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Published in:
IEEE Transactions on Automatic Control

DOI:
[10.1109/TAC.2020.3005156](https://doi.org/10.1109/TAC.2020.3005156)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2021

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Wu, C., Van Der Schaft, A., & Chen, J. (2021). Stabilization of Port-Hamiltonian Systems Based on Shifted Passivity via Feedback. *IEEE Transactions on Automatic Control*, 66(5), 2219-2226. [9127091].
<https://doi.org/10.1109/TAC.2020.3005156>

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Technical Notes and Correspondence

Stabilization of Port-Hamiltonian Systems Based on Shifted Passivity via Feedback

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Abstract—In this article, stabilization of port-Hamiltonian (pH) systems is studied in the context of shifted passivity. The pH systems, in general, are not shifted passive with respect to the given Hamiltonian and output. Therefore, we consider enforcing the property of shifted passivity by designing a state feedback, namely, shifted passivity via feedback. This introduced concept brings more insight into the stabilization of pH systems, and a set of partial differential equations is derived such that its solution specifies a state feedback achieving shifted passivity via feedback. Based on the framework of shifted passivity via feedback, a group of sufficient conditions is given such that a proposed proportional control with respect to a designed power shaping output can achieve closed-loop asymptotic stability. Some extended control design is provided to relax the required conditions.

Index Terms—Feedback equivalence, interconnection and damping assignment passivity-based control (IDA-PBC), port-Hamiltonian (pH) systems, shifted passivity, stabilization.

I. INTRODUCTION

Port-Hamiltonian (pH) systems provide a natural modeling framework for nonlinear multiphysics systems due to its emphasis on energy and interconnection between subsystems [1]. As a result, the stabilization of pH systems is widely studied in control literature [1]–[7]. A range of well-established methods have been proposed, such as, control by interconnection [5], [8], PID passivity-based control [6], power shaping [9], [10], and the interconnection and damping assignment passivity-based control (IDA-PBC) [7], [11], [12]. In general, the aforementioned methods require solving a set of partial differential equations (PDEs), for example, the well-known matching equation arising in IDA-PBC.

Manuscript received February 11, 2020; accepted June 21, 2020. Date of publication June 26, 2020; date of current version April 26, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 61973272, in part by the Fundamental Research Funds for the Central Universities of China under Grant 2019QNA5012, and in part by the scholarship from China Scholarship Council (CSC) under Grant 201706320269. Recommended by Associate Editor Y. Le Gorrec. (Corresponding author: Jian Chen.)

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Digital Object Identifier 10.1109/TAC.2020.3005156

In this article, we will study the stabilization problem of pH systems from the point view of *shifted passivity* [2]. As an extension of the standard passivity concept, shifted passivity refers to passivity of the incremental model with respect to the deviations of the input and output from their values at the equilibrium [2].

Due to the fact that shifted passivity is of great importance for the stabilization of pH systems [2], many researchers have focused on how to verify the shifted passivity of pH systems. In [13], the authors showed that a class of pH systems is shifted passive provided that the Hamiltonian is convex, and the interconnection, damping, and input matrices are all constant. This result is generalized in [14] by allowing state-dependent interconnection and damping matrices, and a matrix inequality condition is given to ensure shifted passivity of pH systems. In [15], the authors specified a class of shifted passive pH systems, in which all the system matrices are state dependent. It is worthy to point out that the property of shifted passivity in [13]–[15] is defined with respect to a same storage function referred to as the *shifted Hamiltonian*, which is the Bregman divergence of the Hamiltonian with respect to the desired equilibrium [16]. By designing an alternative storage function provided certain integrability conditions hold, conditions for shifted passivity of pH systems with state-dependent system matrices are reported in [17].

All the aforementioned papers deal with a problem of system analysis. Since this article studies the stabilization of pH systems, we consider the design of a feedback such that the resulting system is shifted passive, which will be referred to as shifted passivity via feedback (SPvF) in this article. Naturally, the conditions given in [13]–[15] and [17] can be served as guidelines for the design of such a state feedback. Unlike [13] and [14], we study the general pH systems in which the interconnection, damping, and input matrices are all state dependent, and a set of PDEs will be given such that its solution admits the aforementioned feedback design. In essence, the proposed framework of SPvF is equivalent to the IDA-PBC method since they both rely on transforming the closed-loop system into a new desired pH system. The introduced SPvF framework allows to define a matrix for the studied pH system to measure its distance from shifted passivity. Furthermore, by exploiting this framework, we derive a group of sufficient conditions such that a controller of simplified form, i.e., a proportional control with respect to a designed power shaping output, can ensure the closed-loop stability. To the best of our knowledge, these conditions have not been reported in the existing literature.

The remainder of this article is organized as follows. Section II gives the definition of SPvF of pH systems and the PDEs to solve. In Section III, we propose a simplified controller for pH systems based on the proposed SPvF framework, and sufficient conditions are derived to ensure the closed-loop stability. Section IV provides some extensions to the proposed controller. In Section V, the proposed

simplified controller is applied to a buck–boost converter with constant power load (CPL) [18] with a performance comparison to the IDA-PBC controller given in [19]. Finally, Section VI concludes this article.

II. SHIFTED PASSIVITY VIA FEEDBACK (SPvF)

This section introduces the SPvF framework of pH systems. In order to streamline the presentation, we first recall some basic definitions and conclusions regarding the standard shifted passivity of pH systems. Consider the pH system

$$\begin{aligned}\dot{x} &= [J(x) - R(x)]\nabla H(x) + G(x)u \\ y &= G^T(x)\nabla H(x)\end{aligned}\quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^m$, with $m \leq n$, are the system state, control input, and output, respectively. The matrices $J, R : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, with $J(x) = -J^T(x)$ and $R(x) = R^T(x) \geq 0$, are the interconnection and damping matrices, respectively, and $G : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ is the input matrix with $\text{rank}\{G(x)\} = m$. $H : \mathbb{R}^n \rightarrow \mathbb{R}$ is the Hamiltonian, and is assumed to be strictly convex [20] throughout this article. To simplify the notation in the sequel, we define the matrix $F : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ as

$$F(x) \triangleq J(x) - R(x). \quad (2)$$

Consider a desired equilibrium x^* contained in the set

$$\mathcal{E} \triangleq \{x \in \mathbb{R}^n | G^\perp(x)F(x)\nabla H(x) = 0\} \quad (3)$$

where $G^\perp : \mathbb{R}^n \rightarrow \mathbb{R}^{(n-m) \times n}$ is a full-rank left annihilator of $G(x)$, that is, $G^\perp(x)G(x) = 0$. According to [2], we define the *steady-state input–output* relation of (1) as

$$\begin{aligned}\{(u^*, y^*) | \exists x^* \in \mathcal{E}, \text{ s.t. } y^* &= G^T(x^*)\nabla H(x^*) \\ F(x^*)\nabla H(x^*) + G(x^*)u^* &= 0\}\end{aligned}\quad (4)$$

where u^* and y^* denote the steady-state input and steady-state output, respectively. Then, the property of shifted passivity is formally defined as follows.

Definition 1 (Shifted passivity): The pH system (1) is shifted passive with respect to the steady-state input u^* and steady-state output y^* if the mapping $(u - u^*) \rightarrow (y - y^*)$ is passive, i.e., there exists a function $H_s : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ such that

$$\dot{H}_s(x) = \nabla^T H_s(x)\dot{x} \leq (u - u^*)^T(y - y^*) \quad (5)$$

for all state-input pairs $(x, u) \in \mathbb{R}^n \times \mathbb{R}^m$.

Theorem 1 ([13]): If the system matrices $F(x)$ and $G(x)$ are constant, the pH system (1) is shifted passive with respect to the following storage function, namely, the shifted Hamiltonian defined as

$$H_s(x) \triangleq H(x) - \tilde{x}^T \nabla H(x^*) - H(x^*) \quad (6)$$

where $\tilde{x} \triangleq x - x^*$, $x^* \in \mathcal{E}$.

The readers are referred to [13] for the detailed proof of Theorem 1. As pointed out in [2], if $F(x)$ and $G(x)$ are state dependent, the mapping $(u - u^*) \rightarrow (y - y^*)$ is generally not passive. This motivates us to consider the problem of transforming a nonshifted-passive system into a shifted passive system by the application of a state-feedback. Therefore, we give the following definition, which is based on a generalization of Definition 1 and Theorem 1.

Definition 2 (SPvF): The pH system (1) is shifted passive via feedback if there exists a feedback

$$u = \alpha(x) + v, \quad \alpha : \mathbb{R}^n \rightarrow \mathbb{R}^m \quad (7)$$

where $v \in \mathbb{R}^m$ is the virtual input such that the closed-loop system reduces to

$$\dot{x} = F_d(x)\nabla H_{ts}(x) + G(x)(v - u^*) \quad (8)$$

for some $F_d : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, $F_d + F_d^T \leq 0$, and $H_{ts} : \mathbb{R}^n \rightarrow \mathbb{R}$ denotes the new shifted Hamiltonian with respect to the total energy function $H_t(x) \triangleq H(x) + H_e(x)$ with $H_e : \mathbb{R}^n \rightarrow \mathbb{R}$ denoting the added energy function, namely

$$H_{ts}(x) \triangleq H_t(x) - \tilde{x}^T \nabla H_t(x^*) - H_t(x^*). \quad (9)$$

Here, the added energy function $H_e(x)$ satisfies $\nabla H_e(x^*) = 0$, $\nabla^2 H(x^*) + \nabla^2 H_e(x^*) > 0$.

Remark 1: If the input matrix $G(x)$ is constant, then by defining a new output $y_t \triangleq G^T \nabla H_t(x)$, the derivative of H_{ts} along the trajectories of (8) satisfies

$$\begin{aligned}\dot{H}_{ts} &= \nabla^T H_{ts} F_d \nabla H_{ts} + (v - u^*)^T G^T \nabla H_{ts}(x) \\ &\leq (v - u^*)^T G^T (\nabla H_t(x) - \nabla H_t(x^*)) \\ &= (v - u^*)^T (y_t - y^*)\end{aligned}\quad (10)$$

where we use the fact that $G^T \nabla H_t(x^*) = G^T \nabla H(x^*) = y^*$. In this case, (10) implies that the mapping $(v - u^*) \rightarrow (y_t - y^*)$ is passive. If $G(x)$ depends on the state, the inequality in (10) still holds, but not the second equality because $y^* \neq G^T(x) \nabla H_t(x^*)$. For this case, we can define the new output as $y_{ts} = G^T(x) \nabla H_{ts}(x)$, with corresponding steady-state output $y_{ts}^* = 0$, and then, the mapping $(v - u^*) \rightarrow (y_{ts} - y_{ts}^*)$ is passive.

Note that $F_d(x^*) \nabla H_{ts}(x^*) = 0$ in (8), together with (4), this implies that $G(x^*) \alpha(x^*) = 0$. Since $G(x)$ is of full rank, we have $\alpha(x^*) = 0$, and thus, there exists a matrix $\Lambda : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times n}$ such that

$$\alpha(x) = \Lambda(x)\tilde{x}. \quad (11)$$

From (11), the following definition is given.

Definition 3 (Distance from shifted passivity): If the pH system (1) with a prior fixed $F_d(x)$ is shifted passive via feedback with respect to the feedback $u = \Lambda(x)\tilde{x} + v$, then the distance of (1) from shifted passivity (DfSP) is defined as $[\Lambda_{ij}(x)]$, which denotes a matrix in $\mathbb{R}^{m \times n}$ whose ij th element is $|\Lambda_{ij}(x)|$. Here, $\Lambda_{ij}(x)$ denotes the ij th element of $\Lambda(x)$.

Remark 2: According to the aforementioned definition, the matrix $\Lambda(x)$ is named as the DfSP matrix throughout this article. Here, it is worth pointing out that this DfSP matrix $\Lambda(x)$ is not uniquely defined for the same $F_d(x)$, and the different selections of $F_d(x)$ will lead to different $\Lambda(x)$.

The next result gives a set of PDEs, such that the solution to which specifies the DfSP matrix $\Lambda(x)$. To facilitate the analysis, the mappings $N, W : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ are defined such that

$$\begin{aligned}\nabla H(x) - \nabla H(x^*) &= N(x)\tilde{x} \\ F(x)\nabla H(x) + G(x)u^* &= W(x)\tilde{x}.\end{aligned}\quad (12)$$

Proposition 1: For the pH system (1), consider a desired equilibrium $x^* \in \mathcal{E}$. If there exists a matrix $F_d : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, $F_d + F_d^T \leq 0$, and a mapping $\Lambda : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times n}$, such that the mapping $P : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ defined in

$$F_d(x)(P(x) + N(x)) = W(x) + G(x)\Lambda(x) \quad (13)$$

satisfies the following conditions:

$$(i) \quad \frac{\partial P(x)\tilde{x}}{\partial x} = \left(\frac{\partial P(x)\tilde{x}}{\partial x} \right)^T \quad (14)$$

$$(ii) \nabla^2 H(x^*) + \frac{\partial P(x)\tilde{x}}{\partial x} \Big|_{x=x^*} > 0. \quad (15)$$

Then, the pH system (1) is shifted passive via feedback with $u = \Lambda(x)\tilde{x} + v$.

Proof: By substituting $u = \Lambda(x)\tilde{x} + v$ into (1), we arrive at

$$\begin{aligned} \dot{x} &= F(x)\nabla H(x) + G(x)(u^* + \Lambda(x)\tilde{x}) + G(x)(v - u^*) \\ &= (W + G\Lambda)\tilde{x} + G(v - u^*) \\ &= F_d(\nabla H(x) - \nabla H(x^*) + P\tilde{x}) + G(v - u^*) \end{aligned} \quad (16)$$

where (12) and (13) are used. By virtue of Poincaré's lemma, (14) ensures that $P(x)\tilde{x}$ is the Jacobian of a function $H_e : \mathbb{R}^n \rightarrow \mathbb{R}$. Hence, (16) can be rewritten as

$$\begin{aligned} \dot{x} &= F_d(x)(\nabla H_s(x) + \nabla H_e(x)) + G(x)(v - u^*) \\ &= F_d(x)\nabla H_{ts}(x) + G(x)(v - u^*) \end{aligned} \quad (17)$$

where $H_{ts}(x)$ is defined in (9). Additionally, note that $\nabla H_s(x^*) + \nabla H_e(x^*) = 0$, together with condition (15), ensures that x^* is a (local) minimum of the function $H_{ts}(x)$. Therefore, the pH system (1) is shifted passive via feedback with respect to the feedback $u = \Lambda(x)\tilde{x} + v$. ■

From Definition 2 and Proposition 1, the stabilization of pH systems is related to finding a feedback term $\Lambda(x)\tilde{x}$ such that the resulting system is shifted passive, and the DfSP matrix $\Lambda(x)$ is obtained by solving the PDEs in (14) subject to the constraint in (15).

Remark 3 (Comparison with the IDA-PBC method): In (17), if $v = u^*$, that is, consider the following control input:

$$u = u^* + \Lambda(x)\tilde{x} \quad (18)$$

then the closed loop of (1) becomes a new pH system, which is also the case for the IDA-PBC method. In essence, the conditions (13) and (14) are equivalent to the matching equation arising in the IDA-PBC [7], [21], i.e.,

$$G^\perp(x)F(x)\nabla H(x) = G^\perp F_d(x)\nabla H_{ts}(x) \quad (19)$$

for some full-rank left annihilator $G^\perp(x)$. The corresponding IDA-PBC controller is

$$u = (G^T G)^{-1} G^T (F_d \nabla H_{ts} - F \nabla H) \quad (20)$$

which can be rewritten as the form of (18) according to the mean value theorem for vector-valued mappings. Therefore, the proposed SPvF framework, in essence, is equivalent to the IDA-PBC method. However, as an advantage of SPvF, it offers an interpretation of the desired matrix $\Lambda(x)$ as shown in Definition 3, and it facilitates a simplified controller given in Section III.

III. CONTROLLER OF SIMPLIFIED FORM

In this section, we study a particular case that the desired system matrix $F_d(x)$ is invertible for all the x in the domain under investigation. A group of sufficient conditions regarding the DfSP matrix $\Lambda(x)$ is derived such that the following control law:

$$u = u^* + (kI_m - G^T F_d^{-T} G)^{-1} G^T F_d^{-T} W \tilde{x} \quad (21)$$

ensures that x^* is asymptotically stable for the pH system (1) satisfying conditions (14) and (15) in Proposition 1. Here, k is a positive constant, and I_m denotes the identity matrix in $\mathbb{R}^{m \times m}$.

Note that the control law (21) does not depend on the information of the DfSP matrix $\Lambda(x)$. Since $\Lambda(x)$ is the solution to the PDEs in Proposition 1, the corresponding SPvF controller (18) may have a complex analytical form, which will stymie its practical application.

Therefore, the proposed controller (21) offers a simple alternative when $\Lambda(x)$ has a complex expression.

To facilitate the presentation, the following mappings are defined:

$$\begin{aligned} \Pi : \mathbb{R}^n &\rightarrow \mathbb{R}^{m \times n} \\ \Pi(x) &\triangleq (kI_m - G^T F_d^{-T} G)^{-1} G^T F_d^{-T} W \\ q : \mathbb{R}^n &\rightarrow \mathbb{R}^m, \quad q \triangleq \Pi(x)\tilde{x} \\ M : \mathbb{R}^n &\rightarrow \mathbb{R}^{n \times n}, \quad M(x) \triangleq F_d^{-1} W \\ S : \mathbb{R}^n &\rightarrow \mathbb{R}^{n \times m}, \quad S(x) \triangleq F_d^{-1} G \\ R_d : \mathbb{R}^n &\rightarrow \mathbb{R}^{n \times n}, \quad R_d(x) \triangleq -\frac{1}{2}(F_d + F_d^T). \end{aligned} \quad (22)$$

To streamline the subsequent discussions, we first consider a restricted case that $R_d(x)$ and $W(x)$ are all of full rank. Then, these restrictions are removed to obtain a more general result.

Remark 4: The proposed stabilization law (21) is, in essence, a proportional control of the form

$$u = -k y_{dps} + u^* \quad (23)$$

where y_{dps} denotes a designed *power shaping output* as

$$y_{dps} \triangleq -G^T(x)F_d^{-T}(x)\dot{x}. \quad (24)$$

Here, we use the subscript “dps” to distinguish (24) from the standard power shaping output [10]

$$y_{ps} \triangleq -G^T(x)F^{-T}(x)\dot{x} \quad (25)$$

which requires the system matrix $F(x)$ to be invertible. It is worth pointing out that k in (21) can be replaced by a positive definite matrix in the general case. Here, with a minor loss of generality, we select it as a constant to simplify the subsequent analysis.

Assumption 1: The pH system (1) is shifted passive via feedback with the mappings $F_d(x)$, $\Lambda(x)$ specified in Proposition 1. There exists a nonopen set $\mathcal{D} \subset \mathbb{R}^n$ containing the desired equilibrium x^* , such that for $x \in \mathcal{D}$, $R_d(x) > 0$, $W(x)$ is of full rank, and the following condition holds:

$$I_m + \frac{G^T W^{-T} \Lambda^T}{2} + \frac{\Lambda W^{-1} G}{2} > 0. \quad (26)$$

Proposition 2: Suppose that the pH system (1) satisfies Assumption 1. Then, there exists a positive constant $\bar{k} \in \mathbb{R}_{>0}$, such that for all $0 < k \leq \bar{k}$, the control law (21) ensures that x^* is an (locally) asymptotically stable equilibrium of the closed-loop dynamics of (1).

Proof: Since $R_d(x) > 0$ for $x \in \mathcal{D}$, the matrix $F_d(x)$ is invertible. Therefore, it is ensured that the control law in (21) is properly defined. From (22), note that $u = u^* + q(x)$ and $\Pi(x)$ can be rewritten as

$$\Pi(x) = (kI_m - S^T F_d S)^{-1} S^T F_d M. \quad (27)$$

By using (21) and (22), the closed-loop dynamics of (1) with the control input (21) is obtained as

$$\dot{x} = F_d(x)(M(x)\tilde{x} + S(x)q(x)). \quad (28)$$

From (13)–(17) in Proposition 1, we can conclude that

$$\nabla H_{ts}(x) = (M(x) + S(x)\Lambda(x))\tilde{x}. \quad (29)$$

That is, if $q = \Lambda\tilde{x}$, (28) is equivalent to $\dot{x} = F_d(x)\nabla H_{ts}(x)$. According to the definition of $q(x)$ in (22), it is concluded that

$$S^T F_d (M\tilde{x} + Sq) - kq = 0. \quad (30)$$

Then, by taking the derivative of $H_{ts}(x)$ along the trajectories of (28) and using (29) and (30), we have

$$\begin{aligned}\dot{H}_{ts} &= -(M\tilde{x} + Sq)^T R_d(M\tilde{x} + Sq) \\ &\quad + (\Lambda\tilde{x} - q)^T S^T F_d(M\tilde{x} + Sq) \\ &= -(M\tilde{x} + Sq)^T R_d(M\tilde{x} + Sq) + k\tilde{x}^T \Lambda^T q - kq^T q \\ &= -[\tilde{x}^T \ q^T] \Psi(x) [\tilde{x}^T \ q^T]^T \\ &= -\tilde{x}^T [I_n \ \Pi^T(x)] \Psi(x) [I_n \ \Pi^T(x)]^T \tilde{x}\end{aligned}\quad (31)$$

where I_n denotes the identity matrix in $\mathbb{R}^{n \times n}$, and

$$\Psi(x) \triangleq \begin{bmatrix} M^T R_d M & M^T R_d S - k\Lambda^T/2 \\ S^T R_d M - k\Lambda/2 & S^T R_d S + kI_m \end{bmatrix}. \quad (32)$$

Since $R_d(x) > 0$ and $W(x)$ is of full rank, this implies $M^T R_d M > 0$ for $x \in \mathcal{D}$. After a straightforward calculation, the Schur complement [22] of $M^T R_d M$ in Ψ is obtained as

$$\begin{aligned}\Psi_{schur} &= k(I_m + S^T M^{-T} \Lambda^T/2 + \Lambda M^{-1} S/2 \\ &\quad - k\Lambda M^{-1} R_d^{-1} M^{-T} \Lambda^T/4) \\ &= k(I_m + G^T W^{-T} \Lambda^T/2 + \Lambda W^{-1} G/2 \\ &\quad - k\Lambda M^{-1} R_d^{-1} M^{-T} \Lambda^T/4)\end{aligned}\quad (33)$$

where the definition of $M(x)$ and $S(x)$ are utilized for the second equality. From (26) and (33), there exists a small enough $\bar{k} > 0$, such that

$$\begin{aligned}I_m + G^T W^{-T} \Lambda^T/2 + \Lambda W^{-1} G/2 \\ > \bar{k}\Lambda M^{-1} R_d^{-1} M^{-T} \Lambda^T/4\end{aligned}\quad (34)$$

for $x \in \mathcal{D}$. Thus, $\Psi_{schur} > 0$ for $0 < k \leq \bar{k}$. Based on the aforementioned analysis, it is concluded that $\Psi(x)$ is positive definite for $0 < k \leq \bar{k}$, and $x \in \mathcal{D}$. Thus, $[I_n \ \Pi^T(x)] \Psi(x) [I_n \ \Pi^T(x)]^T > 0$ for $x \in \mathcal{D}$ because $[I_n \ \Pi^T(x)]^T$ is of full rank. Therefore, $\dot{H}_{ts}(x) < 0$ for $x \in \mathcal{D}/x^*$, and $\dot{H}_{ts}(x^*) = 0$. From Proposition 1, x^* is a (local) minimum of the function $H_{ts}(x)$. Hence, x^* is an asymptotically stable equilibrium. ■

Remark 5: We make the key observation that the condition (26) can be satisfied with a small enough DfSP matrix $\Lambda(x)$, in the sense that every element of $[\Lambda_{ij}(x)]$ is small enough. Therefore, we can claim that the control law (21) is suitable to stabilize the pH systems that are “close” enough to shifted passivity according to Definition 3.

Assumption 2: The pH system (1) is shifted passive via feedback with the mappings $F_d(x)$, $\Lambda(x)$ specified in Proposition 1. There exists a nonopen set $\mathcal{D} \subset \mathbb{R}^n$ containing the desired equilibrium x^* , such that for $x \in \mathcal{D}$, $F_d(x)$ is invertible, and the following condition holds:

$$\begin{aligned}I_m + B^T A^\dagger \Lambda^T + \Lambda A^\dagger B - k\Lambda A^\dagger \Lambda^T/4 &\geq 0 \\ (I_n - AA^\dagger) \Lambda^T/2 &= 0\end{aligned}\quad (35)$$

for some $k > 0$, with $A \triangleq M^T R_d M$, $B \triangleq M^T R_d S$, and $A^\dagger$ denoting the generalized inverse [22] of A .

Proposition 3: Suppose that the pH system (1) satisfies Assumption 2. Then, the control law in (21) ensures that x^* is a stable equilibrium of the closed-loop dynamics of (1), and it is (locally) asymptotically stable if the largest invariant set under the closed-loop dynamics (28) contained in

$$\{x \in \mathbb{R}^n | [\tilde{x}^T \ q^T] \Psi(x) [\tilde{x}^T \ q^T]^T = 0\} \quad (36)$$

equals $\{x^*\}$.

Proof: Based on the proof procedure of Proposition 2 and by utilizing the generalized Schur complement argument [22], the condition (35) ensures that $\Psi(x)$ is semipositive definite for $x \in \mathcal{D}$, which implies $\dot{H}_{ts} \leq 0$. Additionally, x^* is a (local) minimum of the function $H_{ts}(x)$ from Proposition 1. Thus, x^* is a stable equilibrium. Furthermore, if the largest invariant set condition (36) holds, x^* is asymptotically stable by invoking LaSalle’s invariance principle [23]. ■

IV. EXTENDED RESULTS

A. Functional Approximation-Based Controller

This part gives a constructive procedure for complexity reduction of the SPvF controller (18) while relaxing the condition (26) in Assumption 1. This procedure is inspired by Remark 5.

Consider the scenario that an obtained SPvF controller (18) has a complex analytic form and the condition (26) in Assumption 1 does not hold. By adopting some well-established functional approximation methods, we can construct an approximation for $\Lambda(x)$, denoted as $\Lambda_0(x)$, such that $\Lambda_0(x)$ has a simpler expression that suits to be adopted for practical application. That is, $\Lambda(x)$ is split into two terms as

$$\Lambda(x) = \Lambda_0(x) + \Lambda_e(x) \quad (37)$$

where $\Lambda_e(x)$ denotes the approximation error. With a close enough approximation result, it is ensured that the absolute value of every element of $\Lambda_e(x)$ is small enough such that

$$I_m + \frac{G^T(W + G\Lambda_0)^{-T} \Lambda_e^T}{2} + \frac{\Lambda_e(W + G\Lambda_0)^{-1} G}{2} > 0. \quad (38)$$

Proposition 4: Provided that the pH system (1) is shifted passive via feedback with the mappings $F_d(x)$, $\Lambda(x)$, and $H_{ts}(x)$ defined in Proposition 1. For $x \in \mathcal{D}$, $W(x) + G(x)\Lambda_0(x)$ is of full rank, $R_d(x) > 0$, and the condition (38) holds. Then, the control law

$$\begin{aligned}u &= u^* + \Lambda_0(x)\tilde{x} + q_e(x) \\ q_e &= (kI_m - G^T F_d^{-T} G)^{-1} G^T F_d^{-T} (W + G\Lambda_0)\tilde{x}\end{aligned}\quad (39)$$

ensures that there exists $\bar{k} \in \mathbb{R}_{>0}$, such that x^* is an (locally) asymptotically stable equilibrium of the closed-loop dynamics for all $0 < k \leq \bar{k}$.

Proof: The closed-loop dynamics of (1) with the control input (39) is obtained as

$$\dot{x} = F_d(x)(\bar{M}(x)\tilde{x} + S(x)q_e(x)) \quad (40)$$

where $\bar{M}(x) = M(x) + S(x)\Lambda_0(x)$. Based on a similar analysis to Proposition 2, the derivatives of $H_{ts}(x)$ along the trajectories of (40) is

$$\dot{H}_{ts} = -[\tilde{x}^T \ q_e^T] \Psi_e(x) [\tilde{x}^T \ q_e^T]^T \quad (41)$$

with

$$\Psi_e(x) \triangleq \begin{bmatrix} \bar{M}^T R_d \bar{M} & \bar{M}^T R_d S - k\Lambda_e^T/2 \\ S^T R_d \bar{M} - k\Lambda_e/2 & S^T R_d S + kI_m \end{bmatrix}. \quad (42)$$

Note that $\bar{M}^T R_d \bar{M} > 0$, together with the condition in (38), it implies that there exists $\bar{k} \in \mathbb{R}_{>0}$, such that if $0 < k \leq \bar{k}$, then $\Psi_e(x) > 0$ for $x \in \mathcal{D}$. Hence, x^* is asymptotically stable equilibrium. ■

Remark 6: By transforming an IDA-PBC controller into the form of (18), it can also be simplified by applying the aforementioned method. The similar idea has been discussed in [19]. As a contrast, our contribution is related to the design of $q_e(x)$ in (39), which compensates the impact of the approximation error for the closed-loop stability.

B. Generalization to Affine Nonlinear Systems

The proposed controller (21) can also be utilized to stabilize the affine nonlinear systems in general form, i.e.,

$$\dot{x} = f(x) + G(x)u \quad (43)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, the mappings $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$, $G: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ are continuous and sufficiently differentiable.

It is worthy of pointing out that the terminology ‘‘SPvF’’ is not reliable anymore in this case since, unlike the pH system, (43) is not passive in general.

For the system (43), consider to stabilize an assignable equilibrium $x^* \in \mathbb{R}^n$. That is, there exists $u^* \in \mathbb{R}^m$ such that

$$f(x^*) + G(x^*)u^* = 0. \quad (44)$$

By (44), there exists a mapping $\bar{W}: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ such that

$$f(x) + G(x)u^* = \bar{W}(x)\tilde{x}, \quad \tilde{x} \triangleq x - x^*. \quad (45)$$

Based on Propositions 1 and 2, the following corollary can be claimed.

Corollary 1: Consider the system (43) with the desired equilibrium x^* . If there exists a nonopen set $\mathcal{D} \subset \mathbb{R}^n$ containing the desired equilibrium x^* , such that for $x \in \mathcal{D}$, $\bar{W}(x)$ is nonsingular, and there exist mappings $F_d: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, $F_d + F_d^T > 0$, $\Lambda: \mathbb{R}^n \rightarrow \mathbb{R}^{m \times n}$, and $\bar{P}: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ satisfy the following conditions:

$$F_d(x)\bar{P}(x) = \bar{W}(x) + G(x)\Lambda(x) \quad (46)$$

$$\frac{\partial \bar{P}(x)\tilde{x}}{\partial x} = \left(\frac{\partial \bar{P}(x)\tilde{x}}{\partial x} \right)^T \quad (47)$$

$$\frac{\partial \bar{P}(x)\tilde{x}}{\partial x} \Big|_{x=x^*} > 0 \quad (48)$$

$$I_m + \frac{G^T \bar{W}^{-T} \Lambda^T}{2} + \frac{\Lambda \bar{W}^{-1} G}{2} > 0 \quad (49)$$

for all $x \in \mathcal{D}$. Then, there exists a positive constant $\bar{k} \in \mathbb{R}_{>0}$, such that for all $0 < k \leq \bar{k}$, the control law (21) ensures that x^* is an (locally) asymptotically stable equilibrium of the closed-loop dynamics of (43).

The following result specifies a class of single-input affine nonlinear systems such that (49) in Corollary 1 can be verified without knowing the exact information of $\Lambda(x)$.

Proposition 5: Consider the system (43) with the desired equilibrium x^* and $u \in \mathbb{R}$. Provided there exists a nonopen set $\mathcal{D} \subset \mathbb{R}^n$ containing the desired equilibrium x^* , such that for $x \in \mathcal{D}$, $\bar{W}(x)$ is nonsingular and Metzler (i.e., every off-diagonal entry of $\bar{W}(x)$ is nonnegative) [24], and every entry of $G(x) \in \mathbb{R}$ is nonnegative and $G(x) \neq 0$. Then, (49) in Corollary 1 holds if every entry of $\Lambda(x) \in \mathbb{R}^{1 \times n}$ is nonpositive for $x \in \mathcal{D}$.

Proof: Since $\bar{W}(x)$ is Metzler for $x \in \mathcal{D}$, then the matrix $\Theta(x) = I_n + \varepsilon \bar{W}(x)$ is a nonnegative matrix (i.e., every entry of $\Theta(x)$ is nonnegative) [24] for $x \in \mathcal{D}$ with a sufficiently small scalar $\varepsilon > 0$. Here, I_n denotes the identity matrix in $\mathbb{R}^{n \times n}$. Then, according to [25, Lemma 6.2.1], the inverse of $\bar{W}(x)$ can be computed as

$$\bar{W}^{-1}(x) = -\frac{1}{\varepsilon}(I_n - \Theta(x))^{-1} = -\frac{1}{\varepsilon} \sum_{k=0}^{\infty} \Theta^k(x). \quad (50)$$

Hence, $\bar{W}^{-1}(x)$ is a nonpositive matrix for all $x \in \mathcal{D}$.

Since a single input case is considered, $\Lambda(x)\bar{W}^{-1}(x)G(x)$ is a scalar, and satisfies

$$\Lambda(x)\bar{W}^{-1}(x)G(x) \geq 0, \quad \forall x \in \mathcal{D} \quad (51)$$

Fig. 1. Buck-boost converter with a CPL.

due to that $\Lambda(x), \bar{W}^{-1}(x)$ have nonpositive entries, and $G(x)$ has nonnegative entries. Therefore,

$$1 + \Lambda(x)\bar{W}^{-1}(x)G(x) > 0, \quad \forall x \in \mathcal{D}. \quad (52)$$

That is, the condition (49) holds for $x \in \mathcal{D}$. ■

In Proposition 5, the restriction that $\Lambda(x)$ has nonpositive entries is relatively mild. This is because that the control law $u = u^* + \Lambda(x)\tilde{x}$ is a negative feedback in this case due to that $G(x)$ has nonnegative entries. Generally speaking, $\Lambda(x)$ is allowed to have positive entries, while the closed-loop dynamics are still stable. However, this will deteriorate the transient control performance.

Based on the aforementioned analysis, if a controlled system satisfied the conditions in Proposition 5, it is possible to applying the proposed control law (21) without solving the matching equations (46) and (47) since the condition (49) trivially holds with a negative feedback. Toward this end, we can first select a nonsingular $F_d(x)$ according to an involutive condition given in [26, Prop. 3] that ensures the solvability of (46) and (47), and then, choose a k in (21) such that the Jacobian matrix of the closed-loop system is Hurwitz at $x = x^*$. This Hurwitz condition guarantees that the locally stability of the equilibrium x^* , and it is also related to the local minimum condition given in (48).

V. APPLICATION EXAMPLE

In this section, we apply the proposed simplified controllers in Section III to a buck-boost converter with a CPL. Fig. 1 shows the circuit of this converter system. Its dynamics are given by [18]

$$\begin{aligned} L\dot{i} &= -(1-u)v + uE \\ C\dot{v} &= (1-u)i - \frac{P_c}{v} \\ \text{s.t. } i &> 0, v > 0 \end{aligned} \quad (53)$$

where i is the inductor current, v is the capacitor voltage, E is the input voltage, P_c is the rated power of the CPL, and $u(t) \in [0, 1]$ is the control input, i.e., the duty cycle of the converter. Here, L and C are the inductance and capacitance parameters of the converter.

In order to facilitate the control design, the following change of coordinates $x = [x_1, x_2]^T$ is introduced [19]:

$$x_1 \triangleq \frac{1}{E} \sqrt{\frac{L}{C}} i, \quad x_2 \triangleq \frac{v}{E}. \quad (54)$$

Furthermore, similar to [19], we consider the system dynamics in the scaled time $\tau = \frac{t}{\sqrt{LC}}$. Then, (53) is equivalent to the following normalized model:

$$\begin{aligned} \frac{dx_1}{d\tau} &= -(1-u)x_2 + u \\ \frac{dx_2}{d\tau} &= (1-u)x_1 - \frac{D}{x_2} \\ \text{s.t. } x_1 &> 0, x_2 > 0 \end{aligned} \quad (55)$$

where $D = \sqrt{\frac{L}{C} \frac{P_c}{E^2}}$. With a minor abuse of notations, $(\cdot)$ is utilized to denote $\frac{d(\cdot)}{d\tau}$ in the subsequent analysis. Note that the pH structure is preserved in (55), specifically, (55) can be rewritten as

$$\dot{x} = F(x)\nabla H(x) + G(x)u, \text{ s.t. } x_1 > 0, x_2 > 0 \quad (56)$$

where the Hamiltonian $H(x) = \frac{1}{2}x^T x$ is a strictly convex function, and

$$F(x) = \begin{bmatrix} 0 & -1 \\ 1 & -\frac{D}{x_2^2} \end{bmatrix}, \quad G(x) = \begin{bmatrix} 1+x_2 \\ -x_1 \end{bmatrix}. \quad (57)$$

The assignable equilibriums of (56) is in the following set:

$$\mathcal{E} \triangleq \left\{ x^* | x_1^* - \frac{D}{x_2^*} - D = 0 \right\}. \quad (58)$$

The corresponding steady input is $u^* = x_2^*/(x_2^* + 1)$. For the simulation analysis, we choose $D = 1$ according to [19], and the equilibrium to stabilize is $x^* = [1.25, 4]^T$.

Note that both the matrices $F(x)$ and $G(x)$ are state dependent in this case. Furthermore, it is worth pointing out that the desired equilibrium x^* is not stable with the steady-state input $u = u^*$. This means that the controlled system is not shifted passive according to Definition 1.

Therefore, we consider to stabilize the converter system by using the control design in (21). For a comparison purpose, an IDA-PBC controller for this same converter system given in [19] is also analyzed in the subsequent simulation. From Remark 3, the IDA-PBC method is equivalent to the introduced SPvF framework, and an IDA-PBC controller can be transformed into the form of (18), that is, the corresponding DfSP-matrix $\Lambda(x)$ can be obtained. Then, we can verify if Assumption 1 holds for this $\Lambda(x)$ in order to apply (21).

In [19], the designed system matrix $F_d(x)$ is

$$F_d(x) = \begin{bmatrix} -\frac{x_2}{x_1} & -\frac{2x_2}{x_2+1} \\ \frac{2x_2}{x_2+1} & -\frac{2x_1}{(x_2+1)^2} \end{bmatrix}. \quad (59)$$

Note that $F_d(x)$ is invertible, and $F_d + F_d^T < 0$, i.e., $R_d > 0$ for $x \in \mathbb{R}_{>0}^2$. Furthermore, from (12), (56), and (57), some straightforward

calculations yield

$$W(x) = \begin{bmatrix} 0 & -(1-u^*) \\ 1-u^* & \frac{D}{x_2 x_2^*} \end{bmatrix}. \quad (60)$$

Since $u^* = x_2^*/(1+x_2^*) < 1$, $W(x)$ is of full rank for $x \in \mathbb{R}_{>0}^2$. Then, it only remains to show the condition (26) holds. In this case, since we consider a single input system, (26) reduces to

$$1 + \Lambda(x)W^{-1}(x)G(x) > 0. \quad (61)$$

Due to the fact that the corresponding DfSP-matrix $\Lambda(x)$ in [19] has a lengthy and complex expression, we utilized a numerical verification algorithm adopted in [27] to verify the condition (61). Specifically, we calculate the value of $1 + \Lambda(x)W^{-1}(x)G(x)$ for each point $[x_1, x_2]^T$ with $x_1 = 0.01 : 0.01 : 3.5$, and $x_2 = 0.01 : 0.01 : 7$. The verification result is shown in Fig. 2(a), where the gray area indicates the domain $\mathcal{D}_0$ in which the condition (61) holds. Hence, Assumption 1 holds in a neighborhood of the desired equilibrium x^* .

Then, the control law in (21) is applied with the parameter setting $F_d(x)$ in (59) and $k = 1$. The explicit expressions of the proposed controller and the IDA-PBC controller in [19] are presented in (62) and (63), shown at the bottom of the page. Note that the proposed controller has a much simpler form compared to the latter.

Fig. 2(a) presents the vector field of the closed-loop system with the proposed control law in (21) and some system trajectories with different initial conditions. In Fig. 2(b), system trajectories with both control laws (62) and (63) are presented for the same group of initial conditions. It is shown that the IDA-PBC controller cannot stabilize the desired equilibrium with the initial conditions $x(t_0) = [0.01, 10]^T$, $[0.01, 4.5]^T$, $[0.01, 2.5]^T$, $[5, 0.5]^T$, and $[6, 6]^T$. Note that both controllers cannot achieve a stable result when $x(t_0) = [0.01, 2.5]^T$. From the aforementioned comparison, we can conclude that the proposed method admits a larger domain of attraction than the IDA-PBC method.

Remark 7: Note from Fig. 2 that the proposed controller achieves an asymptotic stability result even if $x(t_0) \notin \mathcal{D}_0$. Here, we can offer the following interpretation of this result. First, the condition (26), i.e., (61) in this case, is a sufficient condition rather than a necessary condition for the closed-loop stability. Therefore, there exist scenarios that the desired equilibrium x^* is stable, and meanwhile, the condition (26) does

Proposed controller:

$$u = u^* + \frac{(x_2 + 1)^2}{k(4x_2^2 + 2x_2) + (x_2 + 1)^2(2x_1 + x_1x_2)} (2x_1(x_2 - x_2u^* - u^*) - (1 - u^*)x_1x_2 + D), \quad k = 1. \quad (62)$$

IDA-PBC controller in [19]:

$$\begin{aligned} u = & \frac{1}{(x_1^2 + (x_2 + 1)^2)} \left(x_2(x_2 + 1) + x_1 \left(x_1 - \frac{D}{x_2} \right) - \left(\frac{x_2(x_2 + 1)}{x_1} + \frac{2x_1x_2}{(x_2 + 1)} \right) \left(k_1x_1 \left(2(k_2 + x_1^2) + x_2^2 - \frac{D(1+x_2)}{2x_1^2 + x_2^2} \right) \right. \right. \\ & + \frac{\sqrt{2}Dx_1 \arctan \left(x_1 / \sqrt{x_1^2 + x_2^2/2} \right)}{(2x_1^2 + x_2^2)^{\frac{3}{2}}} \Bigg) + \frac{1}{2x_2(2x_1^2 + x_2^2)^{\frac{3}{2}}} \left(\frac{2x_1^2}{(x_2 + 1)^2} - 2x_2 \right) \left(\sqrt{2x_1^2 + x_2^2} (2Dx_1(1+x_2) \right. \\ & \left. \left. + x_2(2x_1^2 + x_2^2)(-1 + 2k_1x_2(k_2 + x_1^2) + k_1x_2^3)) + \sqrt{2}Dx_2^2 \arctan \frac{x_1}{\sqrt{x_1^2 + x_2^2/2}} \right) \right) \end{aligned} \quad (63)$$

$$\text{where } k_1 = 0.8, \text{ and } k_2 = \frac{1}{k_1} \left(\frac{D(1+x_2^*)}{2x_1^*(2x_1^{*2} + x_2^{*2})} - \frac{\sqrt{2}D \arctan \left(x_1^* / \sqrt{x_1^{*2} + x_2^{*2}/2} \right)}{2(2x_1^{*2} + x_2^{*2})^{\frac{3}{2}}} \right) - \frac{x_2^{*2}}{2} - x_1^{*2}.$$

Fig. 2. System trajectories in phase plane with different initial conditions (green circle), and the desired equilibrium $[1.25, 4]^T$ (green *). (a) Closed-loop vector field and trajectories with the proposed controller (62), and the domain $\mathcal{D}_0$ (gray area) where the condition (26) holds. (b) Comparison of the proposed method (red solid line), and IDA-PBC controller (blue dashed line).

Fig. 3. Stabilization results of the proposed controller and the IDA-PBC controller given in [19].

not hold. Second, the domain $\mathcal{D}_0$ as shown in Fig. 2(a) is obtained for a specific DfSP-matrix $\Lambda(x)$ that corresponds to the IDA-PBC controller in [19]. As pointed out in Remark 2, the DfSP-matrix $\Lambda(x)$ is not uniquely defined by the selection of $F_d(x)$. Thus, there may exist another DfSP matrix for the same $F_d(x)$ such that the corresponding condition (26) holds for a domain larger than $\mathcal{D}_0$ as shown in Fig. 2(a). On the contrary, note that the proposed controller (62) cannot stabilize the desired equilibrium when $x(t_0) = [0.01, 2.5]^T \in \mathcal{D}_0$. This is because the system (55) is not shifted passive via feedback with the corresponding $F_d(x)$ and $\Lambda(x)$ in this neighborhood according to that the IDA-PBC controller (63) also does not stabilize the system in this case.

Fig. 3 depicts the time response of the closed-loop system with both controllers (62) and (63) for the initial condition $x(t_0) = [2, 3]^T$. Note that the IDA-PBC controller is more “aggressive” in the sense that it has a shorter setting time. However, it violates the constraint of $u \in [0, 1]$ at the beginning. For this reason, the IDA-PBC controller is implemented in simulation with actuator saturation. As a contrast, the proposed control input is in the range of $u \in [0, 1]$ for all the time and does not result in an overshoot at the beginning.

VI. CONCLUSION

In this article, the concept of SPvF of pH systems is formally introduced, and a set of partial differential equations is derived such that its solution admits a required feedback design. Our main contribution

is related to a new controller of simplified form for the pH system based on the framework of SPvF. From the perspective of an introduced definition, namely, *distance from shifted passivity*, it is proved that the proposed controller ensures asymptotic stability of the desired equilibrium provided that the studied pH systems are “close” enough to shifted passivity. As a major advantage of the proposed method, it offers a simpler alternative of the original controller based on SPvF, and thus, can reduce the computation complexity for practical applications. Furthermore, in order to relax the required assumptions, some extensions including a functional approximation-based procedure for the controller complexity reduction are provided. The proposed control law is validated via simulation on a buck–boost converter with CPL. It is shown that the proposed stabilization method outperforms a reported IDA-PBC controller in terms of domain of attraction, and has a simpler expression form.

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