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From Multipliers to the Distribution of Income: Connecting Leontief and Sraffa

Albert E. Steenge

1 INTRODUCTION

Wassily Leontief and Piero Sraffa belong to the great innovators of economic thought in the twentieth century. Both have brought entirely new concepts to the field and were at the forefront of a new literature. While both these authors have much in common, there also are substantial differences, sometimes puzzling ones.

In this contribution we shall focus on the role of Leontief and Sraffa in laying the foundations of what is generally referred to as multi-sectoral or input-output (I-O) analysis. This is a field of economic science in which production is described in terms of interconnected sectors that produce a characteristic product thereby employing fixed coefficients production functions. The multi-sector models presented by Leontief and Sraffa have both their similarities and their differences. For example, in Leontief's analysis the famous multipliers play a central role.

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Multipliers are special constructs that enable us to calculate the total cost in terms of sacrificed resources in generating the net product of a region or country. Incomes in this analysis are distributed proportionally to the supplied inputs, with only limited attention being given to alternative income distribution mechanisms. On the other hand, Sraffa spends a major part of his analysis on the distribution of income between wages and profits including the corresponding prices. An example of an entirely new concept introduced by him is the Standard commodity, a commodity basket of specific proportions that is completely lacking in Leontief's models.

In this article we will also try to obtain a clearer picture of the similarities as well as the differences between Leontief and Sraffa. In particular, we will try to understand the contributions of both authors in terms of the analytical structure they have adopted, where we shall confine ourselves to a selection from their work. In the case of Leontief, this concerns his familiar open static model, probably the best known of his many models and sub-models. For Sraffa, our selection deals with the models proposed in Part I of his *Production of Commodities by Means of Commodities*.

We aim to show how the work of both authors can be understood by looking for concepts that they both have in common. More specifically, our goal is to see if a direct path can be found between the Leontief and Sraffa type of approaches at income distribution issues. If we would succeed, the possible gains would be quite clear: Leontief offers a wide range of models and access to a wealth of empirical data and experience, while Sraffa offers in many areas a wider range of insights, particularly a deeper insight into specific distribution mechanisms. We start from Leontief's early model and explore an extension based on a regularity in workers' consumer behaviour, expressed by means of so-called extended input coefficients. Regarding the Leontief based approach we conclude with a dual system of a price and quantity equation (Eqs. 12.17 and 12.18' below) where especially the price equation has a remarkable resemblance to Sraffa's price equation. We conclude with some numerical exercises illustrating the role of the numeraire—including the Standard commodity—in both the Sraffian and the Leontief approach.

2 LEONTIEF'S FORMULAE

In this section we shall briefly discuss Leontief's static open model with a single primary vector, homogeneous labour.¹ This model is probably the best known of all of Leontief's models. The structure is familiar: There are n industries or sectors, each sector producing a single, characteristic product. Input proportions for each sector are fixed, which means that there is no possibility of input substitution following a shift in prices or technology change. All workers are identical irrespective of the sector they are working in or how that sector is performing. The goal is to determine the resources (commodities and labour) to be sacrificed for producing a certain net output plus the corresponding prices.

The model consists of two parts, a real or 'physical' part explaining the production of commodities and a 'dual' part explaining prices. For the real model we have

$$\mathbf{x} = \mathbf{Ax} + \mathbf{f}, \quad (12.1)$$

where $\mathbf{A}$ is the $(n \times n)$ nonnegative and indecomposable matrix of direct input coefficients with dominant eigenvalue less than 1, $\mathbf{f}$ the final demand vector, also known as the final output or net output vector, and $\mathbf{x}$ the gross output vector.² $\mathbf{f}$ is assumed to be exogenously determined with positive demand for all goods, while $\mathbf{x}$ is endogenous. Solving for $\mathbf{x}$ gives

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f}, \quad (12.2)$$

where matrix $(\mathbf{I} - \mathbf{A})^{-1}$ is the so-called multiplier matrix, also known as the Leontief inverse. We see that the columns of this matrix stand for the amounts of each of the n goods required to produce one unit of final output of the good in question. For example, the first column of the inverse gives the quantities of each good required to produce a net output of one unit of good 1. We also have $\mathbf{x} > \mathbf{f}$, where we see the multipliers at work.³

¹ In this context the term 'primary' means that the production of this factor is not explained by the model.

² Matrices and vectors are in bold type, scalars in standard. Vectors are column vectors, row vectors are indicated by an apostrophe.

³ See also the next section on additional properties of the system.

Labour requirements are given by

$$L = \mathbf{l}'\mathbf{x}, \quad (12.3)$$

where L stands for total employment and $\mathbf{l}'$ for the (row) vector of direct labour input coefficients, that is the quantity of labour required per unit of output of each sector.⁴ Together matrix $\mathbf{A}$ and vector $\mathbf{l}'$ define the technology of the system, both constant by assumption during the period under consideration. Because labour is the only primary factor, it receives the entire income or 'surplus', that is the commodity basket (or bundle) $\mathbf{f}$. Occasionally we shall refer to this vector as the real wage or the real wage basket; income per head thus equals $\mathbf{f}/L$.⁵

Corresponding equilibrium prices are given by

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + w\mathbf{l}', \quad (12.4)$$

where $\mathbf{p}'$ is the vector of commodity prices and w the exogenously given wage rate in, say, Euro or \$ per unit of labour. From Eq. (12.4) we have

$$\mathbf{p}' = w\mathbf{l}'(\mathbf{I} - \mathbf{A})^{-1}, \quad (12.5)$$

where we meet the multiplier matrix again. The amounts of labour 'embodied' in each produced good are obtained by pre-multiplying the Leontief inverse by the row vector $\mathbf{l}'$. Equation (12.5) thus informs us that relative prices are given by these amounts of embodied labour while absolute prices are obtained by attributing a particular value to w . We directly have

$$\mathbf{p}'\mathbf{f} = w\mathbf{l}'\mathbf{x} = wL \quad (12.6)$$

That is, the consumption bundle $\mathbf{f}$ can be bought in its entirety out of the wage received by the work force, L . There are no profits; any investments are accounted for as part of final demand. For further details, see Miller and Blair (2009).

⁴We assume that each industry employs labour, thus $\mathbf{l}' > 0$.

⁵Each element of the vector $\mathbf{f}/L$ is equal to the corresponding element of the vector $\mathbf{f}$ divided by L .

3 ANOTHER VIEW OF LEONTIEF'S OPEN STATIC MODEL

Let us stay for a moment with the above model. Firstly, we observe that we have a fully functioning model. The model is complete in the sense that row sums equal corresponding column sums, prices and quantities are well-defined and the entire system is 'in equilibrium'. At the same time, the model is very limited in scope. The focus is on the multiplier matrix and its role in determining gross output and commodity prices. Clearly, many questions cannot be fruitfully addressed by employing the above framework, such as questions concerning the connection between prices and the distribution of income.

Over the years, many extensions of the basic model have been presented.⁶ However, several fields or sub-fields have been neglected in this respect. An example of such a field is constituted by income distribution issues. Since this is a very important field, we will take another look at these issues in this chapter. In fact, we shall try to find out if there may be an extension of the basic model that will help us understand the differences between Leontief and Sraffa in terms of the fundamentals of multi-sector analysis. More concretely, our goal will be to see if the I-O model of Sect. 2 can be extended in such a way as to enable us to address specific distribution issues. As we shall see further below, this is indeed possible; there is a way to extend the model such that distribution issues can be addressed. The procedure involves a certain rearranging of the basic formulas and a close look at a familiar assumption.

The assumption we are referring to here is the so-called homogeneity assumption: the assumption that workers are identical and receive the same wage irrespective of the sector in which they are employed. In this chapter we will interpret this in the sense that workers also have identical tastes. That is, they consume the same commodity bundle, to be called the real wage or real wage basket. As we shall see immediately below, this interpretation of the term 'identical' allows us to work with what may be viewed as 'extended' input coefficients.⁷ In fact, this type of coefficients allows the introduction of a distribution mechanism that embodies elements of both the Sraffa and Leontief tradition and can as such be seen as a bridge between both traditions.

⁶ See Kurz and Salvadori (1995) or Miller and Blair (2009) for comprehensive presentations of historical background and current research lines.

⁷ There are a few examples of work in which these commodity bundles are used. Although used in a different context, Quadrio Curzio (1967, pp. 43–44) provides an example. Seton (1977), in his analysis of Marxian price systems, provides another.

3.1 Extended Input Coefficients

Let us see how this works out. Section 2 answered the question: If $\mathbf{f}$ must be produced, how much gross output is required and what does that mean in terms of overall employment? However, we can look at the equations of Sect. 2 in a quite different way. To see this, we start by taking a second look at Eq. (12.6). We directly have

$$w = (\mathbf{p}'\mathbf{f})/L = \mathbf{p}'(\mathbf{f}/L), \quad (12.7)$$

which is the value of the net income per worker.⁸ Returning to Eq. (12.4) and substituting our new expression for the wage, we obtain successively

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + [w]\mathbf{l}' \quad (12.8)$$

$$= \mathbf{p}'\mathbf{A} + [\mathbf{p}'(\mathbf{f}/L)]\mathbf{l}' \quad (12.8')$$

$$= \mathbf{p}'[\mathbf{A} + (\mathbf{f}/L)\mathbf{l}'] \quad (12.8'')$$

We see here (following Eq. 12.4) that the price of the i -th good consists of two parts. That is, firstly a part consisting of the intermediate inputs used up in its production, represented by the i -th column of matrix $\mathbf{A}$. The second part, following Eq. (12.8), consists of the wage part of the i -th good, here represented by the term $w\mathbf{l}'_i$, where $\mathbf{l}'_i$ stands for the i -th element of the labour input coefficients vector $\mathbf{l}'$.

However, we can say more, as indicated by Eqs. (12.8') and (12.8''). Since workers do not save or borrow, the wage per worker must be equal to the value of the goods *bought and consumed* by that worker. So, the unit price of good i must also cover the value of this basket of consumption goods which (again per unit) is equal to $[\mathbf{p}'(\mathbf{f}/L)]\mathbf{l}'_i$. In physical terms, this means that the price of good i must consist of the price of *two* commodity baskets, i.e. one consisting of commodities as given by the i -th column of matrix $\mathbf{A}$ and one given by the consumption basket $(\mathbf{f}/L)\mathbf{l}'_i$. Equation (12.8') thus also informs us immediately how these two types of inputs can be represented in terms of input coefficients.

A similar interpretation is valid for the real output model of Eq. (12.1). We have

⁸ Note: if $\mathbf{p}'(\mathbf{f}/L) = 1$ and $L = 1$, w stands for the value of the economy's net income.

$$\mathbf{x} = \mathbf{A}\mathbf{x} + (\mathbf{f}/L)L \quad (12.9)$$

$$= \mathbf{A}\mathbf{x} + (\mathbf{f}/L)\mathbf{I}'\mathbf{x} \quad (12.9')$$

$$= [\mathbf{A} + (\mathbf{f}/L)\mathbf{I}']\mathbf{x} \quad (12.9'')$$

We see that matrix $\mathbf{A} + (\mathbf{f}/L)\mathbf{I}'$ can be interpreted as the coefficients matrix of an economic system where commodities are produced (only) by commodities: we see very clearly that the wage in each sector has been replaced by the vector of consumption goods bought by the workers. Since the system is in equilibrium, these entities (i.e. the wage and the consumption expenditure) as said, are equal in value. Because the elements of the i -th column of matrix $(\mathbf{f}/L)\mathbf{I}'$ (as given by Eqs. 12.8" and 12.9'') stand for a special type of input coefficients, we shall designate the elements of matrix $\mathbf{A} + (\mathbf{f}/L)\mathbf{I}'$ as 'extended input coefficients'. Matrix $\mathbf{A} + (\mathbf{f}/L)\mathbf{I}'$ then can be called an 'extended input coefficients matrix', representing in an alternative way the *technology* of the entire system. We should note here that Leontief's fixed coefficients assumption applies also here as long as workers do not diversify their consumption. (If we wish to do so, we can fix the value of the vectors $\mathbf{f}$ or $\mathbf{f}/L$ as we like. This can be 1, but also any value that fits the equations in question.)

The above equations lead to a most interesting interpretation. Looking at Eq. (12.8''), we see that the price vector $\mathbf{p}'$ as introduced in Sect. 2 can be interpreted in an entirely different way, namely as the *left-hand eigenvector* corresponding to the dominant or Perron-Frobenius eigenvalue of the matrix in brackets. Analogously, the gross output vector can be interpreted as the *right-hand eigenvector* of the dominant eigenvalue of the same matrix. Thus, Eqs. (12.8'') and (12.9'') provide an alternative way to view Leontief's open static model, the model we started with in Sect. 2.

3.2 *Properties of the Extended Input Coefficients Matrix*

A substantial part of the I-O literature consists of finding and interpreting properties of matrix $\mathbf{A}$ that guarantee that the multiplier matrix is positive (because otherwise problems of interpretation will arise and $\mathbf{x} > \mathbf{f}$ cannot be guaranteed). We already mentioned that matrix $\mathbf{A}$ has a

dominant eigenvalue less than 1.⁹ However, the extension presented in Eqs. (12.8'') and (12.9'') points to further properties of the matrices involved here.

To simplify notation, let us write

$$\mathbf{M} \equiv \mathbf{A} + (\mathbf{f}/L)\mathbf{I}' \quad (12.10)$$

We observe that this matrix $\mathbf{M}$ has a number of interesting properties. For example, from Eqs. (12.8'') or (12.9'') we have that it has a dominant eigenvalue λ equal to 1. Under the conditions of Sect. 2, we also have that each element of $\mathbf{M}$ is positive, that is $\mathbf{M} > 0$, for short. This means that the eigenvectors corresponding to λ also are positive. So, $\mathbf{p}' > 0$ and $\mathbf{x} > 0$.¹⁰

A question we should ask here is: Why should we be interested in the above extension in the first place? One answer is that the new formulations of Sect. 3 are important because they point to additional relations between the variables concerned. In fact, the 'extended input coefficients' allow us to study entirely different relations between the core variables $\mathbf{x}$, $\mathbf{p}'$ and L . Rewriting Eq. (12.9'') we have

$$\left(\mathbf{I} - [\mathbf{A} + (\mathbf{f}/L)\mathbf{I}']\right)\mathbf{x} = \mathbf{0} \quad (12.11)$$

From this we have the important result that the determinant of the matrix on the left-hand side of Eq. (12.11) must be equal to zero, that is

$$\left|\mathbf{I} - [\mathbf{A} + (\mathbf{f}/L)\mathbf{I}']\right| = 0 \quad (12.12)$$

For us, Eq. (12.12) is particularly important because, given $\mathbf{f}$, it can be interpreted as a single equation in one variable, L . And, once we have obtained L , we can find $\mathbf{x}$ (as the right-hand eigenvector of the matrix in question) and standardize it using $\mathbf{I}'\mathbf{x} = L$.

⁹These properties are basically self-evident. Indecomposability e.g. prevents that there are sub-systems capable of (re)production without inputs, directly or indirectly, from sectors not belonging to that sub-system. The full rank property means that there are no industries using the same technology. A dominant eigenvalue smaller than 1 guarantees that a surplus is possible. For further details, see e.g. Takayama (1974) or Seneta (1981).

¹⁰See also footnote 4. With $\mathbf{f} > 0$ and $\mathbf{I}' > 0$ also matrix $(\mathbf{f}/L)\mathbf{I}' > 0$ and, consequently, matrix $\mathbf{M}$.

There is a second reason why the above extension is important. We have seen that 'equilibrium' in the sense of the first paragraph of Sect. 3 can be modelled employing an input coefficients matrix (i.e. matrix $\mathbf{M}$) that has a dominant eigenvalue equal to 1. However, matrix $\mathbf{M}$ consists of two parts. Suppose now that for some reason certain elements of matrix $\mathbf{A}$ *increase* in size. Adopting the symbol $\mathbf{A}$ for this matrix, that would mean that, based on the continuity rules of standard matrix theory, matrix $\mathbf{A} + (\mathbf{f}/L)\mathbf{I}$ would have a dominant eigenvalue larger than 1 and equilibrium would be destroyed. However, equilibrium might be restored—in the sense of the relevant matrix $\mathbf{M}$ having a dominant eigenvalue equal to 1—if simultaneously some elements of matrix $\mathbf{f}/L\mathbf{I}$ would *decrease*. That is, if final demand for certain goods would decrease. Below, in Sect. 4, we shall further explore this connection.

4 THE DISTRIBUTION OF GROSS OUTPUT

In this section we continue with Eqs. (12.8") and (12.9"). Suppose indeed that at the beginning of a new period, due to some unexpected event, more intermediate inputs (per unit of output) are required than before. Say, the new intermediate input requirements are given by a new matrix $\alpha\mathbf{A}$ with $\alpha > 1$.¹¹ With output $\mathbf{x}$ unchanged and for all intermediate inputs taken together, we then need the quantity $\alpha\mathbf{A}\mathbf{x}$ to be available. However, the right-hand side of Eq. (12.9") (the 'destinations') is now larger than the left-hand side ('production'). So the economy's equilibrium is disturbed.

To see what may happen we will definitely need more context. There are various possibilities; so let us select one, and suppose that workers plan to buy consumption goods in the same quantities as before. The standard model of Sect. 2 informs us that this would imply that both $\mathbf{x}$ and L would increase. However, what if circumstances do not allow that? Suppose that because of a shortage of labour, employment must remain the same. Because more intermediate inputs than before are required and only the same amount of primary inputs (i.e. labour) is available, the only way to proceed is to accept that workers' consumption, in one form or another, must *decrease*.

How might that happen? To proceed, we need a further assumption regarding the consumption behaviour of the workers. If we assume that

¹¹ That is, each element of $\mathbf{A}$ is multiplied by α in the new configuration.

workers consume goods in the *same proportions* as before but in lower quantities, each worker can buy, in the new circumstances, a real wage basket equal to $\beta(\mathbf{f}/L)$, where $0 \leq \beta < 1$ is a new parameter that captures the decrease in real income.

In terms of the ‘extended’ model of the previous section, this would mean that it changes correspondingly to

$$\mathbf{x} = [\alpha \mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}']\mathbf{x}, \quad (12.13)$$

where β now has to be determined such that the matrix on the right-hand side of Eq. (12.13) has dominant eigenvalue equal to 1 (which means that it models a circular flow). Writing

$$\mathbf{M}(\alpha, \beta) \equiv \alpha \mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}', \quad (12.14)$$

the next step is to determine the pairs of α and β that satisfy Eq. (12.14) and then to select the value of β that corresponds to the exogenously given value of α . Following this, gross output $\mathbf{x}$ can be calculated as the right-hand eigenvector of $\mathbf{M}(\alpha, \beta)$ which gives the distribution of output after standardization via $\mathbf{I}'\mathbf{x}$.

Prices follow the same pattern, and will change according to

$$\mathbf{p}' = \mathbf{p}'[\alpha \mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}'], \quad (12.15)$$

$\mathbf{p}'$ here is the left-hand eigenvector corresponding to the dominant eigenvalue. This also means that the system determinant changes to

$$|\mathbf{I} - [\alpha \mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}']| = 0, \quad (12.16)$$

which is an equation in two variables, α and β . A question to be asked is if there is a combination of economically interpretable values of α and β that satisfy Eq. (12.16). In fact, this is guaranteed by the model of Sect. 2. The basic question is about the behaviour of the multiplier matrix $(\mathbf{I} - \alpha \mathbf{A})^{-1}$. If α is such that the Perron-Frobenius eigenvalue of matrix $\alpha \mathbf{A}$ is smaller than 1, there generally will be no problem.¹²

So, summarizing, on the basis of the structure proposed above, we can straightforwardly extend the basic model of Sect. 2 to investigate the

¹²For further background, see Steenge and Serrano (2012), or Steenge (2015).

consequences of different distributions of gross output. The extension was based on the assumption that the consumption per head vector $\mathbf{f}/L$ stays the same and that the decrease in workers' consumption manifests itself in a lower value for β . A vice versa trajectory would apply for $\alpha < 1$. In the next section we shall focus on the distribution of *net output*.¹³

5 THE DISTRIBUTION OF NET OUTPUT

5.1 *Distributing Net Output in an Extended Leontief World*

The above model does not yet give us a clear income distribution picture. However, in line with Sects. 3 and 4, a way forward can be formulated. To this end, we go back to the assumption of homogeneous labour and the assumption that workers have identical preferences.

If we put $\alpha = 1 + r$, we have the beginning of a model with a rate of profit. However, as in the model of the previous section, one more step is required to interpret the model in terms of an income distribution model. That is, we need to determine if workers' consumption patterns will change if their wage increases or decreases. In order to provide a sufficiently comprehensive context, we again need to adopt one or more policy rules. If we assume that consumption proportions *and* employment remain unchanged, we effectively fix the vector $\mathbf{f}/L$ as exogenously given. Following the procedure of Sect. 4, that leaves the parameter β to reflect changes in workers' consumption, and we can interpret shifts in the income distribution as shifts in r and β . With $\beta \in [0, 1]$ we impose that the value of β can fluctuate from 0 to 1. If $\beta = 0$, we have a situation of capital obtaining all profit. If, contrarily, $\beta = 1$, profits are zero.

Substituting $1 + r$ for α we have accordingly, for the real side

$$\mathbf{x} = \left[(1+r)\mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}' \right] \mathbf{x}, \quad (12.17)$$

where the term $r\mathbf{A}\mathbf{x}$ stands for the profit part and $\beta(\mathbf{f}/L)\mathbf{I}'\mathbf{x}$ for the wage part.

We observe that each combination of r and β that satisfies Eq. (12.17), in general, generates a new right-hand side matrix, and, thus, a new gross

¹³ However, we should point out again that the choices we have made in this section only are one example of possible extensions. Future work may be called for to explore further possibilities.

output vector. This vector subsequently can be standardized via the assumption of unchanged total employment.

So, we model our distribution of income as a simultaneous shift in r and β where the dominant eigenvalue of the matrix on the right-hand side of Eq. (12.17) should remain 1, the value we started with. This means, again *ceteris paribus*, that if r increases, β must decrease, and vice versa.

For corresponding prices we obtain

$$\mathbf{p}' = (1+r)\mathbf{p}'\mathbf{A} + \beta[\mathbf{p}'(\mathbf{f}/L)]\mathbf{I}' \quad (12.18)$$

$$= \mathbf{p}'[(1+r)\mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}'] \quad (12.18')$$

With

$$\mathbf{M}(r, \beta) \equiv (1+r)\mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}', \quad (12.19)$$

we have straightforwardly

$$|\mathbf{I} - \mathbf{M}(r, \beta)| = |\mathbf{I} - [(1+r)\mathbf{A} + \beta(\mathbf{f}/L)\mathbf{I}']| = 0, \quad (12.20)$$

which is a single equation in two unknowns, r and β , and can directly be solved for r and β . Next, given these values, we substitute them in (12.19), and calculate the left-hand Perron-Frobenius eigenvector. This vector gives the relative prices which subsequently can be standardized, see also the next sub-section. Thus, the model is quite clear. We first have to calculate the income distribution determining parameters r and β . After that, the corresponding prices can be determined from the left-hand Perron-Frobenius eigenvector of matrix $\mathbf{M}(r, \beta)$.

5.2 *The Rate of Profits and the Wage Rate in the Sraffian Price Model*

In this section we shall juxtapose parts of the Leontief and the Sraffa tradition. We thereby shall focus on the price equation because the Sraffian real model is very much like Leontief's (chs. 1 and 2 of *Production of Commodities by Means of Commodities*).

So far we have followed the Leontief extension of Sects. 3 and 4. In that model we assumed that workers will have the same preferences over the entire range of β , the relevant parameter for workers. A lower wage

(accompanied by a higher rate of profit) means a lower β and thus less consumption, though proportions within the basket of consumed goods remain the same.

Let us now consider the Sraffian price equation (Sraffa, 1960, ch. 5), Kurz and Salvadori (1995, ch. 4) or Abraham-Frois and Berrebi (1997, ch. 2). We have

$$\mathbf{p}' = (1+r)\mathbf{p}'\mathbf{A} + w\mathbf{l}' \quad (12.21)$$

Matrix $\mathbf{A}$ and row vector $\mathbf{l}'$ have the same interpretation as above, i.e. as input coefficients matrix and vector of direct labour input coefficients, respectively.¹⁴ $\mathbf{p}'$ is the price vector, with r the rate of profits and w the wage rate in terms of the national income, determined exogenously. Solving for $\mathbf{p}'$ we find

$$\mathbf{p}' = w\mathbf{l}'[\mathbf{I} - (1+r)\mathbf{A}]^{-1} \quad (12.22)$$

So, if r changes, $\mathbf{p}'$ will change accordingly. However, there is no relation between a change in r and the wage rate w of the type discussed in Sect. 4 and sub-section 5.1. In Eq. (12.21), the scalar w just establishes absolute prices, relative prices being given by the row vector

$$\mathbf{l}^*(r) = \mathbf{l}'[\mathbf{I} - (1+r)\mathbf{A}]^{-1}. \quad (12.22')$$

Now let us return for a moment to the Leontief-based price Eq. (12.18'). If we put

$$\underline{w} = \beta[\mathbf{p}'(\mathbf{f}/L)], \quad (12.23)$$

with β the workers' share of the national income and substitute in Eq. (12.18') we obtain

$$\mathbf{p}' = (1+r)\mathbf{p}'\mathbf{A} + \underline{w}\mathbf{l}', \quad (12.24)$$

¹⁴ Matrix $\mathbf{A}$ in this model is assumed to have the same properties as matrix $\mathbf{A}$ in the Leontief model of Sect. 2.

which is a price equation that is structured around the inverse relation between r and β . As we observe, written in this way the Leontief-based Eq. (12.18'), combined with the wage variable w as in Eq. (12.23), is remarkably similar to the Sraffian price Eq. (12.21). Also, w can be interpreted as a flexible numeraire which can be given any acceptable value. Future work here will have to decide if additional points of similarity can be found, which also concerns the role of standardizations such as income or employment being fixed at unity.

5.3 The Standard Commodity

We have seen above that the relation between r and β can be found by solving Eq. (12.21) for these two variables. In general, this function will be non-linear. However, looking at Eqs. (12.17) and (12.19) we see that if f has the proportions of the Perron-Frobenius eigenvector of matrix A , the relation between r and β is a linear one. In fact, we have, with R the maximum rate of profit and using $\lambda = 1/(1 + R)$, after some manipulation,

$$\beta = 1 - r/R \quad (12.25)$$

This is the case of the well-known Standard system and Standard commodity. So, we meet here, not un-expected, the famous linear relation between rate of profits and the wage rate, here β .¹⁵ In this case, r and β can be found *without* having first to calculate the functional relation as given by Eq. (12.21).

6 NUMERICAL ILLUSTRATION

6.1 The Non-linear Case

We shall illustrate the above-proposed procedure by means of a numerical example with a two-industries case and one primary factor, homogeneous labour, as interpreted before. The data are artificial, but satisfy the standard requirements of Sect. 2.

¹⁵For a derivation of this relation in the Sraffian system, see e.g. Pasinetti (1977, Sect. 12.2). For a recent discussion of Standard commodity and Standard system, see Sinha (2016, esp. ch. 7).

Let us say that the technology is given by the following input coefficients matrix¹⁶

$$\mathbf{A} = \begin{bmatrix} 0.25 & 0.40 \\ 0.54 & 0.42 \end{bmatrix}$$

and vector of direct labour input coefficients

$$\mathbf{l}' = [0.70 \quad 3.20]$$

Suppose further that final demand $\mathbf{f}$ is

$$\mathbf{f} = \begin{bmatrix} 1600 \\ 800 \end{bmatrix}$$

Direct calculation gives¹⁷

$$\mathbf{x} = \begin{bmatrix} 5698.630 \\ 6684.932 \end{bmatrix}$$

and

$$L = 25,380.822$$

Furthermore, for the dominant eigenvalue we have

$$\lambda = 0.807,$$

so,

$$R = (1 - \lambda) / \lambda = 0.238.$$

¹⁶Symbols have the same interpretation as before.

¹⁷We observe that vectors $\mathbf{f}$ and $\mathbf{x}$ are not proportional to each other, so there is no role for the Standard commodity, see the next section.

Calculation gives

$$\varphi(r, \beta) = 0.2190 - 0.8920r - 0.2190\beta - 0.1110r^2 - 0.0740r\beta = 0.$$

Graphically, we have Figs. 12.1 and 12.2. Figure 12.1 gives the relation between r and β for the area relevant for our study. First of all, we observe the *non-linearity* of the curves. We also observe that $\beta = 1$ if $r = 0$. Furthermore, when $\beta = 0$, we find, as expected, $R = 0.238$. The right-hand figure below gives the wider picture of the curves where only a small part is relevant for us.

It may be useful to look at some particular numerical values. Selecting $r = 0$, we have $\beta = 1$. After having obtained matrix $\mathbf{M}(r, \beta)$ we obtain the relative commodity prices from the left-hand Perron-Frobenius eigenvector as $p_2/p_1 = 1.256$. For $r = 0.1$, we find $\beta = 0.568$. Substituting these values in $\mathbf{M}(r, \beta)$ and calculating its left-hand Perron-Frobenius eigenvector we find for the relative prices $p_2/p_1 = 1.154$. For $r = 0.2$, we have $\beta = 0.155$ and $p_2/p_1 = 1.064$. Absolute prices can be obtained by standardizing at any appropriate value.

It is straightforward to obtain the surplus and the part each party receives. For example, for $r = 0.2$ we have for the profits part

$$\alpha \mathbf{A} \mathbf{x} = 0.2 \mathbf{A} \mathbf{x} = \begin{bmatrix} 798.215 \\ 1117.665 \end{bmatrix},$$

Fig. 12.1 Relation between r and β ; non-linear case. Left-hand: relevant area; right-hand: the wider picture.

and for the wage part,

$$\beta(\mathbf{f}/L)(\mathbf{l}'\mathbf{x}) = 0.155(\mathbf{f}/L)(\mathbf{l}'\mathbf{x}) = \begin{bmatrix} 247.456 \\ 123.728 \end{bmatrix},$$

adding up to

$$\begin{bmatrix} 1045.671 \\ 1241.393 \end{bmatrix}.$$

6.2 *The Standard System*

In this sub-section we take a look at the Standard system, where we keep employment L the same, that is 25,380.822. We first calculate the Perron-Frobenius eigenvector of matrix $\mathbf{A}$. If the final demand vector has the same proportions as this eigenvector, also the gross output vector has. With L as above, and the symbol $\mathbf{f}^*$ standing for the Standard commodity, calculation gives

$$\mathbf{f}^* = \begin{bmatrix} 947.0732 \\ 1319.9050 \end{bmatrix}$$

Direct calculation (via Eq. [12.1], substituting $\mathbf{f}^*$ for $\mathbf{f}$) gives

$$\mathbf{x}^* = \begin{bmatrix} 4919.0161 \\ 6855.4720 \end{bmatrix}$$

We now are looking for r and β such that

$$|\mathbf{I} - \mathbf{M}(r, \beta)| = |\mathbf{I} - [(1+r)\mathbf{A} + \beta(\mathbf{f}^*/L)\mathbf{l}']| = 0$$

Direct calculation gives

$$\varphi(r, \beta) = 0.2190 - 0.8920r - 0.2190\beta - 0.1110r^2 - 0.0265r\beta = 0$$

Fig. 12.2 Relation between r and β ; linear case. Left-hand: relevant area; right-hand: the wider picture.

We observe that the curve is *linear*, also in the area $0 \leq r \leq R$, see Fig. 12.2. (Minor differences are due to rounding.)

Also here, once combinations of r and β have been selected (from Eq. 12.24), commodity prices are obtained straightforwardly. For example, for $r = 0.1$, we have $\beta = 0.581$. Substitution in Eq. (12.19) gives $p_2/p_1 = 1.154$. For $r = 0.2$ we find $\beta = 0.161$ and $p_2/p_1 = 1.0641$. We note that prices are the same for the r values as in the non-linear case. This is because relative prices obey Eqs. (12.21) and (12.18') (in an appropriate standardization).

7 FINAL REMARKS AND CONCLUSION

In this chapter we have explored a new approach to understanding the contributions made by Leontief and Sraffa in the field of multi-sector analysis. We focused on Leontief's well-known open static model—probably the best known of his models—and on the model proposed by Sraffa in Part I of *Production of Commodities by Means of Commodities*. We observed that both models have a number of properties in common, but also show significant differences. Our objective in this contribution was to see if both models could be viewed from an alternative perspective from which their similarities and differences could be understood, see Sect. 3.

We showed that such a perspective does in fact exist; in fact, several ones can be found. We decided to concentrate on a particular perspective which is based on Leontief's earlier work. In doing so we particularly focused on the so-called 'homogeneity property' of employed labour. If we interpret this property in terms of identical consumer preferences, we obtain, in a situation of a changing income distribution, one additional parameter, that is the real wage, or, equivalently, the real wage per capita. By including this new parameter in the basic system equations employing the so-called extended input coefficients, we are able to present the distribution mechanism in terms of two equations: a real output equation and a price equation which are dual to each other, i.e. Eqs. (12.17) and (12.18) in Sect. 5. Clearly, this chapter should be seen as a first step. As mentioned, several types of extensions are possible, and future work will be required to elaborate on these.

Concluding, we have presented here an extension of Leontief's early work meant to include basic elements of an income distribution mechanism. We have shown that this extension can be an interesting building block in finding a theoretical background that bridges a number of differences between a Leontief and a Sraffian approach in multi-sector analysis, and may help us to understand the similarities and differences between the two. The main characteristics of this approach can be summarized as follows:

1. Inputs are registered in a new way which leads to a definition of technology in terms of extended input coefficients, i.e. coefficients that include both technical and consumption based elements.
2. Simultaneously, a new coefficients matrix is created with dominant eigenvalue equal to 1 which signals a situation of equilibrium; the Perron-Frobenius eigenvectors of this matrix are the equilibrium price and quantity vectors.
3. A disturbance of this equilibrium is characterized by this dominant eigenvalue becoming greater than 1 or less than 1.
4. Equilibrium can be restored by introducing mechanisms that increase profits and reduce the wage, or vice versa.
5. In this extension, the rate of profits and the wage rate are determined independently of prices, although prices are dependent on them.

As we stated, future work will be needed to explore the further possibilities offered by this approach.

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Comment

A Comment on Steenge's 'From Multipliers to the Distribution of Income: Connecting Leontief and Sraffa'

Roberto Scazzieri

In his contribution to this volume (Steenge 2020) Professor Albert Steenge outlines an ambitious framework aimed at bridging the approaches to the economy as a circular flow due, respectively, to Wassily Leontief (1941, 1991 [1928]) and Piero Sraffa (1960). In so doing, Steenge provides the coordinates of a general scheme for the investigation of a circular economy, of which the contributions of Leontief and Sraffa appear to be special cases. Steenge's aim is 'to understand the contributions of both these authors in terms of the analytical structure they have adopted' by concentrating on Leontief's open static model and on the model that Sraffa outlines in Part One of *Production of Commodities by Means of Commodities*. At the root of Steenge's treatment is the consideration of the relationship $\mathbf{x} = \mathbf{Ax} + \mathbf{f}$, 'where $\mathbf{A}$ is the $(n \times n)$ matrix of direct [intermediate] input coefficients, $\mathbf{f}$ the final demand vector, also known as the final or net output vector, and $\mathbf{x}$ the gross output vector' (Steenge 2020, this volume). In this model, labour requirements per unit of output are given by vector $\mathbf{l}'$, which is 'the (row) vector of direct labour input coefficients' (Steenge 2020, this volume). An important consequence of Steenge's approach is that, since labour is the only primary (non-produced) factor of production, it 'receives the entire income or "surplus", that is the commodity bundle $\mathbf{f}$ ' (Steenge 2020, this volume). If we solve equation $\mathbf{x} = \mathbf{Ax} + \mathbf{f}$ for $\mathbf{x}$, we obtain $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$, in which matrix $(\mathbf{I} - \mathbf{A})^{-1}$ is the mathematical operator known as the 'Leontief inverse'. It is at this point that Steenge's argument introduces an important twist concerning the role of labour in a circular production economy. For Steenge highlights that, if we assume

that ‘workers are identical and receive the same wage irrespective of the sector in which they are employed’, then each unit of the labour input (say, each worker) may be assumed to consume ‘the same commodity bundle, to be called the real wage or real wage basket’ (Steenge 2020, this volume). This procedure entails introducing what Steenge calls ‘extended input coefficients’, that is, input coefficients including both the quantity of each intermediate input that is needed to deliver one unit of any given product as well as the quantity of that input entering the ‘real wage basket’ needed to deliver each unit of that product. This approach leads Steenge, with L total employment, to the following expression for the wage basket: $w = (\mathbf{p}'\mathbf{f}) / L = \mathbf{p}'(\mathbf{f} / L)$, and to the following expression for the price equation: $\mathbf{p}' = \mathbf{p}'\mathbf{A} + w\mathbf{l}' = \mathbf{p}'\mathbf{A} + \mathbf{p}'(\mathbf{f} / L)\mathbf{l}' = \mathbf{p}'[\mathbf{A} + (\mathbf{f} / L)\mathbf{l}']$. In short, ‘the price of the i -th good consists of two parts. That is, first, a part consisting of the intermediate inputs used up in its production, represented by the i -th column of matrix $\mathbf{A}$. The second part consists of the bundle of commodities consumed per unit of the labour input in each sector, here represented by the term $(\mathbf{f} / L)\mathbf{l}'_i$, where $\mathbf{l}'_i$ stands for the i -th element of the labour input coefficients vector $\mathbf{l}'$ ’ (Steenge 2020, this volume). This approach to labour inputs entails that ‘the unit price of good i must cover the value of this basket of consumption goods which (again per unit) is equal to $[\mathbf{p}'(\mathbf{f} / L)\mathbf{l}'_i]$ ’ (Steenge 2020, this volume). In turn, this means that the unit price of each produced commodity *must* cover not only the cost of intermediate inputs needed to deliver any unit of that commodity, but also the costs of what may in fact be considered as the ‘necessary consumption’ needed to produce each unit of that commodity via the unit labour input requirements for that commodity.

As Steenge points out, this treatment of labour inputs has already been followed by a few previous authors, such as Alberto Quadrio Curzio and Francis Seton. Quadrio Curzio introduced extended coefficients in a discussion of a 2-commodity Leontief closed system in which ‘consumption only takes place by way of the labour employed in production’ (Quadrio Curzio 1967, p. 43), so that the following system is obtained:

$$\begin{bmatrix} a_{11} + l_1 c_1 & a_{12} + l_2 c_1 \\ a_{21} + l_1 c_2 & a_{22} + l_2 c_2 \end{bmatrix}' \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

Quadrio Curzio argued that solving the above system would lead to determining a ‘subsistence wage’ $c_1 p_1 + c_2 p_2$, which would be the value of

a wage basket consisting of the necessary consumption of commodities 1 and 2 entering each unit of labour input (Quadrio Curzio 1967, p. 44). In a similar vein, Francis Seton called attention to the possibility of representing the production system by means of an 'augmented technology' including both intermediate input coefficients as well as the coefficients of necessary consumption (Seton 1977). As Steenge noted in a previous contribution, this approach 'opens up the possibility to formulate also this model [the Leontief open static model] in terms of 'commodities being produced by commodities', since 'labour inputs are still present [...] but expressed in terms of the remuneration labour receives in exchange for its productive inputs' (Steenge 2015, p. 113).

The consequences of switching from a 'mixed' representation of production technology separately considering intermediate inputs and labour inputs to an 'augmented' representation of technology embedding labour inputs (via real wage baskets) into the physical representation of commodity flows, are far reaching. First of all, Steenge's extended coefficients allow the introduction of an augmented technology matrix with the remarkable property that a value of 1 of its dominant eigenvalue expresses a condition of uninterrupted circular flow. This means that the viability (self-replicability) condition of the circular economy is now expressed by explicitly considering 'final consumption' (defined as the consumption of final goods entering the workers' real wage basket) as a constitutive component of the circular flow, and therefore as a condition for the economy to be in a self-replacing state (i.e. as a condition for the whole production process to be repeated from one period to another). In addition, by attributing 'weights' α and β to matrices $\mathbf{A}$ and $(\mathbf{f} / L)\mathbf{I}'$, under the condition that the dominant eigenvalue of matrix $\alpha\mathbf{A} + \beta(\mathbf{f} / L)\mathbf{I}'$ must be equal to 1, consequences of varying the distribution of income can be followed straightforwardly in a situation of uninterrupted circular flow.

Steenge's analysis suggests several remarks and several questions. The consideration of extended technical coefficients makes it possible to *generalize the point of view of the circular flow* beyond the domain of conventional intermediate goods (means of production needed for the formation of means of production, including themselves) and to include within the circular flow itself the final consumption goods needed for the economy to be in a self-replacing state. This is clearly a return to the classical concept of 'necessary consumption', which is not in the foreground when considering Leontief's open static input-output model or Sraffa's treatment of 'the whole of the

wage as a variable' (Sraffa 1960, p. 10).¹⁸ Steenge's emphasis on extended technical coefficients leads to a more comprehensive theory of the circular flow that might be of considerable heuristic value. For example, an augmented technology matrix with dominant eigenvalue greater or less than 1 would point to an 'interrupted' circular flow, which could in turn signal that some necessary product flows have been disregarded, or that the economy is following a technical change trajectory associated with bottlenecks and emergent scarcities. This point of view suggests the formulation of a 'flexible' circular model in which multiple different closures of the circular flow may be possible depending on different representations of the circular flow and on different policy questions to be addressed. For example, a stationary or semi-stationary economy may require a type of closure through necessary consumption ('necessary' final demand) that might not be required when considering a dynamic economy growing at a sustained rate triggered by intermediate flows internal to the production system.¹⁹ Or, the closure of a circular flow governed by internal demand may be different from the closure of a circular flow driven by external demand (export-led closure). Steenge's approach to system closure leads to a remarkable twist in the analysis of distribution between wages and profits, which Steenge investigates by focusing on the physical circular system. Here too the starting point of Steenge's contribution is the consideration of extended technical coefficients including workers' necessary consumption. This representation of technology highlights the (aggregate) physical commodity basket going to workers and the (aggregate) physical commodity basket entering the 'take out' (what is left after replacing the means of production and remunerating workers).²⁰ In this case, wages and the aggregate 'take out' (profits in Sraffa's terminology) are

¹⁸ Sraffa is of course aware of the possibility that wages may at least in part consist 'of the necessary subsistence of the workers' and would thus enter 'the system on the same footing as the fuel for the engines or the feed for the cattle' (Sraffa 1960, p. 9). However, he decides to 'follow the usual [post-classical] practice of treating the whole of the wage as variable' (Sraffa 1960, p. 10).

¹⁹ Different types of closure in the two cases of stationary (or semi-stationary) conditions and of sustained growth are discussed in Quadrio Curzio and Scazzieri (1986), who introduce a distinction between the representations of production technology corresponding to the two above states of the economy (*transformation apparatus* and *structural apparatus*, respectively).

²⁰ The concept of 'Take Out' has been introduced in John Hicks's *Capital and Time* as 'the difference between value of output and value of input' (Hicks 1973, p. 30). Here it is used as a general expression to denote what is left after subtracting from the gross product the physical quantities needed to cover the self-replacement of means of production and workers' consumption.

determined independently of prices, and an inverse relationship may be envisaged between the aggregate wage basket and the aggregate 'take out' (aggregate profits), expressed in terms of the physical commodity shares assigned to the workers' wage basket and to the 'take out' respectively. This result should be assessed against Sraffa's own formulation of the relation $r = R(1 - w)$ between wages (as a proportion of the Standard net product) and the rate of profits (Sraffa 1960; Sinha 2016). Steenge does not provide a detailed discussion of this issue but highlights that his analytical formulation leads to an inverse (and generally non-linear) relationship between the workers' share of the national income, here denoted by β , and the rate of profits r .²¹ He also argues that if the final output or net output vector $\mathbf{f}$ 'has the proportions of the Perron-Frobenius eigenvector of matrix $\mathbf{A}$, the relation between r and β is a linear one' (Steenge 2020, this volume). This latter result replicates, in a different setting, Sraffa's own formulation within the Standard system. Steenge's argument suggests a possible generalization, which however raises new questions concerning the interpretation of net output under an 'augmented' production technology (extended technical coefficients) and, correspondingly, the interpretation of the rate of profits under that technology.

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²¹ See Morishima (1966, p. 521) for an alternative proof of this proposition.

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Reply A Response to Scazzieri's Comment

Albert E. Steenge

I have very little to add to Roberto Scazzieri's remarks on my contribution to this volume. The chapter, as pointed out excellently by Scazzieri, tries to find a bridge between the works of the two great contributors to multi-sectoral analysis, Wassily Leontief and Piero Sraffa. Such efforts have been made before, naturally, but a gap has remained, in the sense that none of these efforts has been fully successful in providing an *overall* conceptual framework. The present contribution should be seen as belonging to that same category. However, the premise from which we start is less familiar, since we proceed by endowing the standard model of one of the two, Leontief, with a less well-known attribute, that is the concept of extended input-output (I-O) coefficients. We then proceed to explore the potential of this concept.

First of all, obviously these two directions or 'schools' (if you like) have quite a bit in common. Here we may think of the strict distinction between produced and non-produced goods, and the distinction between economies with a surplus and without a surplus. Nonetheless, the focus of both these schools of thought has been quite different. In Leontief-based research and applied work, the focus has largely been on policy and policy analysis based on multipliers. However, in terms of theoretical underpinnings, quite a few structural aspects still require further study; for instance in cases of multiple primary resources or multiple final destinations for the produced products.

In comparison, the Sraffa-based research direction has a different focus. In addition to structure, much attention is given to the distribution of income between the owners of capital and labour, and the corresponding price movements. Furthermore, Sraffian analysis includes a significant role for standards of value, a concept that is absent in the Leontief world.

A promising research direction here is offered, in my view, by the concept of *extended input coefficients*. Standard input coefficients stand for the amount of produced goods used as intermediate inputs in unit production of produced goods. To these intermediate inputs, extended input coefficients (partly or wholly) add the standardized elements of the consumption bundle that are consumed by the primary non-produced factor—labour—and paid for out of the wage (there are no savings). A shift in this consumption bundle, given some additional conditions, is then interpreted as a shift in income.

As we have shown, these extended input coefficients can straightforwardly be introduced in a Leontief framework. If there is equilibrium in the sense of each industry being in equilibrium, that is being in a state where it sells its goods for the same total amount as it paid for its inputs (including the labour input), all coefficients can be assembled into a coefficients matrix with a number of special properties. In our contribution, this matrix consists of two non-negative matrices to which ‘weights’ can be attributed. In terms of this structure, shifts in these weights can then be interpreted as shifts in the distribution of income, per industry.

Here we are building on the work of authors such as Quadrio Curzio (1967) or Seton (1977), who utilized the concept of the so-called extended I-O coefficients. How to proceed precisely in special cases is basically a matter of choice, and of selecting a required context. In fact, quite a few questions then can be formulated while still remaining in a Leontief-based methodology. The present contribution just provides the outlines of an early exploration in this area. Formal ‘bridges’ can be constructed along the way.

A number of challenges definitely remain. Roberto Scazzieri correctly suggests a closer look at the possible role of ‘flexible’ circular models, thereby partly returning to classical thought. In fact, as he points out, economic theory may suggest several ways of ‘closing’ the model, each of which is motivated by economic or political necessity. Proceeding along these lines may also generate intriguing further links between economic theory and linear algebra, in particular Perron-Frobenius theory, which are now found mainly in multiplier and growth theory.

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