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Verstappen, Roel

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Symmetry-preserving Regularization Modeling of Turbulent Channel Flow

Roel Verstappen

Institute of Mathematics and Computing Science, University of Groningen,
P.O.Box 800, 9700AV Groningen, The Netherlands; R.W.C.P.Verstappen@rug.nl

1 Introduction

Most turbulent flows can not be computed directly from the (incompressible) Navier-Stokes equations,

$$\partial_t u + \mathcal{C}(u, u) + \mathcal{D}(u) + \nabla p = 0, \quad (1)$$

because they possess far too many scales of motion. The computationally almost numberless small scales result from the nonlinear convective term $\mathcal{C}(u, v) = (u \cdot \nabla)v$ which allows for the transfer of energy from scales as large as the flow domain to the smallest scales that can survive viscous dissipation. As the full energy cascade can not be computed, a dynamically less complex mathematical formulation is sought. In the quest for such a formulation, we consider smooth approximations (regularizations) of the convective term:

$$\partial_t u_\epsilon + \tilde{\mathcal{C}}(u_\epsilon, u_\epsilon) + \mathcal{D}(u_\epsilon) + \nabla p_\epsilon = 0, \quad (2)$$

where the variable name is changed from u to u_ϵ to stress that the solution of (2) differs from that of (1). The idea is to smooth the convective term directly to set bounds to the creation of smaller and smaller scales of motion and thus to confine the cascade of energy. Equation (2) provides a basis for a simulation shortcut if the low modes of u_ϵ approximate the corresponding low modes of the Navier-Stokes-solution u , whereas the high modes of u_ϵ vanish faster than those of u [1].

The first outstanding approach in this direction goes back to Leray [2], who took $\tilde{\mathcal{C}}(u, u) = \mathcal{C}(\bar{u}, u)$ and proved that a moderate filtering of the convective velocity is sufficient to regularize a turbulent flow. The Navier-Stokes- α -model forms another example of regularization modeling, see for instance [3],[4]. In this model, the convective term becomes $\tilde{\mathcal{C}}_r(u, u) = \mathcal{C}_r(u, \bar{u})$, where $\mathcal{C}_r$ denotes the convective operator in rotational form: $\mathcal{C}_r(u, v) = (\nabla \times u) \times v$.

The regularization method basically alters the nonlinearity to control the convective energetic exchanges. In doing so, one can preserve certain fundamental properties of (the convective operator in) the Navier-Stokes equations exactly, e.g. symmetries, conservation properties, transformation properties, Kelvin's theorem, Bernoulli's theorem, etc.

2 Symmetry-preserving Approximations

We purpose to consider approximations that conserve the energy, the enstrophy (in 2D) and the helicity in the absence of viscous dissipation, among others because they are intrinsically stable (in the energy-norm; in 2D: enstrophy-norm). The Leray model conserves the energy, but not the enstrophy or helicity, whereas the Navier-Stokes- α -model conserves the enstrophy and helicity, yet not the energy. Since the conservation of energy, enstrophy and helicity results from the symmetries of the convective operator $\mathcal{C}$ (see e.g. [5]), we propose to approximate $\mathcal{C}$ in such manner that the underlying symmetries are preserved. This criterion yields the following class of (energy-, enstrophy-(2D) and helicity-conserving) approximations

$$\partial_t u_\epsilon + \mathcal{C}_n(u_\epsilon, u_\epsilon) + \mathcal{D}(u_\epsilon) + \nabla p_\epsilon = 0, \quad (3)$$

($n = 2, 4, 6$) where the convective term is smoothened according to

$$\mathcal{C}_2(u, v) = \overline{\mathcal{C}(\bar{u}, \bar{v})} \quad (4)$$

$$\mathcal{C}_4(u, v) = \mathcal{C}(\bar{u}, \bar{v}) + \overline{\mathcal{C}(\bar{u}, v')} + \overline{\mathcal{C}(u', \bar{v})} \quad (5)$$

$$\mathcal{C}_6(u, v) = \mathcal{C}(\bar{u}, \bar{v}) + \mathcal{C}(\bar{u}, v') + \mathcal{C}(u', \bar{v}) + \overline{\mathcal{C}(u', v')} \quad (6)$$

Here, a bar denotes a filtered quantity and a prime indicates the residual. The three approximations $\mathcal{C}_n(u, u)$ are consistent with $\mathcal{C}(u, u)$, where the error is of the order of ϵ^n ($n = 2, 4, 6$) for symmetric filters with filter length ϵ . Both the Leray model and the alpha model are second-order accurate in terms of ϵ .

3 Nonlinear Transport Mechanism

The evolution of the vorticity

$$\partial_t \omega_\epsilon + \mathcal{C}_n(u_\epsilon, \omega_\epsilon) + \mathcal{D}(\omega_\epsilon) = \mathcal{C}_n(\omega_\epsilon, u_\epsilon), \quad (7)$$

resembles that of the Navier-Stokes equations: the only difference is that $\mathcal{C}$ is replaced by $\mathcal{C}_n$. The approximations $\mathcal{C}_n$ reduce the high frequencies in the vortex stretching term. The Navier-Stokes equations yield $\mathcal{C}(\omega, u) = \overline{S\omega} + \overline{S\omega'} + S'\bar{\omega} + S'\omega'$ (where $S = \frac{1}{2}(\nabla u + \nabla u^T)$ is the deformation tensor), while here the vortex stretching term becomes $\mathcal{C}_2(\omega, u) = \overline{S\omega}$, $\mathcal{C}_4(\omega, u) = \overline{S\omega} + \overline{S\omega'} + \overline{S'\omega}$,

$\mathcal{C}_6(\omega, u) = \overline{S\omega} + \overline{S'\omega'} + \overline{S'\omega'}$. Qualitatively, vortex stretching leads to the production of smaller and smaller scales, i.e. to a continuous, local increase of both S' and ω' . Consequently, at the positions where vortex stretching occurs, the terms with S' and ω' will eventually amount considerably to $\mathcal{C}(\omega, u)$. Since the approximations $\mathcal{C}_n(\omega, u)$ diminish these terms, the symmetry-preserving regularization counteracts the production of smaller and smaller scales by means of vortex stretching and may eventually stop the continuation of the vortex stretching process. In conclusion, the approximations $\mathcal{C}_n(u, u)$ restrain the convective production of smaller and smaller scales of motion by means of vortex stretching, while ensuring that the solution cannot blow up (in the energy-norm; 2D: enstrophy-norm).

A detailed study of the triadic interactions shows that $\mathcal{C}_4(u, u)$ (for example) approximates the local interactions between large scales of motion (meaning that $\epsilon|k| < 1$) up to fourth-order, i.e. the triadic interactions between large scales are only slightly altered. All interactions involving longer wavevectors (smaller scales of motion) are reduced. The amount by which the interactions between the wavevector-triple (k, p, q) are lessened depends on the length of the legs of the triangle $k = p + q$. Triadic interactions for which at least two legs are (much) longer than $1/\epsilon$ are (strongly) attenuated, whereas interactions for which at least two legs are (much) shorter than $1/\epsilon$ are reduced to a small degree only.

4 Results for Turbulent Channel Flow

As a first step in the application of symmetry-preserving regularization, the approximation $\mathcal{C}_4$ is tested for a turbulent channel flow by means of a comparison with the direct numerical simulations performed by Kim et al. [6]. Based on the channel half-width and the friction velocity the Reynolds number is 180. The numerical algorithm used to solve (3) preserves the considered symmetries, too, [7] and is therefore well-suited to test the proposed simulation shortcut. We consider two, coarse, computational grids consisting of $16 \times 16 \times 8$ and $32 \times 32 \times 16$ grid points, respectively. The filtering is performed by means of a Helmholtz operator. The least to be expected from a simulation shortcut is a good prediction of the mean flow. As can be seen in Fig. 1, the conservative approximation $\mathcal{C}_4$ satisfies that minimal requirement already at the very coarse $16 \times 16 \times 8$ grid (when the filter length ϵ is taken equal to about three times the grid width h). Overall good agreement between the $\mathcal{C}_4$ -calculation at the $32 \times 32 \times 16$ grid and the DNS is observed for both the first- and second-order statistics, see Fig. 1. Heuristic arguments as well as computational results (Fig. 2) show that the energy spectrum of the solution of (3)+(5) follows the DNS for large scales of motion, whereas a much steeper (numerically speaking: more gentle) power law is found for small scales, which is precisely what a simulation shortcut is ought to do.

Fig. 1. The least to be expected from a simulation shortcut is a good prediction of the mean flow. The left-hand figure shows that the symmetry-preserving regularization model satisfies that minimal requirement already at the very coarse $16 \times 16 \times 8$ grid (when the filter length ϵ is about three times the grid width h). Overall good agreement for the second-order statistics is obtained with $32 \times 32 \times 16$ gridpoints (see right-hand figure)

Fig. 2. One-dimensional (streamwise) energy spectra at $y^+ \approx 5$. The spectra illustrate that the regularization improves the agreement with the DNS for the low-modes, whereas the tail vanishes faster

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