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An energy-balancing perspective of interconnection and damping assignment control of nonlinear systems☆

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Abstract

Stabilization of nonlinear feedback passive systems is achieved assigning a storage function with a minimum at the desired equilibrium. For physical systems a natural candidate storage function is the difference between the stored and the supplied energies—leading to the so-called energy-balancing control, whose underlying stabilization mechanism is particularly appealing. Unfortunately, energy-balancing stabilization is stymied by the existence of pervasive dissipation, that appears in many engineering applications. To overcome the dissipation obstacle the method of Interconnection and Damping Assignment, that endows the closed-loop system with a special—port-controlled Hamiltonian—structure, has been proposed. If, as in most practical examples, the open-loop system already has this structure, and the damping is not pervasive, both methods are equivalent. In this brief note we show that the methods are also equivalent, with an alternative definition of the supplied energy, when the damping is pervasive. Instrumental for our developments is the observation that, swapping the damping terms in the classical dissipation inequality, we can establish passivity of port-controlled Hamiltonian systems with respect to some new external variables—but with the same storage function.

Keywords: Passivity; Stabilization; Nonlinear systems; Hamiltonian systems

1. Introduction and background material

It is by now well-understood that equilibria of nonlinear systems of the form [1]

$$\dot{x} = f(x) + g(x)u \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, can be easily stabilized if it is possible to find functions $\alpha, h : \mathbb{R}^n \to \mathbb{R}^m$ such that the system

$$\dot{x} = f(x) + g(x)\alpha(x) + g(x)v,$$

$$y = h(x)$$

is passive with external variables (v, y) and a storage function that has a minimum at the desired equilibrium, say $x_* \in \mathbb{R}^n$. This class of systems is called *feedback passive* and stabilization is achieved feeding-back the "passive output" y with a strictly passive operator—a technique that is generically known as passivity-based control (PBC). (See Byrnes, Isidori, & Willems, 1991; Schaft, 2000; Isidori, 1995, or Astolfi, Ortega, & Sepulchre, 2000 for a recent tutorial that contains most of the background material reviewed in this section). From (Byrnes et al., 1991) it is known that necessary conditions for passification of the system (f, g, h) are that it has relative degree

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[1] Throughout the paper, if not otherwise stated, it is assumed that all functions and mappings are C^∞, and all vectors—including the gradient—are column vectors.

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$\{1, \ldots, 1\}$ and is weakly minimum phase. The process is completed verifying the conditions of the nonlinear Kalman–Yakubovich–Popov lemma. The latter involves the solution of a partial differential equation (PDE)—which is difficult to find, in general. An additional complication stems from the minimum requirement on the storage function that imposes some sort of "boundary conditions" on the PDE.

Designing PBCs can be made more systematic for systems belonging to the following class, which contains many physical examples (Ortega, van der Schaft, Mareels, & Maschke, 2001):

Definition 1. System (1) with output $y = h(x)$ is said to satisfy the energy-balancing (EB) inequality if, for some function $H : \mathbb{R}^n \to \mathbb{R}$,

$$H[x(t)] - H[x(0)] \leqslant \int_0^t u^{\mathrm{T}}(s)y(s)\,\mathrm{d}s, \tag{2}$$

along all trajectories compatible with $u : [0, t] \to \mathbb{R}^m$.[2]

Typically, u, y are conjugated variables, in the sense that their product has units of power, and $H(x)$ is the total stored energy—hence the name EB. The EB inequality reflects a universal property of physical systems and it would be desirable to preserve it in closed-loop. On the other hand, since $H(x)$ does not have (in general) a minimum at x_* it is suggested to look for a control action $u = \alpha(x) + \beta(x)v$ such that the closed-loop system satisfies the new EB inequality

$$H_{\mathrm{d}}[x(t)] - H_{\mathrm{d}}[x(0)] \leqslant \int_0^t v^{\mathrm{T}}(s)\tilde{y}(s)\,\mathrm{d}s, \tag{3}$$

for a new output function $\tilde{y} = \tilde{h}(x)$ (that may be equal to y) and some function $H_{\mathrm{d}} : \mathbb{R}^n \to \mathbb{R}_+$ that has an isolated (local) minimum at x_*. (As discussed in Astolfi et al., 2000; Ortega et al., 2001, see also below, the inclusion of a new output function adds considerable flexibility to the design procedure without loosing the physical insight).

A first, natural, approach to solve the problem above is to try to make $H_{\mathrm{d}}(x)$ equal to the difference between the stored and the supplied energies. For that, we must find a function $\alpha(x)$ such that the energy supplied by the controller can be expressed as a function of the state. Indeed, from (2) we see that if we can find a function $\alpha(x)$ such that, for some function $H_{\mathrm{a}} : \mathbb{R}^n \to \mathbb{R}$ and for all x and all $t \geqslant t_0$, we have

$$H_{\mathrm{a}}[\phi(x,t)] - H_{\mathrm{a}}[\phi(x,t_0)] = -\int_{t_0}^t \alpha^{\mathrm{T}}[\phi(x,s)]h[\phi(x,s)]\,\mathrm{d}s, \tag{4}$$

where $\phi(x, t)$ denotes the trajectory of the system with control $u = \alpha(x) + v$ starting from the initial condition at time t_0, then the closed-loop system satisfies (3) with $y = \tilde{y}$ and *new* energy function

$$H_{\mathrm{d}}(x) = H(x) + H_{\mathrm{a}}(x). \tag{5}$$

Hence, x_* can be easily stabilized with the desired storage (Lyapunov) function, and we refer to this particularly appealing class of PBCs as EB-PBCs.

The design of EB-PBCs also involves the solution of a PDE, namely,[3]

$$[f(x) + g(x)\alpha(x)]^{\mathrm{T}}\nabla H_{\mathrm{a}}(x) = -\alpha^{\mathrm{T}}(x)g^{\mathrm{T}}(x)\nabla H(x) \tag{6}$$

that results taking the limit of (4), that is, $\dot{H}_{\mathrm{a}}(x(t)) = -\alpha^{\mathrm{T}}(x(t))h(x(t))$, and the fact that $h(x) = g^{\mathrm{T}}(x)\nabla H(x)$. However, its applicability is mainly stymied by the presence of *pervasive dissipation* in the system. Indeed, it is clear that a necessary condition for the solvability of the PDE (6) is the implication $f(\bar{x}) + g(\bar{x})\alpha(\bar{x}) = 0 \Rightarrow h^{\mathrm{T}}(\bar{x})\alpha(\bar{x}) = 0$. Evaluating, in particular, for $\bar{x}=x_*$ we see that the power extracted from the controller ($=h^{\mathrm{T}}(x)\alpha(x)$) should be zero at the equilibrium. (The interested reader is referred to Ortega et al., 2001 where the effect of pervasive dissipation is illustrated with simple linear time-invariant RLC circuits).

In order to overcome the dissipation obstacle, the method of interconnection and damping assignment (IDA) PBC, that assigns a special—port-controlled Hamiltonian (PCH)—structure to the closed-loop system, has been proposed in Ortega, van der Schaft, Maschke and Escobar (2002). More specifically, in IDA-PBC we fix the matrices $J_{\mathrm{d}}(x) = -J_{\mathrm{d}}^{\mathrm{T}}(x) \in \mathbb{R}^{n\times n}$ and $R_{\mathrm{d}}(x) = R_{\mathrm{d}}^{\mathrm{T}}(x) \geqslant 0 \in \mathbb{R}^{n\times n}$, that represent the desired interconnection and dissipation structures, respectively, and solve the PDE[4]

$$f(x) + g(x)\alpha(x) = [J_{\mathrm{d}}(x) - R_{\mathrm{d}}(x)]\nabla H_{\mathrm{d}}(x),$$

which implies that

$$g^{\perp}(x)f(x) = g^{\perp}(x)[J_{\mathrm{d}}(x) - R_{\mathrm{d}}(x)]\nabla H_{\mathrm{d}}(x), \tag{7}$$

where $g^{\perp}(x)$ is a left annihilator of $g(x)$, that is, $g^{\perp}(x)g(x)= 0$. The PDE (7) characterizes all energy functions that can be assigned to the closed-loop PCH system with the given interconnection and dissipation matrices, and the control that achieves this objective is

$$\alpha(x) = [g^{\mathrm{T}}(x)g(x)]^{-1}g^{\mathrm{T}}(x) \times\{[J_{\mathrm{d}}(x) - R_{\mathrm{d}}(x)]\nabla H_{\mathrm{d}}(x) - f(x)\},$$

where we have assumed (without loss of generality) that the matrix $g(x)$ is full (column) rank. Taking the derivative of $H_{\mathrm{d}}(x)$ along the closed-loop trajectories yields $\dot{H}_{\mathrm{d}}(x) = -\nabla^{\mathrm{T}}H_{\mathrm{d}}(x)R_{\mathrm{d}}(x)\nabla H_{\mathrm{d}}(x) \leqslant 0$. Again, if we can

[2] Notice that no assumption of nonnegativity on $H(x)$ is imposed. Clearly, if it is nonnegative, then the system is passive with external variables (u, y) and storage function $H(x)$. Also, notice that (2) implies $h(x) = g^{\mathrm{T}}(x)\nabla H(x)$.

[3] Throughout the paper we denote $\nabla_p H(p,q) = \frac{\partial H}{\partial p}(p,q)$. When clear from the context the subindex will be omitted.

[4] IDA-PBC is presented in (Ortega et al., 2002) only for systems in PCH form, but it is clear that all derivations carry on to general (f, g, h) systems.

find a solution for (7) such that $x_* = \operatorname{argmin}[H_d(x)]$ then stability of x_* is ensured. The main interest of IDA-PBC is that, in contrast with EB-PBC, the PDE (7) is still solvable (in principle) when the extracted power is not zero at the equilibrium, hence the method is applicable to systems with pervasive dissipation. Another advantage of IDA-PBC is that the free parameters in the PDE (7), $J_d(x), R_d(x)$, have a clear physical interpretation, while there are no simple guidelines for the selection of $\alpha(x)$ in (6).

Although “Hamiltonianizing” the system may seem like an artifice, there are close connections between IDA-PBC and EB-PBC.[5] Namely, in Ortega et al. (2002) conditions on the damping are given so that IDA-PBC is an EB-PBC. More precisely, it is shown that if

(1) the system is PCH, that is,[6] the vector fields $f(x)$ and $g(x)$ satisfy $f(x) = [J(x) - R(x)]\nabla H(x)$ and $h(x) = g^{\mathrm{T}}(x)\nabla H(x)$ for some $J(x) = -J^{\mathrm{T}}(x) \in \mathbb{R}^{n\times n}$ and $R(x) = R^{\mathrm{T}}(x) \geqslant 0 \in \mathbb{R}^{n\times n}$, respectively;
(2) $R_d(x) = R(x)$, that is, no additional damping is injected to the system;
(3) the assigned energy function $H_d(x)$ and the natural damping satisfy

$$R(x)[\nabla H_d(x) - \nabla H(x)] = 0, \tag{8}$$

(This property was called “dissipation obstacle” in Ortega et al. (2001) and, roughly speaking, states that there is no damping in the coordinates where the energy function is shaped).

then, $\dot{H}_d[x(t)] = \dot{H}[x(t)] - \alpha^{\mathrm{T}}[x(t)]h[x(t)]$, and the storage function $H_d(x)$ is equal to the difference between the stored and the supplied energies.

The main contribution of this note is the establishment of a similar equivalence between IDA-PBC and EB-PBC when the damping is “not admissible”, that is when (8) is not satisfied. Specifically, using an alternative definition of the supplied energy, we prove that the methods are also equivalent when the damping is pervasive. Instrumental for our developments is the observation that, swapping the damping terms in the EB inequality, we can establish passivity of PCH systems with respect to some new external variables.

2. A new passivity property for a class of PCH systems

The following lemma is instrumental for the proof of our main result.

Lemma 1. *Assume the matrices* $J(x) = -J^{\mathrm{T}}(x)$ *and* $R(x) = R^{\mathrm{T}}(x) \geqslant 0$ *are such that* $\operatorname{rank}[J(x) - R(x)] = n$, *then*

$$z^{\mathrm{T}}[J(x) - R(x)]^{-1}z \leqslant 0, \quad \textit{for all } z \in \mathbb{R}^n. \tag{9}$$

[5] See Ortega et al. (2001) for the interpretation of IDA-PBC as controlled by interconnection (Schaft, 2000).

[6] In Ortega et al. (2002) it is shown that all asymptotically stable vector fields admit such a PCH realization.

Proof. The proof is completed with the following observation:

$$z^{\mathrm{T}}[J(x) - R(x)]^{-1}z = -\tilde{z}^{\mathrm{T}}R(x)\tilde{z} \leqslant 0,$$

where we have defined $\tilde{z} \triangleq [J(x) - R(x)]^{-\mathrm{T}}z$, with $(\cdot)^{-\mathrm{T}} = [(\cdot)^{-1}]^{\mathrm{T}}$. □

Notice that, if $J(x) - R(x)$ is rank deficient then the open-loop system has equilibria at points which are not extrema of the energy function. Hence, the assumption $\operatorname{rank}[J(x) - R(x)] = n$ does not seem to be restrictive in applications. For this class of PCH systems the proposition below establishes passivity with respect to a new set of external variables.

Proposition 1. *Consider the PCH system*

$$\dot{x} = [J(x) - R(x)]\nabla H(x) + g(x)u$$

$$y = g^{\mathrm{T}}(x)\nabla H(x). \tag{10}$$

Assume $J(x) - R(x)$ *is full rank. Then, the system satisfies the new EB inequality*

$$H[x(t)] - H[x(0)] \leqslant \int_0^t \tilde{y}^{\mathrm{T}}(s)u(s)\,\mathrm{d}s, \tag{11}$$

where $\tilde{y} = \tilde{h}(x,u)$, *with*

$$\tilde{h}(x,u) = -g^{\mathrm{T}}(x)[J(x) - R(x)]^{-\mathrm{T}} \times \{[J(x) - R(x)]\nabla H(x) + g(x)u\}. \tag{12}$$

Furthermore, if $H(x)$ *is bounded from below, the system is passive with external variables* $(u, \tilde{y})$ *and storage function* $H(x)$.

Proof. Under the assumption that $\operatorname{rank}[J(x) - R(x)] = n$, we can rewrite the PCH system (10) in the following form

$$[J(x) - R(x)]^{-1}\dot{x} = \nabla H(x) + [J(x) - R(x)]^{-1}g(x)u. \tag{13}$$

Premultiplying (13) by $\dot{x}^{\mathrm{T}}$ we obtain

$$\begin{aligned}\dot{H}(x) &= \dot{x}^{\mathrm{T}}\nabla H(x)\\ &= \dot{x}^{\mathrm{T}}[J(x) - R(x)]^{-1}\dot{x} - \dot{x}^{\mathrm{T}}[J(x) - R(x)]^{-1}g(x)u\\ &\leqslant -\dot{x}^{\mathrm{T}}[J(x) - R(x)]^{-1}g(x)u\\ &= \tilde{y}^{\mathrm{T}}u,\end{aligned}$$

where we have invoked Lemma 1 to obtain the inequality, and replaced $\dot{x}$ and used (12) in the last equality. The proof is completed integrating the expression above from 0 to t. □

Remark 1. From the derivations above we have that

$$\dot{H}(x) = -\dot{\tilde{x}}^{\mathrm{T}}R(x)\dot{\tilde{x}} + \tilde{y}^{\mathrm{T}}u,$$

where $\dot{\tilde{x}} \triangleq [J(x) - R(x)]^{-\mathrm{T}}\dot{x}$. Comparing with the classical power balance equation,

$$\dot{H}(x) = -\nabla^{\mathrm{T}}H(x))R(x)\nabla H(x) + y^{\mathrm{T}}u,$$

reveals that the new passivity property is established “swapping the damping”.

Remark 2. Proposition 1 lends itself to an alternative interpretation that reveals the close connections with the results reported in (Jeltsema, Ortega, & Scherpen, 2003b; Ortega, Jeltsema, & Scherpen, 2003). In these papers a new passivity property for RLC circuits is established and used in (Ortega et al., 2003) to propose power-shaping, as an alternative to energy-shaping, to overcome the dissipation obstacle for stabilization of systems with pervasive damping. From the proof of the proposition it is clear that, introducing an input change of coordinates

$$\tilde{u} = [J(x) - R(x)]^{-1} g(x)u, \tag{14}$$

we also have passivity with the external coordinates $(\tilde{u}, \dot{x})$—hence, in some respect, we have "added a differentiation" to the port variables as done in (Jeltsema et al., 2003b). It appears that the input change of coordinates (14) has close relations with the well-known Thevenin–Norton equivalent representation used in circuit theory. See (Jeltsema, Ortega, & Scherpen, 2003a) for a detailed discussion on this subject.

Remark 3. Interestingly, as a kind of partial converse, the new passivity property has no influence on systems that do not suffer from pervasive dissipation (in the sense that the new output $\tilde{y}$ coincides with the original output y, i.e., $\tilde{y}=y$). The interested reader is referred to (Jeltsema et al., 2003a) for a detailed discussion and some illustrative examples.

3. IDA-PBC as an energy-balancing controller

As a corollary of Proposition 1 we prove in this section that, even when the damping is pervasive, IDA-PBC is an EB-PBC with the new definition of supplied power $u^{\mathrm{T}}\tilde{y}$.

Proposition 2. *Consider the PCH system (10), where $J(x) - R(x)$ is full-rank, in closed-loop with an IDA-PBC, $u = \alpha(x)$, that transforms the system into*

$$\dot{x} = [J(x) - R(x)]\nabla H_{\mathrm{d}}(x). \tag{15}$$

Then,

$$H_{\mathrm{d}}[x(t)] = H[x(t)] - \int_0^t u^{\mathrm{T}}(s)\tilde{y}(s)\,\mathrm{d}s + \kappa, \tag{16}$$

where $\tilde{y} = \tilde{h}(x,u)$, with $\tilde{h}(x,u)$ defined in (12), and κ a constant determined by the initial condition.

Proof. The PCH system (10), with $u = \alpha(x)$, matches (15) if and only if $[J(x)-R(x)]^{-1}g(x)\alpha(x) = \nabla H_{\mathrm{a}}(x)$, where we have used (5) and the assumption $\mathrm{rank}[J(x) - R(x)] = n$. Premultiplying the latter equation by $\dot{x}^{\mathrm{T}}$ we obtain $\dot{H}_{\mathrm{a}}(x) = -u^{\mathrm{T}}\tilde{y}$, which upon integration yields the desired result. □

Remark 4. The proposition above is restricted to IDA-PBC designs that do not modify the interconnection and damping matrices of the open-loop system, but only shape the energy function. When $J_{\mathrm{d}}(x) \neq J(x)$ and/or $R_{\mathrm{d}}(x) \neq R(x)$ the matching condition becomes, (see Ortega et al., 2002),

$$[J_{\mathrm{d}}(x) - R_{\mathrm{d}}(x)]^{-1}\{[J_{\mathrm{a}}(x) - R_{\mathrm{a}}(x)]\nabla H(x) + g(x)\alpha(x)\} = \nabla H_{\mathrm{a}}(x),$$

where $J_{\mathrm{d}}(x) = J(x) + J_{\mathrm{a}}(x)$ and $R_{\mathrm{d}}(x) = R(x) + R_{\mathrm{a}}(x)$. Some simple calculations show that a term, that is independent of $\alpha(x)$, appears in $\dot{H}_{\mathrm{a}}$. Therefore, the latter cannot be made equal to some (suitably defined) supplied power.

4. Concluding remarks

Summarizing, we have shown that, for the class of systems with pervasive dissipation, the basic IDA-PBC methodology reduces to an EB-PBC design. Thus, if one accepts a set outputs other than the natural ones, we can give an energy-balancing interpretation of IDA-PBC. Instrumental for our developments is that we swap the damping in the classical power-balance in order to conclude passivity with respect to a different set of external port variables, while using the same storage function. The only necessary condition for swapping the damping is that $J(x) - R(x)$ needs to be full rank. However, if $J(x) - R(x)$ is rank deficient then the open-loop system has equilibria at points which are not extrema of the energy function. Therefore, the full rank condition seems not restrictive in physical applications.

References

Astolfi, A., Ortega, R., & Sepulchre, R. (2000). Stabilization and disturbance attenuation of nonlinear systems using dissipativity theory. In K. Aström, P. Albertos, M. Blanke, & A. Isidori (Eds.), *Control of complex systems* (pp. 39–75). London: Springer.

Byrnes, C., Isidori, A., & Willems, J. C. (1991). Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE Transactions on Automatic Control*, *36*(11), 1228–1240.

Isidori, A. (1995). *Nonlinear control systems* (3rd ed.). Berlin: Springer.

Jeltsema, D., Ortega, R., & Scherpen, J. M. A. (2003a). An energy-balancing perspective of interconnection and damping assignment control of nonlinear systems. *Second IFAC workshop on Lagrangian and Hamiltonian methods for nonlinear control*, Sevilla, Spain.

Jeltsema, D., Ortega, R., & Scherpen, J. M. A. (2003b). On passivity and power-balance inequalities of nonlinear RLC circuits. *IEEE Transactions on Circuits and Systems Part-I Fundamental Theory and Application*, *50*(9), 1174–1179.

Ortega, R., Jeltsema, D., & Scherpen, J. M. A. (2003). Power shaping: A new paradigm for stabilization of nonlinear RLC circuits. *IEEE Transactions on Automatic Control*, *48*(10), 1762–1767.

Ortega, R., van der Schaft, A. J., Mareels, I., & Maschke, B. M. (2001). Putting energy back in control. *IEEE Control Systems Magazine*, *21*(2), 18–33.

Ortega, R., van der Schaft, A. J., Maschke, B. M., & Escobar, G. (2002). Interconnection and damping assignment passivity based control of port-controlled Hamiltonian systems. *Automatica*, *38*(4), 585–596.

van der Schaft, A. J. (2000). *$\mathscr{L}_2$-gain and passivity techniques in nonlinear control*. Berlin: Springer.