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PAPER

Helicity Amplitude Analysis of $Z_c(3900)^\pm$ in $e^+e^- \rightarrow \pi^+\pi^- J/\psi^*$

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Helicity Amplitude Analysis of $Z_c(3900)^\pm$ in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ *

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It is crucial to determine the spin and parity (J^P) of $Z_c(3900)^\pm$ for understanding its structure. We perform a helicity amplitude analysis on $Z_c(3900)^\pm$ in the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ with the hypotheses $J^P = 0^-, 1^+, 1^-, 2^+$ and 2^- . To estimate the significance of $J^P = 1^+$ over other hypotheses, we perform a Monte Carlo simulation study, and we also discuss the prospect of measuring the spin parity in the future experiment with a large data sample.

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The charmonium-like state, $Z_c(3900)^\pm$ (denoted by $Z_c^\pm$), was for the first time observed by the BESIII collaboration^[1] in the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$, then confirmed by the Belle collaboration^[2] and CLEO data.^[3] It was also observed in $e^+e^- \rightarrow (D\bar{D}^*)^\pm \pi^\mp$.^[4] Based on an angular analysis of the $(D\bar{D}^*)^\pm$ system, the spin parity of Z_c favors the 1^+ assignment and disfavors 1^- and 0^- hypotheses.

Many theoretical explanations have been proposed to interpret the Z_c structure and its decay mechanism, such as the tetraquark scenario,^[5,6] the hadronic molecule,^[7] the meson loop,^[8] and the initial pion emission model.^[9] The main conclusion from these theoretical investigations is that further experimental study of Z_c resonance and related processes is important for understanding the dynamics of this multi-quark heavy-light system. It is crucial to establish the spin and parity of Z_c resonance.^[5]

In this Letter, we perform a helicity amplitude analysis on the Z_c state in the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$. We discuss the prospective study on its spin and parity with a large data sample taken at around $\sqrt{s} = 4.26$ GeV.

Considering a process $e^+e^- \rightarrow Y(4260)(M) \rightarrow Z_c^\pm(\lambda_1)\pi^\mp \rightarrow \pi^+\pi^- J/\psi(\lambda_2)$, where M denotes the spin z -projection for $Y(4260)$, and $\lambda_{1/2}$ denote the helicity values for Z_c and J/ψ , respectively, we define helicity angles, as shown in Fig. 1. For the first decay $e^+e^- \rightarrow Y(4260) \rightarrow Z_c^\pm(\lambda_1)\pi^\mp$, the z -axis of the rest frame is taken along the Z_c outgoing direction, and the helicity angles (θ_0, ϕ_0) are defined by rotating the z -axis to the e^- beam direction in the e^+e^- center-of-mass system, and the helicity amplitude for this decay is denoted by $F_{\lambda_1}^Y$. For the second decay chain, the momentum of J/ψ is defined in the rest frame of Z_c and is taken as the z -axis, and its helicity angles (θ_1, ϕ_1) are defined by rotating the J/ψ outgoing di-

rection to Z_c momentum. The helicity amplitude for $Z_c^\pm \rightarrow \pi^\pm J/\psi$ is denoted by $F_{\lambda_2}^{Z_c}$. The sequential decay amplitude squared is reconstructed from individual helicity amplitude in the helicity basis, i.e.,

$$|\mathcal{M}|^2 = \sum_{\text{helicities}} D_{M,\lambda_1}^{1*}(\theta_0, \phi_0) D_{M,\lambda_1'}^1(\theta_0, \phi_0) \times F_{\lambda_1}^Y F_{\lambda_1'}^{Y*} |\text{BW}(m_{J/\psi\pi^\pm})|^2 \times D_{\lambda_1,\lambda_2}^{J*}(\theta_1, \phi_1) D_{\lambda_1',\lambda_2}^J(\theta_1, \phi_1) F_{\lambda_2}^{Z_c*} F_{\lambda_2}^{Z_c}, \quad (1)$$

where $D_{m,n}^J(\theta, \phi) \equiv D_{m,n}^J(0, \theta, \phi)$ is the Wigner-D function; λ_1 and λ_1' are the helicity values for Z_c , and λ_2 is the helicity value for J/ψ . The sum over helicity values and M are implied, where $M = \pm 1$ is due to the $Y(4260)$ production from e^+e^- annihilation, and BW is a Breit-Wigner function.

Fig. 1. Definition of the helicity frame for $e^+e^- \rightarrow Y(4260) \rightarrow Z_c^\pm \pi^\mp \rightarrow \pi^+\pi^- J/\psi$.

For two-body decays, the helicity-coupling amplitudes $F_{\lambda_1}^Y$ and $F_{\lambda_2}^{Z_c}$ are related to the LS -coupling amplitudes^[10] according to

$$F_{\lambda_1}^Y = \sum_{LS} \left(\frac{2L+1}{3} \right)^{1/2} \langle L0S\delta | 1\delta \rangle \langle J\lambda_1 00 | S\delta \rangle \cdot G_{LS}^Y r^L B_L(r), \quad (2)$$

$$F_{\lambda_2}^{Z_c} = \sum_{LS} \left(\frac{2L+1}{2J+1} \right)^{1/2} \langle L0S\delta | J\delta \rangle \langle 1\lambda_2 00 | S\delta \rangle \cdot G_{LS}^{Z_c} r^L B_L(r), \quad (3)$$

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where J is the Z_c spin, G_{LS}^Y and $G_{LS}^{Z_c}$ are the LS -coupling amplitudes for the $Y(4260)$ and Z_c decays, respectively, $r = |\mathbf{r}|$ with $\mathbf{r}$ being the relative momentum of daughter particles, and $B_L(r)$ is the Blatt-Weisskopf factor.^[10]

If Z_c is taken as a state with the spin and parity $J^P = 1^+$, the parity conservation in the $Y(4260)$ hadronic decay leads to relations $F_{-1}^Y = F_1^Y$ and $F_{-1}^{Z_c} = F_1^{Z_c}$. After integrating over the angle ϕ_0 in Eq. (1), one obtains a joint angular distribution

$$\begin{aligned} \frac{d|\mathcal{M}|^2}{d\cos\theta_0 d\cos\theta_1} \propto & 2[3 + \cos(2\theta_0)][2\sin^2\theta_1|F_0^{Z_c}|^2 \\ & + [3 + \cos(2\theta_1)]|F_1^{Z_c}|^2|F_1^Y|^2 \\ & + 16\sin^2\theta_0(\cos^2\theta_1|F_0^{Z_c}|^2 \\ & + \sin^2\theta_1|F_1^{Z_c}|^2)|F_0^Y|^2. \end{aligned} \quad (4)$$

If one further integrates over θ_0 or θ_1 , the angular distributions of Z_c and J/ψ are obtained, which takes the form

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_i} \propto 1 + \alpha_i \cos^2\theta_i \quad (i = 0, 1), \quad (5)$$

where $\alpha_i (i = 0, 1)$ are the angular distribution parameters, and they are taken as $\alpha_0 = \frac{|F_1^Y|^2 - |F_0^Y|^2}{|F_1^Y|^2 + |F_0^Y|^2}$, and

$$\alpha_1 = \frac{(|F_0^Y|^2 - |F_1^Y|^2)(|F_0^{Z_c}|^2 - |F_1^{Z_c}|^2)}{|F_0^Y|^2|F_1^{Z_c}|^2 + |F_1^Y|^2(|F_0^{Z_c}|^2 + |F_1^{Z_c}|^2)}.$$

Under the hypothesis that Z_c has $J^P = 0^-$, Eq. (1) is reduced to a simple form $d|\mathcal{M}|^2 d\cos\theta_0 \propto \sin^2\theta_0$, and the angular distribution for J/ψ is reduced to be isotropic.

If Z_c is taken as a state with $J^P = 1^-$, the parity conservation leads to the relations $F_{-1}^Y = -F_1^Y$, $F_{-1}^{Z_c} = -F_1^{Z_c}$ and $F_0^Y = F_0^{Z_c} = 0$. Equation (1) is reduced to

$$\begin{aligned} \frac{d|\mathcal{M}|^2}{d\cos\theta_0 d\cos\theta_1 d\phi_0 d\phi_1} \propto & [3 + \cos(2\theta_0)][3 + \cos(2\theta_1)] \\ & - 4\cos[2(\phi_0 - \phi_1)]\sin^2\theta_0 \sin^2\theta_1. \end{aligned} \quad (6)$$

If one further integrates over ϕ_0 , ϕ_1 , θ_0 or θ_1 , the angular distributions of Z_c and J/ψ are obtained, which take the form

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_i} \propto 1 + \cos^2\theta_i \quad (i = 0, 1). \quad (7)$$

If Z_c is taken as a state with $J^P = 2^-$, the parity conservation leads to the relations $F_{-1}^Y = F_1^Y$ and $F_{-1}^{Z_c} = F_1^{Z_c}$. Equation (1) is reduced to

$$\begin{aligned} \frac{d|\mathcal{M}|^2}{d\cos\theta_0 d\cos\theta_1} \propto & (3 + \cos\theta_0)|F_1^Y|^2[3\sin^2(2\theta_1)|F_0^{Z_c}|^2 \\ & + 2[2 + \cos(2\theta_1) + \cos(4\theta_1)]|F_1^{Z_c}|^2] \\ & + \sin^2\theta_0|F_0^Y|^2\{[1 + 3\cos(2\theta_1)]^2|F_0^{Z_c}|^2 \\ & + 12\sin^2(2\theta_1)|F_1^{Z_c}|^2\}. \end{aligned} \quad (8)$$

If one integrates over θ_1 , the angular distribution of Z_c is obtained in the form

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \alpha_0 \cos^2\theta_0, \quad (9)$$

where $\alpha_0 = \frac{|F_1^Y|^2 - |F_0^Y|^2}{|F_1^Y|^2 + |F_0^Y|^2}$ is the angular distribution parameter. Similarly, the angular distribution for J/ψ is obtained after integration over θ_0 , i.e.,

$$\begin{aligned} \frac{d|\mathcal{M}|^2}{d\cos\theta_1} \propto & 2|F_1^Y|^2\{6\sin^2(2\theta_1)|F_0^{Z_c}|^2 \\ & + 4[2 + \cos(2\theta_1) + \cos(4\theta_1)]|F_1^{Z_c}|^2\} \\ & + |F_0^Y|^2\{[1 + 3\cos(2\theta_1)]^2|F_0^{Z_c}|^2 \\ & + 12\sin^2(2\theta_1)|F_1^{Z_c}|^2\}. \end{aligned} \quad (10)$$

If Z_c is taken as a state with $J^P = 2^+$, the parity conservation leads to the relations $F_{-1}^Y = -F_1^Y$, $F_{-1}^{Z_c} = -F_1^{Z_c}$ and $F_0^Y = F_0^{Z_c} = 0$. Equation (1) is reduced to

$$\begin{aligned} \frac{d|\mathcal{M}|^2}{d\cos\theta_0 d\cos\theta_1} \propto & [3 + \cos(2\theta_0)][2 + \cos(4\theta_1) \\ & + \cos(2\theta_1)]. \end{aligned} \quad (11)$$

If one integrates over θ_1 , the angular distribution of Z_c is obtained in the form

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \cos^2\theta_0, \quad (12)$$

while the angular distribution of J/ψ is obtained after integration over θ_0 , and it reads

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_1} \propto 2 + \cos(2\theta_1) + \cos(4\theta_1). \quad (13)$$

A comparison between the case with the spin-parity assignment $J^P = 1^-$ and 2^+ shows that the angular distribution of the first decay takes the same form $\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \cos^2\theta_0$. To distinguish these two spin assignments, it is essential to include the angular distribution of second decays. Similarly, if Z_c is taken as a 1^+ or 2^- state, the angular distribution of the first decay takes the same form $\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \alpha_0 \cos^2\theta_0$ with $\alpha_0 = (|F_1^Y|^2 - |F_0^Y|^2)/(|F_1^Y|^2 + |F_0^Y|^2)$, which implies that the quantum numbers of 1^+ and 2^- are distinguishable at the first decay chain.

To estimate the statistical significance of the Z_c spin parity hypothesis 1^+ over other hypotheses $J_{H_0}^P = 0^-, 1^-, 2^-$ or 2^+ , we perform a test of two hypotheses: the null hypothesis H_0 over the alternative hypothesis H_1 , where H_0 stands for the case that the data can be described with the spin hypothesis $e^+e^- \rightarrow Y(4260) \rightarrow Z_c^\pm (J_{H_0}^P)\pi^\mp \rightarrow \pi^+\pi^- J/\psi$, while H_1 means that the data can be described with the spin hypothesis $e^+e^- \rightarrow Z_c^\pm (J_{H_0}^P)\pi^\mp + Z_c^\pm(1^+)\pi^\mp \rightarrow \pi^+\pi^- J/\psi$. To test these two hypotheses, we perform two fits to the pseudo data, and their likely function

values are denoted by $L_{\max}(H_0)$ and $L_{\max}(H_1)$, respectively.

We use an MC simulation to estimate the significance, and produce a pseudo data set by assuming a Z_c with $J^P = 1^+$, and event decays are generated with the amplitude as given from Eqs. (1)–(3). The LS -coupling amplitudes are naively fixed to the values $|G_{01}^Y|/|G_{21}^Y| = 0.05$ and $|G_{01}^{Z_c}|/|G_{21}^{Z_c}| = 0.98$.

Table 1. Statistical significance for the quantum number $J^P = 1^+$ over $0^-, 1^-, 2^-$ and 2^+ , N_{evt} is the number of signal events, and $\int \mathcal{L} dt$ is the corresponding integrated luminosity.

$\int \mathcal{L} dt$ (pb $^{-1}$)	N_{evt}	1^+ over 0^-	1^+ over 1^-	1^+ over 2^-	1^+ over 2^+
711	416	13.2σ	11.6σ	4.2σ	17.4σ
1422	832	18.9σ	17.3σ	4.8σ	27.2σ
2844	1664	24.6σ	25.4σ	9.3σ	37.0σ

The spin hypothesis 1^+ over other spin hypothesis $J_{H_0}^P$ is tested by performing a fit to the selected events with the amplitude squared $|\mathcal{M}|_{J_{H_0}^P}^2$ and $|\mathcal{M}|_{J_{H_0}^P}^2 + |\mathcal{M}|_{1^+}^2$, which correspond to the hypotheses H_0 and H_1 , respectively. After fitting to the pseudo data, one obtains $\ln L_{\max}(H_1)$ and $\ln L_{\max}(H_0)$. Taking consideration of the difference in the number of the degrees of freedom $\delta(ndf)$, one estimates the dependence of statistical significance on the number of observed signal events as listed in Table 1. One can see that for a given statistics, the statistical significance to distinguish the quantum number 1^+ over 2^- is smaller than that for all other cases. This is due to the fact that their angular distributions of the first decay are the same, and they are distinguishable with that of the second decay.

The statistical significance determined for the quantum number 1^+ over $0^-/1^-/2^+$ is sensitive to the number of signal events. A sample of about 400 signal events can give a significance larger than 5σ , which corresponds to a data set with an integrated luminosity of $\sim 700 \text{ pb}^{-1}$. Here the signal yields with respect to the luminosity is simply estimated according to the BESIII experiments,^[13] the $Z_c^\pm$ signal yields, 307 events in 525 pb^{-1} data taken at 4.26 GeV . To achieve a significance of 5σ to distinguish the quantum number 1^+ from 2^- , one needs a data sample with an integrated luminosity of at least 1500 pb^{-1} , and a large data set with luminosity 1.8 fb^{-1} can provide a significance about 9σ . If the luminosity of the future e^+e^- collider can reach up to $10^{34} \text{ cm}^{-2}\text{s}^{-1}$, accumulation of 1 fb^{-1} data at $\sqrt{s} = 4.26 \text{ GeV}$ will only take one day.

In summary, we have reported on a helicity amplitude analysis for the determination of the Z_c spin parity in the $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ process. The re-

sults show that if Z_c is taken as a 1^+ state, it yields the same angular distribution as that by the 2^- hypothesis. To distinguish these two spin parity assignments, one needs to look for the angular distribution of $Z_c^\pm$ decaying into $J/\psi\pi^\pm$. To estimate the statistical significance, an MC simulation is performed with hypotheses of the Z_c spin parity to be $0^-, 1^\pm$ and $2^\pm$. The results show that the statistical significance is sensitive to the number of observed signal events. A few hundred signal events will give the statistical significance larger than 5σ . The prospect of determining the quantum numbers of the Z_c spin parity with an even larger data set in the future is discussed.

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