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Star Formation Timescales of the Halo Populations from Asteroseismology and Chemical Abundances*

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Abstract

We combine asteroseismology, optical high-resolution spectroscopy, and kinematic analysis for 26 halo red giant branch stars in the Kepler field in the range of $-2.5 < [\text{Fe}/\text{H}] < -0.6$. After applying theoretically motivated corrections to the seismic scaling relations, we obtain an average mass of $0.97 \pm 0.03 M_{\odot}$ for our sample of halo stars. Although this maps into an age of ~ 7 Gyr, significantly younger than independent age estimates of the Milky Way stellar halo, we considered this apparently young age to be due to the overestimation of stellar mass in the scaling relations. There is no significant mass dispersion among lower red giant branch stars ($\log g > 2$), which constrains the relative age dispersion to $< 18\%$, corresponding to < 2 Gyr. The precise chemical abundances allow us to separate the stars with $[\text{Fe}/\text{H}] > -1.7$ into two $[\text{Mg}/\text{Fe}]$ groups. While the $[\alpha/\text{Fe}]$ and $[\text{Eu}/\text{Mg}]$ ratios are different between the two subsamples, $[s/\text{Eu}]$, where s stands for Ba, La, Ce, and Nd, does not show a significant difference. These abundance ratios suggest that the chemical evolution of the low-Mg population is contributed by Type Ia supernovae, but not by low- to intermediate-mass asymptotic giant branch stars, providing a constraint on its star formation timescale as $100 \text{ Myr} < \tau < 300 \text{ Myr}$. We also do not detect any significant mass difference between the two $[\text{Mg}/\text{Fe}]$ groups, thus suggesting that their formation epochs are not separated by more than 1.5 Gyr.

Key words: Halo stars – Asteroseismology – Chemical abundances – Stellar kinematics – High resolution spectroscopy – Stellar ages

Supporting material: machine-readable tables

1. Introduction

The environment of the birth place of stars is encoded in their surface chemical composition and their orbital characteristics. The history of the Milky Way can potentially be recovered from such information obtained through extensive measurements of stellar chemical abundances (e.g., Freeman & Bland-Hawthorn 2002; Frebel & Norris 2015) and stellar kinematics (e.g., Helmi et al. 1999) by astrometric and spectroscopic observations (e.g., Perryman et al. 1997; Nordström et al. 2004; Steinmetz et al. 2006; Yanny et al. 2009; Zhao et al. 2012; Gaia Collaboration et al. 2016b; Majewski et al. 2017; Martell et al. 2017). However, measurements of stellar age, another fundamental parameter in the study of the Milky Way history, is challenging (Soderblom 2010). Although ages have been estimated from their locations on the Hertzsprung–Russell diagram (HRD) for nearby turnoff stars, the same method is not applicable to intrinsically more luminous red giant stars because their locations do not depend on the age. The lack of age estimates for distant stars prevents us from directly sketching the history of the Galaxy as a function of time.

The emergence of satellite missions that carried out long-term monitoring of stellar oscillations through their variation in luminosity with high precision, such as Kepler (Koch et al. 2010) and CoRoT (Auvergne et al. 2009), has opened a new window to estimate the mass of a large number of red giant stars (Kallinger et al. 2010a, 2010b; Hekker et al. 2011; Stello et al. 2013; Pinsonneault et al. 2014; Mathur et al. 2016; Pinsonneault et al. 2018; Yu et al. 2018), which is otherwise almost inaccessible (however, see also Feuillet et al. 2016; Martig et al. 2016; Ness et al. 2016; Das et al. 2020). Stars with a thick convective envelope show a particular type of oscillations (solar-like oscillations), which can be used to infer their structure, including mass (asteroseismology; Aerts et al. 2010; Chaplin & Miglio 2013). Having the mass of a red giant is of paramount importance in estimating its age because the time spent on the red giant branch is short compared to that on the main-sequence phase and because the mass is the fundamental parameter that controls the evolution of a star including its main-sequence lifetime, although mapping into an age scale is problematic because it depends on the adopted stellar models and assumed mass loss (e.g., Casagrande et al. 2016). Because red giants are intrinsically luminous, the development of asteroseismology allows us to explore stellar ages

* Based on data collected with the Subaru Telescope, which is operated by the National Astronomical Observatory of Japan.

over a wide volume of the Galaxy. These stellar ages obtained through asteroseismology have revealed the evolution of the Galactic disk (e.g., Casagrande et al. 2016; Anders et al. 2017; Silva Aguirre et al. 2018; Wu et al. 2018, 2019).

However, application of asteroseismology to halo stars has been limited, despite the importance of determining their ages. The Galactic halo contains rich information about the merging history of the Milky Way. Most halo stars are affected by galaxy mergers; they are accreted onto the Milky Way after having formed in satellite dwarf galaxies or heated from the Galactic disk at the time of major mergers (Helmi & White 1999; Bullock & Johnston 2005; McCarthy et al. 2012; Jean-Baptiste et al. 2017; Belokurov et al. 2020). For stars known to have originated from accretion or mergers, their ages constrain the epoch of those events (Gallart et al. 2019). The Gaia mission has provided major recent progress in our understanding of the Galactic halo (e.g., Belokurov et al. 2018; Helmi et al. 2018) by enabling comprehensive studies of stellar kinematics, which leads to the increasing demand for ages of halo stars. Although the bulk of the stellar halo has been known to be composed of old stars ($\gtrsim 10$ Gyr old; Iben 1967; Schuster et al. 2012), investigation of age variation among them has been limited.

Nissen & Schuster (2010) showed that in a photometrically selected sample of nearby halo stars, there are two major stellar populations among halo stars: one has high $[\alpha/\text{Fe}]$ and the other has low $[\alpha/\text{Fe}]$ abundance ratios; these are hereafter referred to as the high- α and low- α populations, respectively. The two populations also differ in stellar kinematics and ages (Schuster et al. 2012), indicating different origins. The low- α population exhibits a younger age on average by 2–3 Gyr, with a tendency toward being on a retrograde orbit, which is consistent with an accretion origin. This result is also confirmed by Hawkins et al. (2014), who use the color of α -rich and α -poor halo turnoff stars in the low-resolution spectroscopic survey, SEGUE/SDSS. Chemodynamical analysis of halo stars by Helmi et al. (2018) and Mackereth et al. (2019) revealed that this low- α population would correspond to stars from a relatively massive dwarf galaxy that was accreted onto the Milky Way (Gaia Enceladus a.k.a. Gaia Sausage; Belokurov et al. 2018).¹² The other population with high- α abundance is suggested to be formed in the Milky Way (in situ formation). Belokurov et al. (2020) proposed that this population would have formed in the ancient Milky Way disk and later been heated to the halo from chemodynamical analysis and the age distribution of halo and disk stars.

The age of halo stars has been examined in several studies for stars in globular clusters (e.g., Marín-Franch et al. 2009; Forbes & Bridges 2010; Dotter et al. 2011) and for field stars (e.g., Jofré & Weiss 2011; Schuster et al. 2012; Hawkins et al. 2014; Carollo et al. 2016; Kilic et al. 2019; Das et al. 2020). Although the ages of globular clusters have provided a wealth of information about the accretion history of the Milky Way (e.g., Kruijssen et al. 2019; Massari et al. 2019), there is no guarantee that they are the representative of the halo stellar population. Previous studies that utilized the combination of chemical abundances and stellar ages for field stars estimate the age of turnoff stars from their locations on the HRD (Schuster et al. 2012). However, they are intrinsically faint, so explorations have been limited to the solar neighborhood.

Considering the fact that the Galactic halo extends to a large distance, the combination of ages and abundances for luminous stars is highly desired. Das et al. (2020) approached this problem by estimating ages from infrared photometry and Gaia parallax measurements using a Bayesian framework, showing the prolonged star formation history of Gaia Enceladus. However, because the color and luminosity of a red giant are not very sensitive to its age, their method requires a large-enough sample for statistical treatment and good prior knowledge about the age distribution of the population for the Bayesian approach. Asteroseismology, on the other hand, directly provides the stellar mass with high precision, which enables discussion based on a smaller number of stars.

There have been two main challenges in the application of asteroseismology to halo stars. The first problem is that asteroseismic mass estimates at low metallicity have not been tested or calibrated well compared to solar-metallicity stars. In asteroseismology, mass estimates rely on the following two scaling relations:

$$\Delta\nu \propto \sqrt{\bar{\rho}} \propto M^{0.5}R^{-1.5}, \quad (1)$$

$$\nu_{\text{max}} \propto g/\sqrt{T_{\text{eff}}} \propto MR^{-2}T_{\text{eff}}^{-0.5}, \quad (2)$$

where $\Delta\nu$ is the mean large-frequency separation between adjacent radial modes, ν_{max} is the frequency at which the oscillation amplitude becomes maximum, and g , $\bar{\rho}$, M , R , and T_{eff} are surface gravity, mean density, mass, radius, and effective temperature, respectively. We immediately obtain $M \propto \nu_{\text{max}}^3 \Delta\nu^{-4} T_{\text{eff}}^{1.5}$ from these two scaling relations, which is used for mass estimates. The scaling relations, Equation (1) and Equation (2), have been tested extensively for a large number of red giants (Huber et al. 2011, 2012; Baines et al. 2014; Brogaard et al. 2016; Gaulme et al. 2016; Themeßl et al. 2018; Hekker 2020). We note that Brogaard et al. (2016), Gaulme et al. (2016), and Themeßl et al. (2018) compared radii and masses for eclipsing binaries and reported small but significant offsets. The majority of the red giants used for the testing are, however, solar-metallicity stars. There is no guarantee that the scaling relations work similarly for low-metallicity stars, and, in fact, a number of observational and theoretical studies have suggested the necessity of applying corrections.

Observationally, Epstein et al. (2014) pointed out that the simple use of the scaling relations tends to overestimate masses for low-metallicity stars based on nine low-metallicity stars ($[\text{Fe}/\text{H}] < -1.0$) in the Apache Point Observatory Galactic Evolution Experiment (APOGEE) of the Sloan Digital Sky Survey (SDSS). Casey et al. (2018) also reached the same conclusion using their photometric selection of metal-poor stars (three stars). These studies motivated theoretical efforts to make corrections to the scaling relations to obtain accurate masses for low-metallicity stars (e.g., Guggenberger et al. 2016; Sharma et al. 2016), which lead to lower mass estimates for stars in Epstein et al. (2014). Miglio et al. (2016) and Valentini et al. (2019) analyzed metal-poor stars observed by the K2 mission (Howell et al. 2014) in the globular cluster M4 (eight stars) and those in the field (four + nine stars from Epstein et al. 2014), respectively. Their mass estimates are based on a theoretical correction to Equation (1) and in good agreement with independent estimates, suggesting that theoretical $\Delta\nu$ estimates work well.

¹² Note that there is a discussion that Gaia Enceladus is not exactly the same as the Gaia Sausage (e.g., Evans 2020).

The other difficulty is in finding halo stars with asteroseismic information. Because asteroseismology requires high-quality continuous light curves over a long period, dedicated observations by space telescopes are desired.¹³ The Kepler mission has observed the same field near the Galactic plane (the so-called Kepler field) over about four years and obtained the highest-quality light curves that are suitable for asteroseismology for about 16,000 stars (Yu et al. 2018). Because of the low galactic latitude, most of the stars in the Kepler field are disk stars, while the fraction of halo stars increases at faint magnitudes (Mathur et al. 2016). Therefore, we need a way to efficiently select halo stars for follow-up high-resolution spectroscopy.

Halo stars are characterized by a large velocity relative to the Sun and low metallicity. Therefore, radial velocity and metallicity measurements by spectroscopic surveys provide an efficient way to select halo stars. Fortunately, spectroscopic surveys such as the Large Sky Area Multi-Object Fibre Spectroscopic Telescope (LAMOST) and APOGEE have extensively observed stars in the Kepler field and have provided radial velocity and metallicity measurements (Pinsonneault et al. 2014; De Cat et al. 2015; Ren et al. 2016; Pinsonneault et al. 2018; Zong et al. 2018).

In this study, we obtain high signal-to-noise ratio (S/N), high-resolution spectra for 26 halo star candidates with asteroseismic information ($S/N \sim 100$ – 200 and $R > 60,000$). The spectra provide us with precise measurements of stellar parameters including effective temperature, which is needed for asteroseismic mass estimates. Chemical abundances are also measured with high precision to investigate the two separate halo populations (high α and low α), and to investigate the possible correlation between abundances and stellar masses, such as those seen among solar-metallicity stars (e.g., Nissen 2015; Bedell et al. 2018; Feuillet et al. 2018; Spina et al. 2018; Buder et al. 2019; Delgado Mena et al. 2019). Compared to previous studies of halo stars with asteroseismology, our study can be characterized by the sample size and the high-precision stellar parameters and chemical abundances. All of the stars in our sample are in the Kepler field, and hence precise oscillation frequency measurements are available from about the four year observation (Yu et al. 2018).

Apart from the formation history of the Galaxy, asteroseismic information of halo stars would constrain the formation mechanism of stars with peculiar abundances. High-resolution spectroscopic surveys of low-metallicity stars have revealed metal-poor stars whose abundance ratio is very different from solar composition (e.g., carbon-enhanced metal-poor stars, Beers & Christlieb 2005; r-process enhanced stars, Sneden et al. 1994; Li-rich stars, Li et al. 2018). Chemical peculiarity can be caused by mass accretion from a binary companion, internal nucleosynthesis, or inhomogeneity in the early universe. Asteroseismic constraints on mass and evolutionary status would constrain the detail of these processes and provides a clue to the origin of stars whose formation mechanism is not yet well understood.

We start this paper with target selection and observation in Section 2, which is followed by the use of asteroseismic data in Section 3, kinematic analysis in Section 4, and abundance analysis in Section 5. We discuss the results in Section 6 combining these three methods. A summary and future prospects are provided in Section 7.

Figure 1. Target selection in the radial velocity—[Fe/H] plane. The contours are disk (black dashed) and halo (blue solid) stars from the Besançon model (each contour contains 39.3, 86.5, and 98.9% of the population.). Stars outside of the black solid line are primarily selected as halo star candidates. Targets are shown with orange circles (LAMOST DR5) and green squares (APOKASC-2). If a target is both in LAMOST DR5 and APOKASC-2, the points are connected by a black line. Gray points are all stars that are both in Yu et al. (2018) and in LAMOST DR5. Note that while LAMOST DR4 was used at the time of selection, here we use updated measurements in LAMOST DR5.

2. Observation

2.1. Target Selection

Targets are selected from the LAMOST DR4 and APOKASC-2 catalogs based on radial velocity (V_r) and metallicity measurements (Figure 1). To make sure all of the targets have frequency measurements, we only select stars from the catalog of asteroseismic analysis of $\sim 16,000$ red giants by Yu et al. (2018). The following selection criteria are invoked to minimize the contamination of disk stars based on the distributions of disk and halo stars in a mock catalog of the Kepler field generated from the Besançon galaxy model (Robin et al. 2003). The boundary used for the selection (the black solid line in Figure 1) connects $V_r = -230$ km s $^{-1}$ for $[Fe/H] > -1.1$, $(V_r, [Fe/H]) = (-230, -1.1)$, $(-160, -1.7)$, and $[Fe/H] = -1.7$ for $V_r > -170$ km s $^{-1}$.

There are two stars that do not exactly satisfy the criteria presented above. The more extreme one is KIC 5858947 and the other is KIC 9696716. These stars are observed because the observing condition does not allow us to study fainter stars that satisfy the criteria. While APOGEE measurements of two other stars (KIC 5184073 and KIC 8350894) do not satisfy the criteria, these stars are selected based on the LAMOST measurements. The radial velocity of KIC 7693833 is very different from other halo stars and more similar to disk stars; however, this star is kept in the sample considering its low metallicity.

2.2. Observations

Observations were conducted with the High Dispersion Spectrograph (HDS; Noguchi et al. 2002) on the Subaru Telescope. Most of the targets were observed in 2018 July,

¹³ There are studies that observe stellar oscillations by radial velocity monitoring. However, space photometric observations have a clear advantage in terms of the precision of measurements of $\Delta\nu$ and $\nu_{\max}$, the multiplicity of targets, and the depth of observations.

Table 1
Observation Log

Object	Date ^a	Exposure (s)	S/N^b	V_r (HDS) (km s ⁻¹)	$V_{r,L}^c$ (km s ⁻¹)	[Fe/H] _L ^c	$V_{r,A}^d$ (km s ⁻¹)	[Fe/H] _A ^d	$V_{r,G}^e$ (km s ⁻¹)	$\sigma(V_{r,G})^e$ (km s ⁻¹)	M_G^e (mag)
KIC 5184073	July 11, 2018	5400	107.0	-136.21	-139.92	-1.78	-135.65	-1.43	...	...	13.21
KIC 5439372	July 10, 2018	1200	135.0	-213.57	-218.27	-2.48	-211.94	...	-212.89	0.58	11.82
KIC 5446927	August 4, 2017	1200	115.0	-272.89	-275.41	-0.80	-272.76	-0.72	-272.90	0.39	11.71
KIC 5698156	July 9 & 11, 2018	1200	246.0	-380.25	-386.80	-1.23	-380.12	-1.27	-379.28	0.72	10.45
KIC 5858947	July 10, 2018	1800	135.0	-104.35	...	...	-115.43	-0.80	-106.91	2.09	11.72
KIC 5953450	July 11, 2018	2400	265.0	-286.53	-292.09	-0.68	-286.19	-0.64	-284.63	0.82	12.67
KIC 6279038	July 9, 2018	2200	118.0	-308.10	-318.93	-2.05	-307.36	-2.15	-308.15	0.75	12.43
KIC 6520576	July 11, 2018	5400	171.0	-365.09	-366.93	-2.33	...	...	...	...	13.18
KIC 6611219	July 11, 2018	1800	113.0	-293.95	...	...	-293.35	-1.20	-293.47	0.72	12.06
KIC 7191496	August 2, 2017	1200	167.0	-290.55	-303.87	-2.00	-294.03	-1.98	-297.34	1.58	11.82
KIC 7693833	August 2, 2017	1200	179.0	-7.10	-13.53	-2.27	-7.02	...	-6.69	0.44	11.74
KIC 7948268	July 9, 2018	2200	155.0	-292.58	-297.72	-1.23	-292.76	-1.27	-290.87	1.79	12.44
KIC 8350894	July 11, 2018	3600	198.0	-219.94	-226.13	-1.30	-219.21	-1.03	-219.83	1.61	12.76
KIC 9335536	July 10, 2018	5400	114.0	-350.52	-355.91	-1.59	-350.49	-1.47	-351.49	1.21	12.97
KIC 9339711	July 9, 2018	1800	178.0	-332.81	-340.41	-1.51	-332.85	-1.49	-332.68	0.56	12.08
KIC 9583607	July 10, 2018	2400	169.0	-309.74	-322.84	-0.78	-309.81	-0.64	-309.86	0.37	11.46
KIC 9696716	July 9, 2018	1200	171.0	-146.71	-154.83	-1.54	-159.20	-1.46	-148.23	4.28	11.73
KIC 10083815	August 4, 2017	1800	136.0	-270.98	-283.10	-0.90	-271.17	-0.90	-270.22	1.08	12.26
KIC 10096113	July 11, 2018	4800	160.0	-241.44	-245.16	-0.71	-241.83	-0.63	-240.37	2.64	13.21
KIC 10328894	July 9, 2018	3600	188.0	-315.65	-323.23	-1.95	...	...	-316.37	1.89	13.02
KIC 10460723	August 2, 2017	1800	152.0	-346.84	-354.70	-1.29	-346.67	-1.28	-347.34	0.62	12.32
KIC 10737052	July 11, 2018	5400	200.0	-242.77	-252.10	-1.29	...	...	-237.99	1.04	13.20
KIC 10992126	July 10, 2018	1200	142.0	-259.55	...	...	-259.38	-0.51	-259.84	0.41	10.85
KIC 11563791	July 11, 2018	600	164.0	-262.45	-269.27	-1.13	-261.56	-1.14	-262.20	0.46	10.95
KIC 11566038	July 10, 2018	1800	122.0	-310.67	-316.79	-1.46	-310.34	-1.40	-309.94	2.11	12.04
KIC 12017985	August 2, 2017	600	225.0	-190.24	-189.72	-1.86	-190.52	-1.84	-184.34	2.03	10.14
	July 9, 2018	600	202.0								
KIC 12253381	July 9, 2018	2200	121.0	-272.44	-261.25	-1.59	-259.55	-1.54	-261.64	1.20	12.51

Notes.^a Observations in 2017 are conducted with $R \sim 60,000$, while those in 2018 are with $R \sim 80,000$.^b S/N are measured from the continuum around 5765 Å per 0.024 Å pixel.^c From the LAMOST DR5 catalog.^d From the APOGEE DR16 catalog.^e From the Gaia DR2 catalog.

during which we occasionally had thin clouds. Spectra were taken with a standard setup of HDS with 2×2 CCD binning which covers from ~ 4000 to ~ 6800 Å (StdYd). Exposures taking longer than 40 minutes are split into shorter exposures, each of which typically takes 20–30 minutes. Image slicer #2 was used to achieve high S/N ($\gtrsim 100$) with $R \sim 80,000$ (Tajitsu et al. 2012). A subset of spectra was taken in 2017 as a backup program for other proposals when the sky was covered with thin clouds. These spectra were taken with the same wavelength coverage but with $R \sim 60,000$ without the image slicer. KIC 12017985 was observed in both 2017 and 2018 to confirm the consistency of our analysis. We mainly adopt parameters from the 2018 observation in the figures for this star and denote the 2017 observation as KIC 12017985–17. We note that KIC 10992126 was observed but not included in the following analysis, because it turned out to have large uncertainties in the measured frequencies.

Spectra were reduced with an IRAF¹⁴ script, `hdsq1`,¹⁵ including linearity correction (Tajitsu et al. 2010), cosmic-ray rejection, scattered-light subtraction, flat-fielding, aperture

extraction, wavelength calibration using a ThAr lamp, heliocentric velocity correction, and continuum placements. Spectra are combined just after heliocentric radial velocity correction by simply adding fluxes in each frame. The S/N was estimated from a continuum region around 5765 Å, and the radial velocity was estimated from the wavelengths of iron lines. Details of the observation, as well as radial velocity and metallicity measurements in surveys, are shown in Table 1. We note that Table 1 shows single values for the radial velocities and metallicities of individual objects for LAMOST and APOGEE. We adopt measurements from the highest- S/N spectrum of each star for LAMOST values and those from the combined spectra for APOGEE values.

The uncertainty in radial velocity measurements is estimated to be 1 km s^{-1} considering the stability of the instrument. The measured radial velocities are compared with previous measurements in Figure 2, where all radial velocity measurements in the surveys are plotted. KIC 5439372, KIC 5858947, KIC 7191496, KIC 9696716, and KIC 12253381 show signatures of radial velocity variation between APOGEE data and our observation, suggesting the possibility of the existence of binary companions. Most of these stars show a large `radial_velocity_error` in Gaia DR2, which is shown as $\sigma(V_{r,G})$ in Table 1, for their magnitude, which supports

¹⁴ IRAF is distributed by the National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy (AURA) under a cooperative agreement with the National Science Foundation.

¹⁵ <https://www.subarutelescope.org/Observing/Instruments/HDS/hdsq1-e.html>

Figure 2. Comparison of measured radial velocities relative to the HDS measurements.

the likelihood of radial velocity variation. In addition, KIC 10737052 shows large offsets between our observation and Gaia measurements, which suggests the binarity of these objects.¹⁶

3. Asteroseismology

All of the targets are in Yu et al. (2018) catalog, which provides the results of asteroseismic analysis of red giant stars using about four years of photometric data in the Kepler field using the SYD asteroseismic pipeline (Huber et al. 2009). The long baseline enabled them to precisely measure ν_{max} and $\Delta\nu$ and to utilize the evolutionary status in the literature. Although they also derived mass and radius, their stellar parameters are based on a collection of values from various literature sources. Therefore, those mass and radius estimates need to be revised using updated stellar parameters from our analysis of high-resolution spectra.

We derive the mass and radius using *Asfgrid* (Sharma et al. 2016), which includes a correction to the $\Delta\nu$ scaling relation taking evolutionary status into account. While ν_{max} and $\Delta\nu$ are taken from Yu et al. (2018), the other necessary input parameters, T_{eff} and $[\text{Fe}/\text{H}]$, are taken from our spectroscopic measurements described in Section 5. For the solar values, we adopt $\nu_{\text{max}} = 3090 \mu\text{Hz}$, $\Delta\nu = 135.1 \mu\text{Hz}$, and $T_{\text{eff}} = 5777 \text{ K}$ (Huber et al. 2011).

In Table 2, we also provide masses obtained from the simple scaling relations without any correction. The magnitude of the correction can be important (as large as $0.24 M_{\odot}$, corresponding to 20%; see the values of KIC 6279038 in Table 2).

The upper mass limit of metal-poor main-sequence stars has been estimated to be $\sim 0.8 M_{\odot}$ (e.g., Iben 1967; Meléndez et al. 2010; VandenBerg et al. 2014). We expect similar masses for our sample because the timescale of the evolution after the turnoff is very short. Two stars are obviously more massive than others (Table 2), which would be treated as outliers (KIC 5446927 and KIC 10096113; separately discussed in Section 6.4.2). The weighted average of the masses excluding

the two outliers is $0.96 \pm 0.02 M_{\odot}$ ($\sigma = 0.10 M_{\odot}$). Although this value is closer to the expected mass than the mass obtained from scaling relations without a correction ($1.04 \pm 0.02 M_{\odot}$, $\sigma = 0.10$), it is still significantly higher than the canonical $0.8 M_{\odot}$ expected for old halo stars. More discussion is presented in Section 6.2.

Four stars in our sample are in common with Epstein et al. (2014), which were then reanalyzed by Valentini et al. (2019), who use frequencies derived with the COR asteroseismic pipeline (Mosser & Appourchaux 2009) and a Bayesian approach for mass (and age) estimation. We compare our derived masses with Valentini et al. (2019) for the four stars in Figure 3. There is a good agreement between our study and Valentini et al. (2019). Figure 3 also includes masses that are derived using the same procedure as in this study but with frequencies adopted in Valentini et al. (2019). The agreement for KIC 12017985 and KIC 11563791 becomes better, indicating the small difference would be due to the use of different pipelines for frequency analysis. Because systematic offsets in measured frequencies should not depend on stellar metallicity, further discussions on the effect of asteroseismic pipelines are beyond the scope of this study. Such discussion is provided in Pinsonneault et al. (2018). We note that they quantified the systematic ν_{max} or $\Delta\nu$ differences between the COR and SYD pipelines to be $\sim 1\%$ at most. The effect of these systematic uncertainties is also indicated in Figure 3.

Figure 4 compares our mass estimates with those in the APOKASC-2 catalog (Pinsonneault et al. 2018). There is a good agreement between the two mass estimates except for the most-luminous (lowest $\log g$) stars, for which frequency measurements become more difficult. After excluding stars with $\log g < 2$, the average mass difference is only $0.016 M_{\odot}$ (scatter is $0.033 M_{\odot}$). The two stars for which we obtain significantly lower mass compared to the APOKASC-2 catalog are KIC 5698156 and KIC 6611219, which also have $\log g < 2.0$ (1.89 and 1.74 respectively). There is one star for which we obtain a higher mass than the APOKASC-2 catalog (KIC 6279038), although the difference is barely above the 1σ measurement uncertainty. This star also has a low surface gravity ($\log g = 1.65$) and has a large uncertainty in the obtained mass. This comparison also confirms that our mass scale is not significantly different from previous studies.

¹⁶ We note that this might also be the case for KIC 12017985. However, because all observations, other than the Gaia, including our two observations and two APOGEE measurements, report consistent radial velocities, we do not conclude that this star is likely in a binary system.

Table 2
Asteroseismic Parameters, Mass, and Radius

Object	$\nu_{\max}^a$ (μHz)	$\sigma_{(\nu_{\max})}^a$ (μHz)	$\Delta\nu^a$ (μHz)	$\sigma_{(\Delta\nu)}^a$ (μHz)	Evo. Stage ^a	M^b ($M_{\odot}$)	$\sigma_{(M)}^b$ ($M_{\odot}$)	R^b ($R_{\odot}$)	$\sigma_{(R)}^b$ ($R_{\odot}$)	M_{sc}^c ($M_{\odot}$)
KIC 5184073	9.25	0.22	1.659	0.031	RGB	0.78	0.08	16.77	0.69	0.92
KIC 5439372	6.39	0.22	1.186	0.054	RGB	0.98	0.23	22.53	2.47	1.17
KIC 5446927	21.88	0.38	2.874	0.048	RC	1.49	0.13	14.97	0.58	1.44
KIC 5698156	9.73	0.31	1.677	0.031	RGB	0.76	0.11	16.38	0.94	0.96
KIC 5858947	168.93	0.89	14.533	0.019	RGB	0.98	0.05	4.36	0.09	1.01
KIC 5953450	140.87	0.83	12.715	0.021	RGB	0.99	0.05	4.81	0.10	1.01
KIC 6279038	5.59	0.28	1.018	0.047	RGB	1.18	0.34	26.50	3.29	1.42
KIC 6520576	17.91	0.50	2.693	0.016	unknown	0.94	0.11	13.19	0.64	0.99
KIC 6611219	6.96	0.21	1.330	0.028	RGB	0.70	0.10	18.61	1.12	0.89
KIC 7191496	16.23	0.24	2.455	0.021	RGB	0.88	0.06	13.50	0.35	1.04
KIC 7693833	31.73	0.32	4.046	0.014	RGB	1.03	0.05	10.35	0.18	1.12
KIC 7948268	120.45	0.82	11.450	0.015	RGB	0.93	0.04	5.03	0.08	0.97
KIC 8350894	12.69	0.29	2.005	0.024	RGB	0.90	0.08	15.47	0.54	1.08
KIC 9335536	11.08	0.82	1.861	0.095	RGB	0.85	0.28	15.84	2.10	1.02
KIC 9339711	20.51	0.31	2.825	0.019	RGB	1.05	0.07	13.05	0.36	1.21
KIC 9583607	25.25	0.51	3.816	0.055	RC	0.76	0.05	9.93	0.25	0.70
KIC 9696716	24.09	0.53	3.305	0.022	RGB	0.92	0.07	11.30	0.33	1.05
KIC 10083815	17.99	0.38	2.605	0.015	RGB	0.91	0.07	13.06	0.39	1.08
KIC 10096113	36.31	0.59	4.168	0.042	RC	1.44	0.10	11.50	0.32	1.42
KIC 10328894	30.72	0.40	3.965	0.016	RGB	0.96	0.05	10.19	0.20	1.07
KIC 10460723	22.97	0.45	3.146	0.013	RC	1.08	0.07	12.56	0.28	1.10
KIC 10737052	26.57	0.40	3.518	0.017	RC	1.10	0.05	11.72	0.18	1.11
KIC 11563791	43.03	0.51	5.005	0.019	RGB	1.01	0.05	8.85	0.19	1.15
KIC 11566038	31.36	0.32	3.954	0.023	RGB	1.03	0.05	10.44	0.23	1.15
KIC 12017985	18.24	0.29	2.620	0.018	RGB	1.00	0.07	13.54	0.36	1.15
KIC 12017985-17	...	...	...	...	...	0.99	0.07	13.48	0.36	1.15
KIC 12253381	22.00	0.40	3.032	0.014	RGB	0.96	0.06	12.07	0.31	1.12

Notes.

^a From Yu et al. (2018).

^b Obtained from the scaling relations with correction.

^c Obtained from the scaling relations without correction.

4. Kinematics

Although our selection of halo stars is based on radial velocity and metallicity measurements without taking astrometric measurements into account (Figure 1), we can confirm that most of our targets have halo-like kinematics thanks to the Gaia mission (Gaia Collaboration et al. 2016a). We adopt proper motions provided in Gaia data release 2 (Gaia DR2; Gaia Collaboration et al. 2018) and the radial velocity measured from our spectra (see Section 2). Distances can be estimated from either astrometric parallax or asteroseismic parallax. For the former, we use parallax measurements in Gaia DR2 (Lindgren et al. 2018) after correcting for the systematic offset of 0.052 mas (Zinn et al. 2019). For the asteroseismic parallax, we first compute the stellar radius using asteroseismic scaling relations with the $\Delta\nu$ correction as described in the previous section. Combined with the effective temperature from our high-resolution spectra (see Section 5), we obtain the luminosity of the stars through $L \propto R^2 T_{\text{eff}}^4$. To derive the parallax, this luminosity is then compared with the bolometric magnitude that is based on The Two Micron All-Sky Survey (2MASS; Skrutskie et al. 2006) K_s -band photometry and the bolometric correction provided by Casagrande & VandenBerg (2014). Interstellar extinction is corrected for using the 3D extinction map provided by Green et al. (2018). The two parallaxes are provided in Table 3 and are compared in Figure 5. Note that Table 3 lists the astrometric parallax without the 0.052 mas correction, but the correction is applied in Figure 5. The agreement between the two sets of values is good with a weighted average

difference of $\langle \pi_{\text{Gaia}} - \pi_{\text{seis}} \rangle = 0.028 \pm 0.007$ mas ($\sigma = 0.038$). For the calculation of kinematics, we adopt the asteroseismic parallax for all stars for consistency.

Table 4 shows the radial, azimuthal, and vertical components of the Galactocentric velocity; v_R , v_ϕ , v_z . We adopt $R_0 = 8.2$ kpc (McMillan 2017) and $z_0 = 0.025$ kpc (Jurić et al. 2008) for the solar position, and $(v_x, v_y, v_z)_\odot = (11.10, 247.97, 7.25) \text{ km s}^{-1}$ for the solar velocity relative to the Galactic center, where v_x is the velocity toward the Galactic center, v_y is in the direction of Galactic rotation, and v_z is toward north. The v_x and v_z come from Schönrich et al. (2010), and v_y comes from the proper motion measurement of Sgr A* (Reid & Brunthaler 2004) and R_0 . Coordinate transformation from the observed quantities to the Galactocentric Cartesian system was conducted with the `astropy.coordinates` package. We note that v_ϕ is taken positive toward the Galactic rotation direction. Uncertainties are estimated by Monte Carlo sampling. We note that all stars are treated as single, although some show radial velocity variations. While the presence of a binary companion might lead to inaccurate astrometric measurements in Gaia DR2, the renormalized unit weight error (*ruwe*), which is an indicator of the goodness of the astrometric solution in Gaia DR2, is smaller than 1.4 except for KIC 5858947, for which *ruwe* is ~ 1.7 .¹⁷

¹⁷ See https://gea.esac.esa.int/archive/documentation/GDR2/Gaia_archive/chap_datamodel/sec_dm_main_tables/ssc_dm_ruwe.html for more details about the *ruwe*.

Figure 3. Comparison of derived masses with Valentini et al. (2019) for four stars (KIC 7191496, KIC 12017985, KIC 11563791, and KIC 5858947, from left to right). Each star has three data points: (1) the mass derived in this work, (2) the mass derived in Valentini et al. (2019), and (3) the mass derived with our method but with $\nu_{\max}$ and $\Delta\nu$ values provided in Valentini et al. (2019). The effect of the 1% systematic uncertainties in $\Delta\nu$ and $\nu_{\max}$ measurements are also indicated with thin error bars for the (1) and (3) cases. Data points are horizontally shifted for visualization purposes.

Figure 4. Comparison of derived masses with the APOKASC-2 catalog (Pinsonneault et al. 2018) for common stars. Note that the error bars reflect only our measurement uncertainties. The median uncertainties provided in the APOKASC-2 catalog are 7.2% and 4.3% for random and systematic uncertainties, respectively.

Figure 6 shows the velocity distribution of stars. It is clear that most of the program stars do not follow the motion of the majority of the stars in the Kepler field, which are shown with gray contours and dots, i.e., they have very different velocities from the Galactic disk stars. The distribution of v_ϕ is particularly different (see the upper two and lower-right panels). This is expected because our selection is partly based on radial velocity and because the Kepler field is centered at $l = 76.32^\circ$, which provides a strong correlation between v_ϕ and radial velocity. The only exception would be again

Table 3
Parallax and Extinction

Object	π_{Gaia} (mas)	$\sigma(\pi_{\text{Gaia}})$ (mas)	π_{seis} (mas)	$\sigma(\pi_{\text{seis}})$ (mas)	$E(B - V)^a$ (mag)
KIC 5184073	0.117	0.015	0.202	0.009	0.06
KIC 5439372	0.289	0.020	0.274	0.032	0.06
KIC 5446927	0.331	0.029	0.379	0.016	0.06
KIC 5698156	0.663	0.024	0.776	0.048	0.06
KIC 5858947	1.156	0.028	1.277	0.046	0.07
KIC 5953450	0.791	0.025	0.755	0.027	0.06
KIC 6279038	0.145	0.027	0.177	0.024	0.04
KIC 6520576	0.202	0.013	0.230	0.011	0.05
KIC 6611219	0.272	0.025	0.334	0.021	0.06
KIC 7191496	0.424	0.019	0.421	0.013	0.05
KIC 7693833	0.636	0.024	0.592	0.015	0.12
KIC 7948268	0.762	0.024	0.748	0.019	0.04
KIC 8350894	0.193	0.020	0.259	0.010	0.05
KIC 9335536	0.203	0.015	0.233	0.032	0.05
KIC 9339711	0.415	0.020	0.404	0.013	0.07
KIC 9583607	0.570	0.024	0.627	0.016	0.04
KIC 9696716	0.429	0.025	0.506	0.018	0.04
KIC 10083815	0.367	0.026	0.409	0.017	0.07
KIC 10096113	0.280	0.015	0.321	0.011	0.18
KIC 10328894	0.271	0.014	0.302	0.007	0.05
KIC 10460723	0.370	0.020	0.363	0.010	0.04
KIC 10737052	0.255	0.013	0.262	0.006	0.06
KIC 11563791	0.975	0.025	0.953	0.029	0.05
KIC 11566038	0.495	0.020	0.471	0.012	0.04
KIC 12017985	0.799	0.027	0.887	0.026	0.04
KIC 12017985-17	0.799	0.027	0.892	0.028	0.04
KIC 12253381	0.354	0.023	0.344	0.011	0.04

Note.

^a Obtained from the relation $E(B - V) = 0.884\alpha$, where α and its relation to $E(B - V)$ are defined and provided in Green et al. (2018). Note that α is proportional to the amount of reddening and normalized to give $E(g - r) = 0.901$ at $\alpha = 1$.

KIC 7693833, which also stands out during the sample selection (Figure 1). It follows the motion of disk stars despite its low metallicity ($[\text{Fe}/\text{H}] \sim -2.3$) and is separately discussed in Section 6.4.3.

5. Abundance Analysis

5.1. Line List

Table 5 shows a list of lines used in this study together with measured equivalent widths. Lines were carefully selected by comparing synthetic spectra and a very high-S/N observed spectrum of the archetypal metal-poor red giant HD 122563. Additional lines were taken from Matsuno et al. (2018) for the analyses of high-metallicity stars. Hyperfine structure splitting was included for Sc II, V I, Mn I, Co I, Cu I, Ba II, and Eu II, assuming solar r -process abundance ratio for isotopic ratios of neutron-capture elements. Line positions and relative strengths were taken from McWilliam (1998) for Ba, Ivans et al. (2006) for Eu, and Robert L. Kurucz's line list for the others.¹⁸

Equivalent widths were measured through fitting Gaussian profiles to absorption lines. Lines were limited to those with reduced equivalent width ($REW = \log(EW/\lambda)$) smaller than -4.5 to avoid significant effects of saturation. Figure 7 compares equivalent widths measured from the 2017 and

¹⁸ <http://kurucz.harvard.edu/linelists.html>

Figure 5. Comparison of Gaia and asteroseismic parallaxes. The zero-point offset of the Gaia parallax is corrected. The lower panel shows the difference as a function of reddening.

2018 observations for KIC 12017985. Despite different spectral resolutions, the two measurements show excellent agreement ($EW = 0.51 \pm 2.31$ mÅ and $\Delta \log(EW) = 0.01 \pm 0.04$; in the direction from 2018 observation minus 2017 observation).

5.2. Stellar Parameter Determination

Stellar parameter determination and subsequent abundance measurements were conducted with a modified version of α^2 (Ramírez et al. 2014; Matsuno et al. 2018), which utilizes the February 2017 version of MOOG (Snedden 1973). We obtain models of the structure of atmospheres from interpolation of MARCS stellar model atmospheres with standard chemical composition (Gustafsson et al. 2008). In the process of stellar parameter determination, we adopt the line-by-line nonlocal thermodynamical equilibrium (NLTE) corrections for Fe abundances provided by Amarsi et al. (2016a).¹⁹ All of the subsequent abundance analysis is conducted under one-dimensional plane-parallel (1D) and local thermodynamical equilibrium (LTE) approximations unless otherwise stated. The solar chemical abundance is adopted from Asplund et al. (2009).

Stellar parameters are determined by requiring excitation balance of Fe I lines, using the asteroseismic scaling relation for $\nu_{\max}$, and minimizing the trend between REW and abundance

derived from each Fe I line. Basically, each condition is to constrain T_{eff} , $\log g$, and microturbulent velocity (ξ), respectively. Because the asteroseismic scaling relation constrains $\log g$ within a range of ~ 0.01 dex at a given temperature, the use of the relation is essentially the same as assuming a tight relation between T_{eff} and $\log g$. Note that, although the scaling relation might suffer from systematic uncertainties (see Section 6.2), $\log g$ is robustly constrained. We also used a prior on ξ as a function of $\log g$ (see Appendix A).

To achieve high precision and to minimize the effects of departures from 1D approximations and those of uncertain atomic data, we adopt a line-by-line differential abundance analysis. Because our targets span over ~ 2 dex in metallicity, we repeated the analysis by adopting two different standard stars. One is the well-studied metal-poor star HD 122563 with $[\text{Fe}/\text{H}] \sim -2.6$. This star is the only metal-poor giant whose T_{eff} and $\log g$ have been measured with high accuracy through interferometric measurements ($T_{\text{eff}} = 4636$ K; Karovicova et al. 2018) and asteroseismology ($\log g = 1.418$; Creevey et al. 2019), respectively. The other standard star is one of the program stars, KIC 9583607, which is more metal rich ($[\text{Fe}/\text{H}] \sim -0.7$) and was also observed by APOGEE. Because APOGEE is carefully calibrated at this metallicity and because this star has an asteroseismic $\log g$ constraint, the use of this star as a standard star ensures us that our parameters are not systematically biased. We adopt the value from the APOGEE DR14 for the temperature of this star ($T_{\text{eff}} = 5059$ K) and an asteroseismic estimate for the surface gravity ($\log g = 2.322$). The microturbulent velocities and metallicities of the standard stars are obtained from a standard analysis of individual iron lines. Using the aforementioned T_{eff} and $\log g$, we minimize the trend between the REWs and the abundances obtained from individual neutral iron lines to determine the microturbulent velocity. The average abundance from Fe II lines is adopted as the metallicity of the standard star because it is less sensitive to the choice of stellar parameters than that from Fe I lines. The stellar parameters of other stars determined relative to HD 122563 and those relative to KIC 9583607 are denoted as X_{MP} and X_{MR} , respectively.

Each star has two sets of parameters, X_{MP} and X_{MR} (see Appendix B), which are combined in such a way that we obtain precise parameters without introducing significant systematic offsets. We here combine the two results following the equation

$$X = X_{\text{MP}} * w_{\text{MP}} + X_{\text{MR}} * w_{\text{MR}}, \quad (3)$$

where w_{MP} and w_{MR} are the weights. We determine w_{MP} and w_{MR} from the comparison of the two sets of analyses as described below.

The comparison is provided in Figure 8 for parameters and their uncertainties as a function of metallicity. When a star and the standard star have a large metallicity difference, there are difficulties in accurate parameter determination. One is that departures from 1D approximations might not act in the same way. Another difficulty is that the number of common lines becomes smaller as the metallicity difference becomes larger. This is because absorption lines of the more metal-rich one suffer from saturation or blending while those of the more metal-poor one might be too weak to be detected. These effects are recognizable in Figure 8. We define the following parameters to compute the weights in Equation (3):

$$x_{\text{MP}} = 1.0 - ([\text{Fe}/\text{H}]_{\text{MP}} - [\text{Fe}/\text{H}]_{\text{standard}}) / s_{\text{MP}} \quad (4)$$

¹⁹ The grid is available at <https://www.astro.uu.se/~amarsi/>.

Table 4
Kinematic Properties

Object	v_R (km s ⁻¹)	$\sigma_{(v_R)}$ (km s ⁻¹)	v_ϕ (km s ⁻¹)	$\sigma_{(v_\phi)}$ (km s ⁻¹)	v_z (km s ⁻¹)	$\sigma_{(v_z)}$ (km s ⁻¹)
KIC 5184073	158.3	5.4	83.6	4.3	-119.6	4.6
KIC 5439372	-71.5	16.6	-5.5	1.9	165.5	24.1
KIC 5446927	-152.0	9.4	-18.2	2.8	-73.8	1.4
KIC 5698156	46.0	3.7	-136.2	1.8	-29.6	3.0
KIC 5858947 ^a	-155.7	6.3	96.7	1.7	7.9	1.0
KIC 5953450	334.3	9.6	-3.3	1.4	-6.1	1.8
KIC 6279038	-34.5	15.4	-73.5	4.5	37.6	14.3
KIC 6520576	-101.3	8.5	-79.5	5.1	-130.7	3.3
KIC 6611219	-267.1	20.6	-23.2	8.4	-22.8	1.6
KIC 7191496	38.0	1.0	-43.6	1.1	-50.9	0.5
KIC 7693833	66.9	2.1	240.1	1.1	-1.3	0.4
KIC 7948268	120.8	1.4	-41.1	1.2	-18.5	1.4
KIC 8350894	-169.2	7.8	61.1	4.7	-45.2	0.8
KIC 9335536	251.6	17.1	-241.9	41.9	151.4	36.7
KIC 9339711	52.4	0.5	-61.6	1.0	-158.7	2.7
KIC 9583607	-139.4	5.1	-83.9	1.0	26.6	2.6
KIC 9696716	-116.3	4.8	107.2	1.4	-49.2	0.6
KIC 10083815	358.3	12.7	-59.9	5.8	-60.9	0.8
KIC 10096113	-54.8	2.4	25.9	1.7	-59.0	1.0
KIC 10328894	-48.6	2.7	-81.8	1.0	20.4	2.5
KIC 10460723	106.8	1.3	-118.5	2.0	-34.1	1.7
KIC 10737052	-334.1	7.8	79.9	5.3	75.2	2.8
KIC 11563791	-28.6	1.7	-42.1	1.3	86.1	4.2
KIC 11566038	-158.6	4.8	-43.6	1.7	-32.9	0.8
KIC 12017985	-140.6	4.4	76.3	1.4	-88.1	1.6
KIC 12253381	-373.6	11.9	49.5	6.1	-37.8	1.4

Note.

^a The r_{ruwe} is ~ 1.7 , and hence the values presented here must be considered with a caution for this star.

$$x_{\text{MR}} = 1.0 - ([\text{Fe}/\text{H}]_{\text{standard}} - [\text{Fe}/\text{H}]_{\text{MR}})/s_{\text{MR}}, \quad (5)$$

where s_{MP} and s_{MR} are free parameters, and define $w_{\text{MP}} = \max\{0.0, \min\{1.0, x_{\text{MP}}\}\}$ (similarly for w_{MR}). The w_{MP} and w_{MR} are then scaled so that their sum becomes 1. The two parameters, s_{MP} and s_{MR} are chosen to be 1.4, but results are insensitive to the exact choice of these parameters. The w_{MP} and w_{MR} are also shown in Figure 8, and the adopted parameters are listed in Table 6. The parameters in Table 6 are used in asteroseismic analysis, which includes mass estimates, and in figures throughout this study.

Uncertainties provided in this paper only reflect random uncertainties that are obtained through a Markov Chain Monte Carlo method. It is important to take systematic uncertainties into consideration when one tries to quantitatively compare our results with other studies. Sources of systematic uncertainties include the uncertainties in stellar parameters and abundances of the standard stars, possible blending with unknown weak lines, and the NLTE and/or 3D effects. Among these, the impact of the first source is studied in Appendix C.

The adopted stellar parameters are compared to the results obtained from spectroscopic surveys (the APOKASC-2 catalog, which is based on APOGEE, and LAMOST) and photometric temperatures in Figure 9. The latter were derived by implementing Gaia BP , RP (Evans et al. 2018), and 2MASS J , H , K_s photometry into the InfraRed Flux Method (Casagrande et al. 2010, 2014), with reddening from Green et al. (2019). There are generally good agreements between this study and the literature, which indicates that our parameters do not have large systematic offsets. The exceptions are the outliers found in the comparison with

LAMOST and the metallicity-dependent systematic offset found in the metallicity comparison with APOGEE. The two outliers found in the comparison with LAMOST are KIC 5184073 and KIC 8350894. Although the reasons for the discrepancy are unclear, we note that these stars do not stand out from the overall trend when compared with APOGEE. The systematic metallicity offset from APOGEE at low metallicity demonstrates the difficulty of determining abundances in the absolute scale. For example, Amarsi et al. (2016b) reported $[\text{Fe}/\text{H}] = -2.49 \pm 0.11$ (stat) ± 0.14 (sys) for the metallicity of HD 122563, our metal-poor standard star, from 3D and NLTE modeling of stellar atmospheres. Our adopted value of $[\text{Fe}/\text{H}] = -2.60$ is well within the range of uncertainties. We note that the good agreement at high metallicity would be mainly thanks to our choice of the effective temperature of the standard star, which was taken from APOGEE.

5.3. Elemental Abundances

Elemental abundances are derived based on equivalent widths for most of the elements studied in the present work. We first derive abundances for the standard stars, then derive abundances of the other stars differentially through a line-by-line analysis. Thanks to this approach, we achieve high precision in relative abundances, although the absolute scale could be affected by systematic uncertainties such as those due to the NLTE effect.

As in the case for the stellar parameters, we obtain two sets of chemical abundance for each star through two analyses with different standard stars. We combine these abundances for each species with Equation (3). However, there are cases where a

Figure 6. Large symbols (black crosses, green circles, orange triangles, and pink diamonds) show the velocities of the program stars with error bars, while gray points and contours are the distribution of stars in the cross-matched catalog of LAMOST DR5 (Zong et al. 2018) and Yu et al. (2018). The different symbols are used according to the Mg- and Fe-based classification made in Section 5 (black crosses: metal poor, green circles: high Mg, orange triangles: low Mg, pink diamonds: very low Mg; see Figure 10). For the LAMOST sample, we calculated velocities in exactly the same way as for the program stars.

Table 5
Line List and Measured Equivalent Widths

λ (Å)	Species	χ (eV)	$\log gf$	Syn ^a	KIC 5184073 (mÅ)
4053.821	Ti II	1.893	−1.070		80.8
4056.187	Ti II	0.607	−3.280		49.5
4082.939	Mn I	2.178	−0.354		...
4086.714	La II	0.000	−0.070	syn	58.9
4099.783	V I	0.275	−0.100		41.6

Note.

^a Lines to which spectral synthesis applied are flagged with “syn” in this column.

(This table is available in its entirety in machine-readable form.)

star has no common line with one of the standard stars for some elements. In this case, we have to rely on the abundance derived from the analysis with the other standard star. Unless the other analysis has a weight of 1, the abundance of the star has to be taken with caution. We show these cases with open symbols in the figures.

Here we take into account two sources of uncertainty in abundance measurements. One is due to the noise present in the spectra, which affects measured equivalent widths. We denote this component σ_1 and estimate it from the line-to-line scatter (σ_{sct}) in derived abundance for each species as $\sigma_1 = \sigma_{\text{sct}} / \sqrt{N}$, where N is the number of lines used for the abundance measurements. When N is smaller than 4 and when σ_{sct} is smaller than σ_{sct} for Fe I lines, we substitute σ_{sct} with the latter. The other source is the uncertainties of stellar parameters (σ_2). This component is estimated by repeating the analyses, changing stellar parameters by the same amount as their uncertainties. We quadratically sum σ_1 and σ_2 to obtain final uncertainty estimates.

Abundances from CH (molecule), and O I, Na I, Mg I, Si I, Co I, Cu I, Zn I, Ba II, La II, Ce II, Nd II, Sm II, and Eu II are analyzed through spectral synthesis. The spectral synthesis is necessary for species that produce significantly asymmetric lines, such as CH, Co, and Eu. In addition, these elements have a small number of available lines, and the abundance measurement has to rely on partially blended lines for which

Figure 7. Comparison of equivalent widths for KIC 12017985 from 2017 and 2018 observations. Mean difference and line-to-line scatter are shown in the figure. The dashed lines indicate the one-to-one relation.

we need spectral synthesis. Using the line list from VALD3,²⁰ series of synthetic spectra were generated with varying abundance of the single element of interest to determine the best-fit abundance by minimizing χ^2 . After the abundance is determined, we measure the equivalent width of the line produced by the element of interest by creating a synthetic spectrum without taking other lines into account. To estimate the measurement uncertainty due to the noise and stellar parameters, the measured equivalent width is then fed into the same process as that applied for species analyzed through equivalent width analysis.

5.4. Abundances: Results

Abundances are provided in Table 7 and shown as a function of metallicity in Figures 10–14. Our stars are compared with turnoff halo stars studied by Nissen & Schuster (2010, 2011), Fishlock et al. (2017), and Reggiani et al. (2017) to qualitatively compare our results with already-studied halo populations. It is necessary to account for systematic uncertainties in addition to those reported in this study when our abundances are quantitatively compared

with other studies (see Appendix C). We also note that there could be systematic offsets in the absolute abundance scale because of differences in the line list or in spectral types of the targets.

Following Nissen & Schuster (2010), we divide our sample using Mg abundance (Figure 10). First, our sample is divided by the metallicity at $[\text{Fe}/\text{H}] \sim -1.7$ (metal poor/others). For the metal-rich sample, we further divide the sample by $[\text{Mg}/\text{Fe}]$: stars having $[\text{Mg}/\text{Fe}]$ comparable to the metal-poor subsample (high α), those having lower $[\text{Mg}/\text{Fe}]$ (low α), and those having even lower $[\text{Mg}/\text{Fe}]$ (very low α). While $[\text{Mg}/\text{Fe}]$ spreads among each of the three metal-rich subsamples are small (0.03 and 0.02 dex for the high- and low- α subsamples) and comparable to the measurement uncertainties (typically 0.03 dex), differences in average $[\text{Mg}/\text{Fe}]$ between different subsamples are significantly larger than measurement uncertainties (Table 8).²¹ However, this division is arbitrary, and we quantify the difference and investigate if the three subsamples also show differences in other element abundances or in kinematics in Section 6.1.

There are a handful of Na-enhanced objects at $[\text{Fe}/\text{H}] < -1$ (bottom panel of Figure 11; KIC 8350894, KIC 5184073, and KIC 6520576), for which spectra around Na I 5688 Å are shown in Figure 15. The Na enhancements are clear and cannot be attributable to cosmic rays or bad pixels. While tabulated Na abundances in Table 7 are in 1D/LTE, it is known that Na suffers from a large NLTE effect (Lind et al. 2011). NLTE corrections (NLTE – LTE) from Lind et al. (2011)²² are -0.114 , -0.141 , and -0.232 dex for KIC 6520576, KIC 5184073, and KIC 8350894, while those for their comparison stars in Figure 15 are -0.06 , -0.115 , and -0.145 dex, respectively. Therefore, the NLTE effect cannot be the cause of the large Na abundance. The origin of these stars is discussed in Section 6.4.1. Two other objects (KIC 5446927 and KIC 10096113) at $[\text{Fe}/\text{H}] \sim -0.7$ also have $[\text{Na}/\text{Fe}] \sim 0.4$, which is similar to the value found for one of the previously mentioned Na-enhanced objects. However, their $[\text{Na}/\text{Fe}]$ values are not distinctly high compared to other stars at similar metallicity. It is still interesting that these two stars are significantly more massive than the rest of the stars in our sample (see Section 6.4.2).

The Si abundance ratio exhibits a large scatter and an increasing trend toward low metallicity (top-left panel of Figure 12), which is not seen in previous studies (e.g., Zhang et al. 2011). The NLTE corrections for Si are not expected to be large for giants according to Shi et al. (2009), who derived exactly the same abundance in LTE and NLTE calculations for a metal-poor giant. The high Si abundance at low metallicity would be related to the Si abundance of HD 122563 adopted in this work. Our adopted value of $[\text{Si}/\text{Fe}] = 0.557$ falls within the range reported in literature, which spans from 0.30 to 0.72 dex after correcting for a different assumed solar abundance (Fulbright 2000; Westin et al. 2000; Mishenina & Kovtyukh 2001; Johnson 2002; Barbuy et al. 2003; Cayrel et al. 2004; Honda et al. 2004; Aoki et al. 2005, 2007; Roederer et al. 2014; Sakari et al. 2018).²³ On the other hand, stars with high Si abundance at $[\text{Fe}/\text{H}] \sim -1.2$ are not affected by the choice of the Si abundance of HD 122563. Typically, their Si abundance relies on four Si I lines, and all of the lines consistently provide

²¹ We note that the low- α -stars in Table 8 and in subsequent similar tables do not include the very low- α stars.

²² <http://www.inspect-stars.com>

²³ Data are collected via the SAGA database (Suda et al. 2008).

²⁰ <http://vald.astro.uu.se/>

Figure 8. Comparison of stellar parameters from two analyses. One analysis uses the metal-poor standard star (HD 122563) and the other uses the metal-rich one (KIC 9583607). Because the latter is one of the program stars, we show its location with an open circle in the bottom four panels. Considering the trends in the ξ uncertainties, we adopt stellar parameters with the weights shown in the bottom panel.

high Si abundance. Therefore, their Si abundance could be astrophysical rather than artificial. Fernández-Trincado et al. (2020) recently suggested that there is a population of stars in the inner halo with enhanced $[\text{Si}/\text{Fe}]$ ratios and that the population is remnants of globular clusters being disrupted.

Small offsets between our results and the literature are also found for other elements, e.g., in Ti, Sc, and Ni (Figures 12 and 13). The offsets might reflect systematic uncertainties in both our study and the literature. Our discussion is not significantly affected by systematic uncertainties, because our main interest is to compare the abundance ratios of different stellar populations at a given metallicity within our sample.

We investigate the ionization balance for elements for which absorption lines of both neutral and singly-ionized species are detected, namely Ti, Cr, and Fe (Figure 16). The derived abundance difference, $[\text{X}/\text{H}]_{\text{I}} - [\text{X}/\text{H}]_{\text{II}}$, shows nonzero offsets for all three elements. The offsets are in the sense that abundances derived from neutral species are smaller than those from ionized species, which might indicate that they could be due to the NLTE effect. The differences correlate with metallicity, indicating that the amplitude of the NLTE effect mainly depends on the metallicity. These features are consistent with results of NLTE calculations (e.g., Bergemann & Cescutti 2010; Bergemann 2011; Amarsi et al. 2016b), which predict that neutral species are

overabundant in LTE calculations and that lower abundances are obtained from neutral species rather than from ionized species. While grids of NLTE corrections are available for a subset of stars, most of them do not cover the entire range of stellar parameters spanned by our targets. We therefore stick to 1D/LTE analysis in order to keep consistency within our sample.

6. Discussion

6.1. Halo Subpopulations

We divided the sample into four subsamples in Section 5. We investigate the kinematics and abundances of the four subsamples in this subsection.

We first focus on our high- α and low- α subsamples. There are hints of abundance differences between the high- α and low- α subsamples in $[\text{X}/\text{Fe}]$ of many elements (C, O, Na, Si, Ca, Sc, Ti, V, Co, Ni, Cu, Zn, and Y; Figures 11–14). The abundance differences and their significance are quantified in Table 8. We note that, although the abundance differences listed in Table 8 may be compared with Table 5 of Nissen & Schuster (2011), a direct comparison is not possible due to the difference in metallicity coverage. While the majority of our sample is at $[\text{Fe}/\text{H}] \lesssim -1.1$, Nissen & Schuster (2011) define the difference at $-1.1 < [\text{Fe}/\text{H}] < -0.7$. This relatively lower metallicity of our

Figure 9. Comparison of stellar parameters with literature values. We note that error bars only reflect internal uncertainties in our analysis and surveys.

Table 6
Adopted Stellar Parameters

Object	T_{eff} (K)	$\sigma(T_{\text{eff}})$ (K)	$\log g$	$\sigma(\log g)$	ξ (km s^{-1})	$\sigma(\xi)$ (km s^{-1})	$[\text{Fe}/\text{H}]$	$\sigma([\text{Fe}/\text{H}])$
KIC 5184073	4864	44	1.876	0.010	1.581	0.044	-1.422	0.028
KIC 5439372	4835	34	1.716	0.016	1.915	0.045	-2.479	0.029
KIC 5446927	5102	37	2.261	0.008	1.530	0.047	-0.742	0.022
KIC 5698156	4644	43	1.888	0.015	1.533	0.049	-1.289	0.016
KIC 5858947	5105	72	3.149	0.004	1.221	0.117	-0.775	0.047
KIC 5953450	5127	74	3.071	0.005	1.234	0.115	-0.623	0.057
KIC 6279038	4761	31	1.654	0.025	1.784	0.044	-2.055	0.027
KIC 6520576	4971	35	2.169	0.014	1.602	0.042	-2.289	0.029
KIC 6611219	4652	45	1.742	0.016	1.566	0.052	-1.201	0.018
KIC 7191496	4903	34	2.122	0.007	1.672	0.038	-2.076	0.028
KIC 7693833	5094	46	2.422	0.006	1.664	0.061	-2.265	0.038
KIC 7948268	5154	51	3.004	0.004	1.230	0.062	-1.199	0.028
KIC 8350894	4797	45	2.012	0.012	1.491	0.054	-1.091	0.025
KIC 9335536	4817	32	1.951	0.026	1.527	0.038	-1.528	0.026
KIC 9339711	4937	33	2.226	0.006	1.509	0.037	-1.469	0.020
KIC 9583607	5059	...	2.322	...	1.590	...	-0.700	...
KIC 9696716	4962	41	2.297	0.009	1.453	0.041	-1.440	0.024
KIC 10083815	4784	74	2.161	0.013	1.476	0.097	-0.936	0.060
KIC 10096113	4948	48	2.475	0.007	1.443	0.070	-0.717	0.018
KIC 10328894	5018	33	2.405	0.006	1.528	0.039	-1.850	0.026
KIC 10460723	4922	40	2.275	0.010	1.466	0.046	-1.272	0.021
KIC 10737052	4973	42	2.340	0.008	1.478	0.047	-1.255	0.021
KIC 11563791	4974	54	2.550	0.006	1.326	0.069	-1.114	0.023
KIC 11566038	4999	38	2.413	0.005	1.451	0.046	-1.423	0.025
KIC 12017985	4945	33	2.175	0.006	1.628	0.036	-1.844	0.024
KIC 12017985-17	4932	33	2.175	0.007	1.637	0.034	-1.870	0.024
KIC 12253381	4922	37	2.256	0.007	1.512	0.037	-1.556	0.027

Figure 10. $[\text{Mg}/\text{Fe}]$ vs. $[\text{Fe}/\text{H}]$ with measurements from Mg I. Our targets are shown with large symbols with error bars and are divided into four subgroups depending on their Mg and Fe abundance: metal poor (black crosses), α rich (green circles), α poor (orange triangles), and very α poor (pink diamonds). The solid black line is the linear fit for the metal-poor subsample. Small green circles and orange triangles are the high-/low- α populations from Nissen & Schuster (2010). Small gray crosses are stars from Reggiani et al. (2017).

Table 7
Abundances

Object	Species	$[\text{X}/\text{H}]$	$\sigma_{([\text{X}/\text{H}])}$	$[\text{X}/\text{Fe}]$	$\sigma_{([\text{X}/\text{Fe}])}$
KIC 5184073	CH	-1.92	0.12	-0.38	0.09
KIC 5184073	O I	-1.20	0.09	0.35	0.09
KIC 5184073	Na I	-1.10	0.09	0.44	0.09
KIC 5184073	Mg I	-1.26	0.05	0.27	0.04
KIC 5184073	Si I	-1.23	0.02	0.31	0.03

(This table is available in its entirety in machine-readable form.)

sample leads to a smaller abundance difference in general (e.g., $\Delta[\text{Mg}/\text{Fe}] = [\text{Mg}/\text{Fe}]_{\text{high-}\alpha} - [\text{Mg}/\text{Fe}]_{\text{low-}\alpha}$ is 0.145 and 0.219 in our study and in their study, respectively).

For most of these elements, except for O, Na, and Ca, the probability that the two subsamples have the same mean abundance is small ($P < 0.05$; Table 8), where the P value is calculated by the t test for the means of two independent samples. The limited sample size of O abundances (middle panel of Figure 11), which is due to telluric absorption lines, prevents us from establishing the statistical significance. The effect of the outlier is evident for Na (bottom panel of Figure 11) in Table 8, where the square root of the variance among each subsample is much larger than the typical measurement uncertainties. The Ca abundance difference is neither statistically significant nor as clear as that seen in Nissen & Schuster (2010; top-right panel of Figure 12 and Table 8). Although the Ca abundance difference is clear, Nissen & Schuster (2010, 2011) reported the difference to be small ($\Delta[\text{Ca}/\text{Fe}] = 0.074$ at $-1.1 \lesssim [\text{Fe}/\text{H}] \lesssim -0.7$). Recall that our sample provides a smaller $\Delta[\text{X}/\text{Fe}]$ for other elements because of lower metallicity.

Figure 11. Same as Figure 10 but for light elements, namely, C (CH), O (O I), and Na (Na I). Open symbols indicate that while the abundance should be derived by weighting two analyses with different standard stars, only one of them can be used because the other standard does not have common lines with the target star.

Figure 12. Same as Figure 10 but for other α elements, namely, Si (Si I), Ca (Ca I), and Ti (Ti I and Ti II). Note that the vertical scale is different only for Si.

Other elements do not exhibit clear differences. Cr and Mn abundance differences (Figure 13) are reported by Nissen & Schuster (2011) to be small (0.03–0.04 dex); therefore, the lack of difference would be due to the lack of intrinsic difference. The abundance differences in neutron-capture elements (Ba, La, Ce, Nd, Sm) are not detectable (Figure 14) partly because of the large intrinsic dispersion in each of the two subsamples as can be seen in the square root of variances and small intrinsic abundance difference ($\Delta\langle[X/\text{Fe}]\rangle$) in Table 8.

These results are consistent with the abundance differences between turnoff high- α and low- α populations reported in the literature (Nissen & Schuster 2010, 2011; Nissen et al. 2014). This suggests that our high-/low- α subsamples correspond to the two distinct halo populations reported by Nissen & Schuster (2010).

These two subsamples also differ in stellar kinematics (Figure 6). The radial component of the velocity (v_R) of stars in our low- α subsample shows completely different distributions having large absolute values when compared to the high- α subsample. This suggests that the low- α subsample has a more radial orbit, which is a signature of the Gaia Sausage (Belokurov et al. 2018). Because the Gaia Sausage is considered to correspond to the low- α population of Nissen & Schuster (2010), kinematics also support the correspondence of our low- α subsample and the low- α population of Nissen & Schuster (2010). Although Nissen & Schuster (2010) reported that the

high- α and low- α populations tend to be prograde and retrograde, respectively, we do not detect such differences in v_ϕ . However, we caution that the v_ϕ distribution is strongly affected by our sample selection. The selection is partly based on the observed heliocentric radial velocity, which highly correlates with v_ϕ because of the galactic longitude of the Kepler field. Our selection has a bias against stars on a prograde orbit.

We here discuss the very low- α subsample. This subsample consists of three stars, KIC 5953450, KIC 9335536, and KIC 9583607, among which KIC 9335536 is located at $[\text{Fe}/\text{H}] \sim -1.5$ and the others are at $[\text{Fe}/\text{H}] \sim -0.6$ (Figure 10). While these three stars are selected to be the very low- α subsample based on Mg abundance, other element abundances also seem to behave differently from other subsamples, with a tendency of being extreme cases of the low- α subsample discussed above. The most metal-poor star among this subsample, KIC 9335536 shows a large retrograde motion as well as a relatively large $\sqrt{v_R^2 + v_z^2}$. Its metallicity, low Mg and Ca abundances (Figure 10 and top right panel of Figure 12), and kinematics suggest its association with Sequoia (Matsuno et al. 2019; Myeong et al. 2019), a kinematic substructure suggested to be a signature of dwarf galaxy accretion. Kinematics of the other two stars characterized by high v_R basically follow the trend of the low- α subsample (Figure 6). While these results indicate that the very low- α subsample is

Figure 13. Same as Figure 11 but for iron-peak elements, namely, V (V I), Cr (Cr I and Cr II), Mn (Mn I), Co (Co I), Ni (Ni I), Zn (Zn I) and Cu (Cu I). Note that the vertical scale is different only for Cu.

clearly different from the high- α population, it is still unclear if it can be regarded as a separate component from the low- α subsample.

The metal-poor subsample is chemically homogeneous to some extent; there are tight trends of $[X/Fe]$ with $[Fe/H]$ for many of the elements studied (Figures 10–14). The dispersions around the linear fit in $[X/Fe]$ – $[Fe/H]$ are significant at more than 3σ only for CH (probably because of evolutionary effects), Na (because of the outlier) in Figure 11, and neutron-capture elements, Y, Ba, Nd, and Eu (Figure 14). This is consistent with the result reported by Reggiani et al. (2017), who conducted high-precision abundance analysis for metal-poor turnoff stars, concluding that there is no significant scatter in abundances in most of the elements for the main halo population.

It is not clear from chemical abundances if the metal-poor subsample corresponds to the low-metallicity extension of other subsamples. On the other hand, kinematics are more similar to the high- α subsample, which might be related to the fraction of metal-poor stars in the low- α or high- α populations. We note, however, that radial velocity was not taken into consideration in the sample selection for the lowest metallicity stars (Figure 1), which would introduce a bias in their distribution in velocity space.

6.2. How Reliable Is the Asteroseismic Mass?

In this subsection, we discuss the reliability of mass estimates from the scaling relations of asteroseismology. Previous studies on this for low-metallicity stars include Epstein et al. (2014), Casey et al. (2018), Miglio et al. (2016), and Valentini et al. (2019). Here we have 26 halo stars, five of which are below $[Fe/H] < -2$ and additional 16 are below $[Fe/H] < -1$. As far as we know, this is the largest sample of metal-poor stars for which asteroseismology and high-resolution spectroscopy are combined. We take advantage of our sample to revisit the asteroseismology at low metallicity. We first study a possible systematic bias in the estimated mass that is dependent on the evolutionary status by separating stars into luminous giants ($\log g < 2$), less-luminous giants ($\log g > 2$), and red clump stars. We also discuss how accurately the scaling relations can estimate stellar masses by comparing the derived masses and the expected masses for our sample using theoretical stellar evolution models.

In Figure 17, we investigate if the estimated mass correlates with metallicity or surface gravity. We also present the range of initial stellar masses within which stars are on the red giant branch phase at given age and metallicity using MESA Isochrones and Stellar Tracks (MIST; Choi et al. 2016; Dotter 2016), which is based on Modules for Experiments in

Figure 14. Same as Figure 10 but for neutron-capture elements, namely, Y (Y II), Zr (Zr I), Ba (Ba II), La (La II), Ce (Ce II), Nd (Nd II), Sm (Sm II), and Eu (Eu II).

Stellar Astrophysics (MESA; Paxton et al. 2011). Because halo stars are generally considered to be old from independent studies ($\gtrsim 10$ Gyr; e.g., Jofré & Weiss 2011; Schuster et al. 2012; VandenBerg et al. 2013; Carollo et al. 2016; Kilic et al. 2019), we present the mass as a function of metallicity for 6, 8, 10, 12, and 14 Gyr isochrones. Red giant stars are selected based on surface gravity ($\log g < 3.3$) and the phase parameter as they are on either the red giant branch phase, core He-burning phase, early AGB phase, or thermal-pulsing AGB phase (phase = 2, 3, 4, and 5). The range of initial masses of red giant stars is measured for each isochrone and shown in Figure 17. Although mass loss is not considered, its effect remains small ($\lesssim 0.01 M_{\odot}$) until a hydrogen-shell-burning low-mass star reaches $\log g = 2.0$ while ascending the red giant branch in the case of MIST stellar evolutionary tracks, which assume $\eta = 0.1$ as Reimer’s mass-loss efficiency parameter. The mass lost remains small ($< 0.015 M_{\odot}$) even when considering another stellar evolution model, BaSTI (Pietrinferni et al. 2004, 2006), which assumes higher mass-loss efficiency, $\eta = 0.4$. This is because mass loss is assumed to become important only when stars evolved close to the tip of the RGB.

In the left panel of Figure 17, the large scatter and/or the presence of outliers is evident in particular at high metallicity ($[\text{Fe}/\text{H}] > -1.7$), while at low metallicity, the scatter seems to

be mostly due to the measurement uncertainties.²⁴ Although the scatter might indicate the presence of a significant age dispersion, it could also be caused by other effects, such as systematic bias as a function of evolutionary status.

In order to inspect if this spread is due to the evolutionary status, the right panel of Figure 17 visualizes the correlation between mass and surface gravity. This panel suggests that masses of luminous giants ($\log g \lesssim 2.0$, equivalently $\Delta\nu \lesssim 2.0 \mu\text{Hz}$) are systematically lower than or underestimated ($\langle M \rangle = 0.78 M_{\odot}$) compared to less-luminous hydrogen-shell-burning red giant branch stars ($\langle M \rangle = 0.97 M_{\odot}$), although uncertainties are large for some luminous stars due to low oscillation frequencies. Red clump stars also show a large dispersion in mass including the two obviously overmassive stars, whose origin remains unclear (see Section 6.4.2). Possible systematic mass offsets for these luminous giants and red clump stars are also suggested by, e.g., Pinsonneault et al. (2018). This offset could be due to mass loss or systematic uncertainties that only affect these stars.

Considering these possible effects of the evolutionary status on the mass estimates and uncertainties in modeling of mass

²⁴ This can be confirmed by calculating the reduced χ^2 (χ_{ν}). We obtain $\chi_{\nu} = 0.73$ (7 stars) for the metal-poor sample and $\chi_{\nu} = 1.65$ (14 stars) for the metal-rich stars. Red clump stars are excluded in this χ_{ν} calculation.

Figure 15. Spectra of Na-enhanced objects (red solid) and those of comparison stars (black dashed) with vertical offsets for visualization purposes. Stellar parameters and 1D LTE abundances are shown as ($T_{\text{eff}}/\log g/[Fe/H]/[Na/Fe]$).

Table 8
Abundance Differences

Element	High α			Low α			$\Delta\langle[X/Fe]\rangle$	$\sigma([X/Fe])_{\text{median}}$	P Value
	$\langle[X/Fe]\rangle$	$\sqrt{\text{Var}}$	N	$\langle[X/Fe]\rangle$	$\sqrt{\text{Var}}$	N			
Mg I	0.404	0.030	7	0.258	0.021	9	0.146	0.035	0.000
CH	-0.052	0.054	7	-0.260	0.228	9	0.207	0.082	0.027
O I	0.644	0.068	3	0.443	0.157	4	0.201	0.083	0.079
Na I	0.086	0.153	7	0.112	0.341	9	-0.025	0.081	0.845
Si I	0.479	0.105	7	0.310	0.063	9	0.169	0.036	0.004
Ca I	0.335	0.028	7	0.305	0.039	9	0.030	0.022	0.097
Ti I	0.245	0.040	7	0.152	0.035	9	0.093	0.024	0.000
Ti II	0.395	0.029	7	0.340	0.025	9	0.055	0.035	0.002
Sc II	0.275	0.078	7	0.193	0.054	9	0.082	0.036	0.039
V I	0.101	0.069	7	0.017	0.069	9	0.085	0.051	0.029
Cr I	-0.073	0.039	7	-0.123	0.050	9	0.050	0.032	0.042
Cr II	0.149	0.060	7	0.162	0.090	9	-0.013	0.041	0.730
Mn I	-0.302	0.100	7	-0.330	0.082	9	0.028	0.028	0.561
Co I	0.166	0.079	7	0.030	0.093	9	0.136	0.080	0.007
Ni I	0.075	0.055	7	-0.027	0.039	9	0.102	0.019	0.002
Zn I	0.163	0.059	7	0.074	0.035	9	0.089	0.096	0.006
Cu I	-0.567	0.177	7	-0.810	0.147	9	0.242	0.083	0.013
Y II	0.198	0.083	7	-0.030	0.091	9	0.229	0.042	0.000
Ba II	0.213	0.107	7	0.171	0.165	9	0.042	0.064	0.548
La II	0.329	0.174	6	0.294	0.156	9	0.034	0.089	0.704
Ce II	0.196	0.109	6	0.177	0.121	9	0.019	0.081	0.757
Nd II	0.310	0.073	7	0.398	0.138	9	-0.088	0.061	0.125
Sm II	0.409	0.076	7	0.476	0.099	9	-0.067	0.093	0.147
Eu II	0.459	0.099	7	0.577	0.129	9	-0.119	0.083	0.056

Figure 16. Abundance difference between those derived from neutral species and ionized species. The color shows the temperature of the stars.

Figure 17. Left: mass as a function of metallicity. Red clump stars, luminous stars, and the other red giant branch stars are separated. Small symbols are overplotted according to the Mg and Fe classification in Section 5.4. We present the initial mass range for stars to be on the red giant branch at a given age and metallicity using MIST isochrones (Choi et al. 2016; Dotter 2016; see text for more details.). Note that while each theoretical line has a width reflecting the mass range of stars along the red giant branch, the width is too narrow to be visible in most cases. Right: mass as a function of surface gravity (the points are color-coded by metallicity). Crosses are overplotted for red clump stars.

loss, we separate red clump stars and luminous giants ($\log g < 2.0$) from the other 15 red giant branch stars. The estimated masses of less-luminous red giant branch stars show a smaller dispersion. The χ^2 test for the 15 stars indicates that the mass dispersion is insignificant ($P = 0.65$), whereas there is a significant dispersion in mass ($P < 0.0001$) when we consider all the stars (26 stars). The distribution of mass is also shown in Figure 18, which illustrates a large scatter when all stars are considered and a small scatter when we focus on less-luminous red giant branch stars. These results indicate that the scaling relations provide consistent mass estimates within

less-luminous giants ($\log g > 2$); otherwise, the dispersion should be significantly larger than measurement uncertainties. We hereafter focus on less-luminous giants when discussing stellar masses otherwise stated and consider that a comparison within this subset is not significantly affected by the systematic bias influenced by evolutionary status.

We now discuss the absolute accuracy of our estimated masses. Even though mass dispersion disappears by restricting the sample to less-luminous giants, the average mass ($0.97 M_\odot$) is still higher by $\sim 10\%$ than the value expected for the age of halo stars ($0.8\text{--}0.9 M_\odot$; see the left panel of Figure 17). This

Figure 18. The distribution of mass of our program stars. The distribution functions are the sum of Gaussian functions, each of which corresponds to one star. Each Gaussian is normalized to 1 and centered on the median estimate of the mass and has a width of the corresponding uncertainty.

offset cannot be attributed to stellar parameters. In order to reduce the derived mass by 10%, we would need to decrease T_{eff} by 15%, which corresponds to ~ 750 K. Recall that our stellar parameters were obtained in a differential manner with respect to well-calibrated stars, and the median uncertainties in T_{eff} are 40 K for this subset of stars. Although the mass loss does not affect the discussion, it makes the situation worse if taken into account.

We note that the offset found in the present study might not be unique to low-metallicity stars. Several studies reported that the asteroseismic scaling relations provide systematically larger masses for red giants in eclipsing binary systems than dynamical mass estimates (Brogaard et al. 2016; Gaulme et al. 2016; Themeßl et al. 2018; Hekker 2020). Gaulme et al. (2016) found that the asteroseismic mass estimates are systematically larger by 13%–17% even when corrections, including the one used in the present study, are applied to the scaling relations. Although the correction suggested by Rodrigues et al. (2017) is not discussed in Gaulme et al. (2016), we note that we find consistent mass between Valentini et al. (2019), who make use of the correction of Rodrigues et al. (2017), and our study when the same frequencies are used. Themeßl et al. (2018) suggested using a lower reference large-frequency separation of $\Delta\nu = 130.8 \mu\text{Hz}$ instead of the measured solar value of $\Delta\nu_{\odot} = 135.1 \mu\text{Hz}$ from an empirical approach, which would also lower the derived masses by $\sim 14\%$. If our derived masses are lowered by $\sim 14\%$, the average mass would be consistent with the expected mass for red giants in the Milky Way halo. These results do not contradict Zinn et al. (2019), who also showed that the scaling relations overestimate stellar masses for low-metallicity stars compared to the mass estimates from Equation (1), where R is from $L \propto R^2 T_{\text{eff}}$ and L is partly based on the Gaia DR2 parallax.

Although there seems to be a systematic offset, the small scatter in masses obtained for stars limited to less-luminous red

giants indicates that the mass of low-metallicity stars can be estimated with high internal precision. Therefore, we should be able to explore the history of the Milky Way halo with accurate stellar ages of red giants once the systematic offset is resolved by future studies.

6.3. Formation Timescales of the Halo

In this subsection, we discuss the formation timescale of the Galactic halo focusing on our high- α and low- α subsamples. We first discuss constraints from chemical abundances and then discuss those from stellar mass. Note that the chemical abundance is discussed with the whole sample, while the stellar mass is discussed based on a subsample of stars because of possible evolutionary-status-dependent offsets in the measurements. Appendix D verifies that chemical abundance is not correlated with evolutionary status, which ensures our use of the entire sample for its discussion. Because we have shown that these two subsamples correspond to the low- α and high- α populations of Nissen & Schuster (2010), we refer to each subsample as a population.

6.3.1. Constraints from Abundances

The $[\alpha/\text{Fe}]$ difference, such as seen in Figure 10, is usually interpreted as a result of different contributions from Type Ia supernovae (SNe Ia), which is a result of a different star formation timescale (Nissen & Schuster 2010). Differences in other elemental abundances could also be due to SN Ia contributions. For example, Na and Sc, which are not α -elements, in the bottom panel of Figure 11 and in the top-left panel of Figure 13, are mostly synthesized by massive stars and ejected by Type II supernovae (SNe II) having similar nucleosynthesis origins to α elements. On the other hand, some other elements, especially neutron-capture elements, could have different nucleosynthesis origins, which would deliver independent information for estimates of the formation timescale of stellar systems.

Figure 19 shows the abundance ratios between neutron-capture elements (see also Fishlock et al. 2017) as a function of $[\text{Mg}/\text{H}]$. The associated abundance difference and significance are quantified in Table 9 in a similar manner to Table 8. Y is a light neutron-capture element, which is considered to be formed by the weak s -process in massive stars (e.g., Pignatari et al. 2010). The $[\text{Y}/\text{Fe}]$ ratio is generally low in the low- α population (top left of Figure 14). Abundance ratios relative to other elements, $[\text{Y}/\text{Ba}]$, $[\text{Y}/s]$,²⁵ and $[\text{Y}/\text{Eu}]$ (top row of Figure 19), are also low, indicating underproduction of Y within the progenitor of the low- α population. A similar behavior is also observed at low metallicity in dwarf galaxies (Skúladóttir et al. 2019).

The low abundance of Y, as well as that of Cu (bottom right of Figure 13) in the low- α population, is interpreted as a result of the low efficiency of the weak s -process as discussed in Nissen & Schuster (2011). The $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ reaction is the source of neutrons in the weak s -process. Because ^{22}Ne is produced from CNO elements through H burning and α captures to ^{14}N , this process is dependent on CNO abundance and only efficient at $[\text{C}, \text{N}, \text{O}/\text{H}] \gtrsim -2$ (e.g., Prantzos et al. 1990). Hence, the products of the weak s -process like Cu and Y have a secondary nature. Figure 20 shows the chemical

²⁵ The s abundance is the average of Ba, La, Ce, and Nd.

Figure 19. Abundance trends of neutron-capture elements as a function of $[Mg/H]$. Here, s is the average of Ba, La, Ce, and Nd. The predicted solar abundance is $[Eu/Ba] = 0.94$ and $[Eu/s] = 0.70$ for pure r -process and $[Eu/Ba] = -1.26$ and $[Eu/s] = -1.20$ for pure s -process (Prantzos et al. 2020).

Table 9
Differences of Abundance Ratios in Figure 19

Ratio	High α			Low α			$\Delta\langle[X/Y]\rangle$	$\sigma([X/Y])_{\text{median}}$	P Value
	$\langle[X/Y]\rangle$	$\sqrt{\text{Var}}$	N	$\langle[X/Y]\rangle$	$\sqrt{\text{Var}}$	N			
$[Y/Ba]$	-0.003	0.055	7	-0.202	0.111	9	0.198	0.081	0.000
$[Y/s]$	-0.057	0.060	5	-0.272	0.087	9	0.215	0.058	0.000
$[Y/Eu]$	-0.251	0.094	7	-0.592	0.148	9	0.341	0.093	0.000
$[Eu/Ba]$	0.250	0.115	7	0.393	0.197	9	-0.143	0.103	0.092
$[Eu/s]$	0.237	0.093	5	0.322	0.133	9	-0.085	0.097	0.187
$[Eu/Mg]$	0.065	0.117	7	0.321	0.140	9	-0.256	0.091	0.001

evolution of these two elements in relation to Mg. We here choose Mg instead of Fe because both CNO and Mg are mostly produced by SNe II, whereas Fe is also produced by SNe Ia. Figure 20 demonstrates that Cu and Y abundances show tighter correlations with Mg abundance than with Fe abundance. In addition, the abundance ratios of both low- α and high- α stars are distributed on the same line. These results suggest that the low Cu and Y abundances of the low- α population could be due to their low yield in the weak s -process, which may be the result of the low CNO and Mg abundances at a given $[Fe/H]$.²⁶ The two elements show different behavior at high metallicity ($[Mg/H] \gtrsim -1$), where $[Y/Mg]$ seems to be constant or turns over while $[Cu/Mg]$ keeps increasing. This behavior might be explained by the primary-like behavior of Y at high metallicity when nucleosynthesis by rotating massive stars are considered. (Prantzos et al. 2018). Therefore, we tentatively conclude that the star formation timescales of the low-/high- α populations and abundance differences of Cu and Y in Figures 13, 14, and 19 could be indirectly related through different $[C, N, O/Fe]$ abundances.

We note that Cu abundance can significantly be affected by the NLTE effect (Andrievsky et al. 2018; Shi et al. 2018). For

example, Shi et al. (2018) reported that the $[Cu/Fe]$ abundance of HD 122563 is underestimated in LTE analysis by 0.3–0.4 dex compared to NLTE analysis when the absorption line at 5105 Å is used. Although the trend with metallicity could change when studied with an NLTE analysis, the discussion presented here is mostly based on the relative abundance difference between the low- α and high- α populations at a given metallicity, which should be less affected by the NLTE effect (e.g., Yan et al. 2016).

We now discuss heavy neutron-capture elements presented in Figure 14, starting from an r -process element, Eu, and then moving to s -process elements. Eu is an almost pure r -process element. Eu is significantly enhanced relative to Y or Mg in the low- α population by ~ 0.3 dex (bottom row of Figure 19 and Table 9), as suggested in Fishlock et al. (2017) and Ishigaki et al. (2013). The difference in Eu abundance ratios between the low- α and high- α populations could be understood if there is a delay in r -process enrichments; the long star formation timescale of the low- α population might have allowed the delayed r -process enrichments to contribute to the chemical evolution. The delay time of r -process events is, however, still under debate (Hotokezaka et al. 2018; Côté et al. 2019; Lin et al. 2020; Skúladóttir & Salvadori 2020). While neutron star mergers, which are associated with short gamma-ray bursts (GRBs), are promising r -process sites, these studies suggested that there should be more r -process production events in the

²⁶ We note that we have not measured N abundances for the targets. We assumed that the relative $[N/Fe]$ values between the low-/high- α populations follow the trend of $[C/Fe]$ and $[O/Fe]$.

Figure 20. Abundance trends of Cu and Y as a function of $[\text{Mg}/\text{H}]$.

early universe than the rate predicted based on the observations of GRBs. An additional r -process source with a shorter delay time, such as a rare type of supernova, is suggested as a solution. Because we discuss low-metallicity stars, this “quick” source would matter. Our results might indicate the presence of a non-negligible delay time for the quick source. Although the uncertain delay time of the r -process enrichments prevents us from obtaining a quantitative constraint on star formation timescales, the results demonstrate that the star formation timescale is constrained by elements whose origin is totally different from α elements or Fe.

At solar metallicity, the dominant nucleosynthesis sites of heavy s -process elements, including Ba, La, Ce, and Nd, are considered to be low- to intermediate-mass AGB stars (the so-called main- s ; Prantzos et al. 2020). Because these AGB progenitors have long main-sequence lifetimes, their contributions are expected to occur at late times. The abundances of these elements could consequently differ between systems with different star formation timescales. However, the heavy s -process element abundances, especially relative to Eu, do not show differences between our high- α /low- α populations (Figures 14 and 19, see also Table 9). This suggests that the contribution of low- to intermediate-mass AGB stars would be small for both populations. The almost-flat trend of $[\text{Eu}/s]$ over all of the target stars supports the lack of low- to intermediate-mass star contributions. Previous studies (e.g., Ishigaki et al. 2013; Fishlock et al. 2017) also pointed out a similar lack of $[\text{Eu}/s]$ abundance difference between the outer/inner halo or low-/high- α halo.

It is worth noting that the $[\text{Eu}/s]$ ratio is slightly lower than the pure r -process ratio (bottom middle panel of Figure 19). This might indicate that massive stars contribute to the heavy s -process element enrichments. It has been suggested that massive stars with a high rotation speed can produce heavy s -process elements (e.g., Frischknecht et al. 2012; Choplin et al. 2018).

It is surprising that the s -process contribution is small in both populations while SNe Ia appear to have already started significantly contributing at least for the low- α population. The maximum initial mass of white dwarf progenitors is expected to be higher than that of AGB progenitors that

produce a significant amount of heavy s -process elements. However, SNe Ia would need additional time after the formation of the first white dwarfs. Our results suggest that the minimum delay time for white dwarfs to explode might be shorter than the timescale of the evolution of low- to intermediate-mass stars. There are observational and theoretical results supporting the minimum delay time of SNe Ia being shorter than a few $\times 100$ Myr (e.g., Totani et al. 2008; Maoz et al. 2014).

Although the low- α population is considered to be an accreted massive dwarf galaxy, the behavior of s -process elements, i.e., the lack of significant s -process enrichments, is different from that in dwarf galaxies currently found around the Milky Way. They tend to show signatures of heavy s -process enhancements (e.g., Letarte et al. 2010; Hill et al. 2019). This suggests that the low- α population has experienced faster chemical evolution than stars in the dwarf galaxies.

We note that our low- α population does not include many stars at $[\text{Fe}/\text{H}] \gtrsim -1.0$ (see Figure 10), where the low- α population has been known to show the main s -process contribution (Ishigaki et al. 2013; Fishlock et al. 2017). We expect that more dedicated observations of halo stars at high metallicity in the Kepler field would reveal the s -process contribution in the low- α population. Discussions presented here are applicable only to the well-covered metallicity range in the present study ($-1.5 \lesssim [\text{Fe}/\text{H}] \lesssim -1.0$).

We have investigated contributions of SNe Ia, SNe II, weak s -process in massive stars, r -process, and heavy s -process in the low- α and high- α populations. They give constraints on the star formation timescale of the halo populations. First, SNe Ia need at least ~ 100 Myr after star formation, which corresponds to the lifetime of the most massive white dwarf progenitor (Maoz et al. 2014). This sets a lower limit on the star formation timescale of the low- α population as it shows signatures of SN Ia contribution (e.g., low $[\text{Mg}/\text{Fe}]$ in Figure 10). The lack of main- s contribution provides an additional constraint (bottom middle panel of Figure 19). Heavy s -process elements are mainly produced by stars less massive than $\sim 3 M_{\odot}$ (Straniero et al. 2014). The lifetime of a $3 M_{\odot}$ star is ~ 200 – 300 Myr. Therefore, both the low- α and the high- α populations have to be evolved up to $[\text{Mg}/\text{H}] \sim -1$ within a few $\times 100$ Myr before

$\sim 3 M_{\odot}$ stars start to contribute to the chemical evolution. This constraint imposes an upper limit on the star formation timescale of a few $\times 100$ Myr. In principle, the r -process enhancement of the low- α population can also give an independent constraint. However, because there is a large uncertainty in the delay time of r -process enrichments, we do not aim to put quantitative constraints on r -process abundances (Hotokezaka et al. 2018; Côté et al. 2019).

Based on these estimates, we conclude that the star formation timescale τ can be constrained to $100 \lesssim \tau / \text{Myr} \lesssim 300$ for the low- α population. Although the timescale is not well constrained for the high- α population in the present study, it is clear from the smaller amount of SN Ia contribution that it has a shorter timescale than the low- α population.

6.3.2. Constraints from Mass

Stellar age is a fundamental parameter when we retrieve information about the Galaxy formation history from stars. Because we have systematic offsets in mass determination, we refrain from using age directly. However, as shown in Figure 17, the red giant mass does not depend on the metallicity significantly. Therefore, we here use stellar mass as an indicator of relative stellar age. Discussions presented in this subsection are based on the sample of red giant branch stars with $\log g > 2.0$ following Section 6.2. The numbers of stars are 15 for the entire halo population, and 5 and 4 for the high- α and low- α populations, respectively.

We obtained the average age of 6–7 Gyr for the entire halo population from 15 less-luminous giants with $\log g > 2$. This contradicts the known old age ($\gtrsim 10$ Gyr) of halo stars (e.g., Marín-Franch et al. 2009; Forbes & Bridges 2010; Dotter et al. 2011; Jofré & Weiss 2011; Schuster et al. 2012; Carollo et al. 2016; Kilic et al. 2019). As discussed in Section 6.2, this is likely due to the systematic offset in the scaling relations of asteroseismology. In this subsection, we focus on the age spread of halo stars and age differences between the two halo populations. These are equivalent to studying the relative age among halo stars, which is feasible even in the presence of systematic offsets in estimated ages if the internal precision is sufficiently high (see Section 6.2). In addition, the information obtained from age estimates is complementary to chemical abundances because it does not rely on theoretical nucleosynthesis yields and because it enables us to discuss not only the formation timescale but also the difference in the formation epoch between systems.

Although chemical abundances and kinematics span a significantly wide range with an amplitude much larger than the measurement uncertainties, we do not expect significant correlations between masses and other properties of stars because we did not find a significant dispersion in mass. We have, in fact, explored possible correlations between masses and abundances, and those between masses and kinematics, and found no correlation. For example, abundance ratios such as $[\alpha/\text{Fe}]$ or $[\text{Y}/\text{Mg}]$ are suggested to show correlations with stellar age for disk stars at around solar metallicity (da Silva et al. 2012; Nissen 2016; Tucci Maia et al. 2016; Feltzing et al. 2017; Spina et al. 2018; but see also Silva Aguirre et al. 2018). Figure 21 shows the results of our investigation on $[\alpha/\text{Fe}]$, $[\text{Y}/\alpha]$, and $[\text{Ba}/\alpha]$.²⁷ The absence of a correlation indicates that

chemical evolution has to proceed rapidly compared to the uncertainty of the age estimate, which is not inconsistent with the discussion on chemical abundances.

The lack of mass dispersion among our subsample of halo stars with $\log g > 2$ (Figure 18) provides an upper limit on the intrinsic mass dispersion, which can be translated into the dispersion of relative age. Currently, the weighted sample standard deviation of the mass is $0.05 M_{\odot}$ (or 5% of the average mass) for the 15 stars on the lower red giant branch phase. Because this dispersion can be explained by measurement uncertainties (Section 6.2), the intrinsic mass dispersion should be smaller than this value. The age of red giant branch stars is roughly proportional to $\sim M^{-3.5}$ in this metallicity and mass range according to MIST isochrones. Therefore, we obtain an upper limit on the relative age dispersion of $< 18\%$ for the entire halo population. Considering typical ages of halo stars provided in the literature of 10 Gyr,²⁸ 18% corresponds to ~ 2 Gyr.

The mass difference between our high- α and low- α stars is also insignificant ($\langle M \rangle = 0.958$ and $0.972 M_{\odot}$, respectively; the probability that these two values are different is 0.87). The 1σ upper limits on the mass difference are $0.04 M_{\odot}$ (or 4% of the average). Following the same argument as for the dispersion, the relative age difference between the two populations is < 1.5 Gyr (15%; 1σ upper limit).

This result is not inconsistent with Schuster et al. (2012), who reported the age difference to be 2–3 Gyr using nearby turnoff stars. We note that they obtained a smaller age difference at the lower metallicity bin ($[\text{Fe}/\text{H}] < -1.2$). A similar conclusion is also obtained by Hawkins et al. (2014), who statistically studied age–metallicity relations for the two populations using low-resolution spectroscopic data following Jofré & Weiss (2011). Das et al. (2020) reached the conclusion that the ages of metal-poor stars are similar irrespective of their abundance ratios, which is also consistent with our conclusion.

The nondetection of any age dispersion or age difference is due to the combined effect of limited age (mass) precision, the metallicity range of the targets, and limited sample size. Improvements in age precision are definitely desired. Although there is room for improvements in modeling stellar oscillations as we saw in Section 6.2, it might be difficult to achieve revolutionary improvements in terms of precision. We note that most of the uncertainties in the derived mass stem from the uncertainties in oscillation frequencies. We used oscillation frequencies that are measured from the best available light curves obtained from the about four years of continuous observation by the Kepler mission. No near-future mission is planned to obtain higher-quality or longer-term light curves. One of the ways to improve the mass measurement precision is to take luminosity into consideration. If the luminosity is measured with sufficiently high precision, it enables us to achieve about twice smaller uncertainty (Rodrigues et al. 2017). This is becoming possible thanks to the Gaia mission. On the other hand, it is necessary to resolve the systematic mass offsets found in Section 6.2. Unless we confirm there are no systematic offsets, combining asteroseismic information with luminosity does not necessarily lead to a narrower posterior distribution. Individual frequency modeling recently

²⁷ Here we define α abundance as the average of Mg, Ca, and Ti. For Ti, we adopt the average of Ti I and Ti II.

²⁸ We note that the average mass we obtained is converted to 6–7 Gyr if the systematic offset is not considered. However, because this apparent younger age is likely due to the systematic offset present in the mass estimates from the scaling relations, we stick to the conventional old age of the stellar halo.

Figure 21. Investigation of correlations between mass and abundances. Symbols follow Figure 17.

conducted for a halo star by Chaplin et al. (2020) would help to resolve the issue.

The metallicity range is another issue. In Schuster et al. (2012) and in Hawkins et al. (2014), the age difference between the high- α and low- α stars is larger in bins of higher metallicity. Because our targets are on the more metal-poor side compared to their study, age differences might be detected using asteroseismology by focusing on the high-metallicity end of the halo. Increasing the sample size, in general, would allow us to more tightly constrain the average mass and age for each

population, which would enable us to detect a small age difference if present. For example, a three times larger sample would enable us to detect an age difference of ~ 0.8 Gyr.

6.4. Peculiar Objects

6.4.1. Na-enhanced Stars

As noted in Section 5, there are three Na-enhanced objects (KIC 8350894, KIC 5184073, and KIC 6520576). There is no

Figure 22. The relation between $[O/Mg]$ and $[Na/Mg]$ (upper) and that between mass and $[Na/Mg]$. Na-enhanced stars are shown with red diamonds while others are shown with black symbols. The shape of the symbols follows Figure 17.

anomaly in their elemental abundances except for possible low O abundances found in two stars (Figure 22; KIC 8350894 and KIC 5184073) and the high s -process element abundance in one star (KIC 8350894; Ba, La, Ce, and Nd abundances are 0.50, 0.57, 0.40, and 0.64 in $[X/Fe]$). The O absorption $[O\text{ I}]$ 6300 Å is blended with telluric lines for KIC 6520576. The mass of the three stars are low (Figure 22), and KIC 5184073 and KIC 8350894 are in the red giant branch phase. Unfortunately, KIC 6520576 is the only star among our sample whose evolutionary status is not available in Yu et al. (2018). KIC 5184073 has $\log g < 2.0$, while the other two have $\log g > 2.0$. We also note that no radial velocity variations have been detected for Na-enhanced objects (Figure 2 and Table 1).

Large Na enhancements are sometimes seen in globular clusters (e.g., Carretta et al. 2009) and were used to search for metal-poor stars ejected from globular clusters (Pereira et al. 2019). Those Na-enhanced objects in clusters usually accompany O deficiency (known as the Na–O anticorrelation; Carretta et al. 2009). The possibly low O abundances of the two stars might indicate their globular cluster origin. Further investigation of the Mg–Al anticorrelation, and Li, C, and N

abundances are highly desired to confirm their globular cluster origin.

Internal mixing might also cause Na enhancements especially in AGB stars or in massive stars. However, the low mass and evolutionary phase might not be compatible with this hypothesis.

KIC 6520576 resembles CD $-23^\circ 16310$, which was reported to have $[Na/Fe] = +1.09$ at $[Fe/H] = -1.93$ (Pereira et al. 2019). Both giants have large (~ 1 dex) Na enhancements at low metallicity, but do not show peculiar abundance pattern including C. The most notable difference is the luminosity or surface gravity; while KIC 6520576 has $\log g = 2.17$, CD $-23^\circ 16310$ has $\log g = 0.9$. Pereira et al. (2019) suspect CD $-23^\circ 16310$ is in the early AGB phase. It is not yet clear at this stage if KIC 6520576 can also be interpreted to be in the early AGB phase.

The s -process enhancement of KIC 8350894 might be related to its Na enhancement. Although s -process enhancements are often ascribed to the presence of an evolved companion, this star has not shown any sign of binarity. In addition, it is worth noting that C is not enhanced ($[C/Fe] = -0.44$).

6.4.2. Overmassive Stars

Among our sample, two stars are obviously overmassive (KIC 5446927 and KIC 10096113; 1.49 ± 0.13 and $1.44 \pm 0.10 M_\odot$, respectively). The APOKASC-2 catalog also provides similar masses for these objects, 1.451 and $1.396 M_\odot$, respectively. Even though these stars are on the metal-rich side in our halo stars ($[Fe/H] = -0.74$ and -0.72), their metallicity is still sufficiently low as we can assume they are very old and low mass. They also have supersolar $[\alpha/Fe]$ and large velocity relative to the Sun, supporting their old age.

Therefore, these two stars are too massive for their chemical and kinematic properties. Similar stars have been found in the Galactic disk (the so-called young α -rich stars; Chiappini et al. 2015; Martig et al. 2015). The young α -rich stars are massive but their chemical abundances are just the same as normal old disk stars (Yong et al. 2016; Matsuno et al. 2018). An important property of the young α -rich stars is that they show a high fraction of radial velocity variation (Jofré et al. 2016; Matsuno et al. 2018), suggesting binary interaction is the key to the formation of these stars. Theoretical work also supports this scenario (Izzard et al. 2018).

Radial velocity measurements of KIC 5446927 and KIC 10096113 do not reveal their binary nature at this stage (Figure 2 and Table 1). KIC 5446927 actually has precise radial velocity measurements from our observation, APOGEE, and Gaia, all of which perfectly agree. Although not every young α -rich star shows a radial velocity variation, the absence of radial velocity variation might indicate the origin of these stars is different from the majority of young α -rich stars. These stars could in fact be evolved blue stragglers, which are found to be quite common among kinematically selected halo stars (Casagrande 2020), and a few are thus expected in our sample. Further monitoring of the radial velocities of these stars is obviously welcome to draw a robust conclusion about the origin of these stars.

It is interesting to note that the Na abundance of the two objects appears enhanced (Figure 22), although the level of enhancements is not as significant as the three stars studied in the previous subsection considering the Galactic chemical

evolution (bottom panel of Figure 11). Smiljanic et al. (2016) showed that massive giants have high Na abundance due to internal mixing. However, they concluded that the effects appear in stars more massive than $2.0 M_{\odot}$, which is larger than the mass of our two overmassive stars. Another interesting aspect of these stars is their evolutionary status; both stars are red clump stars, which would support the binary merger scenario (Miglio et al. 2021). Additional mixing near the tip of the red giant branch might affect the Na abundance of these stars (Fujimoto et al. 1999; Weiss et al. 2000).

6.4.3. KIC 7693833

As noted in Section 4, KIC 7693833 is one of the most metal-poor stars while its velocity is similar to disk stars. A population of stars with disk-like kinematics is found among very metal-poor stars (Carollo et al. 2019; Sestito et al. 2019; Venn et al. 2020), whose formation mechanism is still under debate. The property of such stars including their ages would shed light on their origin. KIC 7693833 would belong to this population; therefore, we here briefly summarize its property.

Compared to stars with similar metallicity in our sample, the chemical abundance of KIC 7693833 is characterized by high [C/Fe], low [Na/Fe], high [Co/Fe], and low [Eu/Fe]. We also note that other heavy neutron-capture elements, except for Ba, are not detected. The mass of this star is $1.04 \pm 0.05 M_{\odot}$, which is higher than the average. Although these chemical abundances and mass might be peculiar, a study of a large sample of stars is clearly needed to make a further interpretation.

7. Conclusion

We obtain precise stellar parameters and precise chemical abundances for 26 halo stars in the Kepler field, for which asteroseismic information is available. The sample is selected based on radial velocity and metallicity estimated from spectroscopic surveys. The kinematics of the selected stars are later investigated with the Gaia DR2 data, confirming that they have halo-like kinematics. Stellar parameters are determined precisely and accurately by adopting a differential abundance analysis and using standard stars with accurate T_{eff} and $\log g$. Subsequently, we obtain precise chemical abundances.

Our study is so far the largest sample of low-metallicity stars with asteroseismic information and precise stellar parameters. We investigate the reliability of asteroseismic mass estimates at low metallicity with this sample. The average mass obtained from asteroseismology is $1.03 M_{\odot}$ (without correction) and $0.96 M_{\odot}$ (with $\Delta\nu$ correction). This result indicates that, although correction to the $\Delta\nu$ scaling relation helps to reduce inconsistency between asteroseismic and expected masses ($\sim 0.8 M_{\odot}$), it is not capable of completely resolving the issue.

We also show that luminous red giant stars and red clump stars could suffer from systematic uncertainties in asteroseismic masses. After excluding these stars, there is no significant mass dispersion among our sample. This fact indicates that, despite the systematic offset, the mass of halo stars can be estimated with high precision if we focus on less-evolved red giants.

The precise chemical abundances allow us to separate our targets into high- α and low- α halo populations based on the Mg abundance. The two populations also differ in other element abundances, consistent with Nissen & Schuster (2010, 2011) and Fishlock et al. (2017). The low- α population

shows low values of [X/Fe], where X is C, O, Na, Ca, Sc, Ti, V, Ni, Cu, Zn, and Y. Most of these elements are ejected from massive stars, and thus the low values are understood to be a result of SN Ia contribution. The Y abundance in the low- α population is lower compared to heavy s -elements or Eu. This, together with the Cu abundance, is understood as a result of their secondary nature in the production by the weak s -process. There is no significant difference in the Eu to heavy s -process abundance ratio, indicating that the main s -process from low- to intermediate-mass AGB stars does not contribute significantly to either of the low- α or high- α populations. Eu abundance relative to Mg seems enhanced in the low- α population, suggesting that the delay time of r -process enrichments plays a role.

These chemical abundances provide constraints on the timescale of star formation (τ). Because the low- α population is enriched by SNe Ia, τ should be longer than $\gtrsim 100$ Myr, which comes from the lifetime of the most massive white dwarf progenitor. The lack of main s -process contribution, on the other hand, provides the upper limit on the timescale of $\tau \lesssim 300$ Myr. The low- α /high- α populations are now understood as accretion and in situ origin, respectively (Nissen & Schuster 2010; Helmi et al. 2018). Therefore, our study indicates that the accreted population formed on a timescale of $100 \lesssim \tau/\text{Myr} \lesssim 300$, and the in situ population formed on a shorter timescale.

The asteroseismic information independently constrains the formation timescale and additionally provides a constraint on the relative formation epochs of the two populations. The lack of significant mass dispersion among less-luminous red giant branch stars gives an upper limit on the intrinsic mass dispersion of $< 0.05 M_{\odot}$. This can be translated into an age dispersion of $\lesssim 2$ Gyr. The average mass of less-luminous red giant branch stars is 0.958 and $0.972 M_{\odot}$ for the high- α and low- α populations. This difference is not significant and constrains a relative age difference of < 1.5 Gyr. These results do not contradict previous works (Schuster et al. 2012; Hawkins et al. 2014; Das et al. 2020) or with chemical abundances. The star formation timescale in the early Galaxy is too short for asteroseismology to provide a meaningful constraint at this stage.

Our study is the first that combines chemical abundances and stellar mass (age) estimates for more than 10 field halo stars beyond the solar neighborhood. However, we did not detect age spreads among our sample or age difference between populations unlike Schuster et al. (2012). The reason for the nondetection would be limited precision, metallicity range, and/or sample size. A factor of 2–3 improvement in the precision would make it comparable to the precision that was achieved for nearby turnoff stars (Schuster et al. 2012). This level of improvement might be achieved by incorporating luminosities into mass estimates (Rodrigues et al. 2017), after resolving the systematic offset in the masses we obtained in this study. Instead, a study that focuses on the high-metallicity end of the halo with a larger sample size might be able to reveal the age difference. Such a sample will be provided by space-based photometric monitoring observations with a wider field coverage, such as K2, TESS, or PLATO.

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Software: Asfgrid (Sharma & Stello 2016), Astropy (Astropy Collaboration et al. 2013), Corner (Foreman-Mackey 2016), emcee (Foreman-Mackey et al. 2013), Matplotlib (Hunter 2007), NumPy (van der Walt et al. 2011), Pandas (McKinney 2010), q2 (Ramírez et al. 2014), Scipy (Virtanen et al. 2020).

Appendix A

Microturbulent Velocity Prior as a Function of $\log g$

The microturbulent velocity is determined spectroscopically such that abundances derived from individual neutral iron lines do not show correlation with line strengths. However, due to the wide parameter range of our sample, microturbulent velocities do not converge well for some of the stars. Poorly constrained microturbulent velocity affects the precision of the

temperature and the abundances. In order to mitigate this problem, we use a nonflat prior on microturbulent velocity during stellar parameter determination.

Here we describe how we chose the ξ prior (Figure 23). We first run a stellar parameter determination process with a flat prior on microturbulent velocity. We then carry out a linear fit to the relation between the microturbulent velocity and the surface gravity using stars whose microturbulent velocity is determined with high precision ($\sigma(\xi) < 0.1 \text{ km s}^{-1}$). Note that the microturbulent velocity of the other stars with large uncertainties agree with the best-fit linear relation at the $< 2\sigma$ level. Based on this fit, we put the prior on ξ as

$$p(\xi | \log g) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp(-(\xi - \mu(\log g))^2 / 2\sigma^2), \quad (\text{A1})$$

where $\mu(\log g)$ is the expected value of ξ at a given $\log g$ from the linear fit, σ is obtained from $\sigma = \sigma_{\text{fit}}^2(\log g) + \sigma_{\text{resid}}^2$, and $\sigma_{\text{fit}}(\log g)$ is the expected standard deviation at a given $\log g$ due to the uncertainties in the fitting parameters. σ_{resid} is a constant expressed as $\sigma_{\text{resid}}^2 = \sum_i (\xi_i - \mu(\log g_i))^2 / (N - \nu)$, where N and ν are the numbers of stars used for the fit and the degree of freedom ($\nu = 2$ in this case), respectively.

The inclusion of the ξ prior does not affect stars whose ξ is determined precisely in the first step. For the other stars, it provides better convergence in the stellar parameter determination.

Figure 23. The relation between microturbulent velocity and surface gravity. Large black symbols are used to determine the prior, and small gray circles are other objects. The solid orange line, thick filled region, and thin filled region are the best-fit linear relation, uncertainties in the fit, and 1σ interval of the prior at a given $\log g$. The left and right panels show the results of the analysis relative to HD 122563 and KIC 9583607, respectively.

Appendix B

Stellar Parameters from Analyses with Different Standard Stars

In this study, we carried out two sets of analyses with two reference stars, HD 122563 and KIC 9583607. Table 10 shows

the stellar parameters derived from each of the analyses. The final stellar parameters adopted for, e.g., mass estimates, are the combination of two results following the weights (w_{MP} and w_{MR}) provided in the table.

Table 10
Stellar Parameters from the Two Analyses

Object	Analysis with HD 122563 (MP)							Analysis with KIC 958367 (MR)						
	T_{eff} (K)	$\sigma_{(T_{\text{eff}})}$ (K)	ξ (km s ⁻¹)	$\sigma_{(\xi)}$ (km s ⁻¹)	[Fe/H]	$\sigma_{([\text{Fe}/\text{H}])}$	w_{MP}	T_{eff} (K)	$\sigma_{(T_{\text{eff}})}$ (K)	ξ (km s ⁻¹)	$\sigma_{(\xi)}$ (km s ⁻¹)	[Fe/H]	$\sigma_{([\text{Fe}/\text{H}])}$	w_{MR}
KIC 5184073	4894	51	1.600	0.056	-1.380	0.030	0.213	4856	54	1.576	0.054	-1.433	0.035	0.787
KIC 5439372	4835	34	1.915	0.045	-2.479	0.029	1.000	4724	90	1.673	0.101	-2.591	0.094	0.000
KIC 5446927	5175	53	1.519	0.071	-0.629	0.035	0.000	5102	37	1.530	0.047	-0.742	0.022	1.000
KIC 5698156	4701	79	1.653	0.131	-1.274	0.053	0.085	4639	46	1.522	0.052	-1.290	0.017	0.915
KIC 5858947	5154	69	1.089	0.140	-0.669	0.045	0.000	5105	72	1.221	0.117	-0.775	0.047	1.000
KIC 5953450	5236	81	1.100	0.157	-0.457	0.067	0.000	5127	74	1.234	0.115	-0.623	0.057	1.000
KIC 6279038	4761	31	1.784	0.044	-2.055	0.027	1.000	4719	50	1.713	0.064	-2.123	0.042	0.000
KIC 6520576	4971	35	1.602	0.042	-2.289	0.029	1.000	4958	66	1.565	0.082	-2.320	0.059	0.000
KIC 6611219	4719	90	1.728	0.142	-1.166	0.068	0.000	4652	45	1.566	0.052	-1.201	0.018	1.000
KIC 7191496	4903	34	1.672	0.038	-2.076	0.028	1.000	4826	70	1.619	0.093	-2.179	0.067	0.000
KIC 7693833	5094	46	1.664	0.061	-2.265	0.038	1.000	5034	81	1.493	0.096	-2.334	0.074	0.000
KIC 7948268	5166	66	1.140	0.093	-1.135	0.033	0.000	5154	51	1.230	0.062	-1.199	0.028	1.000
KIC 8350894	4854	92	1.626	0.119	-1.069	0.075	0.000	4797	45	1.491	0.054	-1.091	0.025	1.000
KIC 9335536	4835	40	1.526	0.051	-1.493	0.027	0.347	4807	45	1.527	0.052	-1.547	0.037	0.653
KIC 9339711	4928	39	1.497	0.046	-1.454	0.025	0.290	4940	43	1.514	0.048	-1.475	0.026	0.710
KIC 9583607	5086	61	1.543	0.084	-0.618	0.039	0.000	5059	...	1.590	...	-0.700	...	1.000
KIC 9696716	4962	43	1.433	0.049	-1.414	0.023	0.248	4962	52	1.460	0.052	-1.448	0.031	0.752
KIC 10083815	4828	90	1.525	0.122	-0.890	0.066	0.000	4784	74	1.476	0.097	-0.936	0.060	1.000
KIC 10096113	4943	80	1.417	0.116	-0.661	0.054	0.000	4948	48	1.443	0.070	-0.717	0.018	1.000
KIC 10328894	5031	39	1.525	0.047	-1.832	0.031	0.771	4973	59	1.539	0.067	-1.912	0.048	0.229
KIC 10460723	4938	49	1.412	0.068	-1.213	0.025	0.017	4922	41	1.467	0.047	-1.273	0.021	0.983
KIC 10737052	4998	50	1.454	0.064	-1.190	0.029	0.000	4973	42	1.478	0.047	-1.255	0.021	1.000
KIC 11563791	4996	75	1.368	0.128	-1.070	0.046	0.000	4974	54	1.326	0.069	-1.114	0.023	1.000
KIC 11566038	5002	45	1.417	0.052	-1.391	0.030	0.223	4998	47	1.461	0.057	-1.432	0.031	0.777
KIC 12017985	4962	39	1.639	0.043	-1.823	0.028	0.770	4889	56	1.591	0.066	-1.914	0.044	0.230
KIC 12017985-17	4947	38	1.646	0.038	-1.852	0.027	0.809	4868	61	1.601	0.076	-1.946	0.050	0.191
KIC 12253381	4947	46	1.511	0.047	-1.519	0.027	0.380	4906	53	1.513	0.052	-1.579	0.040	0.620
HD 122563 ^a	4636	...	2.05	...	-2.599	...	...	4434	62	3.613	0.060	-2.851	0.029	...

Note.

^a HD 122563 is included for completeness. It is not used for the other figures.

Appendix C

Uncertainties of the Stellar Parameters of the Standard Stars

Abundances and stellar parameters are precisely determined in this study by adopting differential abundance analyses. While the relative abundance/parameters among our sample should not be significantly affected by the uncertainties of the stellar parameters of the standard stars, the absolute scale totally depends on the abundance/stellar parameters of the standard stars. In this section, we explore how the change in stellar parameters of the standard stars affects the absolute scale of our program stars. One has to take into account these effects when trying to quantitatively compare our results with different studies.

We again carry out analyses for a subset of stars by adopting stellar parameters of the standard stars with offsets that correspond to their uncertainties (Tables 11 & 12). We select

three stars as the subset for each reference star to cover the wide metallicity range. It is clear that the absolute scale does depend on the adopted stellar parameters of the standard star. However, we emphasize that because the change is systematic, we are almost free from this effect as long as we discuss abundance trends among our sample. These effects are only important when one tries to compare our results with other studies.

Tables 11 and 12 also include the number of lines used to derive the abundance of the reference star and the scatter of abundances derived from individual lines. Readers may derive uncertainties of the abundance of the reference star by quadratically summing $\sigma_{\text{sct}}/\sqrt{N_{\text{line}}}$ and shifts in abundances caused by those in stellar parameters (“ref” columns). Because the abundance of the reference star determines our abundance scale, readers may add the obtained values to the reported uncertainties in Table 7 for the purpose of comparison.

Table 11
Change of the Stellar Parameters/Abundances due to the Stellar Parameters of HD 122563

	$n_{\text{line}}^{\text{a}}$	$\sigma_{\text{sct}}^{\text{a}}$	$\Delta T_{\text{eff}} = 36 \text{ K}^{\text{b}}$				$\Delta \log g = 0.007^{\text{a}}$				$\Delta \xi = 0.08^{\text{c}}$				$\Delta [\text{Fe}/\text{H}] = 0.016^{\text{c}}$			
			ref	(1)	(2)	(3)	ref	(1)	(2)	(3)	ref	(1)	(2)	(3)	ref	(1)	(2)	(3)
T_{eff}			36	48	51	46	0	3	4	0	0	-5	-1	2	0	5	3	1
$\log g$			0.00	0.00	0.00	0.00	0.01	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ξ			0.00	0.00	0.01	0.02	0.00	0.00	0.01	0.00	0.08	0.09	0.06	0.06	0.00	0.00	0.00	0.00
[Fe/H]			0.00	0.01	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	-0.00	-0.01	-0.01	0.02	0.02	0.02	0.02
CH	1	0.00	0.06	0.09	0.07	0.05	-0.00	0.01	0.01	-0.00	-0.00	-0.01	-0.00	-0.01	0.01	0.02	0.01	0.01
Na I	1	0.00	0.02	...	0.03	0.03	0.00	...	0.00	0.00	-0.00	...	-0.00	0.00	0.00	...	0.00	-0.00
Mg I	4	0.10	0.02	0.03	0.04	0.04	-0.00	0.00	0.00	-0.00	-0.01	-0.01	-0.01	-0.01	-0.00	0.00	0.00	0.00
Si I	4	0.12	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00
Ca I	28	0.07	0.03	0.03	0.03	0.04	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.01	0.00	0.00	0.00	-0.00
Sc II	14	0.10	0.01	0.03	0.01	0.01	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.01	-0.01	0.00	0.00	0.00	0.00
Ti I	46	0.08	0.06	0.07	0.07	0.07	0.00	0.00	0.01	-0.00	-0.00	-0.01	-0.01	-0.01	-0.00	0.00	0.00	-0.00
Ti II	26	0.08	0.00	0.01	0.00	0.01	0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00
V I	6	0.12	0.06	...	0.08	0.07	0.00	...	0.01	0.00	0.00	...	-0.00	0.00	-0.00	...	0.00	-0.00
Cr I	13	0.10	0.05	0.06	0.06	0.06	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.01	-0.01	-0.00	0.00	0.00	-0.00
Cr II	10	0.10	-0.01	-0.01	-0.01	-0.01	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.01	0.00	0.00	0.00	0.00
Mn I	7	0.05	0.04	0.04	0.05	0.05	-0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.00	-0.01	-0.00	0.00	0.00	0.00
Fe I	158	0.12	0.04	0.05	0.05	0.05	0.00	0.00	0.00	0.00	-0.01	-0.02	-0.02	-0.02	0.00	0.00	0.00	0.00
Fe II	35	0.05	-0.01	0.00	-0.01	-0.01	0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.01	-0.02	0.00	0.00	0.00	0.00
Co I	2	0.03	0.06	0.06	0.07	0.06	0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	0.00	-0.00	0.01	0.00	0.00
Ni I	27	0.06	0.03	0.04	0.04	0.04	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.00	-0.00	0.00	-0.00	0.00
Cu I	1	0.00	0.05	...	0.07	0.06	0.00	...	0.01	0.00	0.00	...	-0.00	0.00	0.00	...	0.00	0.00
Zn I	2	0.01	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.01	0.00	0.00	0.00	0.00
Y II	9	0.09	0.01	0.03	0.02	0.01	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	-0.01	0.00	0.00	0.00	0.00
Ba II	3	0.08	0.02	0.03	0.02	0.02	0.00	0.00	0.00	-0.00	-0.00	-0.02	-0.04	-0.03	0.00	0.01	0.00	0.01
La II	1	0.00	0.02	0.03	0.02	0.02	0.00	0.00	0.00	0.00	-0.00	-0.01	-0.01	-0.01	0.00	0.01	0.01	0.00
Ce II	1	0.00	0.02	...	0.02	0.02	0.00	...	0.00	0.00	0.00	...	-0.00	-0.01	0.00	...	0.00	0.01
Nd II	2	0.01	0.02	...	0.03	0.02	0.00	...	0.00	0.00	-0.00	...	-0.00	-0.01	0.00	...	0.00	0.01
Sm II	1	0.00	0.02	...	0.02	0.02	0.00	...	0.00	0.00	0.00	...	-0.00	-0.00	0.00	...	0.01	0.01
Eu II	1	0.00	0.02	0.03	0.03	0.02	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	0.01	0.01	0.00

Notes. (1): KIC 5439372 ([Fe/H] = -2.53), (2): KIC 12017985 ([Fe/H] = -1.87), (3): KIC 11566038 ([Fe/H] = -1.36). For the elemental abundances we show the results for [X/H].

^a The number of lines used and the scatter among abundances derived from individual lines.

^b Karovicova et al. (2018) and Creevey et al. (2019).

^c Derived in this study. We consider the effects of the uncertainties of the other parameters.

Table 12
Change of the Stellar Parameters/Abundances due to the Stellar Parameters of KIC 9583607

	$n_{\text{line}}^{\text{a}}$	$\sigma_{\text{sct}}^{\text{a}}$	$\Delta T_{\text{eff}} = 89 \text{ K}^{\text{b}}$				$\Delta \log g = 0.010^{\text{c}}$				$\Delta \xi = 0.09^{\text{c}}$				$\Delta [\text{Fe}/\text{H}] = 0.065^{\text{c}}$			
			ref	(1)	(2)	(3)	ref	(1)	(2)	(3)	ref	(1)	(2)	(3)	ref	(1)	(2)	(3)
T_{eff}			89	85	84	79	0	5	1	9	0	6	-5	-2	0	-1	0	-5
$\log g$			0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ξ			0.00	0.00	0.02	-0.01	0.00	0.01	0.00	0.01	0.09	0.08	0.07	0.07	0.00	0.00	-0.00	-0.02
[Fe/H]			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.06	0.07	0.06	0.06
CH	1	0.00	0.07	0.06	0.10	0.10	0.00	0.00	0.00	0.01	-0.00	0.01	0.01	0.01	0.04	0.06	0.04	0.03
O I	1	0.00	0.02	0.02	0.04	...	0.00	0.00	0.00	...	-0.00	0.00	0.00	...	0.02	0.02	0.02	...
Na I	1	0.00	0.06	0.08	0.05	0.04	-0.00	0.00	0.00	0.01	-0.02	-0.02	-0.01	-0.00	-0.00	0.00	-0.00	-0.00
Mg I	5	0.16	0.04	0.05	0.04	0.03	0.00	0.00	0.00	0.01	-0.00	0.01	0.01	-0.00	0.00	0.00	-0.00	-0.00
Si I	4	0.01	0.03	0.02	0.04	0.03	0.00	0.00	0.00	0.00	-0.01	-0.00	-0.01	0.00	0.00	0.01	0.00	-0.00
Ca I	25	0.14	0.08	0.09	0.08	0.06	0.00	0.00	0.00	0.01	-0.03	-0.02	-0.02	-0.00	-0.01	-0.01	-0.01	-0.01
Sc II	9	0.07	-0.01	-0.00	0.01	0.00	0.00	-0.00	0.00	0.00	-0.03	-0.02	-0.01	0.01	0.01	0.01	0.01	0.01
Ti I	41	0.17	0.13	0.15	0.12	0.11	0.00	0.00	0.00	0.01	-0.02	-0.01	-0.01	0.01	-0.00	-0.00	-0.01	-0.01
Ti II	12	0.17	0.00	0.01	0.02	0.01	0.00	-0.00	0.00	0.00	-0.02	-0.02	0.00	0.01	0.01	0.02	0.02	0.01
V I	10	0.16	0.13	0.14	0.12	0.11	0.00	0.01	0.00	0.01	-0.00	0.01	-0.00	0.01	-0.00	0.00	-0.01	-0.01
Cr I	9	0.07	0.13	0.14	0.12	0.11	0.00	-0.00	0.00	0.01	-0.06	-0.05	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01
Cr II	11	0.24	-0.04	-0.04	-0.01	-0.03	0.00	0.00	0.00	-0.00	-0.03	-0.03	0.00	0.00	0.01	0.01	0.01	0.01
Mn I	6	0.07	0.10	0.11	0.09	0.10	0.00	0.00	0.00	0.01	-0.04	-0.02	0.00	0.01	-0.01	-0.00	-0.00	-0.01
Fe I	121	0.14	0.10	0.10	0.10	0.09	0.00	0.00	0.00	0.01	-0.03	-0.02	-0.01	-0.00	-0.00	0.00	-0.00	-0.01
Fe II	23	0.10	-0.03	-0.03	-0.01	-0.01	0.00	-0.00	0.00	-0.00	-0.03	-0.02	0.00	-0.01	0.01	0.02	0.02	0.01
Co I	3	0.14	0.10	0.11	0.10	0.10	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.01	-0.01	-0.01
Ni I	32	0.15	0.10	0.10	0.10	0.10	0.00	0.00	0.00	0.01	-0.03	-0.01	-0.01	0.00	-0.00	0.01	-0.01	-0.01
Cu I	1	0.00	0.12	0.11	0.11	0.10	0.00	0.00	0.00	0.01	-0.03	-0.01	-0.01	-0.01	-0.00	0.01	-0.00	-0.01
Zn I	2	0.01	-0.00	-0.01	0.02	0.01	0.00	-0.01	0.00	0.00	-0.04	-0.03	-0.01	-0.00	0.01	0.01	0.01	0.01
Y II	6	0.09	0.01	0.02	0.02	0.01	0.00	-0.00	0.00	0.00	-0.06	-0.05	-0.00	-0.00	0.01	0.02	0.02	0.02
Ba II	3	0.09	0.02	0.03	0.04	0.04	0.00	-0.00	0.00	0.01	-0.07	-0.05	-0.04	-0.06	0.01	0.02	0.03	0.00
La II	3	0.04	0.02	0.02	0.04	0.03	0.00	0.00	0.00	0.01	-0.00	-0.00	0.00	0.01	0.02	0.02	0.02	0.01
Ce II	4	0.05	0.02	0.02	0.03	0.03	0.00	-0.00	0.00	0.00	-0.01	-0.02	0.01	0.00	0.02	0.02	0.02	0.01
Nd II	4	0.02	0.03	0.04	0.05	0.04	0.00	-0.00	0.00	0.01	-0.01	-0.03	0.00	0.00	0.02	0.02	0.02	0.01
Sm II	1	0.00	0.02	0.03	0.04	0.03	0.00	0.00	0.00	0.01	-0.01	-0.01	0.00	0.00	0.02	0.02	0.02	0.01
Eu II	3	0.06	-0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.01

Notes. (1): KIC 8350894 ([Fe/H] = -0.85), (2): KIC 11566038 ([Fe/H] = -1.36), (3): KIC 12017985 ([Fe/H] = -1.87).

^a The number of lines used and the scatter among abundances derived from individual lines.

^b From the APOGEE DR14 catalog.

^c Derived in this study. We consider the effects of the uncertainties of the other parameters. For the elemental abundances, we show the results for [X/H].

Appendix D

Chemical Abundances of Red Giant Branch and Red Clump Stars

Because while the whole sample is used to discuss chemical abundance, only a subsample is used for the interpretation of asteroseismic data, we here verify that the evolutionary status does not affect the discussion. There is no astrophysical reason why the evolutionary status affects the abundance of most of the elements. The exceptional elements are light elements, for which the surface abundances can be altered because of internal nucleosynthesis and mixing. This could be the case for C and O among the elements investigated in the present study.

Another concern is that the systematic uncertainties in the abundance analysis act differently between red giant branch stars and red clump stars. For example, Masseron & Hawkins (2017) reported that the offset between spectroscopic and asteroseismic $\log g$ varies between the two classes of stars. We note, however, that the measurements by Takeda & Tajitsu (2015) and Takeda et al. (2016) do not show such difference. This different result might be due to the different methods for stellar parameter determination; while the spectroscopic stellar parameters used in Masseron & Hawkins (2017) are derived by globally fitting synthetic spectra to the observed ones, Takeda & Tajitsu (2015)

and Takeda et al. (2016) derived theirs based on equivalent widths of iron lines. Our approach is more similar to that adopted by Takeda & Tajitsu (2015) and Takeda et al. (2016). We also note that red giant branch stars and red clump stars have almost identical spectra except for the regions that are sensitive to light-element abundances (e.g., Hawkins et al. 2018).

Nonetheless, it is worth an investigation to verify that the sample of red clump stars and luminous giants yields a result consistent with less-luminous giants before combining the results for the discussion. Figures 24 and 25 show the abundance trends with metallicity and $\log g$, respectively. Red clump stars and stars with $\log g < 2.0$ are marked with open squares. There is no obvious trend between abundance and evolutionary status. Table 13 quantifies the consistency of the results. It shows the average $[X/Fe]$ for stars in different evolutionary statuses and in different subsamples, and should be compared to Table 8. The abundance differences found in Section 6.1, such as the low $[\alpha/Fe]$, low $[Cu/Fe]$, and low $[Y/Fe]$ of the low- α populations, are confirmed both in a sample of red clump and luminous giants and that of less-luminous giants. These results assure us in combining stars in both evolutionary statuses for the discussion of chemical abundances without unnecessarily reducing the sample size.

Figure 24. Same as Figures 10–14 but with red clump stars and luminous ($\log g < 2.0$) stars being marked with open squares.

Figure 25. Same as Figure 24 but plotted against $\log g$.

Table 13
Average [X/Fe] for Groups of Stars

Elem	All		Metal-poor		High- α		Low- α	
	Group 1 ^a	Group 2 ^b	Group 1	Group 2	Group 1	Group 2	Group 1	Group 2
CH	-0.212	-0.069	-0.151	0.048	-0.098	-0.023	-0.348	-0.188
OI	0.441	0.528	...	0.677	...	0.634	0.454	0.475
Na I	0.108	0.050	0.013	0.075	0.178	0.051	0.144	0.083
Mg I	0.312	0.312	0.431	0.361	0.415	0.381	0.257	0.258
Si I	0.407	0.383	0.589	0.509	0.521	0.399	0.313	0.306
Ca I	0.308	0.313	0.364	0.353	0.338	0.328	0.317	0.292
Sc II	0.202	0.149	0.104	0.048	0.312	0.257	0.227	0.170
Ti I	0.172	0.200	0.243	0.224	0.245	0.252	0.153	0.162
Ti II	0.326	0.342	0.329	0.316	0.384	0.410	0.346	0.336
V I	0.027	0.050	-0.008	0.003	0.093	0.130	0.020	0.026
Cr I	-0.117	-0.129	-0.195	-0.180	-0.082	-0.067	-0.088	-0.152
Cr II	0.139	0.106	0.090	0.044	0.190	0.114	0.176	0.150
Mn I	-0.314	-0.371	-0.457	-0.442	-0.266	-0.321	-0.296	-0.375
Co I	0.062	-0.001	-0.072	-0.048	0.207	0.130	0.076	-0.025
Ni I	0.005	-0.020	-0.028	-0.023	0.103	0.036	-0.017	-0.046
Zn I	0.138	0.097	0.253	0.167	0.205	0.134	0.086	0.058
Cu I	-0.688	-0.821	-0.936	-1.058	-0.490	-0.593	-0.763	-0.842
Y II	-0.006	-0.040	-0.211	-0.244	0.215	0.196	-0.015	-0.024
Ba II	0.082	0.020	-0.355	-0.314	0.186	0.220	0.156	0.180
La II	0.193	0.212	-0.127	0.043	0.231	0.267	0.308	0.266
Ce II	0.115	0.150	-0.010	-0.015	0.149	0.219	0.130	0.192
Nd II	0.293	0.280	0.164	0.030	0.277	0.342	0.348	0.406
Sm II	0.407	0.451	0.267	0.315	0.368	0.445	0.477	0.483
Eu II	0.457	0.410	0.178	0.156	0.458	0.490	0.571	0.551
# of stars	11	15	2	5	3	4	4	5

Notes.

^a Group 1 is a group of red clump stars and luminous ($\log g < 2.0$) red giant branch stars.

^b Group 2 is a group of less-luminous ($\log g > 2.0$) red giant stars.

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References

- Aerts, C., Christensen-Dalsgaard, J., & Kurtz, D. W. 2010, *Asteroseismology* (Dordrecht: Springer Science+Business Media)
- Amarsi, A. M., Asplund, M., Collet, R., & Leenaarts, J. 2016a, *MNRAS*, **455**, 3735
- Amarsi, A. M., Lind, K., Asplund, M., Barklem, P. S., & Collet, R. 2016b, *MNRAS*, **463**, 1518
- Anders, F., Chiappini, C., Rodrigues, T. S., et al. 2017, *A&A*, **597**, A30
- Andrievsky, S., Bonifacio, P., Caffau, E., et al. 2018, *MNRAS*, **473**, 3377
- Aoki, W., Honda, S., Beers, T. C., et al. 2005, *ApJ*, **632**, 611
- Aoki, W., Honda, S., Beers, T. C., et al. 2007, *ApJ*, **660**, 747
- Asplund, M., Grevesse, N., Sauval, A. J., & Scott, P. 2009, *ARA&A*, **47**, 481
- Astropy Collaboration, Robitaille, T. P., Tollerud, E. J., et al. 2013, *A&A*, **558**, A33
- Auvergne, M., Bodin, P., Boissard, L., et al. 2009, *A&A*, **506**, 411
- Baines, E. K., Armstrong, J. T., Schmitt, H. R., et al. 2014, *ApJ*, **781**, 90
- Barbuy, B., Meléndez, J., Spite, M., et al. 2003, *ApJ*, **588**, 1072
- Bedell, M., Bean, J. L., Meléndez, J., et al. 2018, *ApJ*, **865**, 68
- Beers, T. C., & Christlieb, N. 2005, *ARA&A*, **43**, 531
- Belokurov, V., Erkal, D., Evans, N. W., Koposov, S. E., & Deason, A. J. 2018, *MNRAS*, **478**, 611
- Belokurov, V., Sanders, J. L., Fattahi, A., et al. 2020, *MNRAS*, **494**, 3880
- Bergemann, M. 2011, *MNRAS*, **413**, 2184
- Bergemann, M., & Cescutti, G. 2010, *A&A*, **522**, A9
- Brogaard, K., Jessen-Hansen, J., Handberg, R., et al. 2016, *AN*, **337**, 793
- Buder, S., Lind, K., Ness, M. K., et al. 2019, *A&A*, **624**, A19
- Bullock, J. S., & Johnston, K. V. 2005, *ApJ*, **635**, 931
- Carollo, D., Beers, T. C., Placco, V. M., et al. 2016, *NatPh*, **12**, 1170
- Carollo, D., Chiba, M., Ishigaki, M., et al. 2019, *ApJ*, **887**, 22
- Carretta, E., Bragaglia, A., Gratton, R. G., et al. 2009, *A&A*, **505**, 117
- Casagrande, L. 2020, *ApJ*, **896**, 26
- Casagrande, L., Portinari, L., Glass, I. S., et al. 2014, *MNRAS*, **439**, 2060
- Casagrande, L., Ramírez, I., Meléndez, J., Bessell, M., & Asplund, M. 2010, *A&A*, **512**, A54
- Casagrande, L., Silva Aguirre, V., Schlesinger, K. J., et al. 2016, *MNRAS*, **455**, 987
- Casagrande, L., & VandenBerg, D. A. 2014, *MNRAS*, **444**, 392
- Casey, A. R., Kennedy, G. M., Hartle, T. R., & Schlafman, K. C. 2018, *MNRAS*, **478**, 2812
- Cayrel, R., Depagne, E., Spite, M., et al. 2004, *A&A*, **416**, 1117
- Chaplin, W. J., & Miglio, A. 2013, *ARA&A*, **51**, 353
- Chaplin, W. J., Serenelli, A. M., Miglio, A., et al. 2020, *NatAs*, **4**, 382
- Chiappini, C., Anders, F., Rodrigues, T. S., et al. 2015, *A&A*, **576**, L12
- Choi, J., Dotter, A., Conroy, C., et al. 2016, *ApJ*, **823**, 102
- Chopin, A., Hirschi, R., Meynet, G., et al. 2018, *A&A*, **618**, A133
- Côté, B., Eichler, M., Arcones, A., et al. 2019, *ApJ*, **875**, 106
- Creevey, O., Grundahl, F., Thévenin, F., et al. 2019, *A&A*, **625**, A33
- da Silva, R., Porto de Mello, G. F., Milone, A. C., et al. 2012, *A&A*, **542**, A84
- Das, P., Hawkins, K., & Jofré, P. 2020, *MNRAS*, **493**, 5195
- De Cat, P., Fu, J. N., Ren, A. B., et al. 2015, *ApJS*, **220**, 19
- Delgado Mena, E., Moya, A., Adibekyan, V., et al. 2019, *A&A*, **624**, A78
- Dotter, A. 2016, *ApJS*, **222**, 8
- Dotter, A., Sarajedini, A., & Anderson, J. 2011, *ApJ*, **738**, 74

- Epstein, C. R., Elsworth, Y. P., Johnson, J. A., et al. 2014, *ApJL*, **785**, L28
- Evans, D. W., Riello, M., De Angeli, F., et al. 2018, *A&A*, **616**, A4
- Evans, N. W. 2020, in IAU Symp. 353, Galactic Dynamics in the Era of Large Surveys, ed. M. Valluri & J. A. Sellwood (Cambridge: Cambridge Univ. Press)
- Feltzing, S., Howes, L. M., McMillan, P. J., & Stonkutė, E. 2017, *MNRAS*, **465**, L109
- Fernández-Trincado, J. G., Beers, T. C., & Minniti, D. 2020, *A&A*, **644**, A83
- Feuillet, D. K., Bovy, J., Holtzman, J., et al. 2016, *ApJ*, **817**, 40
- Feuillet, D. K., Bovy, J., Holtzman, J., et al. 2018, *MNRAS*, **477**, 2326
- Fishlock, C. K., Yong, D., Karakas, A. I., et al. 2017, *MNRAS*, **466**, 4672
- Forbes, D. A., & Bridges, T. 2010, *MNRAS*, **404**, 1203
- Foreman-Mackey, D. 2016, *JOSS*, **1**, 24
- Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, *PASP*, **125**, 306
- Frebel, A., & Norris, J. E. 2015, *ARA&A*, **53**, 631
- Freeman, K., & Bland-Hawthorn, J. 2002, *ARA&A*, **40**, 487
- Frischke, U., Hirschi, R., & Thielemann, F. K. 2012, *A&A*, **538**, L2
- Fujimoto, M. Y., Aikawa, M., & Kato, K. 1999, *ApJ*, **519**, 733
- Fulbright, J. P. 2000, *AJ*, **120**, 1841
- Gaia Collaboration, Brown, A. G. A., Vallenari, A., et al. 2016a, *A&A*, **595**, A2
- Gaia Collaboration, Brown, A. G. A., Vallenari, A., et al. 2018, *A&A*, **616**, A1
- Gaia Collaboration, Prusti, T., de Bruijne, J. H. J., et al. 2016b, *A&A*, **595**, A1
- Gallart, C., Bernard, E. J., Brook, C. B., et al. 2019, *NatAs*, **3**, 932
- Gaulme, P., McKeever, J., Jackiewicz, J., et al. 2016, *ApJ*, **832**, 121
- Green, G. M., Schlafly, E., Zucker, C., Speagle, J. S., & Finkbeiner, D. 2019, *ApJ*, **887**, 93
- Green, G. M., Schlafly, E. F., Finkbeiner, D., et al. 2018, *MNRAS*, **478**, 651
- Guggenberger, E., Hekker, S., Basu, S., & Bellinger, E. 2016, *MNRAS*, **460**, 4277
- Gustafsson, B., Edvardsson, B., Eriksson, K., et al. 2008, *A&A*, **486**, 951
- Hawkins, K., Jofré, P., Gilmore, G., & Masseron, T. 2014, *MNRAS*, **445**, 2575
- Hawkins, K., Ting, Y.-S., & Walter-Rix, H. 2018, *ApJ*, **853**, 20
- Hekker, S. 2020, *FRASS*, **7**, 3
- Hekker, S., Gilliland, R. L., Elsworth, Y., et al. 2011, *MNRAS*, **414**, 2594
- Helmi, A., Babusiaux, C., Koppelman, H. H., et al. 2018, *Natur*, **563**, 85
- Helmi, A., & White, S. D. M. 1999, *MNRAS*, **307**, 495
- Helmi, A., White, S. D. M., de Zeeuw, P. T., & Zhao, H. 1999, *Natur*, **402**, 53
- Hill, V., Skúladóttir, Á., Tolstoy, E., et al. 2019, *A&A*, **626**, A15
- Honda, S., Aoki, W., Kajino, T., et al. 2004, *ApJ*, **607**, 474
- Hotokezaka, K., Beniamini, P., & Piran, T. 2018, *IJMPD*, **27**, 1842005
- Howell, S. B., Sobek, C., Haas, M., et al. 2014, *PASP*, **126**, 398
- Huber, D., Bedding, T. R., Stello, D., et al. 2011, *ApJ*, **743**, 143
- Huber, D., Ireland, M. J., Bedding, T. R., et al. 2012, *ApJ*, **760**, 32
- Huber, D., Stello, D., Bedding, T. R., et al. 2009, *CoAst*, **160**, 74
- Hunter, J. D. 2007, *CSE*, **9**, 90
- Iben, I. J. 1967, *ARA&A*, **5**, 571
- Ishigaki, M. N., Aoki, W., & Chiba, M. 2013, *ApJ*, **771**, 67
- Ivans, I. I., Simmerer, J., Sneden, C., et al. 2006, *ApJ*, **645**, 613
- Izzard, R. G., Preece, H., Jofré, P., et al. 2018, *MNRAS*, **473**, 2984
- Jean-Baptiste, I., Di Matteo, P., Haywood, M., et al. 2017, *A&A*, **604**, A106
- Jofré, P., Jorissen, A., Van Eck, S., et al. 2016, *A&A*, **595**, A60
- Jofré, P., & Weiss, A. 2011, *A&A*, **533**, A59
- Johnson, J. A. 2002, *ApJS*, **139**, 219
- Jurić, M., Ivezić, Ž., Brooks, A., et al. 2008, *ApJ*, **673**, 864
- Kallinger, T., Mosser, B., Hekker, S., et al. 2010b, *A&A*, **522**, A1
- Kallinger, T., Weiss, W. W., Barban, C., et al. 2010a, *A&A*, **509**, A77
- Karovicova, I., White, T. R., Nordlander, T., et al. 2018, *MNRAS*, **475**, L81
- Kilic, M., Bergeron, P., Dame, K., et al. 2019, *MNRAS*, **482**, 965
- Koch, D. G., Borucki, W. J., Basri, G., et al. 2010, *ApJL*, **713**, L79
- Kruijssen, J. M. D., Pfeffer, J. L., Reina-Campos, M., Crain, R. A., & Bastian, N. 2019, *MNRAS*, **486**, 3180
- Letarte, B., Hill, V., Tolstoy, E., et al. 2010, *A&A*, **523**, A17
- Li, H., Aoki, W., Matsuno, T., et al. 2018, *ApJL*, **852**, L31
- Lin, J., Asplund, M., Ting, Y.-S., et al. 2020, *MNRAS*, **491**, 2043
- Lind, K., Asplund, M., Barklem, P. S., & Belyaev, A. K. 2011, *A&A*, **528**, A103
- Lindgren, L., Hernandez, J., Bombrun, A., et al. 2018, *A&A*, **616**, A2
- Mackereth, J. T., Schiavon, R. P., Pfeffer, J., et al. 2019, *MNRAS*, **482**, 3426
- Majewski, S. R., Schiavon, R. P., Frinchaboy, P. M., et al. 2017, *AJ*, **154**, 94
- Maoz, D., Mannucci, F., & Nelemans, G. 2014, *ARA&A*, **52**, 107
- Marín-Franch, A., Aparicio, A., Piotto, G., et al. 2009, *ApJ*, **694**, 1498
- Martell, S. L., Sharma, S., Buder, S., et al. 2017, *MNRAS*, **465**, 3203
- Martig, M., Fouesneau, M., Rix, H.-W., et al. 2016, *MNRAS*, **456**, 3655
- Martig, M., Rix, H.-W., Aguirre, V. S., et al. 2015, *MNRAS*, **451**, 2230
- Massari, D., Koppelman, H. H., & Helmi, A. 2019, *A&A*, **630**, L4
- Masseron, T., & Hawkins, K. 2017, *A&A*, **597**, L3
- Mathur, S., García, R. A., Huber, D., et al. 2016, *ApJ*, **827**, 50
- Matsuno, T., Aoki, W., & Suda, T. 2019, *ApJL*, **874**, L35
- Matsuno, T., Yong, D., Aoki, W., & Ishigaki, M. N. 2018, *ApJ*, **860**, 49
- McCarthy, I. G., Font, A. S., Crain, R. A., et al. 2012, *MNRAS*, **420**, 2245
- McKinney, W. 2010, in Proc. 9th Python in Science Conf., ed. S. van der Walt & J. Millman, 56, <https://conference.scipy.org/proceedings/scipy2010/mckinney.html>
- McMillan, P. J. 2017, *MNRAS*, **465**, 76
- McWilliam, A. 1998, *AJ*, **115**, 1640
- Meléndez, J., Casagrande, L., Ramírez, I., Asplund, M., & Schuster, W. J. 2010, *A&A*, **515**, L3
- Miglio, A., Chaplin, W. J., Brogaard, K., et al. 2016, *MNRAS*, **461**, 760
- Miglio, A., Chiappini, C., Mackereth, T., et al. 2021, *A&A*, **645**, A85
- Mishenina, T. V., & Kovtyukh, V. V. 2001, *A&A*, **370**, 951
- Mosser, B., & Appourchaux, T. 2009, *A&A*, **508**, 877
- Myeong, G. C., Vasiliev, E., Iorio, G., Evans, N. W., & Belokurov, V. 2019, *MNRAS*, **488**, 1235
- Ness, M., Hogg, D. W., Rix, H. W., et al. 2016, *ApJ*, **823**, 114
- Nissen, P. E. 2015, *A&A*, **579**, A52
- Nissen, P. E. 2016, *A&A*, **593**, A65
- Nissen, P. E., Chen, Y. Q., Carigi, L., Schuster, W. J., & Zhao, G. 2014, *A&A*, **568**, A25
- Nissen, P. E., & Schuster, W. J. 2010, *A&A*, **511**, L10
- Nissen, P. E., & Schuster, W. J. 2011, *A&A*, **530**, A15
- Noguchi, K., Aoki, W., Kawanomoto, S., et al. 2002, *PASJ*, **54**, 855
- Nordström, B., Mayor, M., Andersen, J., et al. 2004, *A&A*, **418**, 989
- Paxton, B., Bildsten, L., Dotter, A., et al. 2011, *ApJS*, **192**, 3
- Pereira, C. B., Holanda, N., Drake, N. A., & Roig, F. 2019, *AJ*, **157**, 70
- Perryman, M. A. C., Lindgren, L., Kovalevsky, J., et al. 1997, *A&A*, **500**, 501
- Pietrinferni, A., Cassisi, S., Salaris, M., & Castellì, F. 2004, *ApJ*, **612**, 168
- Pietrinferni, A., Cassisi, S., Salaris, M., & Castellì, F. 2006, *ApJ*, **642**, 797
- Pignatari, M., Gallino, R., Heil, M., et al. 2010, *ApJ*, **710**, 1557
- Pinsonneault, M. H., Elsworth, Y., Epstein, C., et al. 2014, *ApJS*, **215**, 19
- Pinsonneault, M. H., Elsworth, Y. P., Tayar, J., et al. 2018, *ApJS*, **239**, 32
- Prantzos, N., Abia, C., Cristallo, S., Limongi, M., & Chieffi, A. 2020, *MNRAS*, **491**, 1832
- Prantzos, N., Abia, C., Limongi, M., Chieffi, A., & Cristallo, S. 2018, *MNRAS*, **476**, 3432
- Prantzos, N., Hashimoto, M., & Nomoto, K. 1990, *A&A*, **234**, 211
- Ramírez, I., Meléndez, J., Bean, J., et al. 2014, *A&A*, **572**, A48
- Reggiani, H., Meléndez, J., Kobayashi, C., Karakas, A., & Placco, V. 2017, *A&A*, **608**, A46
- Reid, M. J., & Brunthaler, A. 2004, *ApJ*, **616**, 872
- Ren, A., Fu, J., De Cat, P., et al. 2016, *ApJS*, **225**, 28
- Robin, A. C., Reylé, C., Derrière, S., & Picaud, S. 2003, *A&A*, **409**, 523
- Rodrigues, T. S., Bossini, D., Miglio, A., et al. 2017, *MNRAS*, **467**, 1433
- Roederer, I. U., Preston, G. W., Thompson, I. B., et al. 2014, *AJ*, **147**, 136
- Sakari, C. M., Placco, V. M., Farrell, E. M., et al. 2018, *ApJ*, **868**, 110
- Skúladóttir, Á., & Salvadori, S. 2020, *A&A*, **634**, L2
- Schönrich, R., Binney, J., & Dehnen, W. 2010, *MNRAS*, **403**, 1829
- Schuster, W. J., Moreno, E., Nissen, P. E., & Pichardo, B. 2012, *A&A*, **538**, A21
- Sestito, F., Longeard, N., Martin, N. F., et al. 2019, *MNRAS*, **484**, 2166
- Sharma, S., & Stello, D. 2016, Asfgrid: Asteroseismic Parameters for a Star, Astrophysics Source Code Library, ascl:1603.009
- Sharma, S., Stello, D., Bland-Hawthorn, J., Huber, D., & Bedding, T. R. 2016, *ApJ*, **822**, 15
- Shi, J. R., Gehren, T., Mashonkina, L., & Zhao, G. 2009, *A&A*, **503**, 533
- Shi, J. R., Yan, H. L., Zhou, Z. M., & Zhao, G. 2018, *ApJ*, **862**, 71
- Silva Aguirre, V., Bojsen-Hansen, M., Slumstrup, D., et al. 2018, *MNRAS*, **475**, 5487
- Skrutskie, M. F., Cutri, R. M., Stiening, R., et al. 2006, *AJ*, **131**, 1163
- Skúladóttir, Á., Hansen, C. J., Salvadori, S., & Choplin, A. 2019, *A&A*, **631**, A171
- Smiljanic, R., Romano, D., Bragaglia, A., et al. 2016, *A&A*, **589**, A115
- Sneden, C. 1973, *ApJ*, **184**, 839
- Sneden, C., Preston, G. W., McWilliam, A., & Searle, L. 1994, *ApJL*, **431**, L27
- Soderblom, D. R. 2010, *ARA&A*, **48**, 581
- Spina, L., Meléndez, J., Karakas, A. I., et al. 2018, *MNRAS*, **474**, 2580
- Steinmetz, M., Zwitter, T., Siebert, A., et al. 2006, *AJ*, **132**, 1645
- Stello, D., Huber, D., Bedding, T. R., et al. 2013, *ApJL*, **765**, L41
- Straniero, O., Cristallo, S., & Piersanti, L. 2014, *ApJ*, **785**, 77
- Suda, T., Katsuta, Y., Yamada, S., et al. 2008, *PASJ*, **60**, 1159
- Tajitsu, A., Aoki, W., Kawanomoto, S., & Narita, N. 2010, *PNAOJ*, **13**, 1

- Tajitsu, A., Aoki, W., & Yamamuro, T. 2012, *PASJ*, 64, 77
- Takeda, Y., & Tajitsu, A. 2015, *MNRAS*, 450, 397
- Takeda, Y., Tajitsu, A., Sato, B., et al. 2016, *MNRAS*, 457, 4454
- Thiemeßl, N., Hekker, S., Southworth, J., et al. 2018, *MNRAS*, 478, 4669
- Totani, T., Morokuma, T., Oda, T., Doi, M., & Yasuda, N. 2008, *PASJ*, 60, 1327
- Tucci Maia, M., Ramírez, I., Meléndez, J., et al. 2016, *A&A*, 590, A32
- Valentini, M., Chiappini, C., Bossini, D., et al. 2019, *A&A*, 627, A173
- van der Walt, S., Colbert, S. C., & Varoquaux, G. 2011, *CSE*, 13, 22
- VandenBerg, D. A., Bond, H. E., Nelan, E. P., et al. 2014, *ApJ*, 792, 110
- VandenBerg, D. A., Brogaard, K., Leaman, R., & Casagrande, L. 2013, *ApJ*, 775, 134
- Venn, K. A., Kelt, C. L., Sestito, F., et al. 2020, *MNRAS*, 492, 3241
- Virtanen, P., Gommers, R., & Oliphant, T. E. 2020, *NatMe*, 17, 261
- Weiss, A., Denissenkov, P. A., & Charbonnel, C. 2000, *A&A*, 356, 181
- Westin, J., Sneden, C., Gustafsson, B., & Cowan, J. J. 2000, *ApJ*, 530, 783
- Wu, Y., Xiang, M., Bi, S., et al. 2018, *MNRAS*, 475, 3633
- Wu, Y., Xiang, M., Zhao, G., et al. 2019, *MNRAS*, 484, 5315
- Yan, H. L., Shi, J. R., Nissen, P. E., & Zhao, G. 2016, *A&A*, 585, A102
- Yanny, B., Rockosi, C., Newberg, H. J., et al. 2009, *AJ*, 137, 4377
- Yong, D., Casagrande, L., Venn, K. A., et al. 2016, *MNRAS*, 459, 487
- Yu, J., Huber, D., Bedding, T. R., et al. 2018, *ApJS*, 236, 42
- Zhang, L., Karlsson, T., Christlieb, N., et al. 2011, *A&A*, 528, A92
- Zhao, G., Zhao, Y.-H., Chu, Y.-Q., Jing, Y.-P., & Deng, L.-C. 2012, *RAA*, 12, 723
- Zinn, J. C., Pinsonneault, M. H., Huber, D., et al. 2019, *ApJ*, 885, 166
- Zinn, J. C., Pinsonneault, M. H., Huber, D., & Stello, D. 2019, *ApJ*, 878, 136
- Zong, W., Fu, J.-N., De Cat, P., et al. 2018, *ApJS*, 238, 30