = \partial_q H_1 \partial_\varphi H_0 - \partial_\varphi H_1 \partial_q H_2 \end{aligned}$$

which reduces trivially to $y^T u$. Assume it is true for $i = 1 \dots n$ such that at $i = m$ the total energy is

$$\frac{dH}{dt} = \partial_q H_i \partial_\varphi H_{i-1} - \partial_\varphi H_i \partial_q H_{i+1}$$

Since there is a cancellation of any individual intermediate product, it can be seen that the only residual terms are those at the boundary, resulting in the previous equation, which concludes the proof. \blacksquare

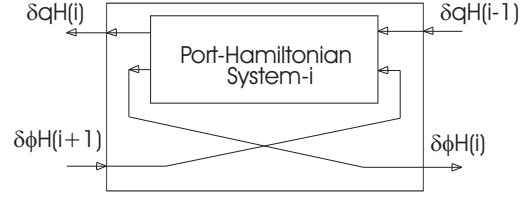


Fig. 5. A block of distributed approximation of DPHS in B form

Consider the following alternative discrete approximation $d(\delta_\varphi H) \approx \Delta^{-1}(\delta_\varphi H_i - \delta_\varphi H_{i-1})$ and $d(\delta_q H) \approx \Delta^{-1}(\delta_q H_{i+1} - \delta_q H_i)$ in the structure (3). Then an alternative form follows

$$(B) \left\{ \begin{bmatrix} \dot{q}_i \\ \dot{\varphi}_i \\ -\partial_\varphi H_i \\ \partial_q H_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H_i \\ \partial_\varphi H_i \\ \partial_q H_{i-1} \\ \partial_\varphi H_{i+1} \end{bmatrix} \right.$$

and represented in collocated form in figure 5. Since both lumped blocks A and B are based on a first-order discretization algorithms, one may conceive the use of a higher order approximation in order to increase the precision or the resulting models. Consider finally the following discrete approximation $d(\delta_\varphi H) \approx (2\Delta)^{-1}(\delta_\varphi H_{i+1} - \delta_\varphi H_{i-1})$ and $d(\delta_q H) \approx (2\Delta)^{-1}(\delta_q H_{i+1} - \delta_q H_{i-1})$ in the structure (3). For a Hamiltonian defined as $H_i = H_i/2\Delta_i$ one such block is presented as follows

$$(C) \left\{ \begin{bmatrix} \dot{q}_i \\ \dot{\varphi}_i \\ \partial_\varphi H_i \\ \partial_q H_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H_i \\ \partial_\varphi H_i \\ \partial_q H_{i+1} \\ \partial_\varphi H_{i+1} \\ \partial_q H_{i-1} \\ \partial_\varphi H_{i-1} \end{bmatrix} \right.$$

which is based on a central difference approximation and provides a second order error in approximating the derivative. Higher difference approximation could be probably conceived, but in the end there does not seem to be of much benefit.

4. SIMULATION RESULTS AND COMPARISONS

The efficiency of the method is tested in the problem posed and described in (Golo *et al.*, 2003) which consists of an ideal transmission line terminated by a lumped resistor, whose parameters $C(x)$ and $L(x)$ are varying along its position x in the transmission line, in the interval $[0, \ell]$. Assuming initial conditions zero, an input voltage source $u = \sin(t)$ in this form results in a voltage distribution in the form $v(x, t) = \sin(t - \ln(x + 1))$ which results in a voltage at the terminal of the resistor in the waveform $v(\ell, t) = \sin(t - 1)$.

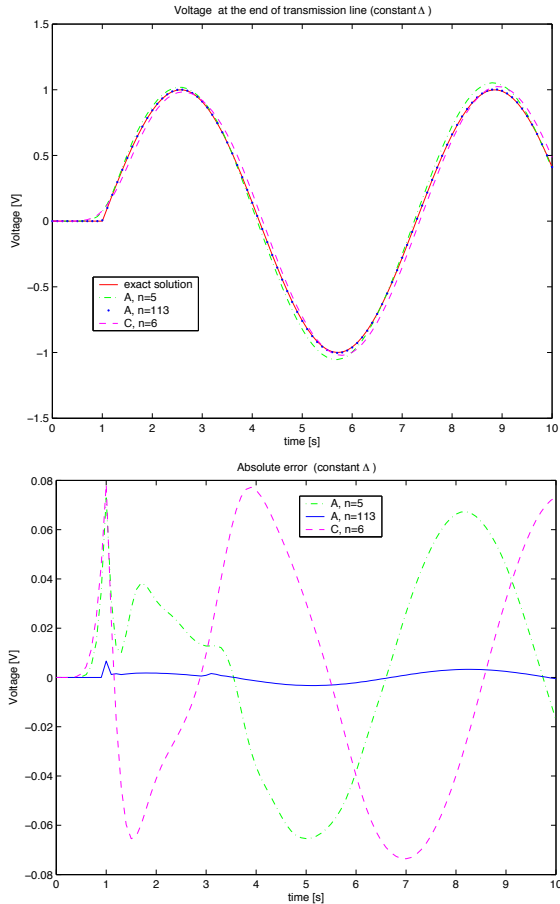


Fig. 6. Output voltage and absolute error.

This last waveform is to be simulated in order to assert the precision of the lumping method. In their test (Golo *et al.*, 2003), a fixed and a variable grid are used with the Runge-Kutta 4 integration technique and step size of 0.01 s. For our method two adequate lumping methods are available, the method A presented in the previous section, and the method C based on central differences. This method of discretization has an error of second-order and therefore it could be expected to perform possibly better than A which is constructed from a combination of backward and forward differences. The figure 6 shows the different waveform obtained by the methods. While it can be seen that actually method A performs better than C, both results are still far from the exact solution as can be seen on the plots of their error. Even though method C was used for $n = 6$, (since

Table 1. Specifications of the transmission line

Variable		Value	Units
Total length	ℓ	$e - 1$	m
Spatial position	x	$x \in [0, \ell]$	m
Sp. Capacitance	C	$1/(x + 1)$	H/m
Sp. Inductance	L	$1/(x + 1)$	F/m
Lumped Resistance	R	1	Ω
Number of elements	n	5	
Fixed Grid interval	Δ	ℓ/n	m
Var. Grid interval	Δ_i	$e^{\frac{i}{n}} - e^{\frac{i-1}{n}}$	m

Table 2. Comparison of simulation results

Method	n	Peak error	Nom. error
A	5	0.07291	0.0673
A	7	0.05554	0.04943
A	10	0.04169	0.03534
A	113	0.00666	0.0033
A	500	0.002348	0.0008
C	5	0.585	0.585
C	6	0.0784	0.0772
C	10	0.0535	0.0525
C	114	0.00932	0.00474
C	166	0.00721	0.0033
Exp. 1 *	5	-0.055, 0.051	0.0033
Exp. 2 *	5	-0.055, 0.051	0.004138
Exp. 3 *	5	-0.055, 0.051	0.00662
Exp. 4 *	5	-0.06, 0.055	0.00828
Exp. 1 *	10	?	0.00084

* From simulation experiments in (Golo *et al.*, 2003).

its response is only comparably good for an even numbers of elements), its error is still higher than that obtained by the A-method.

As can be seen in Table 2, three parameters were used in order to assert on the precision and performance of the method. Since the number of port-Hamiltonian blocks is typically associated to the precision, after considering the number of elements n , two parameters of error were considered: the peak error and the steady state (or nominal) error. While the first one appears typically at the simulations around $t = 1$, the second parameter was considered as the extreme value (higher or lower) of the absolute error during the rest of the simulation (say, after $t = 2$). Notice that in the curves plotted in our example, fig. 6, the approximation of the time delay is performed smoothly, while in the figures presented in (Golo *et al.*, 2003) the error swings with a peak error relatively larger than the nominal error. Nevertheless in all their simulations its peak error is still smaller than the peak error of ours for the same number of elements. A slight increase of the order of our methods ($n = 7$ for A and $n = 10$ for C) seems to be helpful in order to obtain the same peak error. That is not the case for their remarkable nominal error at experiment 1, $n = 5$. In order to reach the same nominal error, we increase the number of our blocks. The resulting simulation required the order of $n = 113$ for method A and $n = 166$ for method C. Finally while trying to attain their nominal error of Exp. 1, $n = 10$, a chain of up to 500 elements were needed in our simulations.

5. CONCLUSIONS

In this paper, several improvements of the procedure of discretization of Stokes-Dirac structures presented in (Clemente-Gallardo *et al.*, 2002) were shown to be necessary in order to preserve the port-Hamiltonian structure of the system. Addi-

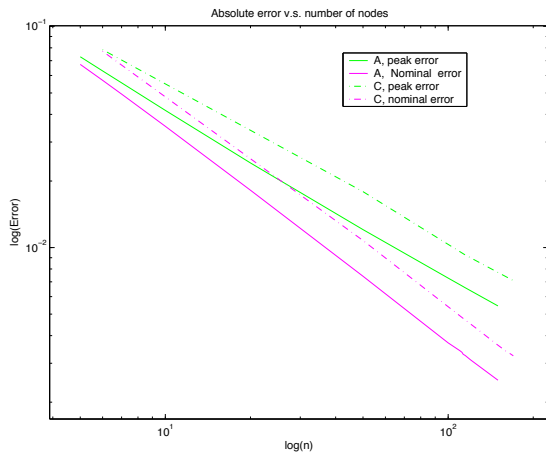


Fig. 7. Chart of Error at the end of transmission line for blocks A and C.

tional modifications were necessary in order to ensure the integrability by quadratures of the resulting system. The Stokes-Dirac approximation procedure,– which is mainly based on a finite difference philosophy,– was shown to be useful for the dynamic simulation of a one-dimensional electromagnetic transmission line and comparisons with another approach (Golo *et al.*, 2003),– mainly based on a finite element philosophy,– were provided. The use of finite differences provides quite direct and simple structures, which are prone for further analysis with control purposes. But simplicity seems to bear the price of precision. The simulation results presented in this note show that in all cases they provide a higher peak and nominal errors compared with the lumping procedures presented in (Golo *et al.*, 2003). Despite having a higher total error in contrast to the charts presented in (Golo *et al.*, 2003), in our results the transient peak error at $t = 1$ tends to be almost comparable to its associated nominal error, resulting in smoother transitions. According to (Golo *et al.*, 2003), the accuracy of their method is conjectured to be of the order of $1/n^2$, while in our procedure it can be asserted to follow an exponential law as seen in the chart of figure 7. In (Golo *et al.*, 2003) the case of a non-uniform grid was also considered. Since our approach assumes the existence of a uniform grid of points, and the finite difference method assumes uniformity of the grid, its application on a nonuniform grid destroys any precision that the method may have. There exists though finite difference methods especially designed for non-uniform grids. Such methods may provide some improved performance but certainly such result will not be better than those obtained with a fixed grid.

Nevertheless for simple applications like the one presented, the margins of error provided by this method can be considered acceptable with a still low computational effort. The simplicity of the method provides a way to analyze its stability and

storage properties based on its constitutive blocks. These advantages are attractive for the model reduction procedures like the one presented in (Lopezlena *et al.*, 2003). It could be argued that the search of reduction methods for distributed systems is unnecessary with the availability of procedures like (Golo *et al.*, 2003) or the one presented here, which provide accurate operational low-order models from the model equations. This may be especially true for conservative systems. The justification of the additional use of reduction procedures can be found on the need of having a deeper understanding of such methods in terms of control properties like stability, controllability, observability, passivity or dissipativity inherited to the reduced system.

6. ACKNOWLEDGEMENT

The first author appreciates the interesting talks and comments of Dr. G. Golo and Prof. A.J. van der Schaft regarding their discretization procedure in (Golo *et al.*, 2003) during his short visit at Twente University.

REFERENCES

- Clemente-Gallardo, J., R. Lopezlena and J.M.A. Scherpen (2002). Geometric discretization of fluid dynamics. In: *Proc. 41st Conf. on Decis. and Control*. Las Vegas, Nevada:U.S.A.. pp. 923–966.
- Golo, G., V. Talasila, A. J. van der Schaft and B. M. Maschke (2003). Hamiltonian discretization of boundary control systems. submitted to *Automatica*.
- Golo, G., V. Talasila and A. J. van der Schaft (2002). Approximation of the telegrapher’s equations. In: *Proc. 41st Conf. on Decision and Control*. Las Vegas, Nevada: U.S.A.
- Lopezlena, R. and J.M.A. Scherpen (2004). On factorization, interconnection and reduction of collocated port-hamiltonian systems. Submitted to this Symposium.
- Lopezlena, R., J.M.A. Scherpen and K. Fujimoto (2003). Energy-storage balanced reduction of port-hamiltonian systems. In: *2nd IFAC Workshop on Lagr. and Ham. Meth. for Nonl. Cont.*. Seville, Spain. pp. 79–84.
- Maschke, B. and A. J. van der Schaft (2000). Port-controlled hamiltonian representation of distributed parameter systems. In: *Proc. IFAC Workshop on Lagrangian and Hamiltonian methods for nonlinear control* (N. Leonard and R. Ortega, Eds.). Princeton Univ.,USA. pp. 28–38.
- van der Schaft, A. J. and B. M. Maschke (2001). Hamiltonian formulation of distributed - parameter systems with boundary energy flow. *J. of Geometry and Physics* **775**, 1–29.