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Input-to-State Stabilizing Control Under Denial-of-Service

Claudio De Persis and Pietro Tesi

***Abstract*—The issue of cyber-security has become ever more prevalent in the analysis and design of networked systems. In this paper, we analyze networked control systems in the presence of denial-of-service (DoS) attacks, namely attacks that prevent transmissions over the network. We characterize frequency and duration of the DoS attacks under which input-to-state stability (ISS) of the closed-loop system can be preserved. To achieve ISS, a suitable scheduling of the transmission times is determined. It is shown that the considered framework is flexible enough so as to allow the designer to choose from several implementation options that can be used for trading-off performance versus communication resources. Examples are given to substantiate the analysis.**

***Index Terms*— Cyber-physical systems , networked control systems , switched systems.**

I. Introduction

RECENT years have witnessed a growing interest towards *cyber-physical systems* (CPSs), i.e., systems with a tight conjoining of computational and physical resources. Their field of application is immense, ranging from autonomous vehicles and supply chains to power and transportation networks. Many of these applications are safety-critical. This has triggered considerable attention to networked systems in the presence of attacks, bringing the question of *cyber-security* into filtering and control theories [1], [2].

As argued in [1] and [2], security in CPSs drastically differs from security in general-purpose computing systems. In CPSs, attacks can in fact cause disruptions that transcend the cyber realm and affect the physical world. For instance, if a critical process is open-loop unstable, failures in the plant-controller communication network can result in environmental damages. Control theory, on the other hand, is typically concerned with well-defined uncertainties or faults. In fact, most of networked control approaches assume that the communication failures follow a given class of probability distributions [3], [4], which is hardly justified in case of a malicious adversary.

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In a networked control system, attacks to the communication links can be classified as either *deception* attacks or *denial-of-service* (DoS) attacks. The former affect the trustworthiness of data by manipulating the packets transmitted over the network; see [5]–[9] and the references therein. DoS attacks are instead primarily intended to affect the timeliness of the information exchange, i.e., to cause packet losses; see for instance [10], [11] for an introduction to the topic.

This paper is concerned with DoS attacks We consider a sampled-data control system in which the plant-controller communication is networked; the attacker objective is to induce instability in the control system by denying communication on measurement (sensor-to-controller) and control (controller-to-actuator) channels. Under DoS attacks, the process evolves in open-loop according to the last transmitted control sample. The problem of interest is that of finding conditions under which closed-loop stability, in some suitably defined sense, can be preserved.

A basic question for this problem is concerned with the modeling of the DoS attacks. As previously noted, it is hard to justify the incentive for an attacker to follow a probabilistic packet drop model. In this paper, no assumption is made regarding the DoS attack underlying strategy. We consider a general attack model that only constrains the attacker action in time by posing limitations on the frequency of DoS attacks and their duration. This makes it possible to capture many different types of DoS attacks, including *trivial, periodic, random* and *protocol-aware jamming* attacks [11]–[14].

One contribution of this paper is an explicit characterization of the frequency and duration of DoS attacks under which closed-loop stability can be preserved. The result is intuitive as it relates stability with the ratio between the *on/off* periods of jamming. The analysis taken here is reminiscent of stability problems for switching systems [15], a modeling tool which has already proved effective in networked systems [16]–[18]. In this paper, however, the peculiarity of the problem under study leads to specific design solutions.

In this paper, in fact, the design of the transmission times turns out to be key. To get stability, the transmission times are selected in such a way that, whenever communication is possible, the closed-loop trajectories satisfy a suitable norm bound. Such a choice has two main advantages: i) it can ensure *global exponential input-to-state stability* (ISS) with respect to process disturbances even in the presence of DoS; and ii) it is flexible enough so as to allow the designer to choose from several implementation options that can be used for trading-off performance versus communication resources. The design of the network transmission times has interesting and perhaps

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surprising connections with the *event-based* sampling approach of [19], though substantial modifications are needed to account for the presence of DoS and disturbances. More specifically, the adoption of sampling rules that suitably constrain the closed-loop trajectories is crucial for achieving a simple Lyapunov-based analysis of the ISS property during the *on/off* periods of DoS.

In the control literature, contributions to this research topic have been reported in [20]–[25]. In [20] and [21], the authors consider the problem of finding optimal control policies when DoS attacks either evolve according to a Bernoulli process or follow a hidden Markov process model. As noted, such a problem is however more close to classical networked control systems literature. A scenario more similar to the present one is considered in [22] and [23], where the problem is to find optimal control and attack strategies assuming a maximum number of jamming actions over a prescribed (finite) control horizon. There are two main differences with respect to our framework: in [22] and [23], the authors consider a pure discrete-time setting, while here we deal with sampled-data networked systems and the performance analysis is concerned with the continuous-time process state. Second, we do not formulate the problem as an optimal control design problem. The controller can be designed according to any suitable design method, robustness against DoS attacks being achieved thanks to the design of the network transmission times.

Perhaps, the closest references to our work are [24] and [25]. In that papers, the authors consider DoS attacks in the form of pulse-width modulated signals. The goal is to *identify* salient features of the DoS signal such as maximum *on/off* cycle in order to suitably scheduling the transmission times. For the case of *periodic* jamming (of unknown period and duration), an identification algorithm is derived which makes it possible to de-synchronize the transmission times from the *on* periods of DoS. This framework should be therefore looked at as complementary more than alternative to the present one, since dealing with cases where the jamming signal is "well-structured" so that de-synchronization from attacks can be achieved. Such a feature is conceptually impossible to achieve in scenarios such as the one considered in this paper, where the jamming strategy is not prefixed (the attacker can modify on-line the attack strategy).

The remainder of this paper is organized as follows. In Section II, we describe the framework of interest and formulate the control problem. In Section III, we introduce a class of sampling logics that achieve ISS in the absence of DoS. The main results along with a characterization of the considered class of DoS signals are given in Section IV. Simplifications that arise in the disturbance-free case are also discussed. In Section V, we discuss implementation issues and present a number of sampling strategies that can be used for trading-off performance versus communication resources. A discussion of the results is given in Section VI along with a number of examples. Section VII ends the paper with concluding remarks.

Notation: We denote by $\mathbb{R}$ the set of reals. Given $\alpha \in \mathbb{R}$, we let $\mathbb{R}_{>\alpha}(\mathbb{R}_{\geq\alpha})$ denote the set of reals greater than (greater than or equal to) α. We let $\mathbb{N}$ denote the set of natural numbers and define $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. Given a vector $v \in \mathbb{R}^n$, $\|v\|$ is its Euclidean norm. Given a matrix M, $M^\top$ is its transpose and $\|M\|$ is its spectral norm. Given two sets A and B, we denote by $B\backslash A$ the relative complement of A in B, i.e., the set of all elements belonging to B, but not to A. Given a set A and a function $f : A \mapsto \mathbb{R}_{\geq 0}$, we use the convention $\sup_{x\in A} f(x) = 0$ when A is empty. Given a measurable time function $f : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$ and a time interval $[0, t)$ we denote the $\mathcal{L}_\infty$ norm of $f(\cdot)$ on $[0, t)$ by $\|f_t\|_\infty := \text{ess sup}_{s\in[0,t)} \|f(s)\|$. Finally, we denote by $\mathcal{L}_\infty(\mathbb{R}_{\geq 0})$ the set of measurable and essentially bounded time functions on $\mathbb{R}_{\geq 0}$.

TABLE I
TABLE OF NOTATIONS

t_k	$\triangleq$	k-th sampling time; it corresponds to the k-th time instant at which communication is attempted
H_n	$\triangleq$	n-th interval of Denial-of-Service (DoS)
τ_D	$\triangleq$	parameter whose inverse provides an upper bound on the average frequency of DoS *off/on* transitions, *i.e.*, average number of DoS *off/on* transitions per unit time
T	$\triangleq$	parameter whose inverse provides an upper bound on the average duration of DoS per unit time
$n(\tau, t)$	$\triangleq$	number of DoS *off/on* transitions occurring in the interval $[\tau, t[$
$\Xi(\tau, t)$	$\triangleq$	union of DoS intervals over $[\tau, t]$
$\Theta(\tau, t)$	$\triangleq$	relative complement of $\Xi(\tau, t)$ in $[\tau, t]$; hence, $\Theta(0, t)$ represents the set of time instants up to the time t where communication is possible
$\bar{\Delta}_\sigma$	$\triangleq$	upper bound on the inter-sampling rate, where σ is a design parameter
Δ_*	$\triangleq$	upper bound on the inter-sampling rate during DoS
$\underline{\Delta}$	$\triangleq$	lower bound on the inter-sampling rate
ω_1	$\triangleq$	parameter related to the convergence rate of the closed-loop trajectories in the absence of DoS
ω_2	$\triangleq$	parameter related to the divergence rate of the closed-loop trajectories in the presence of DoS

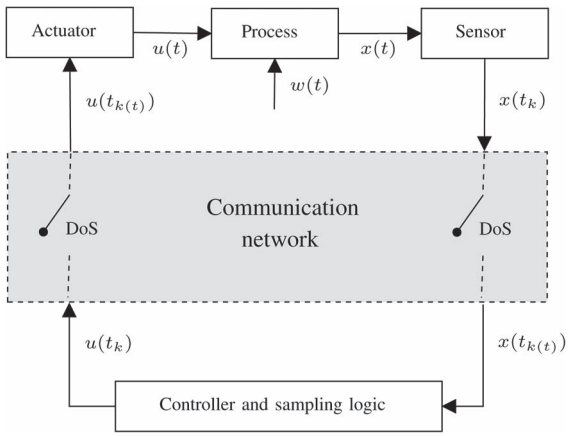

Fig. 1. Block diagram of the closed-loop system.

Table I summarizes the notation most frequently used throughout the remainder of the paper.

II. The Framework

A. Process Dynamics and Ideal Control Action

The framework of interest is schematically represented in Fig. 1. The process to be controlled is described by the differential equation

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t) + w(t) \tag{1}$$

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where $t \in \mathbb{R}_{\geq 0}$; $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the control input; A and B are matrices of appropriate size; $w \in \mathbb{R}^n$ is an unknown disturbance: it accounts for process input disturbances as well as noises on control (controller-to-actuator) and measurement (sensor-to-controller) channels.

The control action is implemented over a sensor/actuator network. We assume that (A, B) is stabilizable and that a state-feedback matrix K has been designed in such a way that all the eigenvalues of $A + BK$ have negative real part. The control signal is sampled using a *sample-and-hold* device. Let $\{t_k\}_{k\in\mathbb{N}_0}$, where $t_0 := 0$ by convention, represent the sequence of time instants at which it is desired to update the control action. Accordingly, whatever the logic generating the sequence $\{t_k\}_{k\in\mathbb{N}_0}$, in the ideal situation where data can be sent and received at any desired instant of time, the control input applied to the process is given by

$$u_{\text{ideal}}(t) = Kx(t_k) \tag{2}$$

for all $t \in \mathcal{I}_k := [t_k, t_{k+1}[$.

B. DoS and Actual Control Action

We refer to *Denial-of-Service* (DoS) as the phenomenon that may prevent (2) from being executed at each desired time. In principle, this phenomenon can affect measurement and control channels separately. In this paper, we consider the case of DoS simultaneously affecting both measurement and control channels. This amounts to assuming that, in the presence of DoS, data can be *neither sent nor received*. Specifically, let $\{h_n\}_{n\in\mathbb{N}_0}$, where $h_0 \geq 0$, denote the sequence of DoS *off/on* transitions, i.e., the time instants at which DoS exhibits a transition from zero (communication is possible) to one (communication is interrupted). Then

$$H_n := \{h_n\} \cup [h_n, h_n + \tau_n[\tag{3}$$

represents the nth DoS time-interval, of a length $\tau_n \in \mathbb{R}_{\geq 0}$, over which communication is not possible. If $\tau_n = 0$, the nth DoS takes the form of a single pulse at time h_n.

In the presence of DoS, the actuator generates an input that is based on the *most recently received* control signal. Given $\tau,\ t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$, let

$$\Xi(\tau, t) := \bigcup_{n\in\mathbb{N}_0} H_n \bigcap [\tau, t] \tag{4}$$

$$\Theta(\tau, t) := [\tau, t] \setminus \Xi(\tau, t). \tag{5}$$

In words, for each interval $[\tau, t]$, $\Xi(\tau, t)$ and $\Theta(\tau, t)$ represent the sets of time instants where communication is denied and allowed, respectively. The reason for considering generic intervals $[\tau, t]$ rather than simply $[0, t]$ will become clear in Section IV. Accordingly, for each $t \in \mathbb{R}_{\geq 0}$, the control input applied to the process can be expressed as

$$u(t) = Kx\left(t_{k(t)}\right) \tag{6}$$

where

$$k(t) := \begin{cases} -1, & \text{if } \Theta(0, t) = \emptyset \\ \sup\{k \in \mathbb{N}_0 \mid t_k \in \Theta(0, t)\}, & \text{otherwise.} \end{cases} \tag{7}$$

In words, for each $t \in \mathbb{R}_{\geq 0}$, $k(t)$ represents the last successful control update. Notice that $h_0 = 0$ implies $k(0) = -1$, which raises the question of assigning a value to the control input when communication is not possible at the process start-up. In this respect, we assume that when $h_0 = 0$ then $u(0) = 0$, and let $x(t_{-1}) := 0$ for notational consistency.

C. Control Objectives

The problem of interest is that of finding sampling logics that achieve robustness against DoS, while ensuring that the control inter-execution times are bounded away from zero. While robustness is concerned with stability and performance of the closed-loop system, positive inter-execution times are required for the control scheme to be physically implementable over a network.

The following definitions reflect the stated goals.

Definition 1: (cf. [26]) Let Σ be the control system resulting from (1) under a control signal as in (6). System Σ is said to be *input-to-state stable* (ISS) if there exist a $\mathcal{KL}$-function β and a $\mathcal{K}_\infty$-function γ such that, for each $w \in \mathcal{L}_\infty(\mathbb{R}_{\geq 0})$ and $x(0) \in \mathbb{R}^n$

$$\|x(t)\| \leq \beta\left(\|x(0)\|, t\right) + \gamma\left(\|w_t\|_\infty\right) \tag{8}$$

for all $t \in \mathbb{R}_{\geq 0}$. If (8) holds when $w \equiv 0$, then Σ is said to be *globally asymptotically stable* (GAS). ■

Definition 2: A control update sequence $\{t_k\}_{k\in\mathbb{N}_0}$ is said to have the *finite sampling rate* property if there exists $\underline{\Delta} \in \mathbb{R}_{>0}$ such that

$$\Delta_k := t_{k+1} - t_k \geq \underline{\Delta} \tag{9}$$

for all $k \in \mathbb{N}_0$. ■

Here and in the sequel, it is understood that the network can send information at the sample rate induced by $\underline{\Delta}$.

III. Stabilizing Control Update Policies

We first introduce a class of control update policies ensuring ISS in the absence of DoS. The results will serve as a basis for the developments of Section IV.

Consider the closed-loop system resulting from (1) under a control signal as in (6). As a first step, we rewrite it in a form that is better suited for analysis purposes. Let

$$e(t) := x\left(t_{k(t)}\right) - x(t) \tag{10}$$

where $t \in \mathbb{R}_{\geq 0}$, represent the error between the value of the process state at the last successful control update and the value of the process state at the current time. The closed-loop system can be therefore rewritten as

$$\frac{d}{dt}x(t) = \Phi x(t) + BKe(t) + w(t) \tag{11}$$

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where $\Phi := A + BK$.

The closed-loop system now depends on the control update rule through e, which enters the dynamics as an additional disturbance term. It is then intuitively clear that stability will not be destroyed if one adopts control update rules that keep e small in a suitable sense. The notion of "smallness" here considered, which characterizes the control update rules of interest, is expressed in terms of the following boundedness inequality:

$$\|e(t)\| \leq \sigma \|x(t)\| + \sigma \|w_t\|_\infty \tag{12}$$

where $\sigma \in \mathbb{R}_{>0}$ is a suitable design parameter. We anticipate that (12) is not the control update rule we are going to implement, because of its dependence on the supremum norm of the disturbance w, that in general unknown. We will rather adopt different update rules that guarantee that (12) is always satisfied. Such different update rules, motivated by Lemma 1 below, are discussed in detail in Section V.

As next result shows, provided that σ is suitably chosen, any control update rule that restricts e to satisfy (12) is stabilizing. This can be proved by resorting to standard Lyapunov arguments. Given any positive definite matrix $Q = Q^\top \in \mathbb{R}^{n\times n}$, let P be the unique solution of the Lyapunov equation

$$\Phi^\top P + P\Phi + Q = 0. \tag{13}$$

Then, by taking $V(x) = x^\top P x$ as a Lyapunov function, and computing it along the solution of (11), it is simple to verify that

$$\alpha_1 \|x(t)\|^2 \leq V(x(t)) \leq \alpha_2 \|x(t)\|^2 \tag{14a}$$

$$\frac{d}{dt} V(x(t)) \leq -\gamma_1 \|x(t)\|^2 + \gamma_2 \|x(t)\| \|e(t)\| + \gamma_3 \|x(t)\| \|w(t)\| \tag{14b}$$

hold for all $t \in \mathbb{R}_{\geq 0}$, with α_1 and α_2 equal to the smallest and largest eigenvalue of P, respectively, γ_1 equal to the smallest eigenvalue of Q, $\gamma_2 := \|2PBK\|$ and $\gamma_3 := \|2P\|$. It is then immediate to see that, under (12), the second of (14) does always satisfy a dissipation-like inequality whenever σ is chosen small enough.

Theorem 1: Consider the control system Σ composed of (1) and control input (6), where K is such that all the eigenvalues of $\Phi = A + BK$ have negative real part. Given any positive symmetric definite matrix $Q \in \mathbb{R}^{n\times n}$, let P be the unique solution of the Lyapunov equation $\Phi^\top P + P\Phi + Q = 0$. Let $V(x) = x^\top P x$. Consider any control update sequence occurring at a finite sampling rate and satisfying (12) for all $t \in \mathbb{R}_{\geq 0}$, with σ such that

$$\gamma_1 - \sigma\gamma_2 > 0 \tag{15}$$

where γ_1 and γ_2 are as in (14b). Then, Σ is ISS.

Proof: See the Appendix. ■

Inequality (15) can always be satisfied by selecting σ sufficiently small in that $\gamma_1 > 0$. Given any σ satisfying (15), the only question that arises is on the possibility of designing sampling logics that guarantee (12) with finite-sampling rate. As next result shows, in the absence of DoS, this is always possible. Given a matrix $M \in \mathbb{R}^{n\times n}$, let

$$\mu_M := \max\left\{\lambda |\ \lambda \in \text{spectrum}\left\{\frac{(M + M^\top)}{2}\right\}\right\} \tag{16}$$

denote the logarithmic norm of M [27].

Lemma 1: Consider the same notation as in Theorem 1. Then, in the absence of DoS, any control update rule with inter-sampling times smaller than or equal to

$$\bar{\Delta}_\sigma := \left(\frac{\sigma}{1+\sigma}\right) \frac{1}{\max\{\|\Phi\|, 1\}} \tag{17}$$

when $\mu_A \leq 0$, and

$$\bar{\Delta}_\sigma := \frac{1}{\mu_A} \log\left[\left(\frac{\sigma}{1+\sigma}\right) \frac{1}{\max\{\|\Phi\|, 1\}} \mu_A + 1\right] \tag{18}$$

when $\mu_A > 0$, satisfies (12) for all $t \in \mathbb{R}_{\geq 0}$.

Proof: See the Appendix. ■

Remark 1: A boundedness inequality similar to (12) was considered by [19] in the context of event-based control for disturbance-free processes. The difference here lies in the bound imposed on e, which is dictated by the need to take into account w. With Lemma 1, we explicitly determine inter-sampling times that ensure (12) without prior knowledge of an upper-bound on w. A detailed discussion on sampling logic based on Lemma 1 is given in Section V. ■

IV. Input-to-State Stability Under Denial-of-Service

The analysis of Section III relies on the possibility to satisfy condition (12) *for all* $t \in \mathbb{R}_{\geq 0}$. According to Lemma 1, in the absence of DoS, this is always possible. In the presence of DoS the analysis becomes more involved since certain control update attempts need not be successful, no matter how we sample. Failing to reset e may cause (12) to be violated and stability can be lost in that (14b) need no longer satisfy a dissipation-like inequality. Thus, a natural question arises on how the conclusions of Theorem 1 can be extended so as to account for the presence of DoS. The remainder of this section is devoted to address this question. In Section IV-A, we introduce and discuss the class of DoS signals under consideration. The main result is given in Section IV-B. An interesting simplification arising in the disturbance-free case is discussed in Section IV-C.

A. Assumptions—Time-Constrained Denial-of-Service

The first question to be addressed is that of determining the amount of DoS that a system can tolerate before undergoing instability. In this respect, it is simple to see that such an amount is not arbitrary, and that suitable conditions must be imposed on both DoS *frequency* and *duration*. To maintain continuity, a detailed discussion on the assumptions to follow is postponed to Section VI.

1) DoS Frequency: Consider first the *frequency* at which DoS can occur, and let $\Lambda_n := h_{n+1} - h_n$, $n \in \mathbb{N}_0$, denote the time elapsing between any two successive DoS triggering. One immediately sees that if $\Lambda_n \leq \underline{\Delta}$ for all $n \in \mathbb{N}_0$ (DoS can

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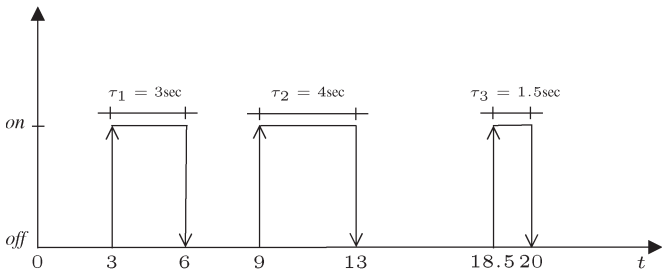

Fig. 2. Example of DoS signal. *Off/on* transitions are represented as $\uparrow$, while *on/off* transitions are represented as $\downarrow$. *Off/on* transitions occur at 3 s, 9 s, and 18.5 s and the corresponding DoS intervals have duration 3 s, 4 s, and 1.5 s, respectively. This yields for instance: $n(0,1) = 0$, $n(1,10) = 2$ and $n(10,20) = 1$, while $\Xi(0,1) = \emptyset$, $\Xi(1,10) = [3,6[\cup [9,10[$, and $\Xi(10,20) = [10,13[\cup [18.5,20[$.

occur at the same rate as the minimum possible sampling rate $\underline{\Delta}$) then stability can be lost regardless of the adopted control update policy. It is intuitively clear that, in order to get stability, the frequency at which DoS can occur must be sufficiently small compared to the minimum sampling rate. As discussed in Section VI, a natural way to express this requirement is via the concept of *average dwell-time*, as introduced by [28]. Given $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$, let $n(\tau, t)$ denote the number of DoS *off/on* transitions occurring on the interval $[\tau, t[$.

Assumption 1—(DoS Frequency): There exist $\eta \in \mathbb{R}_{\geq 0}$ and $\tau_D \in \mathbb{R}_{> \underline{\Delta}}$ such that

$$n(\tau, t) \leq \eta + \frac{t - \tau}{\tau_D} \tag{19}$$

for all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. ■

2) DoS Duration: In addition to the DoS frequency, one also need to constrain the DoS *duration*, namely the length of the intervals over which communication is interrupted. To see this, consider for example a DoS sequence consisting of the singleton $\{h_0\}$. Assumption 1 is clearly satisfied with $\eta \geq 1$. However, if $H_0 = \mathbb{R}_{\geq 0}$ (communication is never possible) then stability is lost regardless of the adopted control update policy. Recalling the definition of $\Xi(\tau, t)$ in (4), the assumption that follows provides a quite natural counterpart of Assumption 1 with respect to the DoS duration.

Assumption 2—(DoS Duration): There exist $\kappa \in \mathbb{R}_{\geq 0}$ and $T \in \mathbb{R}_{>1}$ such that

$$|\Xi(\tau, t)| \leq \kappa + \frac{t - \tau}{T} \tag{20}$$

for all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. ■

Fig. 2 exemplifies values of $n(\tau, t)$ and $\Xi(\tau, t)$ for a given DoS pattern.

Remark 2: Assumption 1 and 2 specify the class of DoS signals that will be considered throughout the remainder of this paper. It is worth noting that no assumption is made on the information available to the attacker about the process dynamics, state-feedback matrix and sampling logic. The assumptions do only constrain DoS in terms of its frequency and duration. Limiting the DoS frequency and duration, in addition to render the control problem meaningful, does also have a practical motivation. In fact, there are several provisions that can be taken in order to *mitigate* DoS attacks, including spreading techniques and high-pass filtering; e.g., see [10], [13], [14]. These provisions decrease the chance that a DoS attack will be successful, and, as such, limit in practice the frequency and duration of the time intervals over which communication is effectively denied. ■

B. ISS Under Denial-of-Service

We are now in position to derive the main result of this section, which can be expressed in words as follows: *any control update rule attaining the conditions of Lemma 1 preserves ISS for any DoS signal that satisfies Assumption 1 and 2 with τ_D and T sufficiently large*. Although the proof of this result is rather involved, the underlying approach is very intuitive. We decompose the time axis into intervals where it possible to satisfy (12) and intervals where, due to the occurrence of DoS, (12) need not hold. We then analyze the closed-loop dynamics as a system switching between stable and unstable modes, and determine values of τ_D and T under which the stable behavior is predominant with respect to the unstable one.

Consider a sequence $\{t_k\}_{k \in \mathbb{N}_0}$ of sampling times, along with a DoS sequence $\{h_n\}_{n \in \mathbb{N}_0}$. Let

$$\mathscr{S} := \left\{ k \in \mathbb{N}_0 \mid t_k \in \bigcup_{n \in \mathbb{N}_0} H_n \right\} \tag{21}$$

denote the set of integers related to a control update attempt occurring under DoS.

The following result holds.

Theorem 2: Consider the control system Σ composed of (1) and control input (6), where K is such that all the eigenvalues of $\Phi = A + BK$ have negative real part. Given any positive symmetric definite matrix $Q \in \mathbb{R}^{n \times n}$, let P be the unique solution of the Lyapunov equation $\Phi^\top P + P\Phi + Q = 0$. Let $V(x) = x^\top P x$. Consider any control update sequence occurring at a finite sampling rate and with inter-sampling times smaller than or equal to $\bar{\Delta}_\sigma$ as in Lemma 1, with σ satisfying (15). Consider any DoS sequence satisfying Assumption 1 and 2 with arbitrary η and κ, and with τ_D and T such that

$$\frac{\Delta_*}{\tau_D} + \frac{1}{T} < \frac{\omega_1}{\omega_1 + \omega_2} \tag{22}$$

where Δ_* is a nonnegative constant satisfying

$$\sup_{k \in \mathscr{S}} \Delta_k \leq \Delta_* \tag{23}$$

Δ_k is as in (9), $\omega_1 := (\gamma_1 - \gamma_2 \sigma)/2\alpha_2$ and $\omega_2 := 2\gamma_2/\alpha_1$, where $\alpha_1, \alpha_2, \gamma_1$ and γ_2 are as in (14). Then, Σ is ISS.

Proof: See the Appendix. ■

In connection with (23), we note that, in accordance with the adopted notation, $\sup_{k \in \mathscr{S}} \Delta_k = 0$ when $\mathscr{S} = \emptyset$.

Remark 3—(Performance Bounds): The considered class of sampling logics provides quite strong stability properties, namely *exponential* ISS with *linear* bounds on the disturbance-to-state map. In particular, from the proof of Theorem 2 one has

$$\|x(t)\| \leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{\frac{\kappa_*(\omega_1+\omega_2)}{2}} e^{-(\frac{\beta_*}{2})t} \|x(0)\| + \sqrt{\delta_*} \left[1 + 2e^{\kappa_*(\omega_1+\omega_2)} \frac{e^{\beta_* \tau_D \eta}}{1 - e^{-\beta_* \tau_D}} \right]^{\frac{1}{2}} \|w_t\|_\infty. \tag{24}$$

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where

$$\kappa_* := \kappa + (1+\eta)\Delta_* \tag{25}$$

$$\beta_* := \omega_1 - (\omega_1+\omega_2)\left(\frac{\Delta_*}{\tau_D}+\frac{1}{T}\right) \tag{26}$$

$$\delta_* := \frac{(\gamma_3+\gamma_2\sigma)^2}{(\gamma_1-\gamma_2\sigma)}\max\left\{\frac{\alpha_2}{\alpha_1(\gamma_1-\gamma_2\sigma)},\frac{1}{4\gamma_2}\right\} \tag{27}$$

with $\alpha_1,\alpha_2,\gamma_1,\gamma_2$ and γ_3 as in (14). One sees that the convergence rate of the closed-loop dynamics as well as the amount of DoS that one can tolerate can be increased by decreasing the sampling rate upon the DoS occurrence, i.e., by decreasing Δ_*. We will discuss a number of sampling logics exploiting this property in Section V. ■

Remark 4—(DoS-Induced Actuation Delay): Decreasing Δ_* can also by viewed as reducing the DoS-induced actuation delay. Assume for instance that the kth sampling instant belongs to $\mathscr{S}$. Since $t_k \in \mathscr{S}$, the process will evolve in open-loop under out-of-date control up to time t_{k+1}, even if DoS ceases before t_{k+1}. Thus decreasing Δ_* has the effect of reducing the delay in the actuation of the new control sample. In general, due to the finite sampling rate, such a delay is *non-zero*. In fact, $\Delta_*=0$ only in two cases. The first one is when $\mathscr{S}=\emptyset$. Such a case is not interesting since if $\mathscr{S}=\emptyset$, then DoS is ineffective and ISS follows directly from Theorem 1. The second case is when $\mathscr{S}\neq\emptyset$ and, for each DoS interval H_n containing a t_k, the control update is scheduled exactly at $h_n+\tau_n$. This case is, however, unrealistic since h_n and τ_n are unknown. The case $\mathscr{S}\neq\emptyset$ and $\Delta_*=0$ becomes of interest when considering control under DoS in a *purely continuous-time* framework. In fact, if the control action is continuous then $\Delta_*=\bar{\Delta}_\sigma=0$. Under such circumstances, (22) reduces to $1/T<\omega_1/(\omega_1+\omega_2)$, which is independent of τ_D. In fact, in a continuous-time setting, slow-on-the-average DoS in the form of pulses has no effect on closed-loop stability. ■

C. Disturbance-Free Case

An interesting simplification arises in the disturbance-free case. Specifically, when $w\equiv 0$ then Assumptions 1 and 2 can be relaxed as follows (to maintain continuity, a discussion on these relaxations is postponed to Section VI).

Assumption 3—(DoS Frequency—Disturbance-Free Case): There exist $\eta\in\mathbb{R}_{\geq 0}$ and $\tau_D\in\mathbb{R}_{>\underline{\Delta}}$ such that

$$n(0,t)\leq \eta+\frac{t}{\tau_D} \tag{28}$$

for all $t\in\mathbb{R}_{\geq 0}$. ■

Assumption 4—(DoS Duration—Disturbance-Free Case): There exist $\kappa\in\mathbb{R}_{\geq 0}$ and $T\in\mathbb{R}_{>1}$ such that

$$|\Xi(0,t)|\leq \kappa+\frac{t}{T} \tag{29}$$

for all $t\in\mathbb{R}_{\geq 0}$. ■

The following result holds.

Corollary 1: Consider the control system Σ composed of (1) and control input (6), where K is such that all the eigenvalues of $\Phi=A+BK$ have negative real part. Given any positive symmetric definite matrix $Q\in\mathbb{R}^{n\times n}$, let P be the unique solution of the Lyapunov equation $\Phi^\top P+P\Phi+Q=0$. Let $V(x)=x^\top Px$. Consider any control update sequence occurring at a finite sampling rate and with inter-sampling times smaller than or equal to $\bar{\Delta}_\sigma$ as in Lemma 1, with σ as in (15). Consider any DoS sequence satisfying Assumption 3 and 4 with arbitrary η and κ, and with τ_D and T satisfying (22). Then, Σ is GAS.

Proof: See the Appendix. ■

Following the same lines as in Remark 3, it is easy to see that the convergence rate of the closed-loop trajectories is exponential.

V. Resilient Control Logics

The considered framework is flexible enough so as to allow the designer to choose from several implementation options that can be used to tradeoff performance versus communication resources. Although the solutions described in the sequel originate from fundamentally different logics, they exhibit the common feature of *resilience*, by which we mean the ability to adapt the sampling rate to the occurrence of DoS and, sometimes, to the closed-loop behavior. Hereafter, only few sampling logics will be discussed. Combinations and variants thereof can be easily envisaged.

A. Periodic Sampling Logics

The simplest architecture one can think of for implementing the control action is through *periodic* sampling. In accordance with Theorem 2, the sampling rate affects the amount of DoS that one can tolerate via (22). Since (22) does only depend on the sampling rate which is adopted during DoS, a convenient strategy consists in making use of a two-mode switching rule, where, upon DoS, the sampling rate is increased so as to reduce the DoS-induced delay.

Proposition 1: Let δ_1 and δ_2 be positive constants such that $\delta_1\leq\delta_2\leq\bar{\Delta}_\sigma$, where $\bar{\Delta}_\sigma$ is as in Lemma 1. For each $k\in\mathbb{N}_0$, let the sampling times be given by

$$t_{k+1}=\begin{cases} t_k+\delta_1, & \text{if } k\in\mathscr{S}\\ t_k+\delta_2, & \text{otherwise}\end{cases} \tag{30}$$

where $\mathscr{S}$ is as in (21). If the conditions of Theorem 2 hold true with $\Delta_*=\delta_1$, then the control system Σ composed of (1) and (6) is ISS. Moreover, the inter-sampling times are bounded from below by $\underline{\Delta}=\delta_1$.

Proof: By (30), the control update occurs at finite time with inter-sampling time not larger than $\delta_2\leq\bar{\Delta}_\sigma$. Hence, ISS descends immediately from Theorem 2. The bounds on Δ_* and $\underline{\Delta}$ hold by construction. ■

B. Event-Based/Periodic Sampling Logics

As discussed in [19], [29], and [30], event-based sampling is a very effective solution for saving communication resources. The basic idea is that periodic sampling can be relaxed by

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triggering the control updates only when certain conditions (events) are met. In its basic formulation, this is achieved by measuring continuously the state x and triggering a control update whenever

$$\|x(t_k) - x(t)\| = \sigma \|x(t)\| \tag{31}$$

where σ is as in (15). In the disturbance-free case, this allows for less frequent sampling, while preserving the existence of a lower bound $\underline{\Delta}$ on the inter-execution times, which can be computed as the time satisfying $\phi(\underline{\Delta}) = \sigma$, where $\phi(t)$ is the unique solution of the generalized scalar Riccati equation

$$\frac{d}{dt}\phi(t) = \|\Phi\| + (\|\Phi\| + \|BK\|)\,\phi(t) + \|BK\|\phi^2(t) \tag{32}$$

with initialization $\phi(0) = 0$. A similar reasoning, though with some noticeable differences, can be adopted also in the present context. Specifically, a suitable strategy still consists of a two-mode switching rule: in the absence of DoS, sampling is based on (31); upon DoS occurrence, the sampling rate is increased so as to reduce the DoS-induced delay. Notice that in this case there is no need to enforce the upper-bound $\bar{\Delta}_\sigma$ on the sampling rate since (31) automatically implies (12). However, since (31) in general guarantees nonzero inter-execution times only in the absence of disturbances, a lower-bound on the sampling rate must be imposed *a priori*.

Let

$$\rho_k := \inf\{t \in \mathbb{R}_{>t_k} : \|x(t_k) - x(t)\| = \sigma \|x(t)\|\} \tag{33}$$

for all $k \in \mathbb{N}_0$. We have at once the following result:

Proposition 2: Let δ_1 and δ_2 be positive constants such that $\delta_1 \leq \delta_2 \leq \bar{\Delta}_\sigma$, where $\bar{\Delta}_\sigma$ is as in Lemma 1. For each $k \in \mathbb{N}_0$, let the sampling times be given by

$$t_{k+1} = \begin{cases} t_k + \delta_1, & \text{if } k \in \mathscr{S} \\ t_k + \delta_2, & \text{if } k \notin \mathscr{S} \vee \rho_k < t_k + \delta_2 \\ \rho_k, & \text{otherwise} \end{cases} \tag{34}$$

where $\mathscr{S}$ is as in (21). If the conditions of Theorem 2 hold true with $\Delta_* = \delta_1$, then the control system Σ composed of (1) and (6) is ISS. Moreover, the inter-sampling times are bounded from below by $\underline{\Delta} = \delta_1$.

Proof: The inter-sampling times associated with (34) are equal to δ_1, δ_2, or $\rho_k - t_k$. By (34), the sampling time t_{k+1} equals ρ_k only if $k \notin \mathscr{S}$. Under such circumstances, since t_k does not belong to any DoS interval, $e(t) = x(t_k) - x(t)$ for all $t \in [t_k, t_{k+1}[$, with $e(t_k) = 0$. Hence, if $t_{k+1} = \rho_k$ we have from (33) that $\|e(t)\| \leq \sigma\|x(t)\|$ for all $t \in [t_k, t_{k+1}[$ since e in continuous on $[t_k, t_{k+1}[$ and $e(t_k) = 0$. Thus (34) guarantees that either the inter-sampling times equal δ_1 or δ_2, in which case they are not larger than $\bar{\Delta}_\sigma$, or they are equal to ρ_k in which case (12) is satisfied. In view of this fact, a minor variation of Theorem 2 shows the thesis. The bounds on Δ_* and $\underline{\Delta}$ hold by construction. ■

Interestingly, a logic similar to (34) has been recently considered in [31] for achieving finite sampling rate in the context of nonlinear output feedback for event-based controllers.

C. Self-Triggering Sampling Logics

Event-based sampling has the positive feature of potentially saving communication resources but requires the continuous monitoring of the process state, and, hence, hardware specifically available for this purpose. An alternative way to relax periodic implementations consists in selecting Δ_k based on the value of the process state at the measurement instants. Logics of this kind are typically referred to as "self-triggering" in that the next update instant is computed directly by the control unit; e.g., see [30], [32].

Given $t, \tau \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$, let

$$\chi(t, \tau) := \left[e^{A(t-\tau)} + \int_\tau^t e^{A(t-s)} BK\, ds \right] x(\tau). \tag{35}$$

Thus, $\chi(t_k, t_{k(t_k)})$ provides a prediction of $x(t_k)$ based on the last successful measurement of the process state. Notice that since the disturbance is not available for measurements, this prediction cannot take into account the influence of the disturbance and is an approximate estimate of the actual state. Define

$$z(t_k) = \begin{cases} \left\|\chi\left(t_k, t_{k(t_k)}\right)\right\|, & \text{if } k \in \mathscr{S} \\ \|x(t_k)\|, & \text{otherwise.} \end{cases} \tag{36}$$

The idea is then to schedule the next control update based on $z(t_k)$, where $\chi(t_k, t_{k(t_k)})$ replaces $x(t_k)$ when the latter is not available: the larger $z(t_k)$ the smaller Δ_k and vice versa, which corresponds to increasing the sampling rate as the distance of the process state from the origin gets larger.[1]

Proposition 3: Let δ_1 and δ_2 be positive constants such that $\delta_1 \leq \delta_2 \leq \bar{\Delta}_\sigma$, where $\bar{\Delta}_\sigma$ is as in Lemma 1. For each $k \in \mathbb{N}_0$, let the sampling times be given by

$$t_{k+1} = t_k + \delta_2 - (\delta_2 - \delta_1)\varphi\left(z(t_k)\right) \tag{37}$$

where $\varphi : \mathbb{R}_{\geq 0} \mapsto [0, 1]$ is an arbitrary class $\mathcal{K}$ function. If the conditions of Theorem 2 hold true with $\Delta_* = \delta_2$, then the control system Σ composed of (1) and (6) is ISS. Moreover, the inter-sampling times are bounded from below by $\underline{\Delta} = \delta_1$.

Proof: By definition, φ takes value in the interval $[0, 1]$ and therefore the inter-sampling time induced by the control update rule (37) ranges in the interval $[\delta_1, \delta_2]$. This implies that the bounds on Δ_* and $\underline{\Delta}$ hold true. Thus ISS descends directly from Theorem 2. ■

Compared with (30), (37) achieves larger bounds on Δ_*, hence larger DoS-induced actuation delays. However, it potentially decreases the number of control updates since, upon DoS, the sampling rate is not set to δ_1 but is chosen depending on the inferred process state magnitude.

[1]We have implicitly assumed that the sampling logic is located at the controller side. In case the sampling logic is located at the process side, then (37) simplifies. In fact, under such circumstances, x can always be assumed to be available for measurements so that $z(t_k)$ reduces to $x(t_k)$.

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VI. Discussion and Examples

In this section, we discuss in more details the considered assumptions and provide examples in order to substantiate the analysis.

A. Slow-on-the-Average DoS: Discussion and Examples

As noted in Section IV-A, both Assumptions 1 and 2 pose constraints on DoS that are reminiscent of *average dwell-time* conditions [28]. In the present context, the rationale behind Assumption 1 is that occasionally DoS can occur at the same rate as (even faster than) $\underline{\Delta}$, i.e., $\Lambda_n = h_{n+1} - h_n \leq \underline{\Delta}$ for some $n \in \mathbb{N}_0$, but the average interval between consecutive DoS triggering is greater than $\underline{\Delta}$. By (19), one may in fact have intervals where $\Lambda_n \leq \underline{\Delta}$; hence, intervals where $n(\tau,t)$ is greater than or equal to the maximum number $\lceil (t-\tau)/\underline{\Delta} \rceil$ of control updates that can be executed over $[\tau, t[$. However, over large time windows, i.e., when the term $(t-\tau)/\tau_D$ is predominant compared to η, the number of DoS triggering is at most of the order of $(t-\tau)/\tau_D$, hence smaller than $\lceil (t-\tau)/\underline{\Delta} \rceil$.

Assumption 2 expresses a similar requirement with respect to the DoS duration. In fact, it expresses the property that, on the average, the time instants over which communication is interrupted do not exceed a certain *fraction* of time, as specified by the constant $T \in \mathbb{R}_{>1}$. Similarly to η, the constant $\kappa \in \mathbb{R}_{\geq 0}$ plays the role of a regularization term. It is needed because during a DoS interval, one has $|\Xi(h_n, h_n + \tau_n)| = \tau_n > \tau_n / T$ since $T > 1$. Hence, κ serves to make (20) consistent.

The considered assumptions are general enough to capture several different situations, as exemplified hereafter. More complex scenarios can be easily envisaged.

Example 1: In analogy with [23], consider the situation where, on every interval which contains N communication attempts, a number $M < N$ of these attempts can be denied. A simple way to account for this situation is to regard the DoS as a train of pulses, M of which are superimposed to the sampling times. This implies that Assumption 2 holds true with $\kappa = 0$ and $T = \infty$. As for Assumption 1, assume a lower bound $\underline{\Delta}$ on the control executions. One sees that

$$n(\tau,t) \leq \left\lceil \frac{t-\tau}{\underline{\Delta} N} \right\rceil M \tag{38}$$

for all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. Then, for each N, (19) holds true with $\eta = M$ and $\tau_D = \underline{\Delta} N / M$. In connection with Theorem 2, this means that stability is preserved whenever $(\underline{\Delta} N)/(\Delta_* M) > (\omega_1 + \omega_2)/\omega_1$. Using logics that sample at rate $\underline{\Delta}$ upon DoS, then stability is preserved whenever $N/M > (\omega_1 + \omega_2)/\omega_1$. For instance, this means that if $(\omega_1 + \omega_2)/\omega_1 = 5$, then up to the 20% of the communication attempts can be denied without destroying stability. The value of N affects both closed-loop performance and robustness against DoS: the larger N is, the larger is the number of consecutive communication attempts that can be denied without destroying stability. However, this potentially results in larger overshoots since $\eta = M$. ■

Example 2: Another interesting scenario is when DoS is sustained. One can account for this situation by modeling the DoS signal as a rectangular wave of a (possibly) variable period and duty cycle [13]. Let P_n and $D_n = \tau_n / P_n$ denote the period and duty cycle of the nth DoS attack, respectively. Also let $P_{\min} := \inf_{n \in \mathbb{N}_0} P_n$, $D_{\max} := \sup_{n \in \mathbb{N}_0} D_n$ and $\tau_{\max} := \sup_{n \in \mathbb{N}_0} \tau_n$. Suppose that $P_{\min} > \underline{\Delta}$, $D_{\max} < 1$ and $\tau_{\max} < \infty$. Since the maximum number of *off/on* transitions of DoS during the interval $[\tau, t[$ can be upper bounded as $n(\tau,t) \leq \lceil (t-\tau)/P_{\min} \rceil$, Assumption 1 holds true with $\eta = 1$ and $\tau_D = P_{\min}$. Let now

$$n(t) := \begin{cases} -1, & \text{if } t < h_0 \\ \sup\{\, n \in \mathbb{N}_0 | h_n \leq t \,\}, & \text{otherwise} \end{cases} \tag{39}$$

where $\tau_{-1} := 0$. For any $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$, it is possible to write

$$\begin{aligned} |\Xi(\tau,t)| &\leq \max\left\{0, h_{n(\tau)} + \tau_{n(\tau)} - \tau\right\} \\ &\quad + \min\left\{t - h_{n(t)}, \tau_{n(t)}\right\} + D_{\max} \sum_{\substack{n \in \mathbb{N}_0; \\ h_{n(\tau)} < h_n; \\ h_n < h_{n(t)}}} P_n. \end{aligned} \tag{40}$$

Hence, one sees that Assumption 2 holds true with $\kappa = 2\tau_{\max}$ and $T = D_{\max}^{-1}$. This example includes as a special case the so-called *periodic jamming* where both period and duty cycle are constant. ■

Example 3: In connection with Example 2, we pointed out in Section IV-A that the requirements $P_{\min} > \underline{\Delta}$ and $D_{\max} < 1$ are in a wide-sense necessary for closed-loop stability unless other conditions are imposed on DoS (as for $\tau_{\max} < \infty$, *cf.* Section IV-C). These conditions, however, rule out for example the possibility of DoS signals with *off/on* transitions occasionally faster than the maximum transmission rate. Consider first Assumption 1 and suppose that $P_{\min} > 0$. Let $f(\tau,t) := (t-\tau)/n(\tau,t)$ represent the average dwell-time of DoS *off/on* transitions on $[\tau, t[$. Assume that for some $\tau_D \in \mathbb{R}_{>\underline{\Delta}}$, there exists a $\gamma \in \mathbb{R}_{>0}$ such that

$$f(\tau, \tau + \gamma) \geq \tau_D \tag{41}$$

for all $\tau \in \mathbb{R}_{\geq 0}$, which implies that the DoS *off/on* transitions are slower than $\underline{\Delta}$ on every sufficiently large time-interval. Then, it is easy to verify that Assumption 1 holds true with $\eta = \lceil \gamma / P_{\min} \rceil$. In fact, if $t - \tau \leq \gamma$ then $n(\tau,t) \leq n(\tau, \tau+\gamma)$ where $n(\tau, \tau+\gamma) \leq \lceil \gamma / P_{\min} \rceil$. If instead $t - \tau > \gamma$, let m denote the largest integer m such that $m\gamma < t - \tau$. Then

$$\begin{aligned} n(\tau,t) &= \sum_{k=0}^{m-1} n\left(\tau + k\gamma, \tau + (k+1)\gamma\right) \\ &\quad + n(\tau + m\gamma, t) \\ &= \sum_{k=0}^{m-1} \gamma / f\left(\tau + k\gamma, \tau + (k+1)\gamma\right) \\ &\quad + n(\tau + m\gamma, t) \leq m\gamma/\tau_D + \lceil \gamma / P_{\min} \rceil \end{aligned} \tag{42}$$

as it follows from (41) and the definition of m. Then the claim follows by recalling that $m\gamma < t - \tau$. As for Assumption 2, let $D(\tau,t) := |\Xi(\tau,t)|/(t-\tau)$, which can be thought of as the

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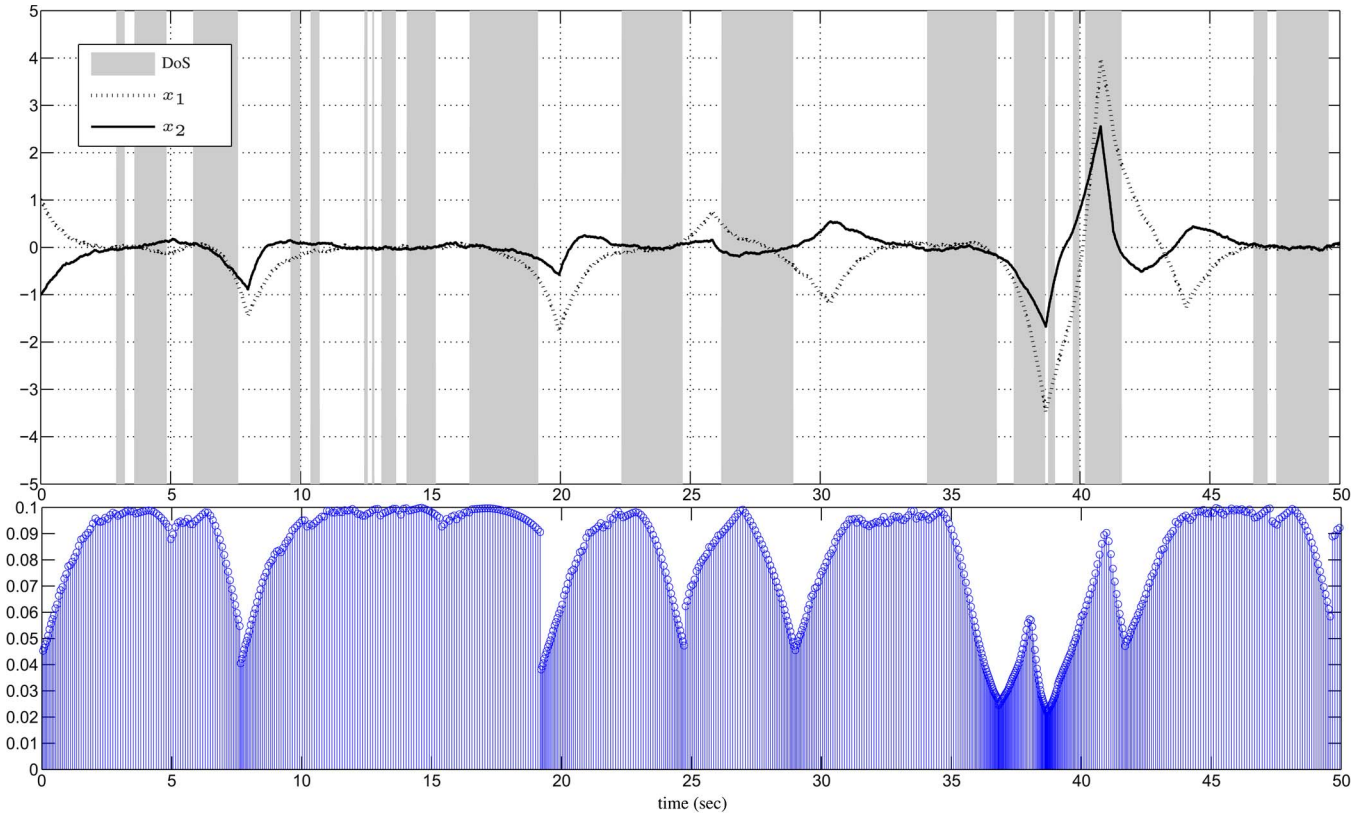

Fig. 3. Simulation example for system (46) under state feedback (48) and sampling logic (37) with $\delta_1 = 0.01$, $\delta_2 = 0.1$ and $\varphi(\cdot) = 2/\pi \arctan(\cdot)$. Top: closed-loop state response under initial conditions $x = [1\ -1]^\top$, disturbance w uniformly distributed in $[-1, 1]$, and sustained DoS attack with variable period and duty cycle, generated randomly, with $P_{\min} = 0.01$ s and $P_{\max} = 10$ s. The resulting DoS signal has an average duty cycle of $\sim$42%. The vertical grey stripes represent the time-intervals over which DoS is active. Bottom: inter-sampling times determined by the logic (37). In terms of regulation performance, very similar results are obtained with the other logics described in Section V.

average DoS duty cycle over $[\tau, t]$. Assume that for some $T \in \mathbb{R}_{>1}$, there exists a $\delta \in \mathbb{R}_{>0}$ such that

$$D(\tau, \tau + \delta) \leq 1/T \tag{43}$$

for all $\tau \in \mathbb{R}_{\geq 0}$. Reasoning as before, it is simple to verify that Assumption 2 holds true with $\kappa = \delta$. In connection with Example 2, the conditions just stated allows one to consider more general DoS classes. For instance, let

$$\tau_{ave} := \lim_{m\to\infty} \frac{1}{m+1} \sum_{n=0}^{m} (h_{n+1} - h_n) \tag{44}$$

$$D_{ave} := \lim_{m\to\infty} \frac{1}{m+1} \sum_{n=0}^{m} D_n \tag{45}$$

denote the average dwell-time of DoS *off/on* transitions and the average DoS duty cycle, respectively. Then (41) and (43) with $\tau_D = \tau_{ave}$ and $T = D_{ave}^{-1}$ are sufficient to conclude that the DoS signal is also slow-on-the-average in the sense of Assumptions 1 and 2 with respect to τ_{ave} and D_{ave}. ■

B. A Numerical Example

For the sake of clarity, a numerical example illustrating the theory as well as the discussion of Section VI-A is reported.

Consider the following open-loop unstable system [33]

$$\frac{d}{dt}x(t) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(t) + u(t) + w(t) \tag{46}$$

under LQR gain

$$K = \begin{bmatrix} -2.1961 & -0.7545 \\ -0.7545 & -2.7146 \end{bmatrix}. \tag{47}$$

Solution of the Lyapunov equation $\Phi^\top P + P\Phi + Q = 0$ with $Q = I_2$ yields $\alpha_1 = 0.2779$, $\alpha_2 = 0.4497$, $\gamma_1 = 1$, and $\gamma_2 = 2.1080$. From this we deduce that we must select σ such that $\sigma < 0.4744$. Picking for instance $\sigma = 0.26$, Lemma 1 yields $\bar{\Delta}_\sigma = 0.1005$, where $\|\Phi\| = 1.9021$ and $\mu_A = 1.5$. $\bar{\Delta}_\sigma$ specifies the inter-sampling time of maximal length that guarantees ISS. Further, $\omega_1/(\omega_1 + \omega_2) = 0.0321$. This latter value determines the DoS signals which are admissible in accordance with the present analysis. In connection with Example 1 in Section VI-A, this means a maximum of $\sim$3% of communication denials on the average. As for Example 2 (Example 3), this implies a maximum (average) duty cycle of $\sim$3% in case of a sustained DoS attack. The value obtained for $\omega_1/(\omega_1 + \omega_2)$ is conservative: as shown in Fig. 3, the bounds can in practice be much smaller than the theoretical one. This was also confirmed by extensive simulations.

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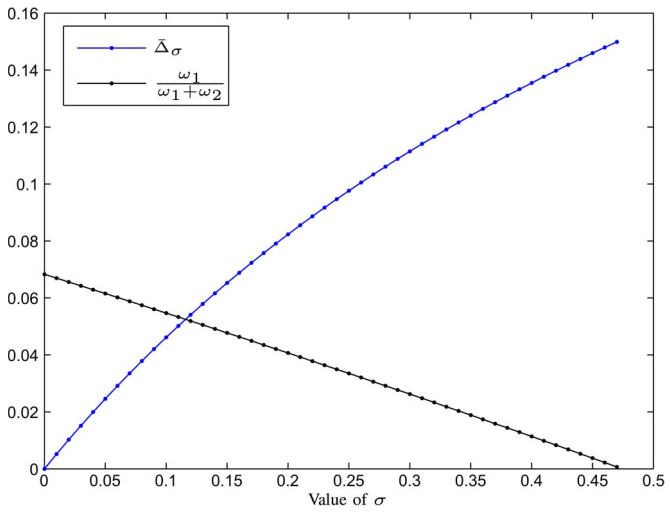

Fig. 4. Maximum inter-sampling time and value of $\omega_1/(\omega_1+\omega_2)$ versus choice of σ using the LQR gain and $Q = I_2$.

The conservativeness of the bound comes from two main sources: i) the bounds on the growth of the Lyapunov function under DoS (*cf.* (72)–(79)). In this respect, the approach in [34] which does not rely on Lyapunov functions (albeit restricted to the disturbance-free case) can provide a possible alternative to the present analysis; and ii) the generality of the considered scenario. In fact, tighter bounds are likely to be obtained when more "structure" is assumed for the DoS. In this respect, interesting results in case of *periodic jamming* have been recently reported in [24], [25].

It is interesting to observe that the value of $\omega_1/(\omega_1+\omega_2)$ also depends on a number of design parameters. In fact, it depends on the Lyapunov equation $\Phi^\top P + P\Phi + Q = 0$, and, as such, on Q and the state-feedback matrix K. For instance, a choice

$$K = \begin{bmatrix} -4.5 & -1 \\ 0 & -6 \end{bmatrix} \tag{48}$$

achieves a bound $\omega_1/(\omega_1+\omega_2) = 0.0971$, thus allowing for an average duty cycle $\sim 10\%$ in case of a sustained attack. This suggests investigation of analytic or numeric methods to find the Q and K that could maximize robustness against DoS. In practice, another possibility for increasing $\omega_1/(\omega_1+\omega_2)$ is to reduce the value of σ. This, however, has to be traded-off against the inter-sampling times. For instance, using the LQR gain and letting $Q = I_2$, the choice $\sigma = 0.1$ is sufficient to increase $\omega_1/(\omega_1+\omega_2)$ to 0.0547. As an offset, $\bar{\Delta}_\sigma$ drops to 0.0462. This phenomenon is illustrated in Fig. 4.

C. Slow-on-the-Average DoS: Disturbance-Free Case

We close with few remarks on the results of Section IV-C. In the disturbance-free case, both Assumptions 1 and 2 can be relaxed. In fact, while Assumptions 3 and 4 are similar in concept, they pose constraints on DoS frequency and duration which must hold on $[0, t)$ only, rather than on each sub-interval $[\tau, t)$ of $[0, t)$. This makes it possible to face more general DoS classes, including DoS signals that deny communication for *unbounded* periods of time.

Consider for instance the example of Section VI-B along with a DoS signal given by

$$\begin{aligned} h_n &= (n+1) + \frac{1}{2}n(n+1) - \frac{1}{\alpha}(n+1) \\ \tau_n &= \frac{1}{\alpha}(n+1) \end{aligned} \tag{49}$$

where $n \in \mathbb{N}_0$ and $\alpha \in \mathbb{R}_{>1}$. It is straightforward to verify that the resulting DoS signal satisfies Assumptions 3 and 4 with $\eta = \tau_D = 1$, $\kappa = 0$, and $T = \alpha$. Picking $\bar{\Delta}_\sigma = 0.01$ and recalling that $\omega_1/(\omega_1+\omega_2) = 0.0321$, one sees that (22) holds true for $\alpha \geq 50$. Since the conditions of Corollary 1 are satisfied, then the closed-loop system is GAS despite the fact that the length of the DoS intervals grows unbounded with n. This is possible since, the closer the state is to the origin, the less the effect of DoS.

In the presence of disturbances, the situation just described is no longer true since w may always cause the state to deviate from its nominal trajectory. It is in fact easy to see that for open-loop unstable systems, no sampling logic exists that achieve ISS in the presence of unbounded DoS. As for Theorem 2, boundedness of the DoS intervals is implicit in Assumption 2. In fact, (4) with $\tau = h_n$ and $t = h_n + \tau_n$ implies $\sup_{n\in\mathbb{N}_0} \tau_n \leq \kappa T/(T-1)$.

VII. Concluding Remarks

We have investigated stability of networked systems in the presence of DoS attacks. One contribution of this paper is an explicit characterization of the frequency and duration of DoS attacks under which closed-loop stability can be preserved. The result is intuitive as it relates stability with the ratio between the *on/off* periods of jamming. An explicit characterization of sampling rules that achieve ISS was given. This characterization is flexible enough so as to allow the designer to choose from several implementation options that can be used for trading-off performance versus communication resources.

The results lend themselves to many possible extensions. As for the framework considered here, identifying optimal attack and counter-attack strategies with respect to some prescribed performance objective, represents an interesting research venue. Moreover, we have not investigated the effect of possible limitations on the information, such quantization and delays. As additional future research topics, we also envision the use of similar techniques to handle output feedback controllers as well as nonlinear systems. As for the latter case, preliminary results have been reported in [35]. Finally, an interesting research line is to address the case where control and measurement channels can be interrupted *asynchronously*. In this respect, the self-triggering logic described in Section VI, which relies on predictions of the process state, appears as a convenient tool for updating the control action in case of DoS of the measurement channel.

One of the main motivations for considering control over networks descends from problems of distributed coordination and control of large-scale systems [36]–[40]. Investigating our approach to control under DoS for self-triggered coordination

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problems such as those in [40] does also represent an interesting research venue.

APPENDIX

Proof of Theorem 1: Substituting (12) into (14b) yields

$$\frac{d}{dt}V(x(t)) \leq -\gamma_4 \|x(t)\|^2 + \gamma_5 \|x(t)\| v(t) \tag{50}$$

where $v(t) := \sup\{\|w(t)\|, \|w_t\|_\infty\}$, $\gamma_4 := (\gamma_1 - \sigma\gamma_2)$ and $\gamma_5 := (\gamma_3 + \sigma\gamma_2)$. Here, we recall that γ_1 is the minimal eigenvalue of Q, $\gamma_2 = \|2PBK\|$, $\gamma_3 = \|2P\|$, and $\sigma < \gamma_1/\gamma_2$. Observe that for any positive real δ, the Young's inequality (e.g., see [41]) yields

$$2\|x(t)\| v(t) \leq \frac{1}{\delta}\|x(t)\|^2 + \delta v^2(t). \tag{51}$$

By letting $\delta := \gamma_5/\gamma_4$, we get

$$\begin{aligned}\frac{d}{dt}V(x(t)) &\leq -\frac{\gamma_4}{2}\|x(t)\|^2 + \gamma_6 v(t)^2 \\ &\leq -\omega_1 V(x(t)) + \gamma_6 v(t)^2 \end{aligned} \tag{52}$$

where $\omega_1 := \gamma_4/(2\alpha_2)$ and $\gamma_6 := \gamma_5^2/(2\gamma_4)$. Note now that $\|v_t\|_\infty = \|w_t\|_\infty$ for any $t \in \mathbb{R}_{>0}$. Thus, standard comparison results for differential inequalities yield

$$V(x(t)) \leq e^{-\omega_1 t} V(x(0)) + \gamma_7 \|w_t\|_\infty^2 \tag{53}$$

where $\gamma_7 := \gamma_6/\omega_1$. Using (14a), we get

$$\|x(t)\|^2 \leq \frac{\alpha_2}{\alpha_1} e^{-\omega_1 t}\|x(0)\|^2 + \frac{\gamma_7}{\alpha_1}\|w_t\|_\infty^2. \tag{54}$$

Since $a^2 + b^2 \leq (a+b)^2$ for any pair of positive reals a and b, we finally get

$$\|x(t)\| \leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{-(\frac{\omega_1}{2})t}\|x(0)\| + \sqrt{\frac{\gamma_7}{\alpha_1}}\|w_t\|_\infty \tag{55}$$

which yields the desired result. ■

Proof of Lemma 1: In the absence of DoS, any control update attempt is successful. Thus, in accordance with (10), the dynamics of e satisfies

$$\begin{aligned}\frac{d}{dt}e(t) &= -Ax(t) - BKx(t_k) - w(t) \\ &= Ae(t) - \Phi x(t_k) - w(t)\end{aligned} \tag{56}$$

for all $t \in \mathcal{I}_k$ and for all $k \in \mathbb{N}_0$, where $e(t_k) = 0$. Recall now that $\|e^{At}\| \leq e^{\mu_A t}$ for all $t \in \mathbb{R}_{\geq 0}$. Using this property we then have

$$\|e(t)\| \leq \kappa_1 \int_{t_k}^{t} e^{\mu_A(t-s)}\left[\|x(t_k)\| + \|w(s)\|\right] ds \tag{57}$$

for all $t \in \mathcal{I}_k$ and all $k \in \mathbb{N}_0$, where $\kappa_1 := \max\{\|\Phi\|, 1\}$. Let $f(t - t_k) := \int_{t_k}^{t} e^{\mu_A(t-s)} ds$. Using the fact that $x(t_k) = e(t) + x(t)$, we obtain

$$\begin{aligned}\|e(t)\| &\leq \kappa_1 f(t-t_k)\|e(t)\| \\ &\quad + \kappa_1 f(t-t_k)\left(\|x(t)\| + \|w_t\|_\infty\right).\end{aligned} \tag{58}$$

Observe now that $f(0) = 0$ and $f(t - t_k)$ is monotonically increasing with t. Accordingly, for any positive real Δ such that

$$f(\Delta) \leq \frac{1}{\kappa_1}\frac{\sigma}{(1+\sigma)} \tag{59}$$

then any control update rule such that $\Delta_k \leq \Delta$ will satisfy (12) for all $t \in \mathbb{R}_{\geq 0}$. To conclude the proof, we derive an explicit expression for Δ. Let first $\mu_A \leq 0$. In this case, $f(\Delta) \leq \Delta$, so that (17) yields the desired result. If instead $\mu_A > 0$, we have

$$f(\Delta) = \frac{1}{\mu_A}(e^{\mu_A \Delta} - 1) \tag{60}$$

and (18) yields the desired result. ■

Proof of Theorem 2: As pointed out in Section III, the idea is to decompose the time axis into intervals where it possible to satisfy (12) and intervals where, due to the occurrence of DoS, (12) need not hold. We then analyze the closed-loop dynamics as a system that switches between stable and unstable modes. For clarity of exposition, the proof is divided into three steps.

Step I. Modeling of the Intervals Related to Stable and Unstable Dynamics: In this step we characterize the intervals of time where (12) holds and those where it need not hold. During these intervals, the closed-loop system evolves obeying to stable and possibly unstable dynamics, respectively. The characterization of these intervals is essential for the Lyapunov-based analysis we carry out in the forthcoming steps and can be formalized as follows:

Lemma 2: For any $\tau, t \in \mathbb{R}_{\geq 0}$, with $0 \leq \tau \leq t$, the interval $[\tau, t]$ is the disjoint union of $\bar{\Theta}(\tau, t)$ and $\bar{\Xi}(\tau, t)$, where $\bar{\Theta}(\tau, t)$ (respectively, $\bar{\Xi}(\tau, t)$) is the union of sub-intervals of $[\tau, t]$ over which (12) holds (respectively, need not hold). Specifically, there exists two sequences of non-negative and positive real numbers $\{\zeta_m\}_{m\in\mathbb{N}_0}, \{v_m\}_{m\in\mathbb{N}_0}$ such that

$$\bar{\Xi}(\tau, t) := \bigcup_{m\in\mathbb{N}_0} Z_m \cap [\tau, t] \tag{61}$$

$$\bar{\Theta}(\tau, t) := \bigcup_{m\in\mathbb{N}_0} W_{m-1} \cap [\tau, t] \tag{62}$$

where

$$Z_m := \{\zeta_m\} \cup [\zeta_m, \zeta_m + v_m[\tag{63}$$

$$W_m := \{\zeta_m + v_m\} \cup [\zeta_m + v_m, \zeta_{m+1}[\tag{64}$$

and where $\zeta_{-1} = v_{-1} := 0$.

Proof of Lemma 2: Let $\mathscr{S}_n := \{k \in \mathbb{N}_0 | t_k \in H_n\}$ denote the set of integers related to a control update attempt occurring over H_n, $n \in \mathbb{N}_0$. Define

$$\lambda_n := \begin{cases} \tau_n, & \text{if } \mathscr{S}_n = \emptyset \\ t_{\sup\{k\in\mathbb{N}_0 : k\in\mathscr{S}_n\}} - h_n, & \text{otherwise} \end{cases} \tag{65}$$

$$\Lambda_n := \begin{cases} 0, & \text{if } \mathscr{S}_n = \emptyset \\ \Delta_{\sup\{k\in\mathbb{N}_0 : k\in\mathscr{S}_n\}}, & \text{otherwise.} \end{cases} \tag{66}$$

Thus,

$$\bar{H}_n := \{h_n\} \cup [h_n, h_n + \lambda_n + \Lambda_n[\tag{67}$$

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specifies the nth time interval where (12) need not hold, which consists of H_n plus the corresponding DoS-induced actuation delay. Note that $\lambda_n + \Lambda_n \geq \tau_n$ for all $n \in \mathbb{N}_0$. Notice now that the intervals $\bar{H}_n$ and $\bar{H}_{n+1}$ may overlap each other in that h_{n+1} may belong to $\bar{H}_n$. For analysis purposes, it is convenient to regard these overlapping intervals as a single interval of the form (63). This can be done by defining an auxiliary sequence $\{\zeta_m\}_{m\in\mathbb{N}_0}$, which is recursively defined from $\{h_n\}_{n\in\mathbb{N}_0}$ as follows:

$$\zeta_0 := h_0 \tag{68}$$

$$\zeta_{m+1} := \inf\{\, h_n > \zeta_m \mid h_n > h_{n-1} + \lambda_{n-1} + \Lambda_{n-1}\} \tag{69}$$

for all $m \in \mathbb{N}$, and letting

$$v_m := \sum_{\substack{n\in\mathbb{N}_0;\\ \zeta_m \leq h_n < \zeta_{m+1}}} |\bar{H}_n \backslash \bar{H}_{n+1}| \tag{70}$$

for all $m \in \mathbb{N}_0$. Given $\tau, t \in \mathbb{R}_{\geq 0}$, define $\bar{\Xi}(\tau, t)$ as in (61). Hence, by construction, $\bar{\Xi}(\tau, t)$ represents the union of sub-intervals of $[\tau, t]$ where, due to the occurrence of DoS, (12) need not hold.

Let W_m be as in (64). Given $\tau, t \in \mathbb{R}_{\geq 0}$, let $\bar{\Theta}(\tau, t)$ as in (62). Note that over each $[\tau, t]$, the sets $\bar{\Xi}(\tau, t)$ and $\bar{\Theta}(\tau, t)$ are complementary since their union equals $[\tau, t]$ and their intersection is the empty set. Also notice that $\bar{\Theta}(\tau, t)$ represents the union of sub-intervals of $[\tau, t]$ where (12) holds true. In fact, by construction, for each $m \in \mathbb{N}_0$, a successful control update necessarily occurs at $\zeta_m + v_m$ and there is no DoS over W_m. ■

Step II. Lypaunov Function Analysis: Consider the intervals W_m, $m \in \mathbb{N}_0$, where (12) holds true by construction. From (53) we get

$$V\left(x(t)\right) \leq e^{-\omega_1(t-\zeta_m-v_m)} V(x\left(\zeta_m + v_m\right)) + \gamma_7 \|w_t\|_\infty^2 \tag{71}$$

for all $t \in W_m$ and all $m \in \mathbb{N}_0$. Here and in the sequel, all the relevant constants are as in the proof of Theorem 1. In particular, we recall that $\gamma_7 = \alpha_2(\gamma_3 + \sigma\gamma_2)^2/(\gamma_1 - \sigma\gamma_2)^2$ with $\sigma < \gamma_1/\gamma_2$.

Consider next the intervals Z_m, $m \in \mathbb{N}_0$, over which the inequality (12) does not necessarily hold. To derive a bound on the growth of $V(x(t))$ in this case, some intermediate steps are needed. We first prove that for each $m \in \mathbb{N}_0$

$$\|e(t)\| \leq (1+\sigma)\,\|x(\zeta_m)\| + \|x(t)\| + \sigma\|w_t\|_\infty \tag{72}$$

for all $t \in Z_m$. Recall that

$$e(t) = x\left(t_{k(\zeta_m)}\right) - x(t) \tag{73}$$

for all $t \in Z_m$, where $x(t_{k(\zeta_m)})$ represents the value of the process state at the last successful control update up to ζ_m. If $\zeta_0 = 0$, then $x(t_{k(\zeta_0)}) = 0$ by definition so that (72) is valid. Consider next the case $\zeta_0 > 0$. By construction, (12) holds true for all $t \in W_m$. Hence, by continuity of x one has

$$\|e(\zeta_m)\| \leq \sigma\,\|x(\zeta_m)\| + \sigma\|w_{\zeta_m}\|_\infty \tag{74}$$

for all $m \in \mathbb{N}_0$. Hence,

$$\left\|x\left(t_{k(\zeta_m)}\right) - x(\zeta_m)\right\| \leq \sigma\,\|x(\zeta_m)\| + \sigma\|w_{\zeta_m}\|_\infty \tag{75}$$

and (72) follows by applying the triangular inequality.

Substituting (72) into (14b) yields

$$\begin{aligned}\frac{d}{dt}V\left(x(t)\right) \leq\ & (\gamma_2 - \gamma_1)\,\|x(t)\|^2 \\ & + \gamma_2(1+\sigma)\,\|x(t)\|\,\|x(\zeta_m)\| \\ & + (\gamma_3 + \sigma\gamma_2)\,\|x(t)\|\,v(t)\end{aligned} \tag{76}$$

where $v(t) := \sup\{\|w(t)\|, \|w_t\|_\infty\}$. We then proceed as in the proof of Theorem 1. Using (51) with $\delta = (\gamma_3 + \gamma_2\sigma)/(\gamma_1 - \sigma\gamma_2)$, simple calculations yield

$$\begin{aligned}\frac{d}{dt}V\left(x(t)\right) \leq\ & \gamma_2(1-\sigma)\,\|x(t)\|^2 \\ & + \gamma_2(1+\sigma)\,\|x(t)\|\,\|x(\zeta_m)\| + \gamma_6 v^2(t)\end{aligned} \tag{77}$$

where we recall that $\gamma_6 = (\gamma_3 + \gamma_2\sigma)^2/(2(\gamma_1 - \sigma\gamma_2))$. Note that

$$\frac{d}{dt}V(x(t)) \leq \omega_2 \max\{V(x(t))\,, V\left(x(\zeta_m)\right)\} + \gamma_6 v^2(t) \tag{78}$$

where $\omega_2 := 2\gamma_2/\alpha_1$. Since $\|v_t\|_\infty = \|w_t\|_\infty$ for any $t \in \mathbb{R}_{\geq 0}$, we then have

$$V\left(x(t)\right) \leq e^{\omega_2(t-\zeta_m)} V\left(x(\zeta_m)\right) + \gamma_8 e^{\omega_2(t-\zeta_m)} \|w_t\|_\infty^2 \tag{79}$$

for all $t \in Z_m$, where $\gamma_8 := \gamma_6/\omega_2$.

Combining (71) and (79), we can prove the following result.

Lemma 3: For all $t \in \mathbb{R}_{\geq 0}$, the Lyapunov function satisfies

$$\begin{aligned}V\left(x(t)\right) \leq\ & e^{-\omega_1|\bar{\Theta}(0,t)|} e^{\omega_2|\bar{\Xi}(0,t)|} V\left(x(0)\right) \\ & + \gamma_* \left[1 + 2 \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-\omega_1|\bar{\Theta}(\zeta_m+v_m,t)|} e^{\omega_2|\bar{\Xi}(\zeta_m,t)|}\right] \|w_t\|_\infty^2\end{aligned} \tag{80}$$

where $\gamma_* := \max\{\gamma_7, \gamma_8\}$.

Hereafter, in accordance with (62), it is understood that $|\bar{\Theta}(\zeta_m + v_m, t)| = 0$ whenever $t < \zeta_m + v_m$.

Proof of Lemma 3: We use an induction argument. First, we show that the inequality holds true over $W_{-1} = [0, \zeta_0]$. If $\zeta_0 = 0$, the claim trivially holds. Suppose $\zeta_0 > 0$. Over W_{-1} the Lyapunov function obeys (71); thus, (80) follows by noting that $|\bar{\Theta}(0,t)| = t$ and $|\bar{\Xi}(0,t)| = 0$ for all $t \in W_{-1}$ and the sum term in (80) is zero.

Assume next that (80) holds true over the interval $[0, \zeta_p]$, where $p \in \mathbb{N}_0$. By hypothesis, and since $V(x)$ is continuous, we have

$$\begin{aligned}V(x(\zeta_p)) \leq\ & e^{-\omega_1|\bar{\Theta}(0,\zeta_p)|} e^{\omega_2|\bar{\Xi}(0,\zeta_p)|} V\left(x(0)\right) \\ & + \gamma_* \left[1 + 2 \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m < \zeta_p}} e^{-\omega_1|\bar{\Theta}(\zeta_m+v_m,\zeta_p)|} e^{\omega_2|\bar{\Xi}(\zeta_m,\zeta_p)|}\right] \|w_{\zeta_p}\|_\infty^2.\end{aligned} \tag{81}$$

Then, consider first the interval Z_p. Over Z_p, the Lyapunov function obeys (79) with

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$$V\left(x(t)\right) \leq e^{\omega_2(t-\zeta_p)}V\left(x(\zeta_p)\right) + \gamma_* e^{\omega_2(t-\zeta_p)}\|w_t\|_\infty^2. \quad (82)$$

Notice that $t-\zeta_p = |\bar{\Xi}(\zeta_p,t)|$ for all $t \in Z_p$. Hence,

$$\left|\bar{\Xi}(0,t)\right| = t-\zeta_p + \left|\bar{\Xi}(0,\zeta_p)\right| \quad (83)$$

$$\left|\bar{\Theta}(0,t)\right| = \left|\bar{\Theta}(0,\zeta_p)\right| \quad (84)$$

for all $t \in Z_p$. Thus, multiplying $e^{\omega_2(t-\zeta_p)}$ by the first term on the right-hand side (RHS) of (81) we get the first term on the RHS of (80). In addition, multiplying $e^{\omega_2(t-\zeta_p)}$ by the sum term in (81), we obtain

$$e^{\omega_2(t-\zeta_p)} \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m<\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,\zeta_p)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,\zeta_p)\right|} = \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m<\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \quad (85)$$

for all $t \in Z_p$ because $|\bar{\Theta}(\zeta_m+v_m,\zeta_p)| = |\bar{\Theta}(\zeta_m+v_m,t)|$ and $t-\zeta_p+|\bar{\Xi}(\zeta_m,\zeta_p)| = |\bar{\Xi}(\zeta_m,t)|$ for all $t \in Z_p$. Overall, the factor multiplying the disturbance term can be therefore rewritten as

$$\begin{aligned} &2\gamma_* e^{\omega_2(t-\zeta_p)} + 2\gamma_* \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m<\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \\ &= 2\gamma_* \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m\leq\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \\ &= 2\gamma_* \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m\leq t}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \end{aligned} \quad (86)$$

where the first equality follows from $|\bar{\Theta}(\zeta_p+v_p,t)| = 0$ and $t-\zeta_p = |\bar{\Xi}(\zeta_p,t)|$ for all $t \in Z_p$, while the second equality holds because condition $\zeta_m \leq \zeta_p$ is equivalent to $\zeta_m \leq t$ for all $t \in Z_p$. Hence, (80) holds true for all $t \in [0, \zeta_p+v_p]$.

Consider next the interval W_p. Over W_p, the Lyapunov function obeys (71). In particular,

$$V\left(x(t)\right) \leq e^{-\omega_1(t-\zeta_p-v_p)}e^{\omega_2 v_p}V(x(\zeta_p)) + \gamma_*\left[1+e^{-\omega_1(t-\zeta_p-v_p)}e^{\omega_2 v_p}\right]\|w_t\|_\infty^2 \quad (87)$$

for all $t \in W_p$, with $V(x(\zeta_p))$ as in (81). To see that this implies again (80), notice that $t-\zeta_p-v_p = |\bar{\Theta}(\zeta_p+v_p,t)|$ and $|\bar{\Theta}(0,\zeta_p)| = \bar{\Theta}(0,\zeta_p+v_p)|$. Hence,

$$\left|\bar{\Theta}(0,t)\right| = t-\zeta_p-v_p + \left|\bar{\Theta}(0,\zeta_p)\right| \quad (88)$$

for all $t \in W_p$. Moreover, $v_p = |\bar{\Xi}(\zeta_p,\zeta_p+v_p)|$ and $|\bar{\Xi}(\zeta_p+v_p,t)| = 0$ for all $t \in W_p$. Hence,

$$\left|\bar{\Xi}(0,t)\right| = v_p + \left|\bar{\Xi}(0,\zeta_p)\right| \quad (89)$$

for all $t \in W_p$. Thus, multiplying $e^{-\omega_1(t-\zeta_p-v_p)}e^{\omega_2 v_p}$ by the first term on the RHS of (81) we get the first term on the RHS of (80). Moreover, multiplying $e^{-\omega_1(t-\zeta_p-v_p)}e^{\omega_2 v_p}$ by the sum term in (81), we obtain

$$e^{-\omega_1(t-\zeta_p-v_p)}e^{\omega_2 v_p} \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m<\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,\zeta_p)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,\zeta_p)\right|} = \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m<\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|}. \quad (90)$$

Overall, the factor multiplying the disturbance term can be therefore rewritten as

$$\begin{aligned} &\gamma_*\left[1+2e^{-\omega_1(t-\zeta_p-v_p)}e^{\omega_2 v_p}\right] + 2\gamma_* \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m<\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \\ &= 2\gamma_* \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m\leq\zeta_p}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \\ &= 2\gamma_* \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m\leq t}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|}. \end{aligned} \quad (91)$$

As before, the second equality holds because, for all $t \in W_p$, condition $\zeta_m \leq \zeta_p$ is equivalent to $\zeta_m \leq t$. Thus (80) holds true for all $t \in [0, \zeta_{p+1}]$, which concludes the proof. ■

Step III. Bounds on DoS Frequency and Duration: In order to conclude the proof, we need to bound the sum term in (80). The following can be proven:

Lemma 4: Under Assumptions 1 and 2 and condition (22), the sum

$$\sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m\leq t}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \quad (92)$$

is bounded from above by

$$e^{(\omega_1+\omega_2)\kappa_*}\frac{e^{\beta_*\tau_D\eta}}{1-e^{-\beta_*\tau_D}} \quad (93)$$

where the constants $\omega_1, \omega_2, \tau_D, \kappa_*$ and β_*, are defined in Theorem 2, (25) and (26).

Proof of Lemma 4: First notice that

$$\left|\bar{\Xi}(\tau,t)\right| \leq |\Xi(\tau,t)| + (1+n(\tau,t))\,\Delta_* \quad (94)$$

for all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. In words, $|\bar{\Xi}(\tau,t)|$ can be upper bounded by the total length of DoS over $[\tau,t]$ plus the maximum actuation delay Δ_*, which may occur once at the beginning of the interval $[\tau,t]$ (as a result of a previous DoS) plus $n(\tau,t)$ times, where $n(\tau,t)$ represents the number of *off/on* transitions of DoS occurring over $[\tau,t[$. Using Assumptions 1 and 2 we have

$$\begin{aligned} \left|\bar{\Xi}(\tau,t)\right| &\leq \kappa + \frac{t-\tau}{T} + \left(1+\eta+\frac{t-\tau}{\tau_D}\right)\Delta_* \\ &=: \kappa_* + \frac{t-\tau}{T_*} \end{aligned} \quad (95)$$

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where $\kappa_* := \kappa + (1+\eta)\Delta_*$ and $T_* := \tau_D T/(\tau_D + T\Delta_*)$. With this in mind, we can now analyze the sum term in (92). From the above inequality, we have

$$\left|\bar{\Xi}(\zeta_m, t)\right| \leq \kappa_* + \frac{t-\zeta_m}{T_*} \tag{96}$$

for all $t \in \mathbb{R}_{\geq \zeta_m}$, $m \in \mathbb{N}_0$. Consider next $|\bar{\Theta}(\zeta_m + v_m, t)|$. We have

$$\left|\bar{\Theta}(\zeta_m + v_m, t)\right| = t - \zeta_m - \left|\bar{\Xi}(\zeta_m, t)\right| \tag{97}$$

for all $t \in \mathbb{R}_{\geq \zeta_m}$, $m \in \mathbb{N}_0$. To see this, consider first the interval Z_m. For all $t \in Z_m$, we have $|\bar{\Xi}(\zeta_m, t)| = t - \zeta_m$ so that (97) holds true since, by definition, $|\bar{\Theta}(\zeta_m + v_m, t)| = 0$ whenever $t < \zeta_m + v_m$. For all $t \in \mathbb{R}_{\geq \zeta_m + v_m}$ we have

$$\begin{aligned}\left|\bar{\Theta}(\zeta_m + v_m, t)\right| &= t - \zeta_m - v_m - \left|\bar{\Xi}(\zeta_m + v_m, t)\right| \\ &= t - \zeta_m - \left|\bar{\Xi}(\zeta_m, t)\right| \end{aligned} \tag{98}$$

where the second equality follows because

$$\begin{aligned}\left|\bar{\Xi}(\zeta_m + v_m, t)\right| &= \left|\bar{\Xi}(\zeta_m, t)\right| - \left|\bar{\Xi}(\zeta_m, \zeta_m + v_m)\right| \\ &= \left|\bar{\Xi}(\zeta_m, t)\right| - v_m \end{aligned} \tag{99}$$

for all $t \in \mathbb{R}_{\geq \zeta_m + v_m}$, $m \in \mathbb{N}_0$. Hence,

$$\begin{aligned}&\sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-\omega_1\left|\bar{\Theta}(\zeta_m+v_m,t)\right|} e^{\omega_2\left|\bar{\Xi}(\zeta_m,t)\right|} \\ &\quad \leq e^{(\omega_1+\omega_2)\kappa_*} \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-[\omega_1-(\omega_1+\omega_2)/T_*](t-\zeta_m)} \\ &\quad = e^{(\omega_1+\omega_2)\kappa_*} \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-\beta_*(t-\zeta_m)} \end{aligned} \tag{100}$$

where $\beta_* = \omega_1 - (\omega_1+\omega_2)/T_*$. Under condition (22), we have $\beta_* > 0$. Moreover, Assumption 1 yields

$$t - \zeta_m \geq \tau_D n(\zeta_m, t) - \tau_D \eta. \tag{101}$$

This implies

$$\sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-\beta_*(t-\zeta_m)} \leq e^{\beta_*\tau_D\eta} \sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-\beta_*\tau_D n(\zeta_m,t)}. \tag{102}$$

Let

$$m(t) := \begin{cases} -1, & \text{if } t < \zeta_0 \\ \sup\{m \in \mathbb{N}_0 | \zeta_m \leq t\}, & \text{otherwise.} \end{cases} \tag{103}$$

We then have

$$\sum_{\substack{m\in\mathbb{N}_0;\\ \zeta_m \leq t}} e^{-\beta_*\tau_D n(\zeta_m,t)} = \sum_{m=0}^{m(t)} e^{-\beta_*\tau_D n(\zeta_m,t)}. \tag{104}$$

Since

$$n(\zeta_m, t) \geq m(t) - m \tag{105}$$

then

$$\begin{aligned}\sum_{m=0}^{m(t)} e^{-\beta_*\tau_D n(\zeta_m,t)} &= \sum_{m=0}^{m(t)} e^{-\beta_*\tau_D (m(t)-m)} \\ &= \sum_{m=0}^{m(t)} e^{-\beta_*\tau_D m} \leq \frac{1}{1-e^{-\beta_*\tau_D}}. \end{aligned} \tag{106}$$

The relations (100), (102), (104), (106) yield the desired bound (93). ■

We can now finalize the proof of Theorem 2. First observe that the first term on the RHS of (80) is bounded by

$$e^{\kappa_*(\omega_1+\omega_2)} e^{-(\omega_1 - \frac{\omega_1+\omega_2}{T_*})t} V(x(0)) \tag{107}$$

where we have exploited the equality $|\bar{\Theta}(0,t)| = t - |\bar{\Xi}(0,t)|$, and the inequality (95). Moreover, $\beta_* = \omega_1 - (\omega_1+\omega_2)/T_*$. Hence, from (80) and (93), one sees that the Lyapunov function $V(x)$ computed along any trajectory $x(t)$ satisfies

$$\begin{aligned}V(x(t)) &\leq e^{\kappa_*(\omega_1+\omega_2)} e^{-\beta_* t} V(x(0)) \\ &\quad + \gamma_* \left[1 + 2e^{\kappa_*(\omega_1+\omega_2)} \frac{e^{\beta_*\tau_D\eta}}{1-e^{-\beta*\tau_D}}\right] \|w_t\|_\infty^2. \end{aligned} \tag{108}$$

Letting $\delta_* := \gamma_*/\alpha_1$ we readily get

$$\begin{aligned}\|x(t)\| &\leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{\frac{\kappa_*(\omega_1+\omega_2)}{2}} e^{-(\beta_*/2)t} \|x(0)\| \\ &\quad + \sqrt{\delta_*} \left[1 + 2e^{\kappa_*(\omega_1+\omega_2)} \frac{e^{\beta_*\tau_D\eta}}{1-e^{-\beta_*\tau_D}}\right]^{\frac{1}{2}} \|w_t\|_\infty. \end{aligned} \tag{109}$$

Note that $\delta_* = ((\gamma_3+\gamma_2\sigma)^2/(\gamma_1-\gamma_2\sigma)) \max\{(\alpha_2/(\alpha_1(\gamma_1 - \gamma_2\sigma))), (1/4\gamma_2)\}$ is a positive constant, i.e., independent of the process initial condition and the disturbance w. Since Δ_* is a positive constant, then also $\kappa_* = \kappa + (1+\eta)\Delta_*$ and $\beta_* = \omega_1 - (\omega_1+\omega_2)((\Delta_*/\tau_D) + (1/T))$ are independent of the process initial condition and w. Thus ISS follows at once. ■

Proof of Corollary 1: It follows directly from the one of Theorem 2 by: i) noting that the rightmost term in (80) is zero; and ii) exploiting the equality $|\bar{\Theta}(0,t)| = t - |\bar{\Xi}(0,t)|$, and the inequality (95) with $\tau = 0$, where the sets $\bar{\Xi}(\tau, t)$ and $\bar{\Theta}(\tau, t)$ are defined in (61) and (62), respectively. ■

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