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Passive Compensation of Nonlinear Robot Dynamics

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***Abstract*—In this paper, we derive a coordinate-free formulation of a passive controller that makes a mechanical system track reference curves in a potential field. Contrary to conventional reference tracking, we do not specify a single time-varying trajectory that the system has to track. Instead, we specify a whole curve that the system has to stay on at all times. Using tools from differential geometry, we first derive a controller that makes the system move along arbitrary (smooth enough) reference curves while keeping the kinetic energy constant. We then apply the results to the case of movement in an artificial potential field, in which case, the reference curves are completely determined by the potential field and cannot be chosen arbitrarily. Simulation then shows the performance of the controller on a benchmark robot with two degrees of freedom.**

***Index Terms*—Differential geometry, motion control, passive control, robot dynamics, robustness.**

I. Introduction

Many robot applications demand that a robot interact with an environment that is not exactly known beforehand. For example, when a robotic manipulator arm needs to insert a peg into a hole, the exact position and orientation of these objects may be unknown. If we use traditional position control to force the peg into the hole, then, in case of misalignment, the contact forces could damage the robot and the objects, and even lead to instability of the system.

Instead, we can use a form of so-called impedance control (introduced by Hogan [1]): the motion of the manipulator is governed by a spring-like behavior, i.e., the control torques on the robot mimic a spring that is connected between the end-effector and its desired position. Such compliant behavior can be represented by an artificial (virtual) potential field with a minimum at the desired configuration. The control torques are determined by the differential of the potential field.

In this paper, we consider the movement of a robot in such a potential field. During nominal operation, with only conservative forces from the potential field present, the robot will move, for example, as shown in Fig. 1; the centrifugal and Coriolis forces cause it to oscillate around the minimum. In this paper, we want to improve this behavior. We want to achieve oscillation through the minimum of the potential field along one fixed curve, determined by the initial configuration.

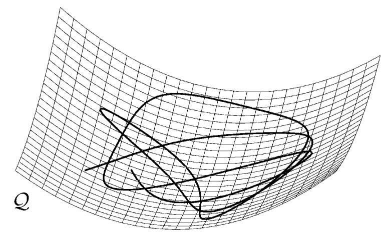

Fig. 1. Uncontrolled movement of a robotic manipulator in a potential field on the configuration manifold $\mathcal{Q}$.

Since we want to augment the controller as defined by the potential field, we do not want to destroy the valuable properties of the potential field. This means we look for control terms that do not change the energy of the system, such that the full closed-loop system (including the potential field) is still passive, i.e., it can only supply a limited amount of energy to the environment, no matter what input it receives.

The results presented in this paper constitute a first step toward passive decoupling of inertial effects in manipulator dynamics, i.e., passively eliminating the nonlinear Coriolis and centrifugal forces present in manipulators; feedback cancellation is, in general, not a passive action, and can lead to instability due to model mismatch.

Many passive control laws have been developed before, e.g., in the form of passivity-based control [2], intrinsically passive control [3], passive extensions to the well-known computed torque algorithm [4] and [5], and, more recently, in the form of interconnection and damping assignment passivity-based control (IDA-PBC) [6]. [3] also briefly discusses the possibilities of using a power-continuous (i.e., energy-conserving) controller to modify the robot dynamics.

Most closely related to this paper is the work of Li and Horowitz [7]–[10]. They developed a passive controller that makes a mechanical system move along the integral curves of a vector field, with the speed of movement equal to a constant times the length of the vectors of the vector field. The constant depends on the initial conditions, and any temporal fluctuations in the kinetic energy of the system are stored in a virtual flywheel (i.e., in the form of virtual kinetic energy), making the whole system passive.

In this paper, we follow a similar strategy, using a vector field to describe the desired movement. However, there are two significant differences: 1) we include a (virtual) potential field instead of (virtual) kinetic energy to shape the energy of the system; 2) the length of the vectors in the vector field has no influence on the movement of the system, i.e., the controller is independent of the parameterization of the vector field; only the direction of the vectors is important.

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The advantage of this approach is that we completely separate curve tracking from energy balancing, i.e., we derive a controller that compensates for Coriolis and centrifugal forces without changing the energy. After that, we include the potential field to shape the energy of the system in a certain way. The approach of Li and Horowitz is more suitable for tasks where a certain velocity (and hence, kinetic energy) is desired, whereas our approach is more suitable for tasks where power continuity of the controller is more important than maintaining a certain velocity.

Section II first explains the mathematical and physical tools that we need in the rest of the paper. Section III formally states the two control problems addressed in this paper, and Sections IV and V derive the controller that solves them. Section VI shows simulation results of the controller on a practical system. Finally, Section VII summarizes the results and gives some remarks on possible future directions of research.

II. Preliminaries

A. Mathematical Preliminaries

In this section, we discuss the mathematical tools and notation we need for the rest of the paper. More precise and detailed information on manifolds and differential geometry can be found in [11]–[15].

We denote a differentiable manifold by $\mathcal{Q}$, its elements (points) by q, and its dimension by $n \in \mathbb{N}$. The tangent bundle $T\mathcal{Q}$ of $\mathcal{Q}$ is the union of the tangent spaces $T_q\mathcal{Q}$ at all points $q \in \mathcal{Q}$. Similarly, we denote the cotangent bundle by $T^*\mathcal{Q}$.

A $\mathcal{C}^\infty$ tensor field $T^{i_1,\ldots,i_k}_{j_1,\ldots,j_l}$ on $\mathcal{Q}$ is a $\mathcal{C}^\infty$ mapping, which assigns to each point $q \in \mathcal{Q}$ a tensor of type (k, l), i.e., order k contravariant and l covariant. We assume the indexes i, j to run from 1 to n, and we use the Einstein summation convention, i.e., repetition of an index (one upper, one lower) indicates summation over that index. Examples of tensor fields on $\mathcal{Q}$ are functions (type (0,0) tensor fields) and vector fields (type (1,0) tensor fields).

Similarly, a Riemannian metric tensor field (denoted by g, or in coordinates by g_{ij}) assigns to each point a positive definite two-covariant tensor. Because g is positive definite, its inverse g^{-1} exists (denoted in coordinates by g^{ij}). A manifold endowed with such a structure is called a Riemannian manifold. Once we have a metric, we define the inner product of two tangent vectors as

$$\langle v, w\rangle_g := g_{ij} v^i w^j, \quad v, w \in T_q\mathcal{Q}. \tag{1}$$

Normally, we leave out the subscript g, so every inner product is understood to be with respect to the appropriate metric. The length of a vector v (in the metric g) is defined as

$$|v| := \sqrt{\langle v, v\rangle} = \sqrt{g_{ij} v^i v^j} \tag{2}$$

and the cosine of the angle α between vectors is defined as

$$\cos(\alpha) = \frac{\langle v, w\rangle}{|v||w|}, \quad v, w \in T_q\mathcal{Q}.$$

Apart from a metric on a manifold, we can define a connection as a map from two vector fields v and w on $\mathcal{Q}$ to a third vector field $\nabla_v w$ on $\mathcal{Q}$, such that $\nabla_v w$ is $\mathbb{R}$-bilinear in v and w, and for any smooth function f on $\mathcal{Q}$

$$\nabla_{fv} w = f\nabla_v w \tag{3}$$

$$\nabla_v fw = f\nabla_v w + w\mathcal{L}_v f \tag{4}$$

where $\mathcal{L}_v f$ denotes the Lie derivative of f along v. The vector field $\nabla_v w$ is called the covariant directional derivative of w along v. It describes the change (at some point $q \in \mathcal{Q}$) of w along the geodesic passing through q with velocity v. In coordinate form, the connection is described by

$$(\nabla_v w)^i = v^j \frac{\partial w^i}{\partial q^j} + \Gamma^i_{jk} v^j w^k$$

where $\Gamma_{ijk} = g_{il}\Gamma^l_{jk}$ and Γ^i_{jk} are called the Christoffel symbols of the first and second kind, respectively [11]. We say a connection is symmetric if and only if $\nabla_w v = \nabla_v w$ whenever v and w commute, i.e., whenever the Jacobi–Lie bracket of v and w vanishes: $[v, w] = 0$.

Using the connection, we define the generalized acceleration of a vector field v as the covariant directional derivative of v along itself. If the generalized acceleration is zero ($\nabla_v v = 0$), then the integral curves of v are the geodesics corresponding to the connection.

From a given metric g, we can derive a unique symmetric connection that is compatible with the metric ($\nabla g \equiv 0$, see also [11]). This connection is called the Levi–Civita connection, and its Christoffel symbols are

$$\Gamma^i_{jk} = g^{il}\Gamma_{ljk} = \frac{1}{2} g^{il}\left(\frac{\partial g_{lj}}{\partial q^k} + \frac{\partial g_{lk}}{\partial q^j} - \frac{\partial g_{lj}}{\partial q^i}\right).$$

In the proofs of the theorems of Sections IV and V, we make frequent use of (3) and (4) and the following identities, which hold when the connection is compatible with the metric g:

$$\mathcal{L}_u\langle v, w\rangle = \langle\nabla_u v, w\rangle + \langle v, \nabla_u w\rangle \tag{5}$$

$$\nabla_v(gw) = g\nabla_v w \tag{6}$$

for any vector fields u, v, $w \in T\mathcal{Q}$.

B. Physical Preliminaries

This section introduces the physical concepts and notation we need in the rest of the paper. Most of them are directly related to the mathematical concepts introduced before, but they also have an intuitive, physical meaning. For an introduction in analytical mechanics, we refer to [16] and [17].

In this paper, we derive a controller for a robot with n degrees of freedom (DOFs). We assume a (more or less accurate) model of this robot, as well as full state measurement, i.e., we can measure the position of each joint, as well as its velocity. We also assume full collocated actuation, i.e., we can directly apply a torque to each joint of the robot at the point where we measure position and velocity.

We describe each configuration (pose) of the robot as a unique point q on an n-dimensional Riemannian manifold $\mathcal{Q}$. The metric g on this manifold arises in a natural way as the inertia tensor of the robot. The Levi–Civita connection corresponding to this metric describes the centrifugal and Coriolis effects (consult [18] for a mathematical introduction to robotics). The metric can be used to define the kinetic energy of the robot as

$$U_k := \frac{1}{2}|\dot{q}|^2 = \frac{1}{2}\langle\dot{q}, \dot{q}\rangle = \frac{1}{2} g_{ij}\dot{q}^i\dot{q}^j. \tag{7}$$

The dynamics of the robot are described by

$$\nabla_{\dot{q}}\dot{q} = g^{-1}\tau \tag{8}$$

where $\tau: T_q\mathcal{Q} \to T_q^*\mathcal{Q}$ are the control torques (we assume that potential fields such as gravity have been compensated for). The metric g^{-1} defines an isomorphism between $T_q^*\mathcal{Q}$ and $T_q\mathcal{Q}$. If we use local coordinates q^l, then (8) reads as

$$\ddot{q}^l + \Gamma^l_{jk}\dot{q}^j\dot{q}^k = g^{lm}\tau_m. \tag{9}$$

Multiplying by g_{il} and using matrix notation (taking $M_{il} = g_{il}$ and $C_{ij} = \Gamma_{ijk}\dot{q}^k$) gives

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau$$

which is the more familiar form of the dynamics equation of an ideal frictionless robot moving in zero gravity.

As indicated in Section I, we want the controlled system to be passive. Consider a system with input $u \in \mathcal{V}$ and output $y \in \mathcal{V}^*$ (with $\mathcal{V}$ a vector space and $\mathcal{V}^*$ its dual), such that $\langle u|y\rangle$ (the intrinsic dual product $\mathcal{V} \times \mathcal{V}^* \to \mathbb{R}$) is equal to the power supplied to that system. Following [19], we say that this system is passive if, for any input signal u_t, the energy U_s supplied to that system is

$$U_s(t_0, t_1) = \int_{t0}^{t1} \langle u_t|y_t\rangle dt > c \tag{10}$$

for every time interval (t_0, t_1) with $t_1 \geq t_0$ and some constant $c \in \mathbb{R}$ that depends on the initial conditions. Equation (10) says that the system cannot supply an infinite amount of energy, no matter how the input changes over time. Note that this is only a lower bound, i.e., the system is allowed to dissipate an infinite amount of energy.

Furthermore, we say a system is power continuous if $\langle u_t|y_t\rangle = 0$ at all times, i.e., if the power flow to the system is zero at all times.

A robot mechanism described by, e.g., (8) with inputs τ and dual outputs $\dot{q}$, is a passive system, since

$$U_s(t_0, t_1) = \int_{t0}^{t1} \langle \tau|\dot{q}\rangle dt = U_k(t_1) - U_k(t_0) > -U_k(t_0)$$

hence, it can only supply a limited amount of energy, its initial kinetic energy.

When we interconnect this robot with a passive controller with input $\dot{q}$ and output τ_c (Fig. 2), then the controlled robot is still passive with respect to environment input torques τ_e, since

$$U_s(t_0, t_1) = \int_{t0}^{t1} \langle \tau_e|\dot{q}\rangle dt = \int_{t0}^{t1} \langle \tau|\dot{q}\rangle dt - \int_{t0}^{t1} \langle \tau_c|\dot{q}\rangle dt > -U_k(t_0) - c_{\text{controller}}$$

where the last step follows from the passivity of the robot and the assumption of passivity for the controller; hence, the existence of $c_{\text{controller}}$.

III. Problem Statement

This section states the two problems that are addressed in this paper: power-continuous movement along specified curves on the configuration manifold; and movement along curves under the influence of a potential field. The controller that solves the first problem is used as a starting point for the solution of the second problem.

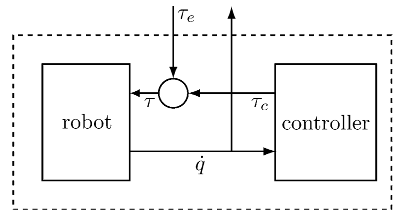

Fig. 2. Input–output configuration of the controlled robot.

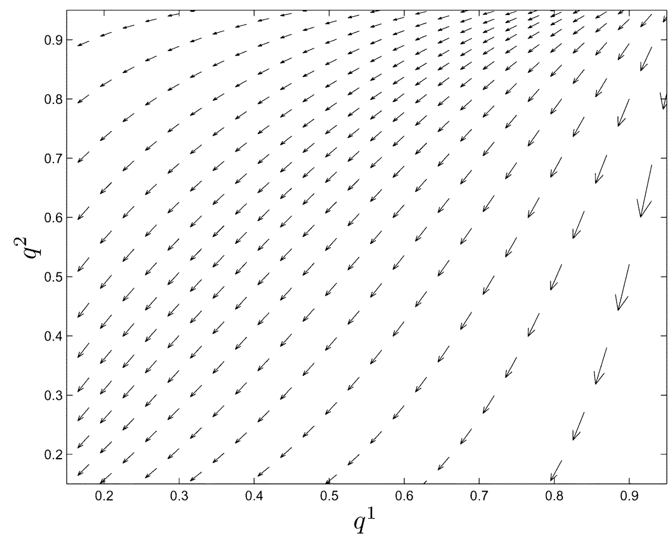

Fig. 3. Example of the vector-field representation of a set of curves on a two-dimensional manifold (one curve going through each point).

A. Movement Along Desired Curves

We want the system to move along a smooth curve determined by the initial configuration. We do not specify one curve that the system must converge to, but instead, we specify a whole set of smooth curves, one going through each point of the manifold.

Furthermore, contrary to conventional trajectory tracking, we do not specify one desired position for each time instant. Instead, we specify a curve (a set of desired positions), and we want the system to stay on that curve at all times. The speed of the motion is determined by the requirement of power continuity of the controller.

We assume the set of desired curves to be specified by a smooth vector field $w: \mathcal{Q} \to T\mathcal{Q}$ (Fig. 3), such that the desired curves are the integral curves of this vector field. We also assume that the vectors $w(q)$ are nonzero and that the covariant derivative of w is bounded in all directions

$$|\nabla_v w(q)| < K_{\max}|v| \quad \forall\, (q,v) \in T\mathcal{Q}$$

for some $K_{\max} < \infty$. Defining a smooth nonzero vector field on the whole manifold is, in general, not possible, so in this paper, we look only at points q at which $w(q) \neq 0$.

Objective 1: Find a power-continuous control law $\tau: T\mathcal{Q} \to T^*\mathcal{Q}$ for the robot mechanism as specified by (8), such that for any initial configuration q_0 and velocity $\dot{q}_0$, the angle between w and $\dot{q}$ monotonically converges to either 0 or π (whichever is closest to the initial conditions), such that the robot moves along the integral curves of the vector field w.

B. Movement Along Curves in a Potential Field

In this case, we include the potential field U to obtain the situation as stated in Section I. We want to compensate for the Coriolis and centrifugal forces and make the robot oscillate along a curve in the potential field. To be more precise:

Objective 2: Find a passive control law $\tau: T\mathcal{Q} \to T^*\mathcal{Q}$ for the robot mechanism as specified by (8), such that for any initial configuration q_0 and velocity $\dot{q}_0$, the robot converges to oscillations along a certain curve (depending on initial conditions), while the sum of (artificial) potential energy U and (real) kinetic energy of the robot is constant over time.

In the derivation of the control law in Section V, it turns out that the feasible curves are determined by the potential field, i.e., the desired vector field w is equal to the gradient vector field of the potential energy function.

IV. Power-Continuous Asymptotic Tracking of Reference Curves

This section discusses the derivation of a controller that makes the robot asymptotically track reference curves, described by the vector field w, while the kinetic energy of the robot remains constant. In other words, we look at solving the first control objective stated in Section III. As noted in Section III, we only look at points q where $w(q) \neq 0$.[1]

A. Nominal Power-Continuous Curve Tracking

We first consider the nominally required control effort, i.e., the control torque τ_n such that if the robot starts out with a velocity in the desired direction, then this torque makes the robot follow the integral curves of the desired vector field.

Theorem 1: Given a smooth vector field w on the configuration manifold $\mathcal{Q}$ of the robot. Suppose $\dot{q}_0 = \alpha_0 w(q_0)$ for some nonzero $\alpha_0 \in \mathbb{R}$. Then the control law (with subscript n for "nominal")

$$\tau_n = g\frac{\langle \dot{q}, w\rangle^2}{\langle w, w\rangle^2}\nabla_w w - g\frac{\langle \dot{q}, w\rangle\langle \nabla_w w, \dot{q}\rangle}{\langle w, w\rangle^2}w \tag{11}$$

makes the robot exactly follow the desired curves, i.e., it keeps the velocity $\dot{q}$ aligned with w. Furthermore, the controller is power continuous, i.e., $\langle \tau_n | \dot{q}\rangle = 0$ at all times.

Proof: First, we check power continuity. Taking the dual product between τ_n and $\dot{q}$ gives

$$\langle \tau_n | \dot{q}\rangle = \frac{\langle \dot{q}, w\rangle^2}{\langle w, w\rangle^2}\langle \nabla_w w, \dot{q}\rangle - \frac{\langle \nabla_w w, \dot{q}\rangle}{\langle w, w\rangle^2}\langle \dot{q}, w\rangle^2 = 0$$

which immediately proves the statement, and implies that all solution curves must have constant kinetic energy.

Next, we want to prove, for some given initial conditions $(q_0, \dot{q}_0)$ with $\dot{q}_0$ aligned with w, that the unique curve (with constant kinetic energy) defined by

$$(q, \dot{q})\big|_{t=0} = (q_0, \dot{q}_0) \quad \dot{q}(t) = \frac{|\dot{q}_0|}{|w(q)|}w(q)$$

is a solution. Indeed, computing $\nabla_{\dot{q}}\dot{q}$ using (3)–(5) gives (writing implicitly $w = w(q)$)

$$\begin{aligned}\nabla_{\dot{q}}\dot{q} &= \frac{|\dot{q}_0|^2}{|w|^2}\nabla_w w + \frac{|\dot{q}_0|^2}{|w|}\mathcal{L}_w\frac{1}{|w|}w \\ &= \frac{|\dot{q}_0|^2}{|w|^2}\nabla_w w - \frac{|\dot{q}_0|^2}{|w|^4}\langle \nabla_w w, w\rangle w.\end{aligned}$$

If we substitute these expressions for $\dot{q}$ and $\nabla_{\dot{q}}\dot{q}$ in the dynamics (8) with $\tau = \tau_n$, then we obtain

$$\begin{aligned}\frac{|\dot{q}_0|^2}{|w|^2}\nabla_w w - &\frac{|\dot{q}_0|^2}{|w|^4}\langle \nabla_w w, w\rangle w \\ &= \nabla_{\dot{q}}\dot{q} = g^{-1}\tau_n \\ &= \frac{|\dot{q}_0|^2}{|w|^2}\nabla_w w - \frac{|\dot{q}_0|^2\langle \nabla_w w, w\rangle}{\langle w, w\rangle^2}w.\end{aligned}$$

This shows that the proposed curve satisfies the closed-loop dynamics equation and is, therefore, a solution of the system. By uniqueness of the solution of (8) for given initial conditions $(q_0, \dot{q}_0)$, this is also the only possible solution, thus completing the proof that for initial conditions aligned with w, the system remains aligned with w. ■

Remark 1: It is important to realize that in the proof of power continuity, we have not used the fact that $\dot{q}$ is collinear with w. This means that the proposed control law is power continuous for all $\dot{q}$.

B. Asymptotic Power-Continuous Curve Tracking

We now extend the results from Section IV-A to the case where $\dot{q}$ is not collinear with w. We propose the following control law.

Theorem 2: Given a smooth vector field w on the configuration manifold $\mathcal{Q}$ of the robot, take the control law (with subscript a for "asymptotic")

$$\tau_a = \tau_n + \beta g\frac{\langle p, p\rangle \mathrm{sign}\langle \dot{q}, w\rangle}{|w|}w - \beta g\frac{|\langle \dot{q}, w\rangle|}{|w|}p \tag{12}$$

where $\beta > 0$ is a design parameter, and $p = p(t)$ is such that $\langle w, p\rangle = 0$ and $\dot{q}(t) = \alpha(t)w(q(t)) + p(t)$ for some function $\alpha: \mathbb{R}^+ \to \mathbb{R}$ at all times. Then, for any initial conditions q_0 and $\dot{q}_0$ and any β such that $\beta > \beta_0$ for a well-defined $0 < \beta_0(q) < \infty$, the velocity of the robot converges asymptotically to the desired velocity, in the sense that $(1/2)\langle p, p\rangle$ decreases monotonically over time. Furthermore, the controller is power continuous, i.e., $\langle \tau_a | \dot{q}\rangle = 0$ at all times.

Proof: We start by proving power continuity. Taking the dual product of τ_a with $\dot{q}$ immediately gives

$$\begin{aligned}\langle \tau_a | \dot{q}\rangle &= \langle \tau_n | \dot{q}\rangle + \beta\frac{\langle p, p\rangle|\langle \dot{q}, w\rangle|}{|w|} - \beta\frac{|\langle \dot{q}, w\rangle|\langle p, \dot{q}\rangle}{|w|} \\ &= 0 + \beta\frac{|\langle \dot{q}, w\rangle|}{|w|}(\langle p, p\rangle - \langle p, \dot{q}\rangle) \\ &= 0\end{aligned}$$

where the second equality follows from power continuity of τ_n for all $\dot{q}$, and the last equality follows from the definition of p. So indeed, this control law is power continuous.

[1]For $w(q) = 0$, the control laws derived in this section are in general unbounded; in case the system accidentally comes very close to a point q_1 with $w(q_1) = 0$, then the controller should be temporarily switched off, such that the system flows through the point q_1 along a geodesic.

To prove asymptotic tracking, we look at how $(1/2)\langle p, p\rangle$ changes as the system moves at velocity $\dot{q}$, i.e., we compute the change of kinetic energy stored in the p direction

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{2}\langle p,p\rangle\right) &= \langle \nabla_{\dot{q}} p, p\rangle \\ &= \langle \nabla_{\dot{q}} (\dot{q} - \alpha w), p\rangle \\ &= \langle \nabla_{\dot{q}}\dot{q}, p\rangle - \alpha\langle \nabla_{\dot{q}} w, p\rangle - \langle w,p\rangle \mathcal{L}_{\dot{q}}\alpha \\ &= \frac{\langle \dot{q}, w\rangle^2}{\langle w,w\rangle^2}\langle \nabla_w w, p\rangle - \beta\frac{|\langle \dot{q},w\rangle|}{|w|}\langle p,p\rangle \\ &\quad - \alpha\langle \nabla_{\dot{q}} w, p\rangle \\ &= -\beta\frac{|\langle \dot{q},w\rangle|}{|w|}\langle p,p\rangle - \frac{\langle \dot{q},w\rangle}{\langle w,w\rangle}\langle \nabla_p w, p\rangle\end{aligned}$$

where we used [14, Ch. 2, Prop. 3.2], (4), (5), (8), (12), and the fact that $\langle w, p\rangle = 0$ by definition of p. It can be seen that if we define β_0 as

$$\beta_0(q) = \max_{\substack{v\in T_q\mathcal{Q}\\ |v|=1}} \frac{|\langle \nabla_v w, v\rangle|}{|w|} \tag{13}$$

then, indeed, $(1/2)\langle p, p\rangle$ decreases monotonically for all $\beta > \beta_0$. ■

Remark 2: An important point to note is that we obtained a control law τ_a with the property that $\langle \tau_a|\dot{q}\rangle = 0$ for *any* model of the robot (any g, ∇). So as long as the velocity $\dot{q}$ is measured correctly, the controller is power continuous; in the proof of power continuity, we only used the modeled g and ∇, not the real ones. This is clearly important for safety reasons in possible applications like human–robot interaction tasks.

V. Movement Along Curves in a Potential Field

In this section, we use the controller derived in Section IV, and apply it to solve the second control objective stated in Section III.

We want the robot to move along curves in a predefined potential field $U\colon \mathcal{Q} \to \mathbb{R}$, starting from any initial condition q_0, $\dot{q}_0$. The following theorem states the control law we propose.

Theorem 3: Consider the mechanical system described by (8) and take as a control law

$$\tau = \tau_a - dU \tag{14}$$

where τ_a is defined in (12), $\beta > \beta_0$ [with β_0 as defined in (13)], the desired curves are chosen as

$$w = g^{-1} dU \tag{15}$$

and the potential field U is such that (for some scalar μ) $(\partial^2 U/\partial q^2)(q) = \mu g(q)$ at all q where $dU(q) = 0$.[2]

Then the sum of kinetic energy of the system and potential energy U is constant at all times, and the velocity of the system converges asymptotically to the desired curves defined by w for any initial velocity $\dot{q}_0$.

Proof: The energy balance follows (using power continuity of $\dot{q}$) as

$$\frac{dU_k}{dt} = \langle \tau|\dot{q}\rangle = \langle \tau_a|\dot{q}\rangle - \langle dU|\dot{q}\rangle = -\frac{dU}{dt}$$

showing that the total energy $U_k + U$ is constant. To prove convergence of the velocity, we look again at how $(1/2)\langle p, p\rangle$ changes

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{2}\langle p,p\rangle\right) &= \langle \nabla_{\dot{q}} p, p\rangle \\ &= -\langle dU|p\rangle + \langle \tau_a|p\rangle - \alpha\langle \nabla_{\dot{q}} w, p\rangle \\ &= -\langle dU|p\rangle - \beta\frac{|\langle \dot{q},w\rangle|}{|w|}\langle p,p\rangle - \frac{\langle \dot{q},w\rangle}{\langle w,w\rangle}\langle \nabla_p w, p\rangle \\ &\leq 0\end{aligned}$$

where the last step follows from using (15) to write $dU = gw$, and taking again $\beta > \beta_0$ with β_0 as defined in (13). This proves that asymptotic stability is obtained.

Unfortunately, the imposed relation between w and dU introduces a new problem, namely, at the points where U has a minimum. At this minimum, the differential of U vanishes, and hence, $|w| \to 0$, which may result in $\tau_a \to \infty$. However, if the potential field is such that at the minimum, its Hessian is equal to a scalar μ times the metric, then in coordinates

$$\begin{aligned}\left(\nabla_w w\right)^i &= \left(g^{-1}\nabla_w dU\right)^i \\ &= g^{ij}\frac{\partial^2 U}{\partial q^j \partial q^k} w^k - g^{ij}\Gamma^l_{jk} dU_l w^k \\ &= \mu w^i - g^{ij}\Gamma^l_{jk} dU_l w^k.\end{aligned}$$

The nominal control law becomes (close to the minimum)

$$\begin{aligned}(\tau_n)_i &= \frac{\langle \dot{q},w\rangle^2}{\langle w,w\rangle^2}\left(\mu dU_i - \Gamma^k_{ij} dU_k w^j\right) \\ &\quad - \frac{\langle \dot{q},w\rangle}{\langle w,w\rangle^2}\left(\langle \mu w, \dot{q}\rangle - \Gamma^k_{lj} dU_k w^j \dot{q}^l\right) dU_i \\ &= -\frac{\langle \dot{q},w\rangle^2}{\langle w,w\rangle^2}\Gamma^k_{ij} dU_k w^j \\ &\quad + \frac{\langle \dot{q},w\rangle}{\langle w,w\rangle^2}\left(\Gamma^k_{lj} dU_k w^j \dot{q}^l\right) dU_i\end{aligned}$$

which is bounded for $w = g^{-1}dU \to 0$. The other control terms in τ_a (12) are also bounded for $w \to 0$ (since $w/|w|$ is bounded for $w \to 0$), so for this choice of Hessian in the minimum, the control torques are bounded, even at points where $|w| \to 0$. ■

Remark 3: The reason that w needs to be chosen as the gradient of U is that for $\dot{q} = 0$, we have $\tau_a = 0$, although, in general, $dU \neq 0$. This means that when the velocity is zero (i.e., at the end of an oscillation), the initial direction of motion is completely determined by the potential field, so we can only obtain oscillation along a desired curve if the direction of departure, determined by U, is the desired direction, so we need $w = g^{-1}dU$ (modulo a constant).

An interesting intuitive explanation for the required shape of U at the minimum can be found by looking at the two-dimensional (2-D) Euclidean case; a point mass m (with metric mI) moving along curves in a potential field. Fig. 4 shows a potential field with Hessian equal to and unequal to the metric. It is intuitively clear that Fig. 4(b) will give problems, since around the minimum, the integral curves change direction very quickly.

[2]It is well known that the Hessian $(\partial^2 U/\partial q^2)(q)$ is a tensor at points q at which $dU(q) = 0$.

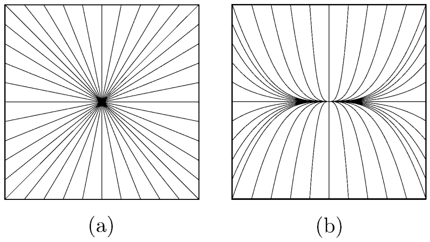

Fig. 4. Integral curves of a potential field with Hessian at the minimum. (a) Equal to the Euclidean metric. (b) Not equal to the Euclidean metric.

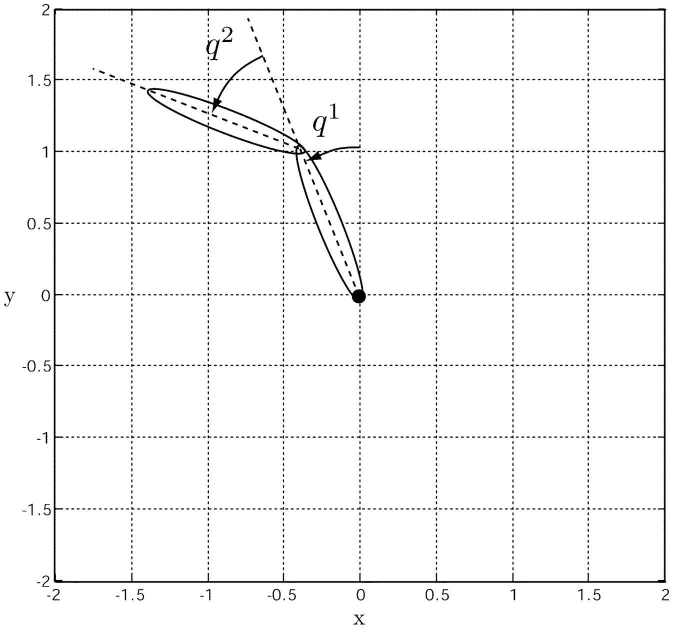

Fig. 5. Schematic of the benchmark robot. Two links (rigid bodies), joined to a fixed base frame by two rotational joints.

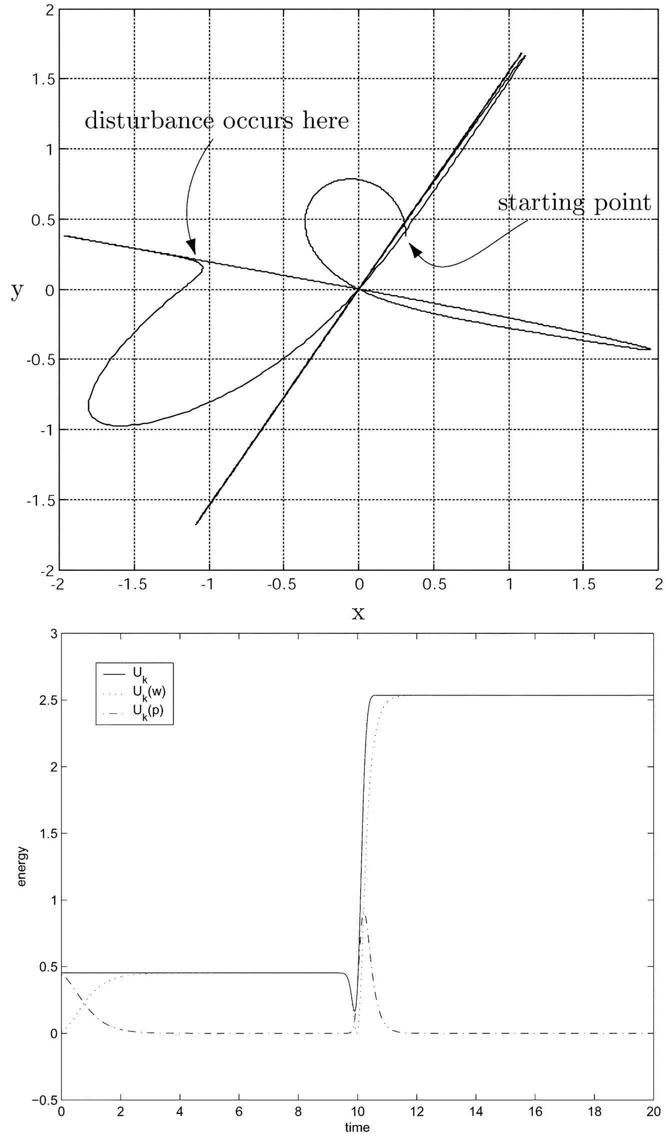

Fig. 6. Curve tracking of straight lines in workspace, with a disturbance torque at $t = 10$. The upper figure shows the path of the tip. The initial position of the end-effector is near $(x, y) = (0.3, 0.4)$. The lower figure shows the total kinetic energy, as well as the energy in the w and p directions.

VI. Simulation

In this section, we illustrate the various parts of the control law (14) by simulation of the robotic manipulator with two DOFs, shown in Fig. 5. We keep the design parameter β in (12) small to be able to observe the effect of disturbances more clearly.

We first want to make the system track (Euclidean) straight lines in work space through the origin, with zero potential field. Since the two links of the robot have equal length, we can encode the lines as a very simple vector field in joint space, namely, as $w = [1 \quad -2]^T$ for all points $q \in \mathcal{Q}$. We apply controller (12) to make the robot track the lines and recover from possible disturbances. At $t = 10$, we apply a short disturbance torque on the second joint, which results in departure from the initial desired curve, and a change of kinetic energy (as described in Section II, the environment is allowed to supply energy). Fig. 6 shows the results. After the disturbance, the total energy has changed, and (after some transients) the system converges to a different line, but the controller is power continuous at all times and the system traces a straight line, before and some time after the disturbance.

As a second test, we use a potential field that has a minimum at $(x, y) = (0, 1)$ and is quadratic in the Euclidean distance from the minimum. Since two configurations are possible that give minimum potential energy, we need to adapt the potential field in two points in joint space to make the Hessian be a scalar multiple of the metric. However, for simplicity, we focus on only one of these configurations, and keep the robot in a small enough neighborhood. We scale and rotate the field such that its Hessian in the minimum is equal to the metric.

We apply controller (14) and drop the robot in the potential field with some initial velocity. Due to the potential field, its kinetic energy increases and the robot starts oscillating along an integral curve of the potential field (Fig. 7).

This result is not exactly what we may have hoped (at first sight) to achieve with a spherical potential field, namely, oscillation along (Euclidean) straight lines through the minimum, just like the behavior of a ball when dropped in the potential field.

Two effects account for the distortion from the intended behavior. The first effect is obviously the scaling and the rotation of the field, which have changed the circles into ellipses. The second effect is the influence of the metric. The metric of a ball

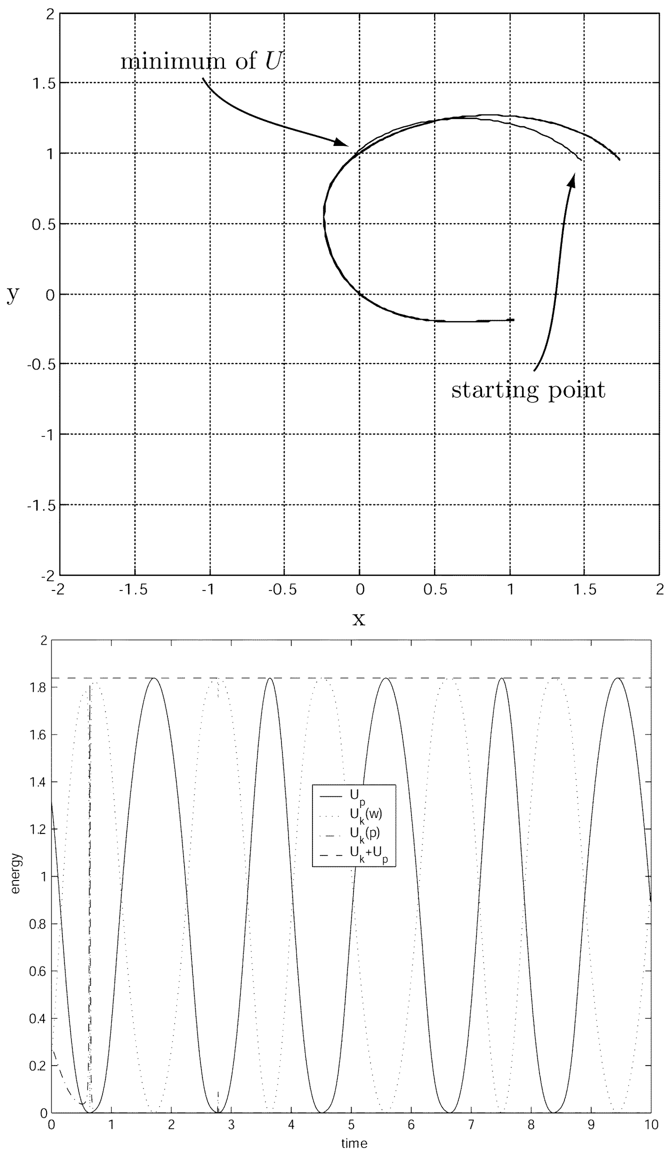

Fig. 7. Curve tracking of integral curves of a potential field. The upper figure shows snapshots of the manipulator (solid), as well as the path of the tip (dashed). The initial position of the end-effector is $(x, y) = (1.5, 0.9)$. The lower figure shows the kinetic energy (decomposed in different directions), and potential and total energy.

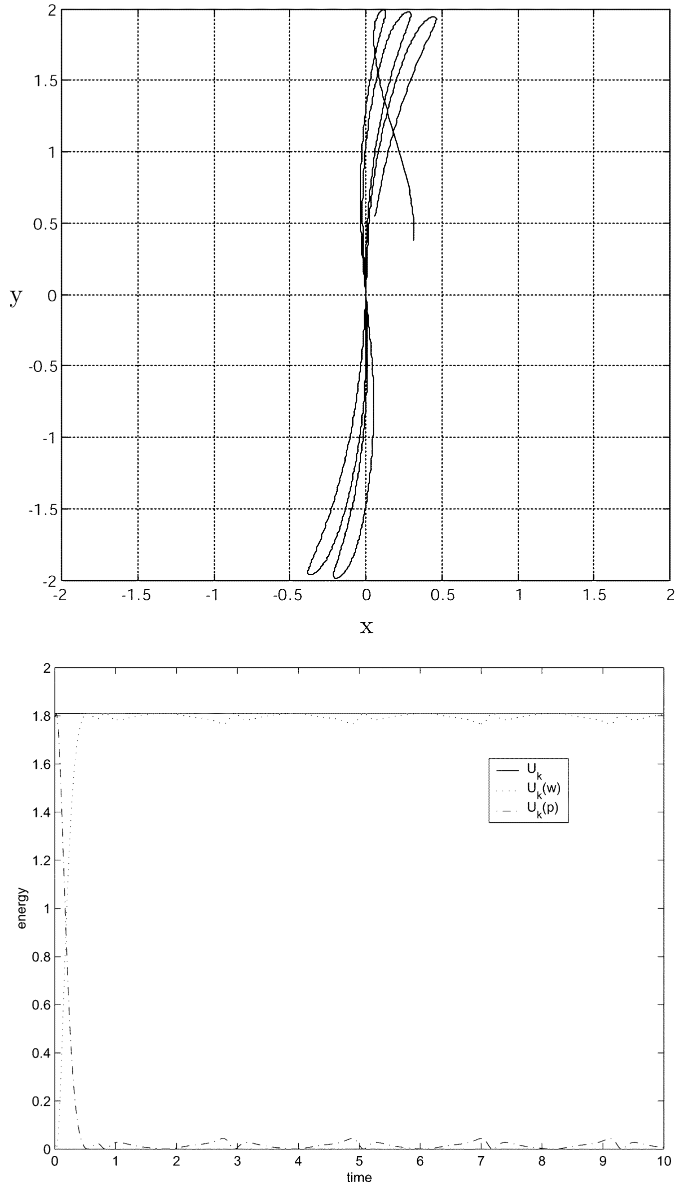

Fig. 8. Curve tracking of straight lines in work space, while suffering from an incorrect model of the robot. The upper figure shows the path of the tip of the robot. The kinetic energy (decomposed in different directions) is shown in the lower figure.

is just a constant scalar, while the metric of the robot is a matrix depending on the position of the robot. Since the controller makes the robot follow the integral curves of $g^{-1}dU$, the metric has a significant influence on these curves.

Finally, we test the robustness of controller (12) against modeling errors, by using an incorrect inertia matrix for the model, and we observe the response of the controller (Fig. 8). The figure shows that the end tip of the robot deviates from the lines, but that the total kinetic energy is always constant, thus ensuring stability.

VII. Conclusions and Future Research

A. Conclusions

We have derived a general, coordinate-free description of a power-continuous controller that makes a robot track curves. We used this controller to make a robot move along curves in a potential field.

To ensure asymptotic tracking for all initial conditions, the desired curves should be the integral curves of the vector field $g^{-1}dU$. Due to the presence of g^{-1} in this expression, the desired curves differ from the "force lines" normally associated with a potential field, i.e., the integral curves of the potential field using a Euclidean metric.

Simulations show that the performance of this controller decreases as model accuracy decreases, but that power continuity is guaranteed under all circumstances, as long as the velocity of the system is measured correctly, and as long as the torque we compute is the same as the torque we actually apply to the motor.

B. Future Research

First of all, it is clear that the controller is no longer passive if there is a mismatch between actual and measured velocity of the system (however, this is a property of every passive controller, and for small mismatches, the controller is still almost passive).

In practice, not all robots have velocity sensors on all joints, and the sensors that *do* exist are not always very accurate. Instead, the velocity is often estimated using observers or dynamic extensions [3]. It is, therefore, interesting to study the effects of using velocity observers or dynamic extensions, and to find conditions on these estimators in terms of speed and accuracy under which the system remains stable, even though passivity may be lost.

Secondly, the condition of full collocated control can not always be satisfied in practice. Some robots have only actuators on a limited number of joints. In other words, for these robots, only a subspace of the cotangent bundle is available for control. Further research should investigate the possibilities of making these robots track trajectories using the theory developed in this paper.

Thirdly, the use of a potential field constrained the desired curves to be the integral curves of $g^{-1}dU$, which were not straight lines through the minimum of the field, which we hoped to achieve. So the question arises: instead of defining the desired curves in terms of the potential field, can we define a potential field in terms of the desired curves? Two problems arise in this situation. First, the Hessian of the potential field in the minimum still must be a constant times the metric, and second, finding the potential field from a vector field involves partial integration of the metric, which may not be possible. However, it may be possible to include this problem already in the design phase of the robot, and optimize the mechanical structure such that the desired curves are more "natural" to implement as a potential field.

Finally, the controller we derived can be described as a set of physical elements, connected by power ports. This suggests a formulation of this controller as a port-controlled Hamiltonian system [20], i.e., as a Dirac structure endowed with an energy function (the Hamiltonian) and interaction ports. The description in terms of power ports allows us to study the interaction between the controlled robot and objects in its environment, i.e., to study the impedance of the controlled robot [3].

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