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Published in:
Physical Review D

DOI:
[10.1103/PhysRevD.104.124010](https://doi.org/10.1103/PhysRevD.104.124010)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2021

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Peng, J., Guo, M., & Feng, X. H. (2021). Observational signature and additional photon rings of an asymmetric thin-shell wormhole. *Physical Review D*, 104(12), [124010].
<https://doi.org/10.1103/PhysRevD.104.124010>

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Observational signature and additional photon rings of an asymmetric thin-shell wormhole

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(Received 15 February 2021; accepted 4 November 2021; published 2 December 2021)

Recently, a distinct shadow mechanism was proposed by Wang *et al.* from the asymmetric thin-shell wormhole (ATW) [X. Wang *et al.*, *Phys. Lett. B* **811**, 135930 (2020)]. On the other hand, Gralla, Holz, and Wald's work [*Phys. Rev. D* **100**, 024018 (2019)] represented a nice description of photon rings in the presence of an accretion disk around a black hole. In this paper, we are inspired to thoroughly investigate the observational appearance of an accretion disk around the ATW. Although the spacetime outside an ATW with a throat could be identical to that containing a black hole with its event horizon, we show evident additional photon rings from the ATW spacetime. Moreover, a potential lensing band between two highly demagnified photon rings is found. Our analysis provides an optically observational signature to distinguish ATWs from black holes.

DOI: [10.1103/PhysRevD.104.124010](https://doi.org/10.1103/PhysRevD.104.124010)

I. INTRODUCTION

The first black hole image of M87* released by the Event Horizon Telescope plays a significant role in the frontier of general relativity [1]. It provides a direct and powerful observational information for general relativity in the strong gravitational regime. As a result, the research of the black hole shadow became very popular thereafter [2–26]. Among these interesting works, Gralla, Holz, and Wald's originality gives an elegant description on black hole shadows, lensing rings, and photon rings considering an emission disk around black holes [21], which naturally led to lots of interesting follow-up works [22–26]. In this context, the term “shadow” refers to a dark area outside the black hole, while the regular shadow denotes the critical curve in the sky of observers, and the critical curve is closely related to the spherical photon orbits which are always radially unstable [27]. To avoid misunderstanding, for the latter we go by the name of critical curve in the rest of this article. Therefore, one can conclude that the shape of the shadow could change if different sources of light are considered for the same black hole. On the contrary, the critical curve is invariant as long as the spacetime geometry is given.

Nevertheless, with the help of the image of M87*, we are still not able to assert that the supermassive object in the

center of M87* must be a black hole. This is because black holes are not the only ones that have photon spheres which have the ability to shade the horizon. It has been found that some ultracompact objects (UCOs) also own photon spheres [27,28]. Furthermore, some of them are found to mimic the optical appearances of black holes including shadows [29–31]. Thus, it is crucial to distinguish black holes from UCOs whose external spacetime geometries are similar or even identical to those of black holes up to the vicinity of horizon, such as gravastar, wormhole, fuzzball, and so on. In fact, one may distinguish black holes from some UCOs in their acoustic properties, such as echo effect [32–35]. Another observational difference between black holes and wormholes is explored in Refs. [36,37]. Roughly speaking, the essential distinction between black holes and UCOs is that the event horizon of black holes is a one-way membrane while UCOs have no horizons. In this sense, black holes can be regarded as absolute black objects with zero reflection, while UCOs have considerable reflection.

Recently, the shadow of an asymmetric thin-shell wormhole (ATW) has been studied in Ref. [38]. The authors showed that due to reflection of photons by the wormhole there exists a novel shadow which is different from that of a black hole in certain parameter space. Then more examples of asymmetric thin-shell wormhole are studied in Refs. [39–41], of which the authors named their results as double shadows [39]. It is worth noting that double shadows should be called double critical curves or photon spheres in the context of our article. On the other hand, the novel shadow proposed in Ref. [38] does refer to the dark

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area; however, its edge is still related to the photon sphere when the throat of the ATW is inside the photon sphere, but the photon sphere here is the one which is located on the other side opposite the observers other than the usual one in the observers' side for an ATW spacetime. So far, there has been no discussion on the observational appearance of an emission disk in ATW spacetime, although no matter novel shadows or double shadows are improper terminologies in practical observation. Our motivation is to give a complete picture of the observational appearance of an accretion disk around an ATW.

The plan of our paper is organized as follows: We first give a brief review of the asymmetric thin-shell wormhole in Sec. II. Then, we analyze the photon trajectories and deflection angles in this wormhole in Sec. III. Next, we discuss the transfer functions and observational appearances of emission disks around this wormhole in Sec. IV. We conclude in Sec. V.

II. NULL GEODESIC IN ASYMMETRIC THIN-SHELL WORMHOLE

In this section, we give a brief review of the asymmetric thin-shell wormhole model presented in Ref. [38]. We consider a thin-shell wormhole using the cut-and-paste method; that is, two distinct spacetimes with different parameters are glued by a thin shell

$$ds_i^2 = -f_i(r_i)dt_i^2 + \frac{dr_i^2}{f_i(r_i)} + r_i^2 d\Omega^2, \quad (1)$$

where $i = 1, 2$, and by focusing on the Schwarzschild case we have

$$f_i(r_i) = 1 - \frac{2M_i}{r_i}, \quad r \geq R, \quad (2)$$

where M_i are the mass parameters and R is the position of the thin shell, i.e., the radius of the throat. Thus, we have

$$R > \max\{2M_1, 2M_2\} \quad (3)$$

(see Fig. 1).

In each spacetime, the radial null geodesic is

$$\left(\frac{dr_i}{d\tau}\right)^2 + V_{i,\text{eff}} = \frac{1}{b_i^2} \quad (4)$$

with the effective potential given by

$$V_{i,\text{eff}} = \frac{f_i(r_i)}{r_i^2}, \quad (5)$$

where $b_i = \frac{L_i}{E_i}$ is called the impact parameter.

Without loss of generality, we suppose that observers are located in spacetime $\mathcal{M}_1$ and set $M_1 = 1$ and $M_2 = k$.

FIG. 1. Asymmetric thin-shell wormhole as a black hole mimicker. We assume that observers are located at spacetime $\mathcal{M}_1$. So the spacetime geometry viewed by observers is identical to that of a black hole up to the thin shell.

The impact parameters in two spacetimes are connected with each other by

$$\frac{b_1}{b_2} = \sqrt{\frac{R-2k}{R-2}} \equiv Z. \quad (6)$$

If there exists a photon sphere in $\mathcal{M}_1$, it is hard to distinguish a wormhole from a black hole based on direct optical observation, because the horizon is shaded by the photon sphere. So we are interested in this case, i.e., $R < 3$. In other words, we want to distinguish a wormhole from a black hole through more abundant information even if they look like each other.

More abundant information indeed exists. Considering the ingoing null geodesics with $b_1 < b_1^c = 3\sqrt{3}$ in spacetime $\mathcal{M}_1$, certain geodesics would turn back, passing through the throat with a necessary condition

$$b_2 = \frac{b_1}{Z} > b_2^c = 3\sqrt{3}k. \quad (7)$$

In summary, the ingoing null geodesic in spacetime $\mathcal{M}_1$ whose impact parameter satisfies

$$3\sqrt{3}kZ < b_1 < 3\sqrt{3} \quad (8)$$

would drop into spacetime $\mathcal{M}_2$ and then turn back to spacetime $\mathcal{M}_1$, passing through the throat. This condition can be satisfied given

$$1 < k < \frac{R}{2} \leq \frac{3}{2}. \quad (9)$$

We show a plot of effective potential in Fig. 2. It can be obviously seen from this plot that the photon in spacetime $\mathcal{M}_1$ with an impact parameter lying in the range $Zb_2^c < b_1 < b_1^c$ can turn back when it reaches the turning point in spacetime $\mathcal{M}_2$. We will see that this reflection

FIG. 2. Plot of the effective potential of radial null geodesic in an asymmetric thin-shell wormhole. We have set $M_1 = 1$, $M_2 = 1.2$, and $R = 2.6$, so $b_1^c = 3\sqrt{3} \approx 5.19615$ and $Zb_2^c = 3.6$ here and in the following. Note that the ticks in spacetime $\mathcal{M}_2$ are scaled by Z^2 , because the impact parameters in two spacetimes are connected with each other by Eq. (6).

mechanism by a wormhole makes an essential distinction in observational appearance between a wormhole and a black hole in the following.

III. TRAJECTORY OF PHOTON AND DEFLECTION ANGLE IN AN ASYMMETRIC THIN-SHELL WORMHOLE

To have a complete understanding of the observational appearance of an accretion disk around an asymmetric thin-shell wormhole through the shadow, photon rings, and lensing rings, we need first to investigate the trajectory and deflection angle of a light ray traveling in the

wormhole. It is convenient to make a coordinate transformation $u_i = 1/r_i$. The trajectory of photon is determined by orbit equation

$$\left(\frac{du_i}{d\phi}\right)^2 = G_i(u_i), \quad (10)$$

where

$$G_i(u_i) = \frac{1}{b_i^2} + 2M_i u_i^3 - u_i^2. \quad (11)$$

All trajectories can be divided into three classes. When $b_1 > b_1^c$, a photon in $\mathcal{M}_1$ from infinity approaches one closest point outside the throat and then moves back to infinity in $\mathcal{M}_1$. When $Zb_2^c < b_1 < b_1^c$, a photon in $\mathcal{M}_1$ from infinity drops into $\mathcal{M}_2$ and then turns back, passing through the throat to infinity in $\mathcal{M}_1$ (see Fig. 3). When $b_1 < Zb_2^c$, a photon in $\mathcal{M}_1$ from infinity drops into $\mathcal{M}_2$ and moves to infinity in $\mathcal{M}_2$.

For $b_1 > b_1^c$, the turning point in spacetime $\mathcal{M}_1$ corresponds to the minimally positive real root of $G_1(u_1) = 0$, which we will denote by $u_1^{\min}$. According to Eq. (10), the total change of azimuthal angle ϕ for a certain trajectory with impact parameter b_1 can be calculated by

$$\phi_1(b_1) = 2 \int_0^{u_1^{\min}} \frac{du_1}{\sqrt{G_1(u_1)}}, \quad b_1 > b_1^c. \quad (12)$$

For

$$Zb_2^c < b_1 < b_1^c, \quad (13)$$

we first focus on the trajectory in spacetime $\mathcal{M}_1$ outside the throat. The total change of azimuthal angle ϕ in spacetime $\mathcal{M}_1$ is obtained by

FIG. 3. Photon trajectories in Euclidean polar coordinates (r, ϕ) with impact parameters lying in the range $Zb_2^c < b_1 < b_1^c$. The photons are coming from far right in spacetime $\mathcal{M}_1$. The red solid lines stand for trajectories in spacetime $\mathcal{M}_1$, and the blue dashed lines stand for trajectories in spacetime $\mathcal{M}_2$.

$$\phi_1(b_1) = \int_0^{1/R} \frac{du_1}{\sqrt{G_1(u_1)}}, \quad b_1 < b_1^c. \quad (14)$$

The turning point in spacetime $\mathcal{M}_2$ corresponds to the maximally positive real root of $G_2(u_2) = 0$, which we will denote by $u_2^{\max}$. According to Eq. (10), the total change of azimuthal angle ϕ for a certain trajectory with impact parameter b_2 in spacetime $\mathcal{M}_2$ can be calculated by

$$\phi_2(b_2) = 2 \int_{u_2^{\max}}^{1/R} \frac{du_2}{\sqrt{G_2(u_2)}}, \quad b_2 > b_2^c. \quad (15)$$

It is convenient to define three orbit numbers

$$\begin{aligned} n_1(b_1) &= \frac{\phi_1(b_1)}{2\pi}, & n_2(b_1) &= \frac{\phi_1(b_1) + \phi_2(b_1/Z)}{2\pi}, \\ n_3(b_1) &= \frac{2\phi_1(b_1) + \phi_2(b_1/Z)}{2\pi} \end{aligned} \quad (16)$$

for later use.

IV. TRANSFER FUNCTIONS AND PHOTON RINGS IN AN ASYMMETRIC THIN-SHELL WORMHOLE

As in Gralla, Holz, and Wald's proposal [21], we focus on the optically and geometrically thin accretion disks for simplicity. Both the static observer and the accretion disk are assumed to locate at spacetime $\mathcal{M}_1$. The static observer is assumed to locate at the north pole, and the accretion disk is on the equatorial plane. The lights emitted from the accretion disk are considered isotropic in the rest frame of the static observer. In view of the spherical symmetry of the spacetime, we also suppose the emitted specific intensity depends only on the radial coordinate, denoted by $I_\nu^{\text{em}}(r)$ with emission frequency ν in a static frame. An observer in infinity will receive the specific intensity I_ν^{obs} with redshifted frequency $\nu' = \sqrt{f}\nu$. Considering I_ν/ν^3 is conserved along a ray, i.e.,

$$\frac{I_\nu^{\text{obs}}}{\nu'^3} = \frac{I_\nu^{\text{em}}}{\nu^3}, \quad (17)$$

we have the observed specific intensity

$$I_\nu^{\text{obs}} = f^{3/2}(r) I_\nu^{\text{em}}(r). \quad (18)$$

So the total observed intensity is an integral over all frequencies:

$$I^{\text{obs}} = \int I_\nu^{\text{obs}} d\nu' = \int f^2 I_\nu^{\text{em}} d\nu = f^2(r) I^{\text{em}}(r), \quad (19)$$

where $I^{\text{em}} = \int I_\nu^{\text{em}} d\nu$ is the total emitted intensity from the accretion disk.

According to the ray-tracing method, if a light ray from the observer intersects with the emission disk, it means the intersecting point as a light source will contribute brightness to the observer. We first consider that a light ray completely travels in spacetime $\mathcal{M}_1$. In the black hole case, a light ray whose orbit number $n_1 > 1/4$ will intersect with the disk on the front side. If n_1 goes larger than $3/4$, the light ray will bend around the wormhole, intersecting with the disk for the second time on the back side. Furthermore, when $n_1 > 5/4$, the light ray will intersect with the disk for the third time on the front side again, and so on. Hence, the observed intensity is a sum of the intensities from each intersection:

$$I^{\text{obs}}(b_1) = \sum_m f^2 I^{\text{em}}|_{r=r_m(b_1)}, \quad (20)$$

where $r_m(b_1)$ is the so-called transfer function which denotes the radial position of the m th intersection with the emission disk.

As discussed in the above sections, for an asymmetric thin-shell wormhole, a photon can turn back, passing through the throat in proper parameter space, so we would have additional transfer functions. According to the definitions of orbit numbers, when $n_2 < 3/4$ and $n_3 > 3/4$, the reflected outgoing trajectories in spacetime $\mathcal{M}_1$ will intersect with the disk on the back side. When $n_2 < 5/4$ and $n_3 > 5/4$, the reflected outgoing trajectories in spacetime $\mathcal{M}_1$ will intersect with the disk on the front side. Thus, the impact parameter range for new second transfer functions is determined by $n_2 < 3/4$ and $n_3 > 3/4$, and the impact parameter range for new third transfer functions is determined by $n_2 < 5/4$ and $n_3 > 5/4$ as illustrated in the middle plot in Fig. 4. We give all transfer functions in the right plot in Fig. 4.

As illustrated in Ref. [21], the first transfer function gives the “direct image” of the disk, which is essentially just the redshift of the source profile. The second transfer function gives a highly demagnified image of the back side of the disk, referred to as a “lensing ring.” The third transfer function gives an extremely demagnified image of the front side of the disk, referred to as a “photon ring.” The images resulting from further transfer functions are so demagnified that they can be neglected. The demagnified scale is determined by the slope of transfer function $dr/d\phi$, called the demagnification factor. We can see from the right plot in Fig. 4 that the new third transfer function near Zb_2^c has a high slope like the usual third transfer function near b_1^c . Another new third transfer function in the left of b_1^c has a smaller slope than the usual third transfer function near b_1^c but a bigger slope than the usual second transfer function in the right of b_1^c . The new second transfer function has a modest slope like the usual first transfer function, so we had better call the resulting image a “lensing band.”

FIG. 4. The ranges of impact parameter and plots for the first three transfer functions. The black, orange, and red ones correspond, respectively, to the first, second, and third transfer functions.

FIG. 5. The left emission profile (model I) is sharply peaked and abruptly ends at the innermost stable circular orbit ($6M_1$). The right emission profile (model II) decays gradually from the photon sphere ($3M_1$) to the innermost stable circular orbit ($6M_1$).

Now we take two typical emission models as examples proposed by Gralla, Holz, and Wald [21] to make physical pictures more clear (see Fig. 5).

The observed intensity can be obtained according to Eq. (20). We give corresponding plots and density plots of

the observed intensities in Fig. 6 for emission model I and Fig. 7 for emission model II. We can see that in both models we have two additional photon rings resulting from corresponding two new third transfer functions for an asymmetric thin-shell wormhole. The new photon ring

FIG. 6. Observed intensity and density plot of emission model I. The top row corresponds to a black hole, and the bottom row corresponds to a wormhole. The left column is the observed intensities. The middle column is the density plots of observed intensities, and the right column is local density plots.

FIG. 7. Observed intensity and density plot of emission model II. The top row corresponds to a black hole, and the bottom row corresponds to a wormhole. The left column is the observed intensities. The middle column is the density plots of observed intensities, and the right column is local density plots.

near critical curve Zb_2^c is highly demagnified like the photon ring near critical curve b_1^c . Another new photon ring which is located at the inside of critical curve b_1^c has a considerable size but is smaller than the lensing ring which is located outside of critical curve b_1^c . However, in emission model I, the new second transfer function makes no contribution to the observed intensity, since this transfer function is out of the domain of the emission model. In emission model II, we have an additional sizable lensing band between critical curves Zb_2^c and b_1^c resulting from the new second transfer function in this emission model.

V. CONCLUSION

In this paper, we studied the trajectories of photons and their deflection angles in an asymmetric thin-shell wormhole connecting two distinct Schwarzschild spacetimes by the throat. Typically, we placed observers in the spacetime $\mathcal{M}_1$ of which the mass parameter is smaller than that of the other side. Our interest is mainly about the photon rings in the observers' sky; we focused on the case that the throat of the ATW is inside the photon sphere of $\mathcal{M}_1$. After giving the formulas of deflection angles in each side, we constructed new orbit numbers counting the total deflection angles and showed the completed trajectories of photons which go through the throat twice; that is, ingoing photons of $\mathcal{M}_1$ pass through the throat and turn back after they reach the turning points in spacetime $\mathcal{M}_2$, and then they go through the throat again. Then, considering optically and geometrically thin accretion disks around the wormhole, we gave the transfer functions and obtained the observed

intensity and density plot in the sky of observers based on some emission models.

From our calculations, we found that the ATW and corresponding Schwarzschild spacetimes differ markedly in the second and third transfer functions. In particular, the second transfer function is no longer a monotonic function; a new segment appears before the monotone increasing part which also exists in corresponding Schwarzschild black hole spacetimes. As for the third transfer function, more structures were found in the ATW. In addition to a new monotone increasing part, the usual one splits into two branches. The underlying reason for these new characteristics is that photons with certain impact parameters could turn back after they go through the throat, while these photons in corresponding Schwarzschild black hole spacetimes would fall into the event horizon and never come back, since the event horizon is a one-way membrane.

As a result, two additional photon rings are found for ATW spacetime, one of which is highly demagnified near Zb_2^c (the critical curve in opposite spacetime viewed by the observers' side) like the usual photon ring near b_1^c (the critical curve in the observers' spacetime) and the other one located inside of critical curve b_1^c is much brighter and has a considerable size, even though it is smaller than the lensing ring which is located outside of critical curve b_1^c . Besides, we also found an additional lensing band when the emission profile overlaps the domain of the new second transfer function. These additional photon rings or lensing rings (or bands) should be exclusive structures for UCOS because of their reflectivity.¹ In this sense, though our

¹In fact, the relevant study of the gravastar as another UCO example has been implemented in Ref. [42].

analysis is based on a rather ideal “toy model,” the results provide a potentially, qualitatively observational signature to distinguish UCOs from black holes. In order to compare with the realistic observation, it is important to consider the case that the observers’ line of view is not perpendicular to the plane of the emission disk, especially when the spacetime owns spin [43]. In such a situation, we must study the null geodesic in general form. Another interesting aspect is to study the observational appearance when there is also an emission disk in the other side of the

throat with respect to the observer. We leave these for future investigations.

ACKNOWLEDGMENTS

J. P. is supported by the China Scholarship Council. M. G. is supported by China Postdoctoral Science Foundation Grants No. 2019M660278 and No. 2020T130020. X.-H. F. is supported by National Natural Science Foundation of China Grants No. 11905157 and No. 11935009.

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