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Pragmatic convex approaches for risk-averse and distributionally robust mixed-integer recourse models

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E. Ruben van Beesten

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