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Adaptive fuzzy control of a quadrotor using disturbance observer

Chuang Li a, b, *, Yujia Wang c, Xuebo Yang c

a The College of Engineering, Bohai University, Jinzhou 121013, Liaoning, China
b Department of Biomedical Engineering, University of Groningen and University Medical Centre Groningen, 9713 GZ Groningen, the Netherlands
c The Research Institute of Intelligent Control and Systems, School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

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In this paper, an adaptive fuzzy tracking control scheme by combining Lyapunov stability theory and backstepping technique is proposed for a quadrotor unmanned aerial vehicle. First, a fuzzy logic system is used to approximate the unmodeled dynamics of the quadrotor system. Second, command filtering is utilized to compute the derivatives of the virtual control signals to avoid the complex analytic derivation of these derivatives. Third, a nonlinear disturbance observer is applied to compensate for external disturbance and approximation error of the fuzzy logic system. Lyapunov stability analysis shows that the quadrotor system can be stabilized by the proposed controller with high control accuracy. Finally, the experimental results are given to verify the effectiveness of the proposed control strategy.

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1. Introduction

Very recently, unmanned aerial vehicles (UAVs) have been widely used in military and civil fields. For example, military strikes, aerial photography, power line inspection, and forest fire prevention [1–4]. Therefore, some research groups have done corresponding researches on UAVs. The quadrotor is a type of UAV that can do aerial hover, low-altitude, low-speed, heading remain unchanged, as well as perform tasks in a confined space. These superior features are not available for other UAVs. However, the quadrotor control system has the characteristics of nonlinearity, uncertainty, underactuated with strong coupling. Moreover, the external disturbance also increases the difficulty of control. Therefore, the design of the quadrotor controller is very challenging.

To guarantee that a quadrotor UAV tracks ideal trajectory within a small tracking error, the attitude loop and the position loop need to be precisely controlled, and then the high precision control of the six degrees of freedom of the quadrotor can be achieved. Among existing literatures, the Proportional-Integral-Derivative (PID) controller [5–8] has been widely used in UAVs due to its simple control structure. In [5], the PID controller has been used to regulate the posture (position and orientation) of the quadrotor. In [6], the authors have proposed a new model design method for a quadrotor, which has been further controlled by a PID controller. In [7], the authors have proposed two algorithms to adjust the gain of PID controller, and then compared the superiority of the two algorithms. In [8], the authors have proposed a PID control method to restrain constant interference for the formation flight of UAVs.

Compared with classical control methods, modern control methods have received much attention in recent decades. A large number of modern control methods have been proposed. For a nonlinear, strongly coupled, multi-input, multi-output quadrotor system with unmodeled dynamics, a robust controller combining backstepping with extended Kalman Bucy filter [9] has been proposed to control the smooth flight of the quadrotor. In [10–15], a control method based on sliding mode technology has been presented to solve the problem of model uncertainty, external disturbance, actuator failure, and time-varying load in a quadrotor control system. In [16,17], for the problem of uncertain disturbances in the quadrotor system, the authors have been presented an error compensation mechanism to enhance the robustness of the system. With the rapid development of intelligent control, intelligent control algorithms have also been widely applied to various systems. Adaptive fuzzy [18] and adaptive neural network [19] control algorithms have been presented to control a 3-DOF helicopter system with model uncertainty, actuator deadzone, and state delay. In [20], the conventional PID controller and fuzzy logic system have been combined to construct a fuzzy self-tuning PID control algorithm to achieve precise control of the flight height of the quadrotor.

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* Corresponding author: The College of Engineering, Bohai University, Jinzhou 121013, Liaoning, China.
E-mail addresses: sever777lee@163.com (C. Li), wangyuusija542@163.com (Y. Wang), xueboyang@hit.edu.cn (X. Yang).

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Most of the control methods mentioned above improve the flight performance of the quadrotor to some extent. However, there are still some problems that should be solved. Linear control methods are commonly used to control a quadrotor, such as PID or LQR. Although linear control methods have been proven to perform well in flight control of the quadrotor, the effectiveness of linear control methods can only be guaranteed within a limited range of operating conditions. For sliding mode control, it has the merits of perfect robustness and better control performance, however, in the process of using sliding mode control, because of the continuous switching logic, there would be chattering, which may affect the control accuracy of the quadrotor. In [21], the authors have realized that the fuzzy logic system can be used to approximate the unmodeled dynamics in the system. In [22,23], a finite-time disturbance observer has been deployed to deal with the uncertainty in the system of a surface vehicle. In addition, a new type of disturbance observer has been designed for a quadrotor in [24], which has solved difficult problems of high-accuracy control for the quadrotor UAV subjected to external disturbance force and unknown disturbance torque.

Inspired by these research results, this paper proposes an adaptive control method based on fuzzy logic system and disturbance observer for a quadrotor system, and the main contributions of this paper are listed as follows:

(a) The unmodeled dynamics presented in the quadrotor system are approximated by the fuzzy logic system, and the external compounded disturbance is estimated by a disturbance observer.

(b) The application of command filtering and error compensation mechanism further improves the tracking accuracy of the quadrotor, and the conservativeness of the controller is reduced and the robustness of the controller is enhanced.

(c) Compared with traditional PID control, the proposed method can achieve full control of the six degrees of freedom of the quadrotor, also, can obtain smaller tracking error.

The remainder of this paper is organized as follows: Section 2 introduces the mathematical model of the quadrotor and the fuzzy logic system. An adaptive controller based on the fuzzy logic system with disturbance observer is designed and the stability analysis of the closed-loop system is given in Section 3. Finally, the experimental results and conclusions are demonstrated in Sections 4 and 5, respectively.

\section{Problem statement}

\subsection{Mathematical model of quadrotor}

The research platform shown in Fig. 1 is a quadrotor UAV by Quanser Innovate Educate Inc. The four brushless motors are symmetrically mounted on the frame, the No. 1 and No. 2 motors rotate counterclockwise, and the No. 3 and No. 4 motors rotate clockwise. Furthermore, motion control of the quadrotor is achieved by adjusting the rotation speeds of the four motors, $F_i$ is the Earth-fixed inertial frame, and $F_b$ is the body-fixed frame with the origin $O_b$, which is the center of mass of the quadrotor UAV. The thrust $F_k$ of each motor is generated by the following a first-order linear transfer function:

$$F_k = K \frac{B_w}{s + B_w} v_k,$$

where $k = 1, 2, 3, 4$, $K$ denotes a positive gain, $B_{w}$ represents the bandwidth of each brushless motor, and the input PWM of the motor is indicated by $v_k$. The total force, torque of the pitch axis, roll axis, and yaw axis can be written as follows:

$$F = F_1 + F_2 + F_3 + F_4$$

$$\tau_\phi = d(F_1 - F_2)$$

$$\tau_\psi = d(F_3 - F_4)$$

$$\tau_y = t_1 + t_2 - t_3 - t_4$$

where $F$ represents the total force, and $\tau_\phi$, $\tau_\psi$, and $\tau_y$ are the torque of the pitch axis, roll axis, and yaw axis, respectively. $d$ indicates the distance from the center $O_b$ of the frame to each motor. $t_k$ is torque produced by rotating of motor, and its relationship with thrust $F_k$ is $t_k = K_k F_k$, and $K_k$ is a constant.

In addition, according to Eq. (1), it can be assumed that $F_k \approx K v_k$. Therefore, Eq. (2) can be written as:

$$F = K(v_1 + v_2 + v_3 + v_4)$$

$$\tau_\phi = Kd(v_1 - v_2)$$

$$\tau_\psi = Kd(v_3 - v_4)$$

$$\tau_y = Kd(v_1 - v_2 - v_3 - v_4)$$

Further, by applying small angle approximation, the attitude dynamics and the position dynamics are expressed in the body-fixed frame and the inertial frame, respectively. They are given as follows [25]:

$$J_y \ddot{\theta} = \tau_\phi + (J_z - J_x) \dot{\psi} \dot{\psi} - J_x \dot{\psi} \dot{\omega} + f_\psi(\Omega) + d_\theta(t)$$

$$J_x \ddot{\psi} = \tau_\psi + (J_y - J_z) \dot{\theta} \dot{\theta} - J_y \dot{\theta} \dot{\omega} + f_\theta(\Omega) + d_\psi(t)$$

$$m \ddot{x} = F \cos \varphi \sin \theta \cos \psi \sin \psi + \sin \psi \sin \psi + f_x(x, \dot{x}) + d_x(t)$$

$$m \ddot{y} = F \cos \varphi \sin \theta \cos \psi \cos \psi + \cos \psi \cos \psi + f_y(y, \dot{y}) + d_y(t)$$

$$m \ddot{z} = F \cos \varphi \cos \theta - mg + f_z(z, \dot{z}) + d_z(t)$$

where $\theta$, $\psi$, and $\psi$ indicate the pitch, roll, and yaw Euler angles of the quadrotor UAV. $x$, $y$, and $z$ represent the coordinates of the quadrotor’s center of mass in the Earth-fixed inertial frame. $Jx$, $Jy$, and $Jz$ denote the moments of inertia along $x$, $y$, and $z$ directions, respectively. $J_x$ represents the moment of inertia of each motor and $\Omega = [\varphi \theta \psi \dot{\psi} \dot{\theta} \dot{\psi}]^T$ is the state vector of the quadrotor. $m$ denotes the total mass of the quadrotor, and $g$ is the gravitational acceleration. $\omega = \omega_1 - \omega_2 + \omega_3 + \omega_4$, and $\omega_k$ indicates the angular speed of each motor. In addition, $f_x$, $f_y$, $f_\phi$, $f_\theta$, $f_\psi$, and $f_z$ represent the uncertainty of dynamic modeling of position and attitude due to installation errors, gyroscope errors, etc. $d_\theta$, $d_\psi$, $d_\psi$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Structure of a quadrotor UAV.}
\end{figure}
\[ d_x, d_y, \text{ and } d_z \text{ indicate the time-dependent external disturbance which can be largely attributed to the exogenous effects, such as wind gusts or other environmental factors.} \]

The desired attitude reference signal \( \Theta_d = [\theta_d \, \psi_d \, \phi_d]^T \) and the desired position reference signal \( X_d = [x_d \, y_d \, z_d]^T \) are given. The control objective is to design a control method such that the attitude and position of the quadrotor can asymptotically track the reference signal with a very small error.

In order to facilitate the establishment of the dynamic model for the quadrotor and to satisfy the implementation of the control algorithm, the following assumptions are given.

**Assumption 1.** The structure of the quadrotor is a symmetrical rigid body.

**Assumption 2.** The desired attitude reference signal and the desired position reference signal are continuous, first order differentiable and bounded.

**Assumption 3.** The external disturbance \( d(t) = [d_0, d_\theta, d_\phi, d_x, d_y, d_z]^T \) has finite energy. The external disturbances function \( d(t) \) and its first-derivative \( \dot{d}(t) \) are continuous functions, and \( \|d(t)\| \leq d_M \), where \( d_M \) is an unknown positive constant.

**Lemma 1.** Defining a continuous and differentiable bounded function \( \Delta(x), \forall t \in [t_0, \, t_1] \), if \( \Delta(x) \) satisfies \( \Delta(x) \leq \pi \), where \( \pi \) is the positive constant, and \( \Delta(x) \) is bounded.

**Lemma 2.** Given a continuous and positive definite Lyapunov function \( V(x) \), and there is bounded initial conditions, which satisfies \( \pi_1(x) \leq V(x) \leq \pi_2(x) \) such that \( \dot{V}(x) \leq -c_1 \, V(x) + c_2 \), where \( \pi_1, \, \pi_2 : \mathbb{R}^n \rightarrow \mathbb{R} \) are class functions, and \( c_1 \) and \( c_2 \) are positive constants, then the solution \( x(t) \) is ultimately uniformly bounded.

### 2.2. Fuzzy logic system

The fuzzy logic system [27] contains the knowledge base, the fuzzifier, the fuzzy inference engine and the defuzzifier parts. If then rules form a knowledge base. \( \text{Ri}^j \): If \( x_1 \) is \( F_1^1 \) and ... and \( x_n \) is \( F_n^1 \) then \( y \) is \( B^j \), where \( x = [x_1, x_2, ..., x_n] \in \mathbb{R}^n \), and \( y \) correspond to the input and output of the fuzzy logic system, \( F_1^1, F_2^2, ..., F_n^i \) and \( B^j \) are fuzzy sets in \( \mathbb{R} \). By applying the form of singleton fuzzification, the product inference, and the center-average defuzzification, the following fuzzy logic system exists:

\[
\begin{align*}
    y(x) &= \sum_{i=1}^{N} \sum_{j=1}^{n} \omega_{ii} \prod_{j=1}^{n} \mu_{F_i^j}(x_j) \\
    &= \sum_{i=1}^{N} \mu_{B_i}(x_i),
\end{align*}
\]

where \( N \) indicates the number of if-then rules, \( \mu_{B_i}(x_i) \) is the fuzzy membership function.

Define \( s_i(x_i) = \prod_{j=1}^{n} \mu_{F_i^j}(x_j) \), \( S(X) = [s_1(x), ..., s_n(x)]^T \), and \( W = [\omega_1, ..., \omega_n]^T \), the fuzzy logic system can be expressed by:

\[
y = W^T S(X).
\]

If any membership is selected as Gaussian functions, there will be the following Lemma 3 set up.

**Lemma 3.** [28] For a continuous function \( f(x) \) defined on a compact set \( a_0 \), define a desired precision function \( |f(x)| \leq \sigma \), then, the fuzzy logic system Eq. (6) satisfies \( |f(x) - W^T S(x)| \leq \sigma \).

**Remark 1.** It should be noted that Lemma 3 is very important for the following controller design process. An arbitrary real continuous function \( f(x) \) can be linearly composed of the basis function vector \( S(X) \) and a bounded error function \( \sigma(X) \), which means that \( f(x) = W^T S(X) + \sigma(X) \), where \( 0 < S^T S \leq 1 \).

### 3. Control design and stability analysis

In this section, an adaptive control based on the fuzzy logic system with disturbance observer is proposed, and then the six degrees of freedom of the quadrotor is precisely controlled. The block diagram of the entire control system is shown in Fig. 2.

#### 3.1. Attitude controller design

For facilitating the design of the attitude controller of the quadrotor, the coordinate transformation is given by:

\[
\Theta_1 = \theta, \quad \Theta_2 = \psi, \quad \Theta_3 = \phi \\
\Theta_{61} = \theta, \quad \Theta_{62} = \psi_d, \quad \Theta_{63} = \psi \\
g_1 = 1 / J_y, \quad g_2 = 1 / J_x, \quad g_3 = 1 / J_z \\
t_1 = \tau_\theta, \quad t_2 = \tau_\psi, \quad t_3 = \tau_\phi \\
G_1 = \left( J_x - J_y \right) \dot{\psi} \dot{\phi} - J_\phi \dot{\psi} \right) / J_y \\
G_2 = \left( J_y - J_z \right) \dot{\phi} \dot{\psi} - J_\phi \dot{\phi} / J_x \\
G_3 = \left( J_z - J_y \right) \dot{\theta} \dot{\phi} / J_z
\]

Furthermore, the attitude dynamics in Eq. (4) can be written as follows:

\[
\dot{\Theta}_i = g_i t_i + G_i + f_i + d_i, \quad i = 1, \, 2, \, 3.
\]

Backstepping technology will be used during the design of the quadrotor’s attitude controller. Therefore, the virtual controller needs to be derived, and in the process of derivation, it will introduce extra signal noise to the controller, which will affect the control of the quadrotor. So the command filtering technique is used to solve this problem, and a command filtering [29] is defined as follows:

\[
\begin{align*}
\dot{\hat{\eta}}_1 &= \sigma T \dot{\eta}_2 \\
\dot{\hat{\eta}}_2 &= -2 \bar{\sigma}_T \sigma \eta_1 - \sigma \left( \eta_1 - a_1 \right)
\end{align*}
\]
where \( \eta_1(0) = \alpha_1(0), \eta_2(0) = 0, \alpha_1 > 0, \) \( \zeta_1 \in (0, 1), \) and \( \alpha_1 = -k_1 e_1 + \Theta \) represents virtual control, which is used to design the controller according to backstepping technology.

Define attitude tracking error as follows:

\[
e_{11} = \Theta_1 - \Theta_{d1},
\]
\[
e_{12} = \dot{\Theta}_1 - \alpha_{1,0},
\]

where \( \alpha_{1,0} = \eta_1 \) represents the output of the command filtering, and the input of the command filtering is virtual control \( \alpha_1 \).

\section*{Remark 2}

The use of the command filtering will bring a small filter error. Hence, an error compensation mechanism is introduced such that the tracking error can be further reduced.

An auxiliary system is given [30] as follows:

\[
\begin{align*}
\dot{\zeta}_{11} &= -k_1 \zeta_{11} + \zeta_{12} + (\alpha_{1,0} - \alpha_1) \\
\dot{\zeta}_{12} &= -k_2 \zeta_{12} - \zeta_{11},
\end{align*}
\]

where \( k_1 \) and \( k_2 \) represent the positive controller parameters to be designed, \( \zeta_{11} \) represents the error compensation signal, and \( ||\zeta_{11}|| \) is bounded [30].

Define the quadrotor attitude compensation tracking errors:

\[
\begin{align*}
v_{11} &= e_{11} - \zeta_{11}, \\
v_{12} &= e_{12} - \zeta_{12}.
\end{align*}
\]

The time derivative of Eqs. (13) and (14) are given as follows:

\[
\begin{align*}
\dot{v}_{11} &= v_{12} - k_1 v_{11}, \\
\dot{v}_{12} &= g_v \tau_1 + g_i + f_1 + d_1 - \dot{\alpha}_{1,0} + k_2 \zeta_{12} + \zeta_{11}.
\end{align*}
\]

By applying fuzzy logic system, Eq. (16) can be written:

\[
\begin{align*}
\dot{v}_{12} &= g_v \tau_1 + g_i + W_1^T S_i(X) + \sigma_i(X) \\
&\quad + d_1 - \dot{\alpha}_{1,0} + k_2 \zeta_{12} + \zeta_{11}.
\end{align*}
\]

Define the following compounded disturbance as:

\[D_i = \sigma_i(X) + d_i,\]

where the compounded disturbance \( D_i \) is unknown because the external disturbances and the fuzzy logic system errors are unknown. However, \( |d_i| \leq \delta_i \) can be obtained from Assumption 3 and Lemma 1. Additionally, based on the approximation theory [31] of fuzzy logic system, the unknown approximation error \( \sigma_i(X) \) satisfies \( |\sigma_i(X)| \leq \delta_i \). Therefore, the following inequality is established:

\[|\dot{D}_i| \leq \delta_i,\]

where \( \delta_i \) is an unknown positive constant.

In order to eliminate the compounded disturbance for the attitude of the quadrotor system, a nonlinear disturbance observer will be utilized [32].

Consider an auxiliary system as follows:

\[Z_i = D_i - l_1 \dot{\Theta}_1,\]

where \( l_1 \) is a positive design parameter. Furthermore, using Eqs. (8) and (18), the derivative of \( Z_i \) concerning time can be obtained:

\[\dot{Z}_i = \dot{D}_i - l_1 \dot{\dot{\Theta}}_1 = \dot{D}_i - l_i (g_v \tau_1 + g_i + W_1^T S_i(X) + D_i).\]

In order to estimate the compounded disturbance in the quadrotor system, the auxiliary variable \( Z_i \) should be estimated:
Proof of Theorem 1. Consider a Lyapunov candidate function as:

\[ V_{11} = \frac{1}{2} v_{11}^2, \]  

where \( i = 1, 2, 3, 4, 5, 6 \), and the derivative of \( V_{11} \) can be obtained from Eq. (15), as follows:

\[ \dot{V}_{11} = v_{11}(v_{12} - k_{11} v_{11}) \]
\[ = v_{11} v_{12} - k_{11} v_{11}^2. \]  

(33)

Choose another a Lyapunov candidate function as:

\[ V_{i2} = V_{12} + \frac{1}{2} v_{12}^2 + \frac{1}{2} \tilde{D}_i^2 + \frac{1}{2} \tilde{W}_i \Gamma_i^{-1} \tilde{W}_i. \]  

(34)

The derivative \( V_{i2} \) is given by:

\[ \dot{V}_{i2} = \dot{V}_{12} + v_{12} \tilde{D}_i \tilde{D}_i - \tilde{W}_i \Gamma_i^{-1} \tilde{W}_i. \]  

(35)

By applying Eqs. (17), (18), (25) and (33), one has:

\[ \dot{V}_{i2} = v_{11} v_{12} - k_{11} v_{11}^2 + v_{12} (\rho_i \Gamma_i + G_i + \tilde{W}_i \Gamma_i^{-1} S_i(X) + \tilde{D}_i \tilde{D}_i - \tilde{D}_i \Gamma_i^{-1} \tilde{W}_i) \]
\[ \leq \frac{1}{2} \rho_i^2 + \frac{1}{2} \tilde{D}_i^2 \]  

(36)

Then, substitute Eqs. (27) and (31) into Eq. (36) leads to:

\[ \dot{V}_{i2} = v_{11} v_{12} - k_{11} v_{11}^2 + v_{12} (-k_{12} v_{12} - v_{11} + \tilde{W}_i \Gamma_i^{-1} S_i(X) + \tilde{D}_i \tilde{D}_i) \]
\[ \leq \frac{1}{2} \rho_i^2 + \frac{1}{2} \tilde{D}_i^2 \]  

(37)

Using the Young inequality and Eq. (19), then the following inequalities exist:

\[ v_{12} \tilde{D}_i \leq \frac{1}{2} \rho_i^2 + \frac{1}{2} \tilde{D}_i^2, \]  

(38)

\[ \tilde{D}_i \Gamma_i \leq \frac{1}{2} \rho_i^2 + \frac{1}{2} \tilde{D}_i^2 \]  

(39)

\[ l_i \tilde{D}_i \tilde{W}_i \Gamma_i^{-1} \tilde{W}_i \leq \frac{1}{2} l_i \tilde{D}_i^2 + \frac{1}{2} l_i \Gamma_i^{-1} \tilde{W}_i^2. \]  

(40)

Using Eqs. (38), (39), and (40), Eq. (37) can be written, as follows:

\[ \dot{V}_{i2} \leq -k_{12} v_{12}^2 - k_{12} v_{12}^2 + v_{12} \tilde{W}_i \Gamma_i^{-1} S_i(X) + \frac{1}{2} \tilde{D}_i^2 \]  

\[ \leq \frac{1}{2} \rho_i^2 + \frac{1}{2} \tilde{D}_i^2 - l_i \tilde{D}_i \Gamma_i^{-1} \tilde{W}_i \]  

(41)

Substituting Eqs. (26) and (30) into Eq. (41) leads to:

\[ \dot{V}_{i2} \leq -k_{11} v_{11}^2 - k_{12} v_{12}^2 + v_{12} \tilde{W}_i \Gamma_i^{-1} S_i(X) - \left( \frac{1}{2} l_i - 1 \right) \tilde{D}_i^2 \]
\[ \leq \frac{1}{2} \rho_i^2 - \frac{1}{2} l_i \tilde{W}_i^2 \]  

(42)

Furthermore, the following inequality exists:

\[ \tilde{W}_i^2 \leq \frac{1}{2} \rho_i^2 + \frac{1}{2} \tilde{D}_i^2 \]  

(43)

Substituting Eq. (43) into Eq. (42) leads to:

\[ \dot{V}_{i2} \leq -k_{11} v_{11}^2 - k_{12} v_{12}^2 - \left( \frac{1}{2} l_i - 1 \right) \tilde{D}_i^2 \]  

\[ \leq \frac{1}{2} (\delta_i - l_i) \tilde{W}_i^2 + \frac{1}{2} \rho_i^2 \]  

(44)

\[ \leq -a_i V_{i2} + b_i, \]  

where \( a_i = \min(2k_{11}, 2k_{12}, l_i - 2, \delta_i - l_i) \) and \( b_i = \frac{1}{2} \rho_i^2 + \frac{1}{2} \delta_i \Gamma_i^{-1} \tilde{W}_i^2 \).

In order to ensure the stability of the quadrotor closed-loop system, the design parameters \( l_i \) and \( \delta_i \) should satisfy \( l_i > 2 \) and \( \delta_i > l_i \). In addition, all signals of the closed-loop system are bounded based on Lemma 2.

The proof is completed here.

Furthermore, the desired position reference signal \( x_d, y_d, \) and \( z_d, \) and desired yaw angle \( \psi_d \) are given according to the actual mission requirements. Since the quadrotor system is underactuated and strongly coupled, the position loop information is used to calculate the attitude loop information \( \theta_d \) and \( \psi_d, \) and at the same time calculate the total lift force, the specific expression is as follows [33]:

\[ \theta_d = \arctan \left( \frac{u_4 \cos \psi_d + u_5 \sin \psi_d}{u_6 + g} \right), \]  

(45)

\[ \psi_d = \arcsin \left( \frac{u_4 \sin \psi_d - u_5 \cos \psi_d}{\sqrt{u_4^2 + u_5^2 + (u_6 + g)^2}} \right), \]  

(46)

\[ F = m \sqrt{u_4^2 + u_5^2 + (u_6 + g)^2}. \]  

(47)

Remark 3. The control method proposed in this paper can theoretically compensate the uncertainty of the quadrotor system model and eliminate the compounded disturbance, but the disturbance observer designed in this paper is more suitable for continuous disturbance due to it taking a certain time to estimate the disturbance. Then, for the instantaneous disturbance, it is difficult to be estimated and eliminated. In addition, sensor noise can also have a certain negative impact on the flight performance of the quadrotor.

4. Experimental results

In order to verify the control method proposed in this paper, the QBall 2 experimental tested is used, and the entire experimental tested system is shown in Fig. 3. In addition to a quadrotor in this experimental tested, there is also a subsystem for measuring the attitude and position of the quadrotor indoors called OptiTrack, which consists of 24 cameras installed indoors. Other than that, the Quanser’s on-board avionics data acquisition card which is a high-resolution inertial measurement unit, and a wireless Gumstix DuoVero embedded computer are used to measure on-board sensors and drive the motors. QUARC, Quanser’s real-time control software is applied, which can be seamlessly connected with MATLAB Simulink. Therefore, we can build the controller in the Simulink with QUARC, and then download, compile, generate code and execute the controller on-board QBall 2 quadrotor [34].
In this experimental environment, in order to improve the safety of the experiment, a safety rope is used to prevent injury to the experimenter and damage to the experimental equipment. Additionally, an electric fan is used which can provide wind as an external disturbance, and the wind speed is about 4 m/s. Therefore, the corresponding external disturbance force and disturbance torque caused by wind will act on the QBall 2 quadrotor.

The initial conditions are: $||\hat{\mathbf{W}}_1|| = ||\hat{\mathbf{W}}_2|| = ||\hat{\mathbf{W}}_3|| = ||\hat{\mathbf{W}}_4|| = ||\hat{\mathbf{W}}_5|| = ||\hat{\mathbf{W}}_6|| = 0$. In the fuzzy logic system, the basis function vectors $S_1(X)$, $S_2(X)$, $S_3(X)$, $S_4(X)$, $S_5(X)$, and $S_6(X)$ use 5 nodes, and the center values $\mu_1$, $\mu_2$, $\mu_3$, $\mu_4$, $\mu_5$, and $\mu_6$ are evenly distributed between $[-1, 1]$, and the width values $\ell_1 = 1.2$, $\ell_2 = 1.2$, $\ell_3 = 1.2$, $\ell_4 = 1.5$, $\ell_5 = 1.5$, and $\ell_6 = 1.5$.

The parameters of the experimental equipment and the parameters of the controller are shown in Table 1 and Table 2, respectively.

**Remark 4.** In order to make QBall 2 quadrotor system stable, the parameters of controller should be selected in strict accordance with the requirements of the controller design. In addition, the parameters of the controller are also related to the uncertainty of the quadrotor system and external disturbances. Therefore, we further adjust the parameters of the controller based on the experimental results.

**Remark 5.** In Table 2, $i = 1, 2, 3$ represents the parameters used in the design of the attitude controller, and $i = 4, 5, 6$ indicates the parameters used in the design of the position control controller.

**Remark 6.** In the following experimental process, the unit of the position of QBall 2 quadrotor is meter, and the unit of the attitude angle of the quadrotor is radian. Additionally, the initial position and the initial yaw angle of QBall 2 quadrotor are slightly different due to the placement of the experimenter during each experiment.

### 4.1. Experiment: set-point tracking

In the first set of experiments, the control method proposed in this paper is first used to make the quadrotor fly to a fixed point for hovering. The coordinates of the hover point are $X_d = [0 0 1]^T$ and the desired yaw angle is $\psi_d = 0$. The initial position and initial attitude of the quadrotor are $X_0 = [0.029 0.015 0.300]^T$ and $\Theta_0 = [0 0 -0.035]^T$, respectively. Since the center height of the quadrotor is 0.3 m, the initial position of the takeoff height is 0.3 m. Second, in order to illustrate the advantages of the proposed method, the conventional PID controller is used for the quadrotor with initial position $X_0 = [0.024 0.005 0.300]^T$ and initial attitude $\Theta_0 = [0 0 -0.050]^T$, respectively.

Fig. 12 and Fig. 17 show the control inputs under the control method proposed in this paper and the conventional PID control method, respectively. Additionally, it can be seen from $x_{afc}$, $y_{afc}$, and $z_{afc}$ in Figs. 4–6 that the quadrotor can hover perfectly at a given reference position. Further, from Figs. 7–9, the tracking of the attitude angle of the quadrotor can be obtained, and Fig. 10 directly reflects the tracking error of the attitude angle. Furthermore, from the tracking error of the attitude angle, the perfect tracking effect of the attitude angle of the quadrotor can also be seen. Additionally, in this set of experiments, Fig. 11 shows the two norms of...
Fig. 7. Pitch angle tracking with the proposed controller.

Fig. 8. Roll angle tracking with the proposed controller.

Fig. 9. Yaw angle tracking with the proposed controller.

Fig. 10. Attitude angle tracking error with the proposed controller.

Fig. 11. Responses of $||\hat{W}_1||$, $||\hat{W}_2||$, $||\hat{W}_3||$, $||\hat{W}_4||$, $||\hat{W}_5||$, and $||\hat{W}_6||$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Fig. 12. Control input with the proposed controller.

Fig. 13. Pitch angle tracking with the PID controller.

Fig. 14. Roll angle tracking with the PID controller.
the weight vector under the control method proposed in this paper. However, when testing the quadrotor using the traditional PID control algorithm, we can obtain from the \( x_{\text{pid}} \), \( y_{\text{pid}} \), and \( z_{\text{pid}} \) in Figs. 4–6 that the quadrotor has a large fluctuation near the given reference point. Besides, Figs. 13–16 also show that the attitude tracking effect of the quadrotor is not ideal under the traditional PID control algorithm.

4.2. Experiment: trajectory tracking

Similar to the first set of experiments, the second set of experiments also used the control algorithm proposed in this article and the traditional PID control algorithm to test the quadrotor. However, the difference is that the quadrotor is required to track a trajectory. The expression of this trajectory is \( X_d = [x_d \ y_d \ 1]^T \), where \( x_d^2 + y_d^2 = 0.25 \), and the desired yaw angle is \( \psi_d = 0 \). The initial position and initial attitude of the quadrotor when using the control algorithm proposed in this paper are \( X_0 = [0.033 \ 0.010 \ 0.300]^T \) and \( \Theta_0 = [0 \ 0 \ -0.020]^T \), respectively, and the initial positions of the quadrotor when using the conventional PID control algorithm are \( X_0 = [0.022 \ 0 \ 0.300]^T \) and \( \Theta_0 = [0 \ 0 \ -0.005]^T \), respectively.

Fig. 24 and Fig. 29 show the control inputs under the control algorithm proposed in this paper and the traditional PID control algorithm. In Fig. 18, compared with the tracking effects of the two control algorithms, we can clearly see that the tracking performance under the control algorithm proposed in this paper is
Fig. 21. Yaw angle tracking with the proposed controller.

Fig. 22. Attitude angle tracking error with the proposed controller.

Fig. 23. Responses of $||\hat{W}_1||$, $||\hat{W}_2||$, $||\hat{W}_3||$, $||\hat{W}_4||$, $||\hat{W}_5||$, and $||\hat{W}_6||$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Fig. 24. Control input with the proposed controller.

Fig. 25. Pitch angle tracking with the PID controller.

Fig. 26. Roll angle tracking with the PID controller.

Fig. 27. Yaw angle tracking with the PID controller.

Fig. 28. Attitude angle tracking error with the PID controller.
better, while the quadrotor tracking effect using the PID control algorithm is not ideal. Further, the attitude angle tracking effect of the quadrotor under the two control algorithms can be seen from Figs. 19–21 and Figs. 25–27. Additionally, in this set of experiments, Fig. 23 indicates the two norms of the weight vector under the control method proposed in this paper. By analyzing Fig. 22 and Fig. 28, we can get the attitude angle tracking performance of the quadrotor under the control algorithm proposed in this paper is satisfactory. However, the PID control algorithm does not make the attitude of the quadrotor have good tracking performance, especially the yaw angle.

Through the analysis and discussion of the above two sets of experiments and comparative experiments, whether the fixed-point hover or the tracking of the trajectory. The tracking performance is relatively worse when the traditional PID control algorithm is used because the PID controller cannot alleviate the influence of the uncertainty of the quadrotor system model and external interference. However, the quadrotor has perfect flight performance and high-precision tracking performance by using the control method proposed in this paper when the system model of the quadrotor is uncertain and subject to external interference.

5. Conclusions

In the paper, an adaptive backstepping control scheme based on a fuzzy logic system, command filtering, and disturbance observer is proposed for a quadrotor UAV. In our method, the fuzzy logic system is used to compensate for uncertainty in the translational dynamics and the rotational dynamics of the quadrotor. Furthermore, we utilize a disturbance observer to approximate the external disturbances in the position loop and the attitude loop and error caused by using the fuzzy logic system. Finally, the proposed control strategy can guarantee the accuracy of the quadrotor full (including translational and rotational) control, and the tracking errors of the position loop and the attitude loop are stabilized. Our future work is to extend this method to the formation control of multiple UAVs. In addition, control problems for UAVs with actuator or sensor failure will also be considered and studied.

Declaration of competing interest

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

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