University of Groningen

Modelling of Comptonisation in low-mass X-ray binaries in the presence of fast variability

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DOI:
[10.33612/diss.192067979](https://doi.org/10.33612/diss.192067979)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2021

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):
Karpouzas, K. (2021). *Modelling of Comptonisation in low-mass X-ray binaries in the presence of fast variability*. [Thesis fully internal (DIV), University of Groningen]. University of Groningen.
<https://doi.org/10.33612/diss.192067979>

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UNIVERSITY OF
Southampton

Modelling of Comptonisation in low-mass X-ray binaries in the presence of fast variability

PhD thesis

to obtain the joint PhD degree at the
University of Groningen and University of Southampton

on the authority of the
Rector Magnificus of the University of Groningen
Prof. C. Wijmenga,
President and Vice-Chancellor of the University of Southampton
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and in accordance with
the decision by the College of Deans of the University of Groningen.

This thesis will be defended in public on
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Thesis introduction

1.1 X-ray binaries

In astronomy, a binary star or binary system is defined as a system of two stars orbiting around a common barycenter, the position of which is defined by the mass ratio of the stellar components. It is believed that a significant fraction of all stars belong to binary systems, therefore their study provide an insight on stellar formation, structure and evolution. Based on the techniques used to detect them, binary stars can be classified in either visual, spectroscopic, eclipsing or non-eclipsing and astrometric. The X-ray binaries (XBs), that are of interest to us, are comprised of a donor star and an accretor, which is a compact object that can be either a neutron star or a black hole. If the mass of the donor star is higher than that of the compact object (the donor star is usually an O-B star with a mass higher than $10M_{\odot}$), then the XB is classified as a high-mass XB (HMXB). In HMXBs the accretion onto the compact object proceeds through stellar winds and the detection is easier since the donor stars emit mainly in the optical. On the other hand, if the donor is an evolved star and its mass is lower than that of the compact object (the stellar mass is less than $\sim 1M_{\odot}$), then the XB is classified as a low-mass XB (LMXB).

In LMXBs, which are the focus of this thesis, the accretion onto the compact object proceeds through Roche lobe overflow. The Roche lobes are imaginary lobes, characteristic for binary systems in circular or nearly circular orbits. The surface of the Roche lobes is defined as the equipotential surface in the non-inertial reference frame that is co-rotating with the binary system. Figure 1.1 shows a cross section of the equipotential surface in the reference frame that is co-rotating with two masses, that represent the compact object and donor star. The Roche lobes are the ones that engulf the two masses, and

intersect at the equilibrium Lagrangian point, L_1 .

Figure 1.1 – Roche lobe overflow and mass transfer onto the compact object through the L_1 Lagrangian point. The Roche lobes are shown as a blue solid line. Figure taken from George, Misra & Ambika (2019).

When the low-mass donor star is evolved enough to reach the L_1 point, matter can be pulled out of L_1 and gravitationally forced onto the compact object. The accreting matter is made mostly of highly ionised hydrogen. We will hereafter refer to the accreting matter as the accretion flow, thus avoiding any further discussion about its composition. During the accretion process, the flow builds up significant angular momentum that prevents it from falling directly onto the compact object. The picture today is that the accretion flow is able to surpass the angular momentum barrier, and move closer to the compact object with the help of magnetic fields. The existence of a magnetic field "binds" fluid elements together, as if they were objects connected via springs with some constant k . With no magnetic field present, a fluid element on a Keplerian orbit at a smaller

radius will have a higher angular velocity than a fluid element on a Keplerian orbit with slightly larger radius, and both fluid elements may exist on their own stable orbits with different angular momentum each. However, since the magnetic field binds those two fluid elements the inner, and thus faster in terms of angular velocity, element will experience the equivalent of a spring force that will delay its nearly-circular motion, thus removing angular momentum from its orbit, while the outer element will experience a force that will increase its angular momentum. The latter mechanism is graphically shown in Figure 1.2.

Figure 1.2 – Schematic representation of the Magneto-Rotational Instability (MRI). The blue and red circles represent fluid elements rotating differentially with larger and smaller angular velocities respectively. The dashed line is their averaged Keplerian orbit around the compact object. The blue and red vectors are the components of spring-like forces due to a magnetic field that help transport angular momentum outwards.

In that sense there is a viable mechanism that can help transfer angular momentum outwards and matter inwards. In reality things are much more complex, but for the needs of a non-technical introduction it suffices to say that if the conditions allow it, an instability can occur to help achieve that outward transfer of angular momentum. Such instability is called the Magneto-Rotational Instability (Balbus & Hawley 1991), or in short the MRI, and is the reason that the flow can, in principle, reach as close to the compact object as the inner-most stable circular orbit (ISCO) will allow. In classical accretion theory, angular momentum transport was hard-coded into the equations as viscosity in the so called α -prescription, even before the MRI was suggested as the physical mechanism for angular momentum transport. Figure 1.3 shows an artistic impression of the process of accretion in a BH LMXB.

The fundamental physics of accretion is common to both LMXBs and HMXBs, albeit the different mass-transfer mechanism in each type. A simple order of

Figure 1.3 – *Artistic impression of a low-mass X-ray binary. The black hole is accreting gas from the donor star through Roche lobe overflow, while the MRI allows the accreting gas to reach the inner regions around the black hole. Credit: Gabriel Pérez Díaz, SMM (IAC)*

magnitude estimate for the typical accretion luminosity, L_{acc} , can be made assuming spherical accretion of proton-electron pairs, and also assuming that all gravitational potential energy is efficiently transformed into thermal (kinetic) energy. Note that taking half of the gravitational energy, following the virial theorem, would not make a difference in the order of magnitude estimate. If that thermal energy could be radiated away from the surface of a hypothetical blackbody of temperature T_b , then using the Stefan-Boltzman law one can write:

$$T_b = \left(\frac{L_{acc}}{4\pi r_*^2 \sigma} \right)^{1/4}, \quad (1.1)$$

where σ is the Stefan-Boltzman constant, and r_* the innermost radius that matter can reach during accretion (Frank, King & Raine 2002). In the limiting case where the flow has the maximum possible (Eddington) Luminosity, L_{edd} , and for a typical NS of $1M_\odot$ accreting gas down to a radius of 10 km, T_b corresponds to a blackbody emitting in the X-ray spectrum (typically $T_b \sim 10^7$ K). In the case of BHs, similar order of magnitude estimates lead to roughly the same result of emission between low-energy (soft) and high-energy (hard) X-ray bands. Thus, the term X-ray binaries seems fair for binary systems with

significant mass transfer through Roche lobe overflow. Furthermore, due to the type of the donor star in LMXBs, the emission in the optical is insignificant compared to the X-rays, which makes the detection more challenging. Classical techniques such as eclipses of the compact object or relativistic (Doppler) beaming in the X-rays can be used to detect LMXBs. Many XBs have been detected to date, while the majority of LMXBs that we know of are concentrated around the galactic center. In this thesis we have used X-ray observations of LMXBs combined with theoretical models of how the variability, imprinted on the observations, is produced and propagated throughout the accretion flow. Our goal was to study the physical and geometrical properties of the accretion flow. By doing the latter we are using LMXBs as small-scale laboratories to understand the physics of accretion which, after proper scaling, can be generalised to accretion phenomena on galactic scales. In the upcoming subsections of this introduction, I explain the two types of observational analyses used to study the physical properties of the accretion flow in LMXBs, namely the time-independent, or time-averaged, spectra, and the time-dependent, or simply timing, analysis that studies the variability in LMXBs.

1.1.1 Spectra of LMXBs

As with nearly any astrophysical source, the shape and features in the spectrum of an LMXB is key for establishing the physical mechanism (or mechanisms) by which the source emits photons. By convention, in high energy astrophysics, we convert wavelength (or equally frequency, ν) into energy, E , by using the famous Planck-Einstein relation ($E = h\nu$), where h is the Planck constant, to identify each photon captured by our detector. X-ray detectors can collect incoming photons from an LMXB within a certain time window. Based on the energy of an incoming photon the detectors place it in a certain energy bin, where photons of similar energies are collected. After enough photons have been collected in every energy bin, one can sort these bins from lower to higher energies. Finally, the energy spectrum of the source can be constructed. In the LMXB literature, energy spectra are also referred to as time-averaged spectra because in constructing them we do not care about the arrival time of each photon, but only about the total number of photons in each energy bin.

A typical energy spectrum of an XB is shown as the black solid line in the bottom panel of Figure 1.4. The black solid line in Figure 1.4 is a combination of many different radiative processes, denoted by different colors in the same Figure. Although the spectra of LMXBs might not extend to as high energies as the one shown in Figure 1.4, we use this figure as reference to discuss the physics of the emission mechanisms involved, since they are quite similar among most XBs. In Fig. 1.4, we immediately notice at least three distinct features of that spectrum, (1) a broad local maximum at low energies (~ 1 -2

Figure 1.4 – Top: A schematic representation of the free electrons in the corona (pink dots) around an LMXB and the different emission mechanisms. The blue thick cone-like structure represents the accretion disc and the black sphere at the center, the compact object. In this picture, the seed photons that enter into the corona come from the disc only, but in reality they could come from the compact object as well in the case of a neutron star. Bottom: The energy spectrum of the accretion disc, thermal Comptonisation and reflection components denoted by blue, red and green color, respectively. The black solid line is the superposition of all the components. The Figure is taken from Gilfanov (2010).

keV), (2) a rather sharp peak at medium energies ($\sim 7\text{-}8$ keV) and (3) another broad local maximum at high energies ($\sim 80\text{-}100$ keV). Typically, to quantify how cold or hot an accretion flow is we express temperature in units of energy by multiplying a characteristic temperature, T , with the Boltzman constant, k , so that an energy of ~ 1 keV corresponds to a temperature of $\sim 10^7$ K. Attempts to explain the aforementioned features with appropriate physical processes give rise to theoretical models about the geometry and radiative mechanisms in LMXBs. The geometry and radiative mechanisms of LMXBs are interconnected, since the geometry is basically a combination of different physical regions around the compact object. The physical regions interact with each other both dynamically, to preserve some stability of the flow, and radiatively, which leads to transfer of radiation from one region to another and out of the system.

Explaining the three features seen in the energy spectrum (black solid line in Figure 1.4), gives rise to the three basic radiative processes that, to date, are believed to drive the spectral properties of LMXBs. It is generally accepted that the lower energy, or simply soft, component that peaks at ~ 1 keV can be attributed to the cool accretion disc, or in the case of a NS LMXB to a combination of an accretion disc and the surface of the NS, or boundary layer. The emission of a NS surface can be modeled as a blackbody of certain radius, a , and surface temperature, T_s , using Planck's law. The cool accretion disc, in the regime where it is geometrically thin and optically thick (Shakura & Sunyaev 1973; Novikov & Thorne 1973; Lynden-Bell & Pringle 1974), can be modeled as a multi-color blackbody (Mitsuda et al. 1984). We usually refer to this model as a Shakura-Sunyaev disc or a disc blackbody model. More specifically, the disc blackbody model assumes that the total disc can be divided in infinitesimally small circular annuli with radii in a range (R_{min}, R_{max}) , where R_{min} could be as small as the ISCO around the compact object, and R_{max} as large as the distance to L1. In the cool (or thin) disc approximation, assuming that the disc is Keplerian, geometrically thin and radiatively efficient, one can solve for the steady state surface dissipation rate, i.e. the energy released on the surface of the disc per unit time and surface area. Assuming that each annulus in the disc emits like a blackbody, one can find an elegant radial dependence for the surface temperature. Under certain assumptions, it can be shown that the surface temperature, T_s , is the highest in the inner-most edge (R_{in}) and decreases, according to a power law $T_s \sim R^{-3/4}$ as the distance from the center of accretion, R , increases. The spectra of a simple blackbody and a disc blackbody differ in shape, and are difficult to distinguish in the observed energy spectra because their original shape is significantly altered by other radiative processes. However, the blackbody and disc-blackbody models are essential components of an additive model for the full energy spectrum, and

their shapes and physical properties can be retrieved, implicitly, by fitting a complete model that includes them to observed energy spectra. The accretion disc around the compact object is shown as the blue thick area in the top panel of Figure 1.4 for the case of a BH XB, and the corresponding spectral component of the disc blackbody model is plotted in the bottom panel of the same figure as a blue solid line.

Although the blackbody and disc blackbody spectra are necessary ingredients to reproduce the overall energy spectrum, alone they can not explain how photons reach energies as high as several tenths of keV, or even hundreds of keV in the case of larger scale accretion processes. The failure of the thin disc to explain higher energies lies in the fact that, even in the extreme case of accretion at the Eddington limit of an optically thick and geometrically thin disc assuming radiative efficiency, the highest temperature that can be achieved is of the order of a few keV (Shapiro, Lightman & Eardley 1976), while the observed spectra imply much higher temperatures by exhibiting significant flux up to one order of magnitude more than one would normally expect. An excellent example of the aforementioned inconsistency, although not in the context of LMXBs, is the case of Cygnus X-1 (Lightman & Shapiro 1975) where the observed energy spectra extend up to 100 keV.

In order to explain the flux contribution at high energies, many theoretical works provided solutions to the equations that describe the accretion flow in the presence of radiative inefficiencies. The aforementioned solutions belong to a class that is called Advection Dominated Accretion Flows (ADAFs; Ichimaru 1977). As their name suggests, ADAFs are solutions in which the energy produced via viscous dissipation can not be emitted thermally, and so a different process is necessary. All ADAF solutions are built upon the idea of a radiative inefficiency in the flow that manifests itself in different ways, depending on the initial density (optical thickness) of the flow. The first kind of ADAF solution is called the slim disc approximation, and occurs at high mass accretion rate, $\dot{M}$, where the flow is optically thick. In the slim disc approximation, the photons that are produced are not able to travel a vertical path of height H fast enough due to the high optical depth τ , and so the cooling timescale, $t_{cool} \sim H\tau/c$, can be larger than the in-fall timescale, $t_{accr} \sim R/v$, where v is the radial velocity component and c the speed of light. As a result, the photons are dragged inwards and partly advected into the black hole, thus making the ADAF radiatively inefficient. In the aforementioned picture, it is reasonable to assume that some form of outflow can be developed at the inner edge of the disc, due to the large thermal energy built up by the advected photons. The second ADAF solution is called the hot flow, and it requires the disc to be optically thin and the accretion rate to be lower than a critical value, $\dot{M}_{crit}$, that depends on the viscosity parameter, α . Shapiro, Lightman & Eardley

(1976) first introduced the idea of the hot flow, and in particular the pioneering idea that the flow turns into a two-temperature plasma, where the ions retain a much higher temperature than that of the free electrons. The two temperatures in the plasma is a necessary assumption to explain the observed spectra. In particular, the assumption is that most of the viscously dissipated energy goes to the ions instead of the electrons. The idea of a two-temperature state is further supported by the physical arguments that (a) electrons can cool more efficiently than ions through synchrotron, Comptonisation and free-free processes and (b) thermal equilibrium in the plasma, which would make it radiatively efficient, can be achieved only through Coulomb interactions between electrons and ions, and in the optically thin regime those are sufficiently inefficient. In particular, the electron-proton collision frequency, ν_{ep} , is lower than the electron-electron collision frequency, ν_{ee} , by a factor of $m_e/m_p \sim 5 \times 10^{-4}$, where m_e and m_p is the electron and proton mass, respectively. The high temperature of the ions that can be maintained through slow Coulomb equilibration, implies significant radiation pressure that can puff up the flow leading to a quasi-spherical hot flow, through convection, that is called the corona. Therefore, it would be more accurate to refer to the hot flow as a convection dominated accretion flow (CDAF), rather than ADAF. For simplicity we refer to all radiatively inefficient solutions as ADAFs. For a detailed qualitative and quantitative review of ADAFs, we refer the reader to Yuan & Narayan (2014) and Narayan & Quataert (2005). Such hot coronae, as we will see, can indeed explain the high-energy tails of the observed spectra.

From all the above, many questions arise. Where exactly is that corona with respect to the compact object? And how does it explain the flux at higher energies? The first question can be answered through General Relativistic (GR) Magnetohydrodynamical (MHD) simulations of the time-dependent structure of an initial flow that is being accreted. The latest and most detailed GR MHD simulation of disc structure in the presence of magnetic fields by Liska et al. (2018, 2019), succeeded in confirming numerous theoretical expectations such as the Bardeen-Petterson alignment, formation of precessing jets and others. Nevertheless, our interest lies with the fact that these simulations also predict an optically thin disc close to the compact object that progressively thickens, as the distance from the central source increases. Based on what we said so far, a corona can be formed in such low density regions through radiative inefficiencies, and thus we would naturally expect that the corona develops, at least initially, in the inner regions of the accretion flow and extends radially as far as rotation and hydrostatic equilibrium allows it. A GR MHD simulation of the density profile of a thin disc (Liska et al. 2019) is shown in Figure 1.5, in the form of a contour plot. The authors have placed their own view of where the corona would be on top of the overall geometry, although a similar picture

Figure 1.5 – The authors’ impression of the position of the corona (hot ADAF flow) in the overall accretion geometry. The corona is shown as the magenta shaded area, while the dashed line represents its boundary. The background contour plot is the most detailed, to date, thin disc GR MHD simulation of the density profile of the disc (Liska et al. 2019). The color scale indicates differences in density of the flow ranging from very low (blue) to very high (red).

has been suggested many times in the past. To conclude answering the first question, we must add that the corona should not be viewed as a stable or compact configuration, at least not in long enough timescales. Thermal instabilities due to inflow/outflow of energy, turbulent eddies from the MRI and cooling processes that take place to radiate away part of its energy, will tend to alter both the geometrical and thermodynamical profile of the corona. For example, we know for a fact that one of the ADAF solutions that allows the corona to form via convection is only valid for $\dot{M} < \dot{M}_{crit}$, while LMXBs go through cycles (as we will see later on) where $\dot{M}$ can increase much beyond the critical value required by that ADAF solution. However, observational evidence suggests that the high energy flux contribution in the spectrum, associated to the corona, has a cyclical behaviour and sometimes persists over a substantial amount of time, which indicates that hot coronae can be both persistent and recurring.

Among various cooling mechanisms for the hot free electrons of the corona, observed energy spectra of LMXBs suggest that inverse Compton (IC) scattering, which is often called Comptonisation, dominates (Gierliński et al. 1997;

Zdziarski et al. 1998; Wardziński et al. 2002; Zdziarski & Gierliński 2004; Done, Gierliński & Kubota 2007; Burke, Gilfanov & Sunyaev 2017). In short, Comptonisation is the process in which a relatively low energy photon interacts with a much higher energy electron via IC scattering, and gains some energy. The higher the number of scatterings, the higher the energy that the photon reaches. Low-energy, seed, photons for Comptonisation can be provided either by other corona cooling processes, e.g. free-free or synchrotron emission, or directly from the cool disc that can exist either outside or partly inside the corona. Nevertheless, the geometry of the corona, or more generally the medium in which Comptonisation occurs, is not clear yet since the high-energy tails in the spectra can be reproduced equally well under the assumption of IC scattering in a spatially resolved jet. In particular, models that treat Comptonisation in spatially resolved jets (Band & Grindlay 1986; Georganopoulos, Aharonian & Kirk 2002; Reig, Kylafis & Giannios 2003; Giannios 2005), have been used in the case of LMXBs to showcase how the high-energy tail of the spectra can naturally occur (e.g. Reig & Kylafis 2016), as well as other observed correlations (Kylafis & Reig 2018; Reig et al. 2018).

Many alternative geometries for the corona exist. In the "sandwich corona" (Haardt & Maraschi 1991, 1993), the corona is assumed to exist as a layer on top of the disc (plane-parallel-like geometry) and feedback of up-scattered photons onto the disc contributes to its steady-state temperature. The "patchy corona" (Galeev, Rosner & Vaiana 1979; Haardt, Maraschi & Ghisellini 1994) is another proposed geometry, where instead of a continuous medium the corona is actually comprised of multiple magnetic loops, created via differential rotation in the disc. When the magnetic loops are Compton-cooled by the soft X-rays from the disc, they can produce a high-energy tail in the spectrum. Finally, simpler geometries such as the "lamp post" (Markoff, Nowak & Wilms 2005; Miller, Homan & Miniutti 2006; Miller et al. 2006), which assume that the corona is a point source located at a certain height above the compact object, or spherically symmetric extended coronae around the compact object (Zdziarski, Johnson & Magdziarz 1996; Życki, Done & Smith 1999; Zdziarski et al. 2020) are used today with success to explain observed spectral and timing features. Throughout this work we have assumed a spherically symmetric corona that can partially or fully cover the soft source, which is either the accretion disc or the NS. In Figure 1.4 (top panel), the corona is represented as a quasi-spherical cloud where the pink particles are the free electrons. Compton up-scattering of disc blackbody photons in the corona leads to an additional right-fat-tailed (right because the tail extends to higher energies) component, which is called the Comptonisation component of the spectrum and is plotted as a red solid line in the bottom panel of Figure 1.4. By fat-tailed we mean that, when plotted against photon energy, the flux, F , of the energy spectrum

at higher energies decreases following a power-law, $F \sim E^{-\Gamma}$, where Γ is called the photon power-law index and is often used as a proxy for how strong the effect of Comptonisation on the overall energy spectrum is.

A combination of a blackbody or disc blackbody spectrum (blue solid line in Figure 1.4) and a Comptonised spectrum (red solid line in Figure 1.4), is still not enough to adequately reproduce the observed spectrum (black solid line in Figure 1.4). There is still a narrow peak around 7-8 keV and a general asymmetry that needs to be accounted for. The aforementioned features can be well reproduced if one assumes that part of the high energy photons that exit the corona, after being Compton up-scattered, illuminate the disc and produce the so called reflection spectrum. The reflection spectrum model (green solid line in Figure 1.4) has two main sub-features. The first feature is a narrow emission line (usually at ~ 6.4 keV) that is the result of fluorescence, when photons with energy above a minimum threshold (>7.1 keV) are able to excite (via photo-electric absorption) electrons of iron atoms or ions that, after decaying to a different energy state, release the photons that construct this narrow feature in the spectrum. This emission line is commonly referred to as the iron-K line. The second feature is a bump that appears above 20 keV, which is called the Compton hump (Lightman & Rybicki 1980; García & Kallman 2010). When the Comptonised component illuminates the disc, photons below 20 keV get absorbed rather than scattered by the material of the disc and so they are missing from the final spectrum that escapes the disc. Similarly, photons that have very high energies (>40 -50 keV) lose significant energy from Compton down-scattering in the disc, and will either escape as photons with much lower energy, or get absorbed in the disc. Naturally one expects that, relative to the incident flux, more photons will escape in between 20-40 keV, which is where the broader peak of the reflection spectrum usually occurs. Fabian (1989) and Ross & Fabian (1993) describe in detail the physics of disc reflection in the case of Active Galactic Nuclei (AGNs), but the same physics apply in the case of LMXBs. The exact shape of the reflection spectrum will depend on the inner radius of the disc, the power-law tail of the incident Comptonised spectrum, the composition and ionisation state of the disc and the exact gravitational field around the compact object. In particular, the iron-K line, which is the most prominent feature in the reflection spectrum, is asymmetrically broadened due to relativistic beaming and gravitational redshift (Fabian 1989).

To summarise, the full energy spectrum of an LMXB can be modelled as a superposition of other energy spectra that constitute sub-components of the full spectrum. Those sub-components are (1) produced by different physical mechanisms that occur in different geometrical regions around the compact object and (2) are interconnected with one another in the sense that a change in the physical properties of one of them will likely cause a change in the observed

properties of another. To make the above statement more clear, one can say that in the case of a BH LMXB soft X-rays are produced in the accretion disc as matter spirals in, via release of gravitational potential energy. A fraction of the soft X-rays produced in the disc can escape without any interaction with the matter and produce the disc blackbody component. Another fraction of the soft X-rays will get Compton up-scattered in a corona or the jet, given that part of the disc is covered by that corona or the soft photons can efficiently be fed to the jet, and reach much higher energies leading to the formation of the Comptonised spectral component. Furthermore, if geometrical conditions allow it, a fraction of the flux of the Comptonised component can irradiate part of the disc producing a reflection component. In the case of NS LMXBs, the general picture is the same except for the fact that there might be direct emission from the NS surface that would add an extra flux component to the spectrum. Due to the strong magnetic field around the NS and the fact that the accreting matter can reach the NS surface, under certain conditions the energy spectra of NS LMXBs can differ from those in BH LMXBs both in shape and evolution as a function of time.

1.1.2 Spectral states and transitions

So far, we have established that the spectra of LMXBs have a complex shape that can be approximated by assuming the existence of certain geometrical regions and radiative mechanisms. A next step would be to understand how those spectra evolve with time. Long term X-ray monitoring of LMXBs has revealed that the majority of NS LMXBs exhibit a more or less constant luminosity, of the order of $\sim 10^{36}$ erg/s, and are thus classified as persistent sources. On the other hand, BH LMXBs go through cycles where the observed luminosity can change by several orders of magnitude from $\sim 10^{30}$ erg/s, at a state that is called quiescence, to $\sim 10^{38}$ erg/s at a hard state that the source usually reaches through an outburst. Assuming that most of the energy is produced via accretion, one can deduce that the changes in source luminosity, and thus the cycles that a source goes through, are related to changes in mass accretion rate (Miyamoto et al. 1992). Source luminosity alone is a poor measure to understand the transition between quiescence and high luminosity states, especially in NS LMXBs where luminosity does not change as much as in BH LMXBs, and so we need another metric that takes into account changes in the shape of the energy spectra during a transition. State transitions in BH and NS LMXBs are studied by implementing slightly different metrics, and so in the following we will describe how state transitions are studied in the case of BH first, and then in the case of NS LMXBs.

In the case of BH LMXBs, the second metric constructed to account for changes in spectral shape is the hardness ratio (HR; Homan et al. 2001; Belloni et al.

2005), which is the integrated flux in a high energy (hard) X-ray band e.g. (6-10 keV) divided by the integrated flux in a low (soft) X-ray band e.g. (1-6 keV). The width, in units of keV, of the bands used to calculate the HR depends on the source and aims at a particular analysis. Plotting the two quantities, HR on the x -axis and source Luminosity (in Eddington units) on the y -axis, in a diagram that is called the Hardness-Intensity diagram (HID; Homan et al. 2001; Belloni et al. 2005; Homan & Belloni 2005), reveals a repeated behaviour which looks like a hysteresis loop. Figure 1.6 shows the HID for the case of the BH LMXB GX 339-4.

Figure 1.6 – The hardness intensity diagram (HID) of the BH LMXB GX 339-4. Different spectral states are specified at their correct position according to hardness ratio and X-ray luminosity. The blue arrows indicate the time sequence in which the transitions happen. Figure taken from Grinberg et al. (2013).

The anti-clockwise cycle, denoted with blue arrows in Figure 1.6, begins when the source moves from quiescence to a state of low luminosity and large HR, which is called the low-hard state and indicates the beginning of an outburst. In the low hard state, the disc is believed to be truncated far from the ISCO,

while the inner flow is believed to be ADAF-like (hot flow). During the outburst, the luminosity of the source increases while the HR remains relatively large and that state is called hard intermediate state (HIMS), top right in Figure 1.6. While still retaining its high luminosity, the HR of the source decreases (horizontal transition at the top of Figure 1.6) and the source passes through the soft intermediate state (SIMS, top left in Figure 1.6), and finally reaches the soft state. During the hard to soft transition, the disc inner radius is believed to move much closer to the ISCO and the corona is expected to be less dominant (even disappear). In the soft state, the HR remains at low values while the luminosity remains relatively high (compared to the hard state). At high luminosities, if $\dot{M}$ is sufficiently high, the inner flow can become radiatively inefficient again leading to radiation driven outflows, and likely the formation of a new corona. Finally, the source can "close the loop" and end up again in the low-hard state, and potentially return to quiescence. Although the aforementioned behaviour is not written in stone for all BH LMXBs and it does not always look the same in the HID, it represents our general understanding for the spectral transitions among the known sources. The geometrical evolution of the accretion flow and corona during the state transitions in BH LMXBs as described above, although still poorly understood, is schematically shown in Figure 1.7 (Müller 2004).

Figure 1.7 – Evolution of the accretion flow during the state transitions in BH LMXBs. Image taken from Müller (2004).

The case of NS LMXBs is more complex than that of BHs, in the sense that NS exhibit larger divergences in their spectral properties. Due to the fact that most NS LMXBs are in a constant state of high luminosity, the HID is not an ideal tool to study state transitions. Therefore, for NS LMXBs we use the color-color diagram (CCD; Hasinger & van der Klis 1989) in which the flux in a soft X-ray band divided by the flux in a softer X-ray band (e.g. $[3.2-4.7 \text{ keV}]/[0.9-3.2 \text{ keV}]$) is plotted on the x -axis as the "soft color", and the flux in a hard X-ray band divided by the flux in a slightly less hard X-ray band (e.g. $[19.1-6.4 \text{ keV}]/[4.7-6.4 \text{ keV}]$) is plotted on the y -axis as the "hard-color", for multiple observations taken at different times. Through the CCD, NS LMXBs can be crudely classified in two categories, namely the Z- and Atoll sources, based on the different shapes that the soft and hard color traces on the CCD. An example of a CCD is shown in Figure 1.8, where a Z-source (left panel) typically goes through three main states, whilst increasing its luminosity, (a) the horizontal branch (HB), (b) the Normal branch (NB) and (c) the flaring branch (FB). On the other hand, in the less luminous Atoll sources the source also goes through three states, namely (a) the island state (IS), (b) lower

banana state (LB), and (c) upper banana state (UB; see Figure 1.8 for the position of each state in the CCD), again with increasing luminosity towards the banana states.

Figure 1.8 – An example of a color-color diagram of Z- and Atoll sources on the left and right panels, respectively. The figure is taken from Wijnands (2001).

The aforementioned states, in the case of NS LMXBs, correspond roughly to the hard to soft transition of BH LMXBs, and the branches in both Z- and Atoll sources show different timing and spectral behaviour, as we will see later. The distinction between Atoll and Z-sources is not absolute and some sources can switch from one type to the other (e.g. Lin et al. 2009).

1.2 Variability in LMXBs

Variability in LMXBs refers to the study of X-ray lightcurves, which are time series of the observed X-ray flux. Analyses that deal with the statistical and physical properties of X-ray lightcurves are often called timing analyses and they can be carried out in either narrow or broad energy ranges. Of course, the best time- and energy-resolution in which variability can be measured are limited by the timing and effective area capabilities of an instrument, respectively. Although the presumed physical origin and statistical properties of each type of variability can differ vastly among NS and BH LMXBs, the mathematical

tools, and therefore the metrics, used to quantify and study that variability are common across the majority of X-ray timing analyses.

Figure 1.9 – Example of a PDS that contains a QPO at a frequency of ~ 0.11 Hz. Note that the y-axis is not in units of power, but power multiplied by Fourier frequency. The PDS is fitted with a Lorentzian mixture model. The red dashed lines are the individual best-fitting Lorentzians plotted separately, while the red solid line is the sum of them. The PDS is from an observation of the Galactic transient Swift J1658.2–4242 with NuSTAR and the figure is taken from Xu et al. (2018).

The search for variability in X-ray lightcurves is almost always done in the frequency rather than the time domain, by using the Fast Fourier Transform (FFT) which is basically an algorithm for calculating the Discrete Fourier Transform (DFT) of the time series (Deeming 1975; Press et al. 1992). The discrete treatment is necessary because a lightcurve, $x(t)$, is always constructed in discrete time intervals such that $t_{n+1} = t_n + \Delta t$, where $n = 1, \dots, N$ is the number of measurements and Δt the time interval between consecutive measurements. The DFT takes as input the full time series, $x(t)$, and returns the Fourier power at some frequency ν_k through the formula:

$$X(\nu_k) = \sum_{n=0}^{N-1} x(t_n) e^{\frac{i2\pi n k}{N}} = \sum_{n=0}^{N-1} x(t_n) e^{i2\pi \nu_k t_n}, \quad (1.2)$$

where $\nu_k = k/(N\Delta t)$ and $t_n = n\Delta t$. In what follows, we use ν instead of ν_k to denote the Fourier frequency. The minimum interval Δt required by the instrument to measure X-ray flux with some statistical significance, places an upper limit to the highest frequency that can be detected, which is called the Nyquist frequency, $\nu_{Nyq} = 1/(2\Delta t)$. The DFT of the X-ray lightcurve, $X(\nu)$, can be multiplied by its complex conjugate, $X^*(\nu)$, and then be properly normalised to give the Power density, $P(\nu)$, of that lightcurve, which plotted against frequency, ν , results in the so called Power Density Spectrum (PDS). The PDS tells us (1) how much the X-ray flux is varying at each Fourier frequency, ν , and (2) if there is any periodicity at some Fourier frequency, ν , or in other words how well the time-dependent profile, or shape, of the observed lightcurve matches that of a sinusoid at that frequency ν . An extreme case of coherent variability would be one with a perfect sinusoidal shape, with constant frequency ν , which would produce a very sharp peak (delta-like function) in the PDS. In reality, the variability in the lightcurves of LMXBs is far from even a moderately well-shaped sinusoid. In Figure 1.9 one can distinguish a peak in the PDS, at around 0.11 Hz, that implies a significant detection of quasi periodic variability. The quasi periodic nature may arise from the fact that the amplitude, or frequency, of the variability change with time. This type of variability is called Quasi Periodic Oscillations (QPOs) and appears in the PDS of both BH and NS LMXB lightcurves, across a range of frequencies that vary by several orders of magnitude. An example of a lightcurve with a QPO, taken from Ingram & Motta (2020), is shown in Figure 1.10.

Figure 1.10 – Raw (black) and filtered (red) lightcurve of GRS 1915+105 during the observation of a QPO at 3.48 Hz. The figure is taken from Ingram & Motta (2020).

The discovery of QPOs dates back to Samimi et al. (1979), although more statistically significant detections of QPOs were presented for the NS LMXB GX 5-1 in van der Klis et al. (1985), and for the BH candidate GX 339-4 in Motch et al. (1983). In the decades that followed, namely from the late 1980s until 2012, observations and analyses regarding QPOs met a profound growth thanks to X-ray satellites such as EXOSAT and GINGA at first, and later with the Rossi X-ray Timing Explorer (RXTE; Bradt, Rothschild & Swank 1993), which operated between 1996 and 2012 and whose data are still considered a goldmine for the study of high energy astrophysical phenomena. The X-ray satellites that came later, e.g. XMM-Newton, Chandra or Swift, were not well suited for studying high-frequency variability phenomena, particularly in NS LMXBs. Finally, much hope is being placed today on the Indian satellite Astrosat, and the NICER instrument on board the International Space Station, for the study of QPOs. Before we discuss about the origin and differences of QPOs in BH and NS LMXBs, we must first define some spectral and timing properties that are necessary for any quantitative study.

The PDS of an X-ray lightcurve can be fitted well with a mixture of N Lorentzian functions, L_i , so that the total Fourier power can be written as

$P(\nu) = \sum_{i=0}^{N-1} L_i(\nu)$, and each Lorentzian is given by the formula

$$L_i(\nu) = \frac{L_{i,0}^2}{\pi/2 + \tan^{-1}(\nu_i/\Delta_i)} \frac{\Delta_i}{\Delta_i^2 + (\nu - \nu_i)^2} , \quad (1.3)$$

(van Straaten et al. 2002) where ν_i is the centroid frequency of the Lorentzian component, Δ_i the full width at half maximum (FWHM), and $L_{i,0}$ a constant whose square is equal to the integral, over all frequencies, of that Lorentzian component. The parameter $L_{i,0}^2$ has units of squared power, since we integrated squared power density over frequency. Through Parseval's theorem one can normalise the PDS such that $\sum \nu_i L_{i,0}^2$ is equal to the square of the fractional root mean square (fractional rms) of the observed time series (Lewin, van Paradijs & van der Klis 1988; Belloni & Hasinger 1990). Parseval's theorem, in the context of X-ray timing, is of two-fold importance; (a) it allows us to study statistical properties of the time series, such as the variance, in the frequency domain through a proper transformation of the Fourier power, and (b) we can study variability in any desired frequency range, through the help of the Lorentzian decomposition technique, which would be otherwise impossible to do through the observed time series.

An example of a multi-Lorentzian model fit is shown in Figure 1.9. The quality factor, $Q = \nu_i/\Delta_i$, which measures the narrowness of a QPO in the PDS, alongside the source intensity, fractional rms, and exposure time determine the significance of a detected QPO. Once the significance of a QPO is established, it is important to know how much of the total variability of the observed lightcurve is attributed to that QPO. Because of the fractional rms normalisation that we introduced earlier, the variability of a QPO with centroid frequency ν_i can be obtained directly from the $L_{i,0}$ parameter of the corresponding Lorentzian, and so $L_{i,0}$ is equal to the fractional rms of the QPO at that frequency, ν_i . All of the above metrics that are constructed to quantify variability in the frequency domain, use as input a time series i.e. a lightcurve. Thus, we can distinguish lightcurves in narrow (as narrow as the exposure limitations allow us) energy bands, and study the energy dependent variability. Of particular interest is the energy-dependent fractional rms, which is simply the best-fitting value of $L_0(E)$ derived from the PDS of a time series in a narrow energy band centered at E , and so we will come back to that later. Now that the notion of distinct lightcurves is introduced, we can define another key metric which is the phase lag between two lightcurves at different energy bands, when the QPO has a specific frequency ν_i . More specifically, after the DFT has been applied to two monochromatic lightcurves, $x(t, E_1)$ and $x(t, E_2)$, of energies E_1, E_2 one can get a value for the complex power spectrum, at each energy, $X(\nu_i, E_1) = A_{1,\nu_i} e^{i\phi_{\nu_i}}$ and $X(\nu_i, E_2) = A_{2,\nu_i} e^{i(\phi_{\nu_i} + \delta\phi_{\nu_i})}$

, respectively, where here we have assumed that the Fourier transform of the two lightcurves at a frequency ν_i can have a difference in phase $\delta\phi_{\nu_i}$, that can be associated to some physical phenomenon. Multiplying $X(\nu_i, E_2)$ by the conjugate of $X(\nu_i, E_1)$ (product of two Fourier transforms) leads to the so-called cross spectrum $C_{E_1, E_2, \nu_i} = A_{1, \nu_i} A_{2, \nu_i} e^{i\delta\phi_{\nu_i}}$, from which the phase-lag between energy bands E_1 and E_2 , at any Fourier frequency, can be estimated. Finally, one can transform the phase-lag into time-lag by dividing $\delta\phi_{\nu_i}$ by an implied angular frequency $\omega_{\nu_i} = 2\pi\nu_i$. For a complete review of Fourier timing techniques, in the context of X-ray binaries, we point the reader to van der Klis (1989) and Uttley et al. (2014). Our current understanding of QPOs, including their dynamical origin, time evolution and difference in properties between NS and BH, has been largely due to the combined study of both their energy-spectra and timing properties such as energy-dependent fractional rms and time lags. Such studies are often referred to as spectral-timing studies.

To summarise, a typical spectral-timing study of a LMXB can be broken down into some basic steps. One would start by constructing the energy-spectrum of the source and calculating the PDS of the lightcurve, from which we can detect the QPOs. The PDS can be created in the full energy band or in a narrow band. From the PDS one can measure the fractional rms of a QPO either in the full band (energy-integrated fractional rms), or in a narrow band (energy-dependent fractional rms). Finally, a time-lag is only meaningful if measured between at least two different energy bands. In particular, we can measure the time-lag by dividing the full energy band into two sub bands, or into more distinct bands (as many as our statistics allows), and measure the time-lag of one (or more) band with respect to some reference band that in general can be arbitrary. Measuring the time delay of distinct energy bands with respect to a reference (energy-dependent time-lags) gives us information about the time-delay between photons of different energies. In turn, the aforementioned time-delay gives us information about the physical scale of the accreting regions, assuming that photons of different energies can either come from different parts of the accretion flow, or undergo additional processes to differentiate their energy. Additionally, the energy-dependent fractional rms tells us which part of the energy spectrum, low- or high-energy, is more variable at the QPO frequency, which in turn allows us to speculate on which region in the accretion flow produces that variability. In the next subsections we briefly discuss some of the phenomenology around QPOs and the connection to their observed spectral and timing properties.

1.2.1 Quasi periodic oscillations in black-hole LMXBs

QPOs in BH LMXBs can be divided into two general categories according to their centroid frequency. More specifically, if the QPO centroid frequency is

above 60 Hz then a QPO is classified as a high-frequency (HF) QPO, whereas if the centroid frequency is below 30 Hz a QPO is classified as a low-frequency (LF) QPO. Although we will not discuss HF QPOs in BH LMXBs here, we mention that those QPOs are weak and therefore appear less frequently in the data (see detections of HF QPOs by Morgan, Remillard & Greiner 1997; Belloni, Sanna & Méndez 2012; Altamirano & Belloni 2012). On the other hand, LF QPOs are divided into three types, A, B and C (Casella et al. 2004), and are much more commonly observed in BH LMXBs partly due to the fact that they are narrow and strong, and are therefore more easily detectable with a good significance. The distinction of LF QPOs in different types is based on differences in their spectral-timing properties. Type-A QPOs usually appear when the source is in the high-soft state, and exhibit low quality factors ($Q \sim 3$), low fractional rms, and frequencies between 6–8 Hz. Observations of type-A QPOs are also infrequent, with roughly ten having been detected with some significance in the RXTE archive. Type-B QPOs appear in the HID (Figure 1.6) during the SIMS (Motta et al. 2011) and exhibit a high quality factor ($Q > 6$), higher fractional rms than the type-A QPOs, and frequencies around 5–6 Hz, although much lower frequencies have been observed (e.g. Motta et al. 2011). Furthermore, type-B QPOs are associated with simultaneous detection of transient radio emission (Fender, Belloni & Gallo 2004; Corbel et al. 2005), which in turn implies a connection between type-B QPOs and jet ejections. Finally, type-C QPOs, which are mainly observed in the low-hard and hard-intermediate state exhibit very high quality factors ($Q > 8$), the highest fractional rms between LF QPOs (up to $\sim 20\%$), and a wide range of frequencies reaching up to even ~ 30 Hz, although with lower fractional rms amplitude, when observed at the softest part of the LHS and in the HIMS (Revnivtsev, Trudolyubov & Borozdin 2000; Motta et al. 2012).

Although the results presented in this thesis are, to some degree, indifferent to the dynamical mechanism that produces the QPO frequencies, we will briefly mention theoretical models that explain the frequencies of LF QPOs. The two component advection flow model (TCAF; Molteni, Ryu & Chakrabarti 1996; Chakrabarti et al. 2008), the accretion-ejection instability model (Tagger & Pellat 1999) and the relativistic precession model (RPM; Stella & Vietri 1998; Stella, Vietri & Morsink 1999) are the most prominent regarding the dynamical origin of LF QPOs. Among the aforementioned models, we briefly discuss the RPM model, and its derivatives, which have led to many testable predictions. The RPM model assumes that the LF QPO is triggered by matter precessing at the Lense-Thirring frequency (LTP; Lense & Thirring 1918), defined as the difference between the orbital (or azimuthal) epicyclic frequency and the vertical epicyclic frequency, which are not equal in General Relativity, of a test particle.

Figure 1.11 – *An optically thick torus (grey structure) precessing at the Lense-Thirring precession frequency, around a compact a object. Figure taken from Ingram, Done & Fragile (2009).*

Ingram, Done & Fragile (2009) studied the LTP of a torus-like structure bounded by the disc inner radius, and were able to explain the observed range of QPO frequencies adequately well. A schematic representation of the precessing torus, taken from Ingram, Done & Fragile (2009), is shown in Figure 1.11. The existence of a torus-like structure in the inner part of the flow is motivated by GR MHD simulations (Fragile & Anninos 2005; Fragile et al. 2007). Geometrically thick regions are expected since the flow transitions to a radiatively unstable regime as mass accretion rate increases towards the HIMs, and some outflows due to increased radiation pressure may form. The fact that the precession is appointed to the torus rather than the disc is motivated by the observed energy-spectrum being more variable at higher energies rather than lower energies. The increased variability of the energy spectrum at higher energies can be quantified and studied through measuring the energy-dependent fractional rms, which we will sometimes refer to as the rms-energy spectrum. Observed rms-energy spectra across different sources reveal a pattern of monotonic increase of fractional rms as a function of energy, implying that most of the variability measured at the QPO frequency comes from the high energy photons. In the case of Ingram, Done & Fragile (2009), if one assumes that this torus represents a hot inner flow that emits high energy photons (e.g. via IC scattering), then precession of that flow can explain the observed variability qualitatively. Ingram et al. (2016) also showed that a modulation of the centroid energy of the iron-K line is observed at half the precession period in the BH LMXB H1743–322, further supporting the idea that LF QPOs are produced via relativistic precession. However, these authors note that precession of the disc, rather than a torus-like hot inner flow, can explain the observations equally well as long as the high fractional rms at high energies is explained somehow. Nevertheless, precession of any spatially resolved structure (disc or

torus) implies also a dependence upon viewing angle or, equivalently, source inclination. Indeed, evidence that the spectral-timing properties of LF QPOs depend upon source inclination has been presented by Motta et al. (2015), Heil, Uttley & Klein-Wolt (2015) and van den Eijnden et al. (2017) by studying the frequency dependence of fractional rms and time lags in a large sample of sources with RXTE. For further information on the observational properties of QPOs and relevant theoretical models for them, mainly in the context of BH LMXBs, we refer the reader to the review papers of Ingram & Motta (2020) and Belloni & Motta (2016). The main purpose of this thesis, regarding LF QPOs in BH LMXBs, is to quantitatively explain both the energy- and frequency-dependent fractional rms and time-lags, which we call radiative properties, of the LF QPOs.

1.2.2 Quasi periodic oscillations in neutron star LMXBs

QPOs in NS LMXBs can also be separated into LF QPOs and HF QPOs, depending on their observed centroid frequency, but it is harder to differentiate between the two based on their position in the CCD diagrams (Figure 1.8). For example, in Z-sources LF QPOs appearing along the horizontal branch (HB) in the CCD are called Horizontal Branch Oscillations (HBOs), LF QPOs appearing alongside the normal branch (NB) are called Normal Branch Oscillations (NBOs) and finally those appearing on the flaring branch (FB) are called Flaring Branch Oscillations (FBOs). However, HF QPOs also appear in some of the branches of the CCD in Z-sources, as well as in Atoll sources where they are more common. In this work we have focussed on studying the spectral-timing properties of HF QPOs in NS LMXBs, and so LF QPOs will not be mentioned further. The detection of HF QPOs (>100 Hz) in NS LMXBs began in 1995 thanks to RXTE. Soon, QPOs at frequencies of a few hundreds of Hz were detected (van der Klis et al. 1996), which are called the kHz QPOs; it was also shown that the kHz QPOs usually appear in pairs (Ford et al. 1997; Strohmayer et al. 1996), consisting of a lower and an upper kHz QPO separated in frequency by a few hundreds of Hz, often called the twin kHz QPOs. The kHz QPOs are observed in the HB or NB of the CCD of Z-sources and in intermediate states (between the island and banana states) of Atoll sources.

Due to their extremely high frequencies, kHz QPOs are often linked to Keplerian frequencies at the ISCO around a NS. We mention that kHz frequencies are consistent with the Keplerian frequency of a test particle orbiting around a $1.5M_{\odot}$ NS at a distance of a few tenths of km. However, crude analogies such as the one presented above, are only helpful to give us an idea of how the observed frequencies relate to the dynamical ones. Self-consistent models are needed to explain the dynamical origin of kHz QPOs and quantitatively

explain their observed spectral-timing properties. Among the theoretical models designed to explain the dynamical origin of the kHz QPOs, two are the most prominent in the literature. The first model that we discuss here is the sonic-point beat-frequency (SPBF) model (Miller, Lamb & Psaltis 1998), which assumes that the upper kHz QPO is due to the Keplerian frequency of matter orbiting at an inner radius (sonic radius), while the lower kHz QPO is due to a beat between the Keplerian frequency and the spin of the NS. The SPBF model faced observational challenges immediately, such as the observation of non-constant separation between the twin kHz QPOs (Méndez et al. 1998), contrary to what the model predicted, which forced the SPBF model to adapt (Lamb & Miller 2001). The second model for kHz QPOs that we discuss here is the RPM model, similar to the one proposed for BH LMXBs. In the context of the RPM model, the upper kHz QPO frequency is the orbital (or azimuthal) epicyclic frequency in the Kerr metric, while the lower kHz QPO frequency is the difference between the orbital and radial epicyclic frequencies, which is called the periastron precession frequency. Note that RPM models can explain both the LF QPOs in BH LMXBs as due to LTP, and the kHz QPOs in NS LMXBs as due to the periastron precession frequency, without assuming the existence of a NS surface. However, it is important to mention that the theoretical explanation of the QPO frequencies in both BH and NS LMXBs that were discussed above, are done on a qualitative level. For a more in-depth discussion of variability in NS LMXBs and the relevant models, we refer the reader to the review by Méndez & Belloni (2021). In the next section we discuss how one can quantitatively study the spectral-timing properties of QPOs in LMXBs, which is the main topic of this thesis.

1.3 Thermal Comptonisation in the presence of variability and feedback

1.3.1 Observed variability and Comptonisation

As we have already discussed, Comptonisation is contributing significantly to the high energy part of the observed energy spectra, and is required for modelling the latter. Furthermore, we have an increasing number of spectral-timing analyses, both for BH and NS LMXBs, that agree on the fact that the high-energy part of the spectrum is oscillating with a larger amplitude than the low-energy part. In other words, the fractional rms is an increasing function of energy for QPOs at many different frequencies (Sobolewska & Życki 2006; Strohmayer et al. 1996; Berger et al. 1996; Zhang et al. 1996; Méndez et al. 1997; Troyer et al. 2018). In particular, the high-energy part of the spectrum can oscillate with an amplitude (20%) that is four times more than the amplitude of the low-energy part (5%) in certain sources and at certain spectral states. Nevertheless, the high fractional rms at high energies where Comptonisation is significant, if not dominant, necessarily connects the QPO properties to those of the Comptonising medium, no matter what the geometry of the latter is. Finally, we must mention that the fractional rms is not only an increasing function of energy but, depending upon QPO centroid frequency, it can increase at a different rate (i.e. different slope at different QPO frequencies) as has been shown by Ribeiro et al. (2019).

Unavoidably, due to the complex nature of the Fourier transform, which we use to extract the fractional rms at different energies, the same transform also provides a measure of the phase lag at each energy band. Comparing the phase lags at different energy bands to some reference energy band, will yield the energy-dependent time lag for a QPO. In other words, while extracting fractional rms as a function of energy we also extract the time lag of every energy band with respect to a reference one, at the same time. If one had to qualitatively guess what the shape of the energy-dependent time lag should be in the context of Comptonisation, a fair answer would be that the time lags are increasing with energy. The latter is because it takes more up-scatterings, thus more time, for the high energy photons to acquire their energy compared to lower energy photons that escape faster with fewer scatterings. Interestingly, spectral-timing analyses in both BH and NS LMXBs (e.g. van den Eijnden et al. 2017 and Troyer et al. 2018) have shown that the observed time lags as a function of energy, in a variety of sources, sometimes increase monotonically (hard lags), sometimes decrease monotonically (soft lags), and sometimes are consistent with a zero slope (flat lags). We show this alternating behaviour of the energy-dependent time lags in Figures 1.12 and 1.13 for NS and BH

LMXBs, respectively.

Figure 1.12 – Typical energy-dependent time-lags for the lower kHz QPO in various NS LMXBs. Figure taken from Troyer et al. (2018).

Figure 1.13 – Typical energy-dependent time-lags for the type-C QPOs in various BH LMXBs. Figure taken from van den Eijnden et al. (2017).

Furthermore, in certain sources the time lags can even switch from hard to soft as the centroid frequency of the QPO changes (Reig et al. 2000; Qu et al. 2010; Pahari et al. 2013; Zhang et al. 2020) and they often do so at some characteristic QPO frequency. From all the observational facts stated above, it becomes clear that both fractional rms and time-lags show a complex behaviour as a function of both energy and QPO centroid frequency. Bearing in mind that our discussion on variability focuses only on either LF or kHz QPOs, one can start thinking of how to connect that variability with the Comptonisation process.

1.3.2 Steady state Comptonised spectrum

Comptonisation can be reduced to studying individual scattering events between photons and electrons in a Monte Carlo framework, where one would calculate the change in energy and direction of a photon, with respect to an observer's frame of reference, after each scattering. Thanks to Kompaneets (1957), we can also study Comptonisation via the Kompaneets equation which is a photon kinetic equation. The Kompaneets equation allows us to study how the photon occupation number of an initial photon number density changes with time as that density travels inside a hot corona and photons scatter multiple times. In simpler terms, if one starts at time $t_0 = 0$ with a photon number density $n_\gamma(E)$ (initial shape of an energy-spectrum), the Kompaneets equation tells us what the shape of the photon number density will be at any future time t (i.e. the shape of the spectrum at a later time). The Kompaneets equation can be written (Psaltis & Lamb 1997; Lee & Miller 1998; Kumar & Misra 2014)

as:

$$t_c \frac{\partial n_\gamma}{\partial t} = \frac{1}{m_e c} \frac{\partial}{\partial E} \left(-4kT_e E n_\gamma + E^2 n_\gamma + kT_e \frac{\partial}{\partial E} (E^2 n_\gamma) \right) + t_c \dot{n}_{s\gamma} - t_c \dot{n}_{esc}, \quad (1.4)$$

where t_c is the Thomson collision time scale (i.e., the mean time between two consecutive collisions), n_γ is the photon number density inside the medium (with units $m^{-3}keV^{-1}$), $\dot{n}_{s\gamma}$ is the seed photon rate, $\dot{n}_{esc}$ is the photon escape rate (number of photons per unit volume that escape per unit time), and T_e is the electron temperature in the assumed corona. One can immediately notice several things about equation 1.4. First of all, it is a partial differential equation in time, t , and energy, E , and so with correct treatment it can reproduce both lightcurves and energy spectra. Secondly, equation 1.4 needs to be solved under the assumption that the medium in which Comptonisation occurs is finite in size, and so a loss term $\dot{n}_{esc}$ is required that will depend on the geometry of that medium. Additionally, equation 1.4 describes how an input photon density is affected when passing through a medium with electrons of temperature T_e , but it does not account for the cooling i.e. losses in that temperature, experienced by the free electrons (constant electron temperature is assumed).

The simplest possible solution to equation 1.4 can be derived via setting $\frac{\partial n_\gamma}{\partial t} = 0$ and getting the steady state solution (SSS), which gives the shape, $n_{\gamma,0}(E)$, of the spectrum emerging from a finite medium at infinite time from when the process started. More specifically, solving for the SSS reduces to solving a second order ordinary differential equation (after some manipulation), as a two point boundary value problem (assuming that the photon occupation number, due to Comptonisation, is zero at high enough and low enough energies). However, to proceed in solving the above problem we need to have an analytic expression for the photon seed and loss terms in equation 1.4. A simple enough photon seed term, $\dot{n}_{s\gamma}$, is that of a blackbody which would be adequate for applications on NS LMXBs (assuming the seed comes from the NS surface) and a rough approximation for the case of BH LMXBs. On the other hand, the photon loss term, $\dot{n}_{esc}$, is more tricky since we need to make some geometrical assumptions about the corona. Obviously, the simplest way to model the corona around a spherical blackbody seed source, is as a spherically symmetric envelope of width L , surrounding the blackbody. The blackbody, with temperature T_s , injects soft photons in the corona with an initially constant rate, $\dot{n}_{s\gamma}$, and then the corona up scatters these photons to higher energies, assuming that the corona is much hotter than the blackbody. The photons that escape the aforementioned configuration constitute the output spectrum. It can be shown, via random walk arguments that a photon initially emitted at a distance L with respect to some boundary from which it

can escape, in a homogenous medium with electron number density n_e where isotropic Thomson scattering with electrons occurs, will escape after an average number of scatterings $N_{esc} \sim \tau^2$, where τ is the corona optical depth defined as $\tau = n_e \sigma_T L$, and σ_T is the Thomson scattering cross section. In the limit of small optical depth $N_{esc} \sim \tau$ and so for back-of-the-envelope calculations one can assume $N_{esc} = \max(\tau, \tau^2)$. The elegance of spherical symmetry becomes apparent in the sense that every photon emitted by the surface of the spherical blackbody effectively "sees" the same number of electrons on its way out of the corona, and so N_{esc} will be the same along any radial direction (assuming that in the simplest case the spherical blackbody represents both the disc and the NS surface sufficiently well). In steady state we can assume that the probability that a photon escapes from the surface of our spherical corona per unit time is constant (otherwise there is already variability introduced), and so the escape probability density, p_{esc} , can be calculated by taking the integral of p_{esc} over the average time interval that a photon needs to escape, t_{esc} , which equals to 1 since the photon will, on average, escape after that time. Thus we can write $p_{esc} = 1/t_{esc} = 1/(N_{esc} t_c)$, where $t_c = L/(c\tau)$ is the average time between two consecutive scatterings. And so the escape probability of photons in the spectrum per unit time is $p_{esc} n_\gamma$. More accurate formulae for the escape probability can be derived by solving the photon diffusion equation for different seed source spectra and boundary conditions (Sunyaev & Titarchuk 1980). Analytical and numerical frameworks that deal with calculating the SSS of the Kompaneets equation have existed for years (Sunyaev & Titarchuk 1980; Zdziarski, Johnson & Magdziarz 1996; Becker 2003; Zdziarski et al. 2020), and models based on those are being extensively used to fit observed energy spectra of XBs in the literature. However, we are interested in explaining the variability imprinted on the SSS and so, in what follows, we assume that one can easily solve for the SSS, $n_{\gamma,0}$, and then build upon that.

1.3.3 Variable Comptonised spectrum

As we mentioned already, equation 1.4 is time-dependent and so once the source of variability, i.e. the QPO, is defined, one can calculate the lightcurve at any time t by solving equation 1.4 in discrete time steps. Variability can be incorporated by assuming any origin for the QPO, such as LTP of the inner disc, temperature oscillations in the disc or NS, temperature oscillations in the corona temperature and even oscillations in the optical depth. All of the aforementioned processes will introduce some form of variability upon the SSS. However, solving equation 1.4 at discrete times is not the most efficient approach, especially when the goal is to create a model that can easily be compared to observations. For that purpose, Lee & Miller (1998) proposed to write the time-dependent solution of equation 1.4 as:

$$n_\gamma(t, E) = n_{\gamma,0}(E)(1 + \delta n_\gamma e^{-i\omega_{qpo}t}), \quad (1.5)$$

which is a sum of the SSS, $n_{\gamma,0}$, and a time dependent part that oscillates coherently at the QPO frequency, ν_{qpo} , with $\omega_{qpo} = 2\pi\nu_{qpo}$ and with a complex fractional amplitude δn_γ that is energy-dependent. The latter expression simply states that since the observed spectrum does have a quasi-periodic variability at the frequency ν_{qpo} , then it only makes sense to write the solution of equation 1.4 as such. An equally valid assumption is that if the photon number density is oscillating while scattering in the medium, then the electron temperature, T_e , can also be oscillating coherently at the same frequency of the QPO, assuming equilibration in the corona (which depends on the electron number density of the corona) happens faster than the QPO frequency. Based on the above, the time-dependent temperature of the corona, T_e , and seed photon source, T_s :

$$T_e(t) = T_{e0}(1 + \delta T_e e^{-i\omega_{qpo}t}), \quad (1.6)$$

$$T_s(t) = T_{s0}(1 + \delta T_s e^{-i\omega_{qpo}t}), \quad (1.7)$$

By replacing n_γ , T_e , T_s in equation 1.4 with their assumed expressions from equations 1.5, 1.6 and 1.7, respectively, the time dependent term $e^{-i\omega_{qpo}t}$ disappears (because all quantities are oscillating at the QPO frequency) and one can get a new second order differential equation for δn_γ which is the complex fractional amplitude of the SSS. The new, perturbed, differential equation for δn_γ can be solved numerically as a two-point boundary value problem, since photon energy is the only independent variable. Since δn_γ is the complex fractional amplitude of the variability, its modulus and argument will give the fractional rms and phase lag, respectively. The added difficulty in the case of the perturbed equation is the existence of the terms δT_e and δT_s . In the initial study, Lee & Miller (1998) assumed that the original oscillation (QPO) should come from one of the quantities T_e , T_s , or the optical depth τ , which can be written as $\tau(t) = \tau_0(1 + \delta\tau e^{-i\omega_{qpo}t})$, and so these authors tried to explore which oscillation was more likely to explain the observed energy-dependent rms and time-lags, in the case of the lower kHz QPO in the NS LMXB 4U 1608–52. For simplicity, Lee & Miller (1998) treated the amplitudes δT_e , δT_s and $\delta\tau$ as free parameters. Unfortunately, the measured energy-dependent time-lags of 4U 1608–52 were erroneously presented as hard lags (Vaughan et al. 1997), thus leading the model of Lee & Miller (1998) to find an oscillation in the optical depth as the possible origin of the QPO. After establishing that the time-lags of the lower kHz QPO in 4U 1608–52 were in fact soft (Vaughan

et al. 1998), a follow up work from Lee, Misra & Taam (2001) used the same framework for solving the perturbed Kompaneets equation, as described above, but introduced the idea of feedback, which is defined as the fraction, η , of up-scattered photons that can impinge back onto the seed photon source and cause both heating and an oscillation in its temperature, even if an oscillation did not already exist. With the addition of feedback, which was treated as a coupling between δT_e and δT_s dependent upon η , Lee, Misra & Taam (2001) were able to explain both the energy-dependent fractional rms and the soft lags assuming that the QPO manifests itself as an oscillation in the corona temperature and, through feedback, induces an oscillation in the seed photon source. These authors found that a large feedback ($\eta \sim 0.5$) was required to satisfactorily explain the observations, while the spherically symmetric corona (envelope) should have a width of about 5km. A comparison of the model of Lee, Misra & Taam (2001) to the observations of the fractional rms and time-lags of the NS LMXB 4U 1608–52 is shown in Figure 1.14 (taken from the original publication).

Figure 1.14 – Fractional rms and phase lag spectrum for the neutron star 4U 1608-52. The Figure is taken from Lee, Misra & Taam (2001), where the solid and dashed lines are predictions of their model.

A few years later Kumar & Misra (2014), following the original idea from Lee & Miller (1998), solved the perturbed (or linearised) Kompaneets equation by self-consistently taking into account Compton cooling and feedback onto the soft source. In the work of Kumar & Misra (2014), Compton cooling was accounted for in terms of the first law of thermodynamics, by assuming that a change in the internal energy of a gas (the corona in our case) in an infinitesimal time should be equal to the energy lost via Compton cooling plus the energy

gained through some external heating at that time,

$$\frac{3}{2}k\frac{\partial T_e}{\partial t} = \dot{H}_{ext} - \langle \Delta \dot{E} \rangle , \quad (1.8)$$

where $\dot{H}_{ext}$ is the external heating per particle per unit time offered to the corona and $\langle \Delta \dot{E} \rangle$ is the Compton cooling rate per electron, given by:

$$\langle \Delta \dot{E} \rangle = \int_{E_{min}}^{E_{max}} (4kT_e - E) \frac{E}{m_e c} n_\gamma \sigma_T dE , \quad (1.9)$$

where m_e is the electron rest mass and k the Boltzman constant. A natural consequence of the energy balance in the corona imposed by equation 1.8 is that the external heating rate, $\dot{H}_{ext}$, should also oscillate; Kumar & Misra (2014) assumed that it also oscillates coherently with the QPO, and so one can write:

$$\dot{H}_{ext}(t) = \dot{H}_{ext,0}(1 + \delta \dot{H}_{ext} e^{-i\omega_{qpo}t}). \quad (1.10)$$

The oscillation of the external heating rate, $\dot{H}_{ext}$, at the QPO frequency is not a necessary assumption, but it greatly simplifies the math. Equations 1.8 and 1.9 can be combined and after substituting T_e , n_γ and $\dot{H}_{ext}$ from the approximations given by equations 1.6, 1.5 and 1.10 one can solve for δT_e , which becomes a rather complex expression (see Appendix 2.A.2). A complication that arises from calculating δT_e is that the differential equation that we need to solve for δn_γ contains δT_e explicitly, and in turn δT_e contains integrals of δn_γ , which is what we are trying to solve for. The situation is the same for δT_s , which appears in the seed source term, and that is because if we want to take feedback self-consistently into account we need to find an expression for the blackbody (the assumed seed photon source) luminosity that includes feedback. Kumar & Misra (2014) assumed that the blackbody luminosity can be written as a sum of a constant luminosity of an initial and non-oscillating blackbody (before feedback had acted upon it) and an additional flux term that the initial blackbody receives per unit time from feedback photons. That luminosity is written as:

$$4\pi a^2 \sigma T_s^4 = 4\pi a^2 \sigma T_{s,init}^4 + \eta V_c \int \frac{n_\gamma}{t_c N_{esc}(E)} E dE , \quad (1.11)$$

where a is the radius of the blackbody used as seed, σ the Stefan-Boltzman constant, V_c the volume of the spherically symmetric shell that represents the

corona around the seed, and $T_{s,init}$ the temperature of the initial constant blackbody. Once T_s and n_γ are substituted from equations 1.7 and 1.5, respectively, the same complication will arise since the resulting δT_s will be an integral function of δn_γ . To deal with the issue of δT_e and δT_s being integral functions of δn_γ , Kumar & Misra (2014) designed a computational scheme where one has to make some initial assumption for the values of δT_e and δT_s , then solve the perturbed Kompaneets equation numerically for δn_γ . After obtaining an initial solution for δn_γ , one can use it to calculate δT_e and δT_s which can then be used to update δn_γ . The latter process is repeated iteratively until the amplitudes δT_e and δT_s have converged. By implementing the aforementioned computational scheme, Kumar & Misra (2014) explained the energy-dependent fractional rms and time-lags of the lower kHz QPO in the NS LMXB 4U 1608–52, by assuming that the QPO appears as an oscillation, $\delta \dot{H}_{ext}$, of the external heating rate, $\dot{H}_{ext}$, that causes an oscillation δT_e in the corona temperature, T_e , which through IC scatterings imprints an oscillation in the spectrum, n_γ , that "travels" in the corona and finally, assuming that part of that spectrum returns to the seed source, there is an additional oscillation, δT_s , imprinted on the seed source temperature, T_s , through feedback. Similarly to Lee, Misra & Taam (2001), Kumar & Misra (2014) found that a large feedback and small corona size can explain the data sufficiently well.

The existence of feedback, η , introduced by equation 2.12, was the key that allowed variable Comptonisation models to explain the observed soft lags. In the context of a variable Comptonisation model with feedback, one can qualitatively explain the soft lag as follows: The soft photons that the observer receives at say 0.5-1 keV, are not entirely those that come from the original blackbody seed source (which could have a much lower temperature initially), but also photons that were emitted from the "final" blackbody after the time needed to heat it up via feedback. In other words, the seed blackbody would not be able to produce as many photons in the range 0.5-1 keV had it not been for high-energy photons going back to heat it, but that takes some finite time to happen. In fact, the high feedback suggested by the data implies that the heating of the initial seed blackbody, through feedback photons, is quite significant. Note that the observed photons between 0.5-1 keV, mentioned above as an example, can be either very hot photons from the seed source (right tail of its Planck spectrum) that leave the corona with little or no scatterings off the electrons, slightly colder seed photons (e.g. 0.1-0.3 keV) that escape with a few scatterings, or very cold seed photons that experience an immense number of up-scatterings. In any case, if feedback photons make up a sufficient amount of the seed source luminosity, a delay can exist between a population of high energy photons that, after a few scatterings, leave the corona and another population of low energy photon that will only be emitted, in quantities significant

to be detected, after the seed source is heated up through feedback. The delay will be of the order of the escape (or diffusion) time t_{esc} . In practise, since the energy dependent lightcurve is represented by equation 1.5, one simply has to transform the phase-lags that come from the term δn_γ into time-lags at distinct energies or energy bands and compare them to each other, after choosing one as reference, to produce the expected energy-dependence of the time-lag. For completeness, we mention that the high feedback values suggested by both Lee, Misra & Taam (2001) and Kumar & Misra (2014) are physically acceptable as was later confirmed via Monte Carlo simulations (Kumar & Misra 2016a) of Comptonisation in various assumed corona geometries around the NS.

The authors of the original idea of the variable Comptonisation model (Lee & Miller 1998; Lee, Misra & Taam 2001; Kumar & Misra 2014), have all speculated more or less on what the origin of the variability, i.e. the QPO, should be. However, the approach of linearising the Kompaneets equation with respect to some perturbations in some physical quantities can be viewed as a linear stability analysis problem rather than an explanation for the origin of the QPO. We do not disagree that an oscillation in any of the physical quantities T_e , T_s , τ or $\delta \dot{H}_{ext}$ can lead to an oscillation of the output spectrum n_γ , but since such oscillation does not occur self-consistently it is safer to remain agnostic about the dynamical origin of the QPO frequency. Equivalently, one can think backwards and start from the fact that there is indeed an oscillation in the output spectrum, n_γ . That oscillation will cause an oscillation in the corona temperature, T_e , which implies an oscillation of the heating rate, $\dot{H}_{esc}$, and also an oscillation of the seed source temperature, T_s , through feedback. In that sense, we can simply ignore the dynamical origin of the QPO and still go ahead and study the physical and geometrical properties of the accretion flow that hosts that QPO, through the observed fractional rms and time-lags. Thus, in this thesis we have adopted an agnostic approach as to what the origin of the QPO is, and by designing a new version of the variable Comptonisation model with an improved numerical scheme, we have been able to efficiently fit observed data of both NS and BH LMXBs and constrain physical and geometrical properties of the accretion flow.

1.4 An overview of the thesis

In this thesis, I have developed a model for variable Comptonisation, to study the geometrical and physical properties of both the corona and seed photon source in NS and BH LMXBs.

In Chapter 2 we present a new mathematical framework that allows efficient numerical solution of the linearised Kompaneets equation, which describes variable Comptonisation. Our approach can easily be implemented into a model that can be fitted to the energy-dependent fractional rms and time-lags of QPOs. We apply our model to the lower kHz QPOs in the NS LMXB 4U 1636+53, and find that a small corona of a few km in width, with high optical depth, can explain the observed lag- and energy-spectra. We also show that the temperature of the corona is oscillating with significantly higher amplitude than that of the seed photon source, indicating that the lower kHz QPO in 4U 1636+53 originates in a small corona.

In Chapter 3 we apply our model to the BH LMXB GRS 1915+105. Although our model was initially developed for NS LMXBs, because it assumes a black-body as a seed photon source, the results in the case of GRS 1915+105 were surprisingly good. In particular, we fitted our model to energy-dependent fractional rms and time-lags of the type-C QPO in this source at multiple different type-C QPO frequencies. Our results imply that as the frequency of the type-C QPO in GRS 1915+105 decreases (and also luminosity decreases), the corona (which is assumed to be spherical) decreases in size from around $100 R_g$ to a few tenths of R_g , until the type-C QPO frequency reaches ~ 2 Hz. Below 2 Hz (where luminosity decreases further) the corona increases abruptly in size to a few hundreds of R_g . In the context of our model the decrease in the corona size is connected to a decrease in feedback efficiency. Finally, we speculate that the abrupt increase in the corona size might be related to a potential change in the geometry of the corona and the appearance of a jet, as indicated by the enhanced radio emission at very low type-C QPO frequencies.

In Chapter 4, inspired by our findings in Chapter 3 that the corona is undergoing significant changes in size, we have extended an existing spectral model to include the effect of a variable corona size. We discuss how the formula for the seed photon source should change to accommodate for a spherical corona covering only part of the accretion disc. We show that our approach re-introduces physical parameters that were previously incorporated into normalisation constants. Finally, by simulating energy spectra for different values of the newly introduced parameters, we show that significant differences in the shape of the spectra are expected at energies below 1 keV.

In Chapter 5 we analyse ~ 1800 observations of the BH LMXB GRS 1915+105

with RXTE that span almost sixteen years, which were accompanied by daily observations of the radio flux at 15 GHz with the Ryle telescope. Although the radio data were not available throughout the full sixteen years, we were able to cross match 410 observations in which a type-C QPO was prominent in the PDS, and at the same time radio data were available within a few days. By analysing those 410 observations, we arrive at the conclusion that as the type-C QPO frequency, repeatedly over the course of many years, decreases from ~ 6 Hz to ~ 0.4 Hz the radio flux systematically increases, and the implied corona temperature systematically decreases. Furthermore, we know from Chapter 3 that the dependence of feedback efficiency upon type-C QPO frequency, suggests a change in the geometry of the corona when the QPO frequency crosses below ~ 2 Hz. Based on the above, we propose that this anti-correlation between radio intensity and corona temperature is due to a potential change in the structure of the magnetic field. The magnetic field allows the corona to exist in the form of a hot extended cloud enshrouding the accretion disc at one time, but at a different time channels the matter into an outflowing jet that emits strongly in radio wavelengths. The latter proposition is supported by additional findings that we present, such as the change in how the flux of the iron-K line is correlated to the total observed X-ray flux below and above a QPO frequency of ~ 2 Hz, and also the fact that there is significant flaring activity in the radio only when the QPO frequency crosses below ~ 2 Hz.

The Comptonising medium of the neutron star in 4U 1636–53 through its lower kilohertz Quasi-Periodic Oscillations

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Submitted 11 November 2019, Accepted 6 December 2019

Published in Monthly Notices of the Royal Astronomical Society, volume 492, pages 1399–1415, 2020.

Abstract

Inverse Compton scattering dominates the high energy part of the spectra of neutron star (NS) low mass X-ray binaries (LMXBs). It has been proposed that inverse Compton scattering also drives the radiative properties of kilohertz quasi periodic oscillations (kHz QPOs). In this work, we construct a model that predicts the energy dependence of the rms amplitude and time lag of the kHz QPOs. Using this model, we fit the rms amplitude and time lag energy spectra of the lower kHz QPO in the NS LMXB 4U 1636–53 over 11 frequency intervals of the QPO and report three important findings: (i) A medium that extends 1–8 km above the NS surface is required to fit the data; this medium can be sustained by the balance between gravity and radiation pressure, without forcing any equilibrium condition. (ii) We predict a time delay between the oscillating NS temperature, due to feedback, and the oscillating electron temperature of the medium which, with the help of phase resolved spectroscopy, can be used as a probe of the geometry and the feedback mechanism. (iii) We show that the observed variability as a function of QPO frequency is mainly driven by the oscillating electron temperature of the medium. This provides strong evidence that the Comptonising medium in LMXBs significantly affects, if not completely drives, the radiative properties of the lower kHz QPOs regardless of the nature of the dynamical mechanism that produces the QPO frequencies.

Keywords: stars: neutron — X-rays: binaries

2.1 Introduction

Kilohertz quasi periodic oscillations (kHz QPOs) represent the fastest variability observed in the X-ray light-curves of neutron star (NS) low mass X-ray binaries (LMXBs) to date. van Doesburgh, van der Klis & Morsink (2018), reported a QPO at 1267 Hz in the NS LMXB 4U 0614+09, which is the highest ever observed in such a system. A QPO is a narrow, albeit with finite width, peak in the power density spectrum (PDS) of a light-curve. See van der Klis (1989) for more information on the timing techniques. QPOs in LMXBs had been previously discovered at lower frequencies, however the timing resolution and large effective area of modern instruments made it possible to also detect them at kHz frequencies. Throughout this paper, we refer to the kHz QPOs when the QPOs appear between approximately 400 Hz and 1200 Hz. The history of kHz QPOs begins with the launch of the Rossi X-ray timing explorer (Bradt, Rothschild & Swank 1993, RXTE). The first detections and analyses of the newly discovered kHz QPOs are presented in van der Klis et al. (1996); van Paradijs et al. (1996); Strohmayer et al. (1996); Berger et al. (1996); Zhang et al. (1996) and Ford et al. (1997). The kHz QPOs in NS LMXBs, that were reported in the aforementioned papers, usually consist of a lower and an upper QPO, separated by a few hundreds of Hz. Later, work done by Wijnands et al. (1997); Wijnands & van der Klis (1997); van der Klis et al. (1997) and

Méndez et al. (1998); Méndez & van der Klis (1999); Zhang et al. (2006) shed more light into the existence of pairs of kHz QPOs, or twin as they are often called. For a detailed review on the first detections and characterization of kHz QPOs we refer the reader to van der Klis (1998). Theoretical models trying to explain the origin of the QPO frequencies date back to 1985, before kHz QPOs were even detected. Among the first attempts to link the low-frequency QPOs to a physical mechanism, was the beat frequency model, as described in Alpar & Shaham (1985), where the QPO frequency is a beat between the spin of the NS and the Keplerian frequency of the accreting material. Later, Hameury, King & Lasota (1985) connected the QPO frequency with the NS spin. More specifically, a magnetic field driven mechanism was thought to be able to produce bright spots on the NS surface, which rotate at the NS spin, causing brightness oscillations to manifest themselves as QPOs. Boyle, Fabian & Guilbert (1986), assumed for the first time a spatially resolved Comptonising medium, which we will hereafter refer to as the corona, whose optical depth was linked to the QPO frequency. In the model of Boyle, Fabian & Guilbert (1986) the corona can exist on top of an accretion disc. The optical depth of this corona has a spatial dependence and thus can be linked to a Keplerian frequency associated to a certain distance inside the corona. The optical depth is regulated by the incident flux, from the central object onto to the outer surface of the corona, and for the first time is linked to the QPO frequency. The aforementioned work models the intensity of a source as a function of a Keplerian frequency, and a fit to the source GX5-1 resulted in reasonable values for the spectral parameters. Another pioneering QPO model was introduced by Fortner, Lamb & Miller (1989), where the frequency of an oscillating radial flow of material is associated to radiation-hydrodynamic overstability that causes the observed QPOs. Shortly after the discovery of kHz QPOs, the statistical properties and dependence of these high-frequency oscillations upon observed physical quantities became too complex to be explained by the models that had been proposed to explain the low-frequency QPOs. Without going into detail, Klein et al. (1996); Kaaret, Ford & Chen (1997); Miller, Lamb & Psaltis (1998); Stella & Vietri (1998); Titarchuk, Lapidus & Muslimov (1998); Osherovich & Titarchuk (1999) and Zhang (2004), proposed a new generation of models that led to a more physically robust explanation of the observed kHz QPO frequencies, based on their latest observational properties (see van der Klis (2006) for a review of these models). At the same time as their discovery, a lot of attention was given to the energy resolved variability of kHz QPOs. By energy resolved variability, we mean the study of how the properties of the kHz QPOs, namely the fractional root mean square amplitude (hereafter rms) and time (or phase) lags, depend on photon energy. For a review on timing techniques, and derivation of rms and lags, we refer the reader to Uttley et al. (2014). The spectral analysis of a source, combined with the study of energy re-

solved variability has recently become famous as the spectral-timing approach. Strohmayer et al. (1996) found, for the first time, that the rms of the lower kHz QPO in the LMXB 4U 1728 – 34 increased with energy from 3%, at energies between 2-6 keV, to $\sim 9.5\%$ at 12 keV. Later, Berger et al. (1996) found that the rms of the $\sim 800\text{Hz}$ QPO in 4U 1608 – 52 increased with energy from 4% at 2 keV to 15% at 12keV, while at higher energies it remained constant within the uncertainties. Zhang et al. (1996) suggested that the high values of rms, of the kHz QPOs at high energies indicate that the QPOs originate either at the NS surface or the corona, since the temperature of both is higher than that of the disc. Méndez et al. (1997) carried out an interesting analysis of the absolute root mean square (RMS) in multiple energy bands for the source 4U 0614+09. More specifically, the absolute RMS in units of counts/(s keV) is a measure of the shape of the oscillating spectrum that would be on top of a non-varying component (e.g. a truncated accretion disc). Méndez et al. (1997) found that a Wien spectrum of temperature ~ 1.56 keV and radius of $\sim 500\text{m}$ (at a distance of 3 kpc) could explain the data. In the same work, the fractional rms spectrum is moderately well fitted by a black body spectrum with a $\sim 2.5\%$ variation in its temperature and by a Comptonised spectrum with a $\sim 5\%$ variation in its optical depth. Later, Revnivtsev, Gilfanov & Churazov (1999) introduced a technique called frequency resolved spectroscopy (FRS), in which the spectrum of the absolute RMS is constructed in separate frequency bands with the help of the energy resolved PDS. An application of the FRS technique was discussed by Gilfanov, Revnivtsev & Molkov (2003), who showed that the spectral shape of the absolute RMS, resolved around the QPO frequency for the sources GX 340 + 0 and 4U 1608 – 52, is in very good agreement with the average spectrum of the source if a disc blackbody component is subtracted from the energy spectrum. These authors claimed that the remaining energy spectrum, after subtracting the disc blackbody, matches that of the boundary layer. A simple Wien spectrum, representing the boundary layer, fits their data well, but they note that, depending on the spectral state of the source, taking into account thermal Comptonisation might improve the fits significantly. At the same time, the study of the energy-dependent time lags was added to the spectral timing approach. In particular, after the discovery that, in the lower kHz QPOs, the soft photons lag behind the hard ones, an effect that we will refer to as soft lags, by Vaughan et al. (1998); Kaaret et al. (1999); Markwardt, Lee & Swank (2000); de Avellar et al. (2013), the study of time lags became an important part of any comprehensive spectral-timing analysis. de Avellar et al. (2013), mentioned for the first time that the energy-dependent time lags of the upper kHz QPOs exhibit a behaviour significantly different to that of the lower kHz QPO, such that the hard photons lag behind the soft ones, producing the so called hard lags. The findings presented in de Avellar et al. (2013), suggest a different origin for the lower and the upper kHz QPO. Later, a detailed

spectral-timing analysis of the NS LMXB 4U 1728 – 34, done by Peille, Barret & Uttley (2015) revealed that although the rms increased with energy, both for the lower and the upper kHz QPOs, the time lags of the lower kHz QPOs were always soft whereas for the upper kHz QPO the time lags were mostly hard. These authors also fitted the RMS, or covariance, spectra of the sources 4U 1608 – 52 and 4U 1728 – 34 with a thermal Comptonisation model and obtained a very good fit. In agreement with de Avellar et al. (2013), Peille, Barret & Uttley (2015) concluded that the physical mechanisms driving the lower and upper kHz QPOs must be different and that the oscillating component in the source spectra, thus the QPO, must be associated with Comptonisation. A systematic study of multiple sources presented by Troyer et al. (2018), also verified the differences between upper and lower kHz QPOs. However, the question of whether the kHz QPOs originate in the accretion disc, the NS surface, the NS boundary layer, or in a surrounding Comptonising region remained unknown. The explanation of the dependence of the time lags and rms upon photon energy is still highly debatable. Predicting the energy dependence of timing properties such as time lags, due to Comptonization, dates back to Wijers, van Paradijs & Lewin (1987), who used a Monte Carlo simulation to calculate the Comptonization of a black-body spectrum inside a spherical homogeneous cloud, highlighting the importance of energy dependent time-delay measurements as a probe for both the source of soft photons and the properties of the Comptonizing cloud. Specific models trying to explain this dependence were introduced by Lee & Miller (1998), Lee, Misra & Taam (2001) and Kumar & Misra (2014). The models described in the aforementioned papers assume that the flux oscillation of the time averaged spectrum, that is modulated at the QPO frequency, is produced by an oscillation in the thermodynamic properties of a Comptonising region, hereafter the corona, or of the seed photon source, which can be either the NS surface, accretion disc, or both. More specifically, these thermodynamic properties are the temperature and external heating rate of the corona, the electron density of the corona and finally the temperature of the soft photon source that provides the seed photons for Comptonisation. Lee & Miller (1998) assumed that, if thermal Comptonization is responsible for the shape of the rms spectrum (Méndez et al. 1997; Gilfanov, Revnivtsev & Molkov 2003; Peille, Barret & Uttley 2015), then thermodynamic properties such as the heating rate and temperature of the corona and seed photon source should be also oscillating. The simplest case would be one in which the latter quantities oscillate exactly at the QPO frequency. The novel idea presented in Lee & Miller (1998) predicts the rms and time lags due to Comptonisation, of a seed photon source in a homogenous and spherically symmetric corona. Temperature oscillations of either the corona or the seed photon source, but also the effect of an oscillating electron density, are used to reproduce the rms and time lag spectra. The authors apply an empirical fit to rms and time lag

spectra of the source 4U 1608 – 52, with a model that uses a superposition of the three oscillations mentioned above, and conclude that a time varying coronal electron density is more likely to drive the observed behaviour. They also find a size of around 6 km for the, assumed spherical, corona, which reinforced the idea that the kHz QPOs are either produced or radiatively enhanced in a region close to the NS. The hard time lags of the source 4U 1608 – 52 initially reported in Vaughan et al. (1997), that were used by Lee & Miller (1998), were later on proven to have the opposite sign. After Vaughan et al. (1998) clarified that the time lags of 4U 1608 – 52 were actually soft, a second work by Lee, Misra & Taam (2001) assumed that an oscillation in the temperature of the corona, that produces the QPO, causes a delayed oscillation in the temperature of the seed photon source through feedback. By feedback we mean that a fraction of the photons Comptonised in the corona return to the seed photon source through random walks, and re-heat it. Because of this delayed response of the seed photon source, the low energy photons are expected to lag behind the hard ones, thus explaining the observed soft lags. However, in Lee, Misra & Taam (2001) the QPO amplitude is approximated as a linear combination of the amplitudes of the corona and seed photon source oscillations. This ad-hoc method is used in order to connect the two amplitudes (corona and seed photon source) through feedback and calculate one as a function of the other. This approach succeeds in explaining, at least qualitatively, the behaviour of the rms and time lags. However, the major disadvantage of this model is that the energy balance of the corona, through Compton cooling, is not taken into account and thus the amplitude of the temperature oscillations do not emerge as natural consequences of a self consistent physical process. A self consistent model, that takes into account the dynamic cooling of the corona and the feedback onto the soft photon source was proposed by Kumar & Misra (2014). Since the work presented here uses, and builds upon, the aforementioned model we will provide a detailed description of the model in the next section. Kumar & Misra (2016b) used this model for a preliminary study of the rms and lag energy spectra of the NS LMXB 4U 1608 – 52. The authors found that a small corona ($\sim 2\text{--}6$ km) and a significant amount of feedback (20–80%) are required to qualitatively explain the observed behaviour. Finally, Kumar & Misra (2016a) used a Monte Carlo method to estimate the expected amount of photons that would impinge back onto the seed photon source during Comptonisation. They simulated different corona geometries and concluded that in all cases a significant amount of feedback is justified. Although self consistent, the model of Kumar & Misra (2014) failed to predict the flattening of the rms energy spectrum above 12 keV that was observed by Berger et al. (1996) in the NS LMXB 4U 1608 – 52. Attempts to explain the aforementioned flattening were made before, initially by Miller, Lamb & Psaltis (1998) and later by Lee & Miller (1998), based on the assumption that the optical depth is oscillat-

ing at the QPO frequency. The assumption of an oscillating electron number density, and thus optical depth, allowed the model of Lee & Miller (1998) to explain the flattening of the rms energy spectrum better. However, to explain the soft time lags of 4U 1608 – 52 the model of Lee, Misra & Taam (2001) introduced the assumption of feedback and an oscillation in the temperature of the corona instead of an oscillating optical depth. Particularly interesting is the NS LMXB 4U 1636 – 53, for which spectral analyses date back to 1986 (see Vacca et al. 1986; Hirano et al. 1987 and Damen et al. 1990). Recently, Ribeiro et al. (2019) showed that the rms energy spectrum of the NS LMXB 4U 1636 – 53 depends highly on the lower kHz QPO frequency and, on average, decreases at energies above 12 keV, a fact that might present an interesting challenge for all of the aforementioned models. Evidence that the properties of the corona undergo changes was already given by Barret (2013). In the aforementioned work the optical depth, corona temperature and seed photon temperature, of the NS LMXB 4U 1608 – 52, appear to have a dependence on QPO frequency. A similar dependence was reported, for the NS LMXB 4U 1636 – 53, in Zhang et al. (2017) and Ribeiro et al. (2017). Furthermore, phase resolved spectroscopy of type B QPOs in the black hole X-ray binary GX 339–4 and of kHz QPOs in the NS LMXB 4U 1608–52, presented in Stevens & Uttley (2016) and Stevens, Uttley & Altamirano (2018), respectively, revealed that the corona temperature and seed photon source temperature oscillate at the QPO frequency. Preliminary results (Stevens, Uttley & Altamirano 2019) indicate that the properties of the corona and soft photon source, apart from oscillating coherently at the QPO frequency, show systematic time lags between them. All of the the above results prove the importance of linking the variability of the photon number density (QPO) to the spectral and geometric properties of the corona. In the present work we reproduced the model of Kumar & Misra (2014) with minor changes in the physical assumptions and a basic change of the solving scheme. These changes are discussed in section 2, with the help of Appendix A. In section 3, we discuss the process of fitting our model to a set of frequency resolved rms and time lag spectra of the lower kHz QPO, in the NS LMXB 4U 1636 – 53 (Ribeiro et al. 2019). Finally, in sections 4 and 5 we summarize our results and discuss our findings on the magnitude of the measured quantities and their dependence upon frequency.

2.2 The model

In this section we give a brief description of the model presented in Kumar & Misra (2014) alongside the changes introduced by us. The model assumes that the surface of a neutron star of radius a and temperature T_s , injects photons inside a spherically symmetric medium of width L , consisting of highly

energetic electrons, the so called corona, centered at the NS, with a rate $\dot{n}_{s\gamma}$. The photons are inverse Compton scattered and, since the medium is finite, after a photon of energy E undergoes certain number of scatterings it will escape the medium with an energy $E' > E$. The escape rate per unit volume is $\dot{n}_{esc}$. This symmetry, and the assumption that the density of the corona is uniform, simplifies the definition of the (Thomson) optical depth, which reduces to $\tau_T = \sigma_T n_e L$, where σ_T is the Thomson cross section and n_e the electron number density of the plasma. Schulz & Wijers (1993) showed that theoretical spectra constructed with a simple geometry, such as the one described above, can yield good fits to the data.

The time dependent evolution of the spectrum, $n_\gamma(E, t)$, that undergoes multiple inverse Compton scatterings is described by the Kompaneets equation (Kompaneets 1957). We use the form of the equation that was first presented in Psaltis & Lamb (1997), then used by Lee & Miller (1998), Lee, Misra & Taam (2001) and later by Kumar & Misra (2014). This equation can be written as:

$$t_c \frac{\partial n_\gamma}{\partial t} = \frac{1}{m_e c^2} \frac{\partial}{\partial E} \left(-4kT_e E n_\gamma + E^2 n_\gamma + kT_e \frac{\partial}{\partial E} (E^2 n_\gamma) \right) + t_c \dot{n}_{s\gamma} - t_c \dot{n}_{esc} , \quad (2.1)$$

where $t_c = L/c\tau_T$ is the Thomson collision time scale, T_e the temperature of the medium, k the Boltzmann constant, m_e the electron rest mass and c the speed of light. Assuming that the soft photon source emits like a blackbody, the photon injection rate, $\dot{n}_{s\gamma}$, is

$$\dot{n}_{s\gamma} = \left[\frac{3a^2}{[(a+L)^3 - a^3]} \right] \left(\frac{2\pi}{h^3 c^2} \frac{E^2}{e^{\frac{E}{kT_s}} - 1} \right) , \quad (2.2)$$

where h is the Planck constant and a the NS radius. We used the same notation and symbols as in Kumar & Misra (2014) so far in order to avoid confusion and built upon their model in a coherent way. Equation (2.1) is subject to specific assumptions, some of which will be summarised now for the sake of clarity. First of all, a uniform and isotropic distribution of electrons and photons is assumed. We thus ignore the spatial dependence of the photon phase-space density and also ignore any effects of electron bulk motion inside the plasma. The seed photon source is the NS surface which, combined with the fact that the medium is a spherically symmetric shell, means that each photon is emitted at equal distance from the surface of a sphere with radius $L + a$.

We further assume that photons escape the medium at a constant rate, $\dot{n}_{esc}$. According to equation (2.1), the number density of photons with energy between E and $E + dE$ at a given time is $n_\gamma(t, E)$. After production, a photon undergoes a number of scatterings, N_{esc} , before it escapes from the medium. Assuming that each scattering is an independent event, we can assign an escape probability per scattering, $P_{esc} = 1/N_{esc}$. For N_{esc} we used the same formulation as in Lightman & Zdziarski (1987), which is based on the solution of the photon diffusion equation, under the assumption that the intensity of the source inside the sphere is sinusoidally distributed with radius, as presented in Sunyaev & Titarchuk (1980). Using the Klein-Nishina optical depth, τ_{KN} , we can write,

$$N_{esc} = \tau_T + \frac{1}{3}\tau_T\tau_{KN}(E)f(E), \quad (2.3)$$

where

$$f(E) = \begin{cases} 1 & \text{if } E \leq 0.1m_e c^2 \\ (1 - \frac{E}{m_e c^2})/0.9 & \text{if } 0.1m_e c^2 < E < m_e c^2 \\ 0 & \text{if } E \geq m_e c^2 \end{cases} \quad (2.4)$$

and given the Klein-Nishina cross section, $\sigma_{KN}(E)$, the Klein-Nishina optical depth is

$$\tau_{KN}(E) = \frac{\tau_T \sigma_{KN}(E)}{\sigma_T}. \quad (2.5)$$

In order to calculate $\dot{n}_{esc}$, which according to equation (2.1) has units of photons per unit volume per unit energy per unit time, we must multiply $n_\gamma(E, t)$ by the escape probability density function, which is assumed as a constant number that represents probability per unit time. Thus, based on the fact that the average time a photon takes to escape the medium is $t_c N_{esc}$, the escape probability per unit time is $\frac{1}{t_{esc} N_{esc}}$ or $\frac{c\tau_T}{LN_{esc}}$ so that

$$\dot{n}_{esc} = \frac{c\tau_T n_\gamma(t, E)}{LN_{esc}} = \frac{c}{L} \frac{n_\gamma(t, E)}{\left[1 + \frac{1}{3}\tau_{KN}(E)f(E)\right]} \quad (2.6)$$

Equation (2.6) is the same as equation (21a) of Lightman & Zdziarski (1987). We must note that in Lightman & Zdziarski (1987) the authors include a relativistic correction term, $\omega(x)$, in equation (22a) of their paper. Given

that the majority of the spectral analyses presented in the literature use *nthcomp* (Lightman & Zdziarski 1987; Zdziarski, Johnson & Magdziarz 1996) to model thermal Comptonization, we want our model for n_γ to resemble that of *nthcomp* as much as possible for a given set of spectral parameters. Therefore, in our approach for $\dot{n}_{esc}$ we include Klein-Nishina corrections and the same form of N_{esc} that is used in Lightman & Zdziarski (1987), because the aforementioned paper is the basis of the *nthcomp* model (Zdziarski, Johnson & Magdziarz 1996) in *XSPEC* (Arnaud 1996). To conclude with the assumptions, the electrons of the plasma are hot, but not relativistic ($kT_e \ll m_e c$) in our case, thus allowing for the use of the Thomson cross section, σ_T , for the scatterings. In spite of this, we use the Klein-Nishina cross section, σ_{KN} , to preserve generality throughout this work, but we note that using σ_T would not change any of the results of this work, because we mainly work with energies $E \ll m_e c^2$. The steady state solution ($\frac{\partial n_\gamma}{\partial t} = 0$, hereafter SSS) of equation (2.1) is the energy averaged spectrum, $n_{\gamma 0}$, i.e. the Comptonised continuum of the observed spectrum. Equation (2.1) is discretised and solved numerically as a boundary value problem in order to obtain $n_{\gamma 0}$. In Appendix 2.A we provide the reader with details on the solving scheme.

The model that we reconstruct in this work is based on the idea that a QPO is modeled as a small oscillation of the SSS. Regardless of whether the QPO is produced on the NS or accretion disc, or whether it is the result of oscillations of thermodynamical and physical properties of the corona, a perturbation of the SSS can be linked to a perturbation in any physical quantity that appears in equation (2.1). This concept was first introduced by Lee & Miller (1998). We can write the perturbation as $n_\gamma = n_{\gamma 0}(1 + \delta n_\gamma e^{-i\omega t})$, where ω is the mean frequency of the QPO and δn_γ the complex fractional amplitude of the QPO. We further assume that this perturbation can be linked to a perturbation in the temperature of the plasma, $T_e = T_{e0}(1 + \delta T_e e^{-i\omega t})$, that could be a product of an oscillating external heating rate, $\dot{H}_{ext}$, which we write as $\dot{H}_{ext} = \dot{H}_{ext0}(1 + \delta \dot{H}_{ext} e^{-i\omega t})$. The external heating rate is required in order to account for the observational fact that the cooling of the medium, through repeated scatterings, does not lead to an extreme drop in the temperature of the medium. Finally it is assumed that a fraction, η , of the Comptonised photons impinge back onto the surface of the NS causing an oscillation in its temperature, $T_s = T_{s0}(1 + \delta T_s e^{-i\omega t})$. In principle the NS, as a black body, could undergo inherent oscillations on its own, on top of the oscillations caused by this feedback loop.

All of the perturbations are written in the form $X_i = X_0(1 + \delta X_i e^{-i\omega t})$ so that δX_i is a fractional variation of the equilibrium value, X_0 , and thus dimensionless. We must also note that in the above, $\delta \dot{H}_{ext}$, δT_e , δT_s and δn_γ are complex quantities, while δn_γ is a function of energy. After linearising equation (2.1)

we can form a new differential equation for the complex amplitude. For the new variables $N = n_\gamma/n_c$ and $x = E/kT_e$ (see Appendix 2.A), the linearised form is:

$$\begin{aligned} \frac{d^2 \delta n_\gamma}{dx^2} = & -\frac{d\delta n_\gamma}{dx} - \frac{2}{N} \frac{dN}{dx} \frac{d\delta n_\gamma}{dx} + \frac{1}{N} \frac{\delta n_\gamma}{e^{xT_e/T_s} - 1} \\ & - \frac{ic_5 \delta n_\gamma}{x^2} + \delta T_e \left(\frac{2}{x^2} - \frac{1}{N} \frac{d^2 N}{dx^2} \right) - \\ & - \delta T_s \frac{1}{N} \frac{T_{e0}}{T_{s0}} \frac{x}{\left(e^{xT_{e0}/T_{s0}} + e^{-xT_{e0}/T_{s0}} - 2 \right)}, \end{aligned} \quad (2.7)$$

where $c_5 = \frac{\omega m_e t_e c^2}{kT_{e0}}$. The complex amplitudes δT_e , δn_γ and $\delta \dot{H}_{ext}$ are connected through the first law of thermodynamics, and δT_s is related to δn_γ through a feedback term added to the blackbody spectrum such that the energy balance of both the corona and seed photon source are taken into account (for more information see Appendix 2.A.2).

In Appendix 2.A.2, we show that δT_s , δT_e are integral functions of δn_γ but are also present in equation (2.7), which we need to solve in order to find δn_γ . The solution of equation (2.7), as presented in Kumar & Misra (2014), requires an initial assumption both for δT_e and also δT_s . The authors use the assumed values to calculate an initial solution for δn_γ , which they afterwards use to update the values of δT_e and δT_s . This process is repeated a few times until the temperature amplitudes converge. The final solution, δn_γ , after the required steps for convergence, is taken as the final QPO complex amplitude whose modulus, $|\delta n_\gamma(E)|$, and argument, $\tan^{-1} \frac{\text{Im}(\delta n_\gamma)}{\text{Re}(\delta n_\gamma)}$, will translate directly into rms and phase lag respectively. If the external heating rate is assumed variable, then $\delta \dot{H}_{ext}$ can be factored out of equation (2.7) and its modulus can be used as a normalization for $|\delta n_\gamma|$, which is the rms. Thus, the argument, or phase, of $\delta \dot{H}_{ext}$ could be arbitrary. In Appendix 2.A.2 we discuss a different mathematical approach to solve for δn_γ , δT_e and δT_s without the need of a converging iterative scheme. Finally, we must note that although our numerical solution, δn_γ , is calculated at each energy value of the selected grid, the actual quantity that we compare to the data, eventually, is the ratio of the integrals $\int \delta n_\gamma n_{\gamma 0} dE / \int n_{\gamma 0} dE$ where the integration is performed within the selected energy band of the corresponding measurement.

2.3 Application to the lower kHz QPO of the NS LMXB 4U 1636 – 53

2.3.1 Data overview

Recently, Ribeiro et al. (2019) presented a detailed analysis of the timing properties of the lower and upper kHz QPOs in the NS LMXB 4U 1636 – 53. In particular, they studied the rms of both the lower and upper kHz QPO as a function of photon energy and frequency. In order to get a good signal to noise ratio for the rms measurements, Ribeiro et al. (2019) averaged RXTE observations with lower kHz QPOs, over narrow frequency intervals centered at frequencies that span from 530 Hz to 940 Hz. The observations that they averaged were taken at different times, but correspond always to the transition from hard to soft state (Zhang et al. 2017). Furthermore, de Avellar et al. (2013) studied the time lags as a function of energy, in 4U 1636 – 53, at different QPO frequencies and they concluded that the time lags of the lower kHz QPO were soft and did not depend on QPO frequency.

Ribeiro et al. (2019) found that, in all frequency intervals, the rms of the lower kHz QPO of 4U 1636 – 53 was an increasing function of energy up to 12 keV and then, in some cases, it decreased. The authors therefore refer to the critical energy of 12 keV as the break energy, E_{break} . In the same work it was also noted that the slope of the increasing ($E < E_{break}$) part, of the rms as a function of energy, initially increased with increasing QPO frequency, until a critical QPO frequency of ~ 770 Hz while for higher frequencies the slope decreased. The frequency at which the slope is maximum coincides, more or less, with the frequency at which the energy averaged rms is also maximum. Ribeiro et al. (2019) also report that the energy averaged rms, measured for the full energy band (nominally 2-60 keV), exhibits the same dependence with frequency as the slope of the rms versus energy for the lower kHz QPO. To conclude with the data overview, for every frequency interval, defined in Ribeiro et al. (2019), we fit the energy dependent rms, given in the aforementioned work, and the energy dependent time lags, given in de Avellar et al. (2013), which according to their results are same for each one of the selected frequency intervals.

2.3.2 Fitting the model

The model has eight free parameters, namely the size, L , of the corona, the amplitude of the external heating rate, $\delta\dot{H}_{ext}$, the feedback parameter, η , the corona temperature, T_e , the Thomson optical depth, τ_T , the seed photon source (NS here) temperature, T_s , the NS radius, a , and the frequency of the QPO, ν_{low} . The spectral parameters T_e, τ_T and T_s are the steady state values corresponding to the time averaged spectrum. In principle any fit of our model to rms and time lag spectra should fit the time averaged spectrum at the same time, so one would get a good constraint on the spectral parameters. In this work, since the rms and time lag spectra were derived by combining obser-

vations and since the time averaged model assumed in Zhang et al. (2017) is different than ours, we do not fix T_e, τ_T and T_s to the values reported in Zhang et al. (2017) and Ribeiro et al. (2017), but instead treat them as free parameters and expect to retrieve reasonable values for them. We kept the NS radius, a , fixed at 10 km in every frequency interval. We note that any value of the NS radius, in the 8-18 km range, does not change the results presented here except for the external heating rate, $\dot{H}_{ext0}$, which increases almost by a factor of 8 at $a = 18$ km.

Equation (2.12) is used to describe the total, blackbody, luminosity of the seed photon source as a superposition of a black body with an inherent temperature $T_{s,inh}$ and a feedback dominated term that is associated with an assumed fraction of photons that would fail to escape the medium and impinge back onto the NS increasing its temperature. For a set of spectral parameters T_s, T_e, τ_T and size L , if feedback dominates the luminosity of the soft component, we can assume the term associated with $T_{s,inh}$ on the right hand side of equation (2.12) is zero and solve for η to find the maximum feedback, η_{max} , that is allowed. Clearly, the value of η_{max} depends on the parameters (T_s, T_e, τ_T and size L) and, therefore, if we use η as a free parameter we would have to dynamically change its upper value (through a uniform prior distribution) based on the values of the other free parameters. Instead of using η directly, we define a feedback fraction, $f_\eta = \eta/\eta_{max}$, which now takes values between zero and unity, and use this as a free parameter, so that the model calculates η internally using f_η and the rest of the free parameters. Since we fit our model to the data of the energy dependent rms and time lag simultaneously, we define two different likelihood functions, one for the rms and one for the time lag. Assuming that the rms and time lag are independent measurements, we define our best fit as the set of parameters that minimize the product of the two likelihoods.

We use the python implementation of the affine-invariant ensemble sampler, introduced by Goodman & Weare (2010), as presented in Foreman-Mackey et al. (2013), to find the best-fitting parameters given the data. We used 200 walkers in the 6-dimensional parameter space of $T_e, T_s, \tau_T, \delta\dot{H}_{ext}, \eta$ and L , and allowed 1000 iterations, or walks, for each walker on average. The QPO frequency, ν_{low} , in our model was fixed to the mean of every frequency interval of the data. After performing a 20-30% burn-in on the total number of iterations, we noticed that in all frequency intervals the posterior distributions were bi-modal. In order to distinguish between the two modes we chose a pair of free parameters, that were different at each frequency interval, for which the two modes in their posterior were well separated. After a pair was chosen, we applied the Density Based Spatial Clustering Algorithm of applications with noise, DBSCAN, (Ester et al. 1996). After the clusters were detected in the

two dimensional density plot of a pair of parameters we assigned an ID to each member of each cluster. The IDs that we assigned correspond to the position in the parameter space of a certain walker at a specific iteration. By identifying the cluster, in any pair of parameters, and assigning IDs to their members, one can distinguish between the modes in the posterior of any parameter simply by isolating the IDs of a single cluster. We chose DBSCAN as a clustering algorithm for two reasons: The first, and most important, reason is DBSCAN's ability to deal with noise-like patterns, that contaminate the two dimensional density plot of a pair of two parameters, caused by the fact that at the end of the MCMC iterations not all of the walkers have converged necessarily. The second reason is that DBSCAN also works well in cases where the clusters are not of the same size, which also happens in our case. We took as the best-fitting value of each parameter the median of the corresponding posterior of each mode. The median was used because the posterior distributions were not strictly normal. Finally, we note that in the cases where the data value of the rms, in a certain energy band, was taken to be zero (due to a large uncertainty as specified in Ribeiro et al. (2019)), we excluded the particular data point from the fit. To get an estimate of the quality of our fits, we computed a common reduced χ^2 by combining the rms and time lag measurements at each frequency and for each one of the two modes, denoted by χ_m^2 . We used the general formula $\chi_m^2 = \frac{\sum_{i=1}^D (d_i - e_i)^2}{D-6}$, where $D-6$ is the total number of measurements of rms and time lag that were used in the fitting process minus the six degrees of freedom of our model, d_i and e_i are, respectively, the data and the model estimation. The subscript m denotes which mode of the posterior was used.

To summarise, at each frequency interval we have two best-fitting models, one for each one of the two modes of the posteriors. This bimodality arises from the fact that, although we use the time averaged spectrum to generate the rms and time lag spectra, we do not directly fit it. As we discuss in the next section, one of the two modes predicts a low temperature for the seed photon source, T_s , while the other one predicts a significantly higher temperature. Therefore, following Kumar & Misra (2016b) and Kumar & Misra (2016a), we will refer to one mode as the cold-seed model and to the other one as the hot-seed model.

2.4 Results

In Figures 2.1 and 2.2, we plot our best-fitting cold- and hot-seed models together with the data of 4U 1636 – 53 (Ribeiro et al. 2019) at each frequency interval. As we described in the previous section, we fit the model assuming that the external heating rate, $\dot{H}_{ext}$, is perturbed with a fractional amplitude $|\delta\dot{H}_{ext}|$. In table 2.1, we summarise the results of our model fit to every frequency interval and in Figure 2.3, we plot the best-fitting parameters versus

the frequency of the lower kHz QPO. In each panel of the aforementioned figure we plot the best-fitting value of each one of the free parameters of both our cold- and hot-seed model, alongside the corresponding 1σ MCMC uncertainty, at each frequency interval. The asymmetric uncertainties reflect the non-symmetric, in some cases, posterior distributions.

The size, L , of the corona is larger in general for the cold-seed model, with a minimum of about 3.9 km at 574 Hz and a maximum of 8.7 km at 765 Hz. In the hot-seed case, the size is systematically smaller, staying below 2 km at almost all frequency intervals. The amplitude of the external heating rate, $\delta\dot{H}_{ext}$, for the cold-seed model increases with QPO frequency from approximately 8 % at 570 Hz, to a maximum of over 10% at 730 Hz and then decreases again to around 5% at 890 Hz. In the hot-seed case although $\delta\dot{H}_{ext}$ has the same behaviour as in the hot-seed case, the maximum value of 13% is found at around 800 Hz. Nevertheless, the relatively large uncertainties of $\delta\dot{H}_{ext}$, in the cold-seed case compared to those of the hot-seed case, do not allow us to place a tight constraint on the exact lower kHz QPO frequency at which $\delta\dot{H}_{ext}$ exhibits its maximum value. The feedback fraction, f_η , is significantly different between the cold- and hot-seed models. More specifically, in the hot-seed model case f_η is almost 100% at all frequency intervals, whereas in the cold-seed model case it stays more or less stable around 50%. In both model cases, the high values of f_η indicate that a significant amount of the NS luminosity is due to feedback.

We also found reasonable values (compared to Zhang et al. 2017) for the average corona temperature, kT_e . In Figure 2.3 we plot the predicted kT_e , for both the cold- and hot-seed model cases, together with the values from Zhang et al. (2017). The electron temperature, kT_e , decreases with QPO frequency in the cold-seed model case, as reported in Ribeiro et al. (2017), but has a more complex behaviour in the hot-seed model case. However, as we explain later, we did not expect a perfect agreement between our fitted kT_e values and the values presented in Zhang et al. (2017), given that our time averaged spectral model is necessarily different from theirs, mainly because we use a simple blackbody as the seed photon source. Nevertheless, our predicted values of kT_e for the cold-seed model case agree at the 1σ level with the observed values from Zhang et al. (2017), contrary to the hot-seed case. The temperature of the NS, kT_s , decreases from around 1.8 keV at 570 Hz, to 1 keV at 610 Hz and then remains quite stable in the hot-seed model case, while in the cold-seed model case kT_s is around 0.3 keV at all QPO frequencies.

Finally, we find that in the cold-seed model case the optical depth, τ_T , slightly decreases from around 10.6 at 610 Hz, to 8.5 at 860 Hz. These values are consistent with the observed values of Zhang et al. (2017), but not with the

increasing behaviour with QPO frequency reported in the latter paper and also in Ribeiro et al. (2017). On the other hand, in the hot-seed model case, the values of τ_T are significantly smaller with no clear behaviour. The photon power-law index, Γ , for the hot-seed model case is found to be above 5, on average. For the cold-seed case model, Γ increases from around 1.5 at 570 Hz to around 2.2 at 860 Hz and then decreased to 1.7 at 920 Hz. In Figure 2.4 we plot the observed Γ values from Zhang et al. (2017) and Ribeiro et al. (2017) together with the predicted values of Γ , given the best-fitting values of kT_e τ_T and their MCMC uncertainties. The values of Γ , alongside their propagated uncertainties, are given in Table 2.2.

Figure 2.1 – Best fit of the cold- and hot-seed model to the lower kHz QPO rms and time lag spectra of 4U 1636 – 53, in the frequency range 574.1–732.1 Hz. In the left hand side panels we plot the measured rms as a function of energy (grey circles and shaded area; taken from Ribeiro et al. (2019) alongside our best model fits. The red dashed lines represent the best-fitting hot-seed model, whereas the blue dotted lines represent the best-fitting cold-seed model. In the right hand side panels we plot the time lag in μs (grey circles and shaded area) alongside the best-fitting models, in the same way as in the left hand side.

Figure 2.2 – Same as Figure 2.1 for the lower kHz QPO in the frequency range 765.2-917.8 Hz

Figure 2.3 – Summary of the MCMC results for the fits to the rms and time lag energy spectra of the lower kHz QPO in 4U 1636 – 53 at all QPO frequency intervals. We plot the best-fitting parameters of the size, L , amplitude of the external heating rate, $|\delta\dot{H}_{ext}|$, feedback fraction, f_η , corona temperature, kT_e , NS temperature, kT_s , and optical depth, τ_T , in panels (a), (b), (c), (d), (e) and (f), respectively, using blue squares for the cold-seed mode and red circles for the hot-seed model, alongside their 1- σ MCMC uncertainties, as a function of frequency of the lower kHz QPO. In the middle and bottom panels, alongside our best-fitting values of kT_e and τ_T , we plot the same best-fitting parameters (grey shaded area) obtained by Zhang et al. (2017), through a slightly different spectral model, as presented in Ribeiro et al. (2017).

Figure 2.4 – Observed (black squares, Ribeiro et al. (2017)) values of the photon power law index, Γ , as a function of the frequency of the lower kHz QPO for 4U 1636 – 53, and the predicted values based on the hot- (red dotted line) and cold-seed (blue dashed line) model cases.

Table 2.1 – Best-fitting values of the model parameters for the rms and time lag energy spectra of the lower kHz QPO in 4U 1636 – 53 as a function of QPO frequency. At each line of the table, the first row corresponds to the best-fitting cold-seed model and the second one to the best-fitting hot-seed model case.

ν_{low} [Hz]	L [km]	$ \delta\dot{H}_{ext} $	f_η	kT_e [keV]	kT_s [keV]	τ_T	$\chi^2/d.o.f.$
574.1	$3.9^{+2.2}_{-2.3}$	0.08 ± 0.04	0.7 ± 0.2	6.7 ± 1.7	0.4 ± 0.45	$10.4^{+2.5}_{-2}$	14.2/6
	2 ± 0.4	0.07 ± 0.2	0.96 ± 0.03	$8.9^{+6.2}_{-4.4}$	$1.8^{+1}_{-0.6}$	$1.4^{+1.2}_{-0.6}$	11.5/6
611.2	4.7 ± 2.4	0.1 ± 0.05	0.5 ± 0.2	$5.5^{+1.4}_{-2.1}$	$0.3^{+0.2}_{-0.06}$	$10.6^{+1}_{-1.5}$	28.2/6
	$1.3^{+1.2}_{-0.6}$	$0.064^{+0.007}_{-0.02}$	0.97 ± 0.04	$7.9^{+1.71}_{-2.2}$	1 ± 0.1	1.1 ± 0.5	16.5/6
651.9	5.3 ± 2.2	0.14 ± 0.05	0.5 ± 0.08	4.9 ± 1.5	$0.3^{+0.05}_{-0.2}$	10.4 ± 1	9.8/8
	1.1 ± 0.3	0.08 ± 0.02	0.97 ± 0.02	5.8 ± 1.5	1 ± 0.3	$1.6^{+0.3}_{-0.5}$	14.8/8
698.3	$6.4^{+3}_{-2.7}$	0.12 ± 0.06	0.5 ± 0.1	4.2 ± 1.4	$0.3^{+0.02}_{-0.04}$	10 ± 1	22.6/8
	1.1 ± 0.3	0.08 ± 0.008	0.97 ± 0.02	$7.4^{+0.9}_{-1.6}$	1 ± 0.2	1.2 ± 0.5	10.2/8
732.1	7.5 ± 2.6	0.13 ± 0.06	0.5 ± 0.1	3.6 ± 1.1	0.3 ± 0.03	10 ± 1	15.1/7
	1.6 ± 0.6	0.1 ± 0.04	0.96 ± 0.03	$8.7^{+6.7}_{-4.3}$	1.5 ± 0.4	$1.2^{+1.8}_{-0.4}$	16.7/7
765.2	8.7 ± 2.8	0.094 ± 0.04	0.6 ± 0.2	2.7 ± 1	$0.3^{+0.04}_{-0.1}$	9.8 ± 1	27.9/8
	1.2 ± 0.5	0.1 ± 0.04	$0.96^{+0.04}_{-0.08}$	5.8 ± 2.6	1.2 ± 0.3	$1.4^{+1.1}_{-0.6}$	31.4/8
809.1	5.4 ± 1.6	0.09 ± 0.03	0.5 ± 0.09	2.9 ± 0.9	0.31 ± 0.03	10.3 ± 0.8	32.5/8
	$1.3^{+0.3}_{-0.4}$	0.13 ± 0.03	0.8 ± 0.09	$4.9^{+1.9}_{-1.6}$	1 ± 0.2	4.9 ± 3	55/8
838.8	7.4 ± 2	0.08 ± 0.03	0.6 ± 0.1	$2.8^{+0.9}_{-0.6}$	$0.3^{+0.02}_{-0.06}$	9.4 ± 1	29.2/8
	1.4 ± 0.4	0.1 ± 0.03	0.95 ± 0.04	$6.6^{+3}_{-3.3}$	$1.3^{+0.3}_{-0.2}$	$1.45^{+1}_{-0.7}$	31.7/8
864.3	5 ± 1	0.06 ± 0.02	0.7 ± 0.08	2.9 ± 0.7	$0.25^{+0.06}_{-0.02}$	8.5 ± 0.7	70/7
	$1.1^{+0.6}_{-0.4}$	0.07 ± 0.02	$0.98^{+0.01}_{-0.03}$	$6.5^{+4.4}_{-2.7}$	1 ± 0.2	$6.1^{+1.6}_{-0.7}$	41/7
894.4	5.2 ± 1.6	0.005 ± 0.003	$0.58^{+0.1}_{-0.08}$	$2.7^{+0.5}_{-1}$	0.3 ± 0.03	9.9 ± 1	53.8/8
	1.2 ± 0.4	$0.074^{+0.03}_{-0.01}$	0.94 ± 0.04	$5.5^{+2.5}_{-1.7}$	1 ± 0.2	$2^{+1.7}_{-0.8}$	52.8/8
917.8	$5.1^{+1.9}_{-1.6}$	0.055 ± 0.03	0.66 ± 0.08	$5^{+1.2}_{-1.4}$	0.25 ± 0.07	9.2 ± 0.9	55.5/8
	1.5 ± 0.5	$0.048^{+0.03}_{-0.01}$	$0.97^{+0.02}_{-0.09}$	$6.7^{+1.8}_{-1.9}$	1.3 ± 0.2	$5.9^{+6}_{-3.8}$	78.8/8

2.4.1 The leading oscillation

Our solving scheme, described in detail in section 2.2 and Appendix 2.A.2, solves for the complex amplitudes $\delta n_\gamma(E)$, δT_e and δT_s self consistently. Since no mathematical information about which one of the two oscillations leads in time is put in equation (2.7), a priori, the solution will dictate which of δn_γ , δT_e or δT_s will be the leading signal at the time of escape from the corona. At all QPO frequencies, we find that the oscillation of the NS temperature, δT_s , lags the oscillation of the corona temperature, δT_e , by $\sim 150 \mu s$ on average, both for the cold- and hot-seed model. In order to visualize our result, we created a mock model for the temperatures of the corona and NS, both for the cold- and hot-seed model cases. We represent both of the temperatures T_e and T_s as simple sinusoidal signals with amplitudes $|\delta T_e|$ and $|\delta T_s|$, and phases ϕ_{T_e} and ϕ_{T_s} , respectively, normalized around zero. All the quantities mentioned above are taken directly from our best-fitting cold- and hot-seed model at the given frequency interval. Our wave-like mock models are presented in Figure 2.5 for three different QPO frequencies, alongside the predicted time lags, Δt_T , between the leading and lagging oscillations for the same QPO frequencies. We plot the time lag, $\Delta t_T = t_{T_e} - t_{T_s}$, between the response of the corona and the NS, in the second panel of Figure 2.5. We summarise the retrieved values of Δt_T in Table 2.2, for each QPO frequency.

Based on our model, we can explain both why the oscillating NS temperature leads in time, and the dependence of the time delay between the two oscillations, Δt_T , upon QPO frequency. On the first question, the high feedback fraction that we find in all our fits means that a significant fraction of the emitted soft photons, from the NS surface, is delayed because of the feedback mechanism. If the temperature of the NS is not oscillating independently, then the only allowed oscillation of the NS temperature would be the one caused by the feedback photons, whose number density oscillates at the QPO frequency. The time that it takes for a photon to impinge back onto the NS surface is hard to calculate using random walk arguments. However, we can assume that it is of the order of $t_c N_{esc}$, which is the average escape time from the corona. Thus, by the time the QPO is produced, the NS temperature, through feedback, responds after $\sim t_c N_{esc}$. If the QPO is produced by an oscillation of the corona temperature, then δT_e would naturally lead in time. However, even if the corona responds to an already existing QPO (δn_γ), through Compton cooling, the extremely short, compared to $t_c N_{esc} \sim 10^{-5} s$, electron-electron collision characteristic timescale ($\sim 10^{-10} s$) would make the corona temperature to respond almost instantaneously to the presence of an oscillating photon number density inside it.

From the above reasoning, one would expect Δt_T to be of the order of $t_c N_{esc}$.

That brings us to the second question, about the measured time delay Δt_T . This is well correlated (in the cold-seed model case) to the time $t_c N_{esc}$, needed for photons, on average, to escape the medium. We plot Δt_T and $t_c N_{esc}$ as a function of QPO frequency in the second and third panels of Figure 2.5, respectively. This agreement between the time difference, Δt_T , and the factor $t_c N_{esc}$, predicted by random walk arguments, was expected and therefore it serves as a sanity check of our model.

2.4.2 Thermodynamic properties of the corona

In our model we can retrieve the average external heating that the corona requires per unit time, per scattering and per electron, $\dot{H}_{ext0}$, in order to maintain equilibrium. This external heating rate can be calculated from equation 2.15 and the dependence upon energy, through the Kein-Nishina cross section, simply means that the cooling is less efficient at higher energies (i.e. less external heating is required). Based on our best-fitting parameters, we can estimate that $\dot{H}_{ext0}$ is, on average, below 2% of the Eddington luminosity both for the cold- and hot-seed model. In the hot-seed model case, we find that $\dot{H}_{ext0}$ is systematically larger than in the cold-seed case by almost an order of magnitude. We plot the dependence of $\dot{H}_{ext0}$ to QPO frequency, for the cold- and hot-seed model case, in Figure 2.6. In the cold-seed case, $\dot{H}_{ext0}$ is on average much smaller than the observed luminosity of 4U 1636 – 53, which is around 0.1 Eddington, at the state where the kHz QPOs are observed. The reason that we find low values of $\dot{H}_{ext0}$ is because we only study the kHz QPOs which have time lags of the order of micro seconds, which in turn means that the corona sizes are expected to be small as we find here. In other words, we only study the inner part of the corona while the full corona could be more extended, thus requiring a much higher heating rate.

On top of the heating rate, $\dot{H}_{ext0}$, an oscillation of this heating rate, $\delta \dot{H}_{ext}$, is used to explain the observations. Since $\delta \dot{H}_{ext}$ is a free parameter in our model, we can assume that its complex phase is zero, since it does not affect the phase (or time) delay between δT_e and δT_s or the QPO time lags, given that they are calculated with respect to a reference energy band. Given the fact that $|\delta \dot{H}_{ext}|$ plays the role of a normalization for the modulus of the QPO amplitude, $|\delta n_\gamma|$, it is not surprising that $|\delta \dot{H}_{ext}|$ is correlated to the energy averaged rms (Figure 2.7), since the latter may be thought of us approximately the area under the curve of rms as a function of energy. A direct comparison between our best-fitting cold- and hot-seed model and the energy averaged rms and time lags presented in Ribeiro et al. (2017) and de Avellar et al. (2013) respectively is shown in Figures 2.7 and 2.8. We calculated the energy averaged rms in the full energy band (nominally 2-60 keV). For the comparison to the time lags of de Avellar et al. (2013), we computed the time lag of the 12-20

Figure 2.5 – Comparison between the oscillating temperature and oscillation amplitude of the corona, T_e , and NS, T_s , in 4U 1636 – 53 as a function of the frequency of the lower kHz QPO for the cold- (left column) and hot-seed model (right column). In the top panel, the solid lines represent the mock oscillations of the corona temperature, T_e , while the dotted lines represent the mock oscillations of the NS temperature, T_s , at two arbitrary lower kHz QPO frequencies, denoted with different colors. In the second panel, we plot the time lag between T_e and T_s as a function of the QPO frequency. In the third panel, we plot the average photon escape time, $t_c N_{esc}$, as a function of QPO frequency. In the bottom panel, we compare the oscillation amplitudes $|\delta T_e|$ and $|\delta T_s|$. We must note that the quantities plotted here are not directly observed but retrieved from the model, after fitting the rms and time lag data.

Figure 2.6 – The retrieved average external heating rate, $\dot{H}_{ext0}$, for 4U 1636 – 53 in units of the Eddington luminosity, as a function of the frequency of the lower kHz QPO for the cold- (blue dashed line) and hot-seed (red dotted line) model cases.

Figure 2.7 – The energy averaged fractional rms amplitude measurements as a function of the frequency of the lower kHz QPO for 4U 1636 – 53 (Ribeiro et al. 2017; grey squares and shaded area), alongside our cold- and hot-seed model predictions with blue dashed and red dotted lines, respectively.

keV band with respect to the 4-12 keV band using the best-fitting parameters in each frequency interval.

Finally, the NS temperature oscillation amplitude, $|\delta T_s|$, remained constant, within uncertainties, at 1% both in the cold- and hot-seed model case, at all QPO frequencies. However, the oscillation amplitude of the corona temper-

Figure 2.8 – The energy averaged time lags of the lower kHz QPO in 4U 1636 – 53 vs QPO frequency (de Avellar et al. 2013; grey squares and shaded area), alongside the cold- and hot-seed model predictions with blue dashed and red dotted lines, respectively.

ature, $|\delta T_e|$, in the cold-seed model case increased from $\sim 2.6\%$ at 574 Hz to around 7% at 732 Hz, and then decreased again to 2.5% at 918 Hz. In the hot-seed model case, $|\delta T_e|$ was almost as low as the NS temperature oscillation amplitude, $|\delta T_s|$, at all frequencies albeit a small range around 809 Hz where a maximum of 4% was found. In the bottom panel of Figure 2.5 we plot these amplitudes as a function of QPO frequency. We present the exact values of the retrieved parameters $|\delta T_e|$, $|\delta T_s|$ and $\dot{H}_{ext0}$ in Table 2.2.

Table 2.2 – Physical parameters retrieved from our model after the fitting to the lower kHz QPO in 4U 1636 – 53, at each QPO frequency interval. Each parameter is accompanied by the 1σ uncertainty around the best fitting, median, value.

ν_{low} [Hz]	$ \delta T_e $	$ \delta T_s $	Δt_T [μs]	$\log \dot{H}_{ext0}$ [L_{edd}]	η	Γ
574.1	$0.026^{+0.005}_{-0.009}$	0.011 ± 0.003	135^{+40}_{-70}	-2.3 ± 1.7	$0.1^{+0.06}_{-0.2}$	1.5 ± 0.2
	0.022 ± 0.01	0.011 ± 0.003	162^{+57}_{-110}	-1.3 ± 0.2	$0.85^{+0.07}_{-0.2}$	$5^{+3.1}_{-2}$
611.2	$0.042^{+0.007}_{-0.03}$	0.012 ± 0.009	128^{+12}_{-24}	-2.8 ± 1.3	$0.1^{+0.1}_{-0.02}$	1.55 ± 0.2
	$0.021^{+0.004}_{-0.01}$	$0.01^{+0.001}_{-0.006}$	135 ± 68	-2.3 ± 0.9	$0.88^{+0.02}_{-0.2}$	6.2 ± 2
651.9	$0.058^{+0.01}_{-0.02}$	$0.014^{+0.005}_{-0.003}$	133^{+16}_{-33}	$-3^{+0.2}_{-0.3}$	$0.1^{+0.08}_{-0.02}$	1.6 ± 0.2
	$0.02^{+0.004}_{-0.008}$	$0.014^{+0.005}_{-0.002}$	165^{+58}_{-54}	-2.3 ± 0.3	$0.88^{+0.07}_{-0.05}$	$5.5^{+1}_{-1.4}$
698.3	0.058 ± 0.01	$0.012^{+0.004}_{-0.003}$	151^{+15}_{-31}	$-3.2^{+0.9}_{-0.7}$	$0.14^{+0.08}_{-0.03}$	1.7 ± 0.2
	$0.022^{+0.003}_{-0.009}$	0.014 ± 0.002	166^{+21}_{-66}	-2.3 ± 0.1	0.89 ± 0.06	6.2 ± 2
732.1	$0.066^{+0.03}_{-0.01}$	$0.011^{+0.006}_{-0.002}$	162^{+35}_{-19}	-3.2 ± 0.2	$0.17^{+0.1}_{-0.03}$	1.8 ± 0.2
	$0.044^{+0.007}_{-0.009}$	$0.02^{+0.002}_{-0.005}$	150^{+22}_{-44}	-1.7 ± 0.06	$0.87^{+0.08}_{-0.02}$	5.6^{+6}_{-2}
765.2	$0.054^{+0.02}_{-0.01}$	$0.008^{+0.005}_{-0.001}$	184^{+18}_{-24}	-3.2 ± 1.8	$0.27^{+0.03}_{-0.1}$	2.1 ± 0.3
	$0.04^{+0.005}_{-0.008}$	$0.016^{+0.007}_{-0.002}$	122^{+31}_{-42}	$-2.2^{+0.2}_{-0.02}$	$0.89^{+0.07}_{-0.5}$	$6.3^{+3.6}_{-2.3}$
809.1	0.05 ± 0.02	0.008 ± 0.002	131^{+24}_{-32}	-3.19 ± 0.1	0.21 ± 0.1	1.9 ± 0.3
	$0.04^{+0.007}_{-0.01}$	0.018 ± 0.005	59^{+12}_{-50}	-1.7 ± 0.005	$0.55^{+0.05}_{-0.1}$	2.7 ± 1.4
838.8	$0.046^{+0.02}_{-0.004}$	$0.008^{+0.001}_{-0.003}$	173^{+25}_{-16}	$-3.4^{+0.3}_{-0.1}$	$0.3^{+0.03}_{-0.08}$	2.14 ± 0.3
	$0.04^{+0.003}_{-0.007}$	$0.018^{+0.006}_{-0.001}$	126^{+13}_{-21}	-1.9 ± 0.01	$0.87^{+0.05}_{-0.2}$	$5.6^{+3.1}_{-2.4}$
864.3	$0.031^{+0.003}_{-0.004}$	$0.007^{+0.0005}_{-0.002}$	155^{+12}_{-30}	$-3.7^{+0.2}_{-0.7}$	$0.33^{+0.2}_{-0.03}$	2.2 ± 0.3
	$0.017^{+0.006}_{-0.002}$	0.013 ± 0.002	178^{+25}_{-42}	$-2.3^{+0.2}_{-0.08}$	$0.88^{+0.02}_{-0.2}$	$5.2^{+2.6}_{-1.9}$
894.4	$0.03^{+0.003}_{-0.02}$	$0.006^{+0.0008}_{-0.002}$	135^{+18}_{-58}	-3.3 ± 1.2	$0.26^{+0.07}_{-0.1}$	2 ± 0.3
	$0.025^{+0.009}_{-0.005}$	$0.01^{+0.003}_{-0.001}$	100^{+21}_{-38}	-2.1 ± 0.01	$0.83^{+0.03}_{-0.1}$	$4.9^{+3}_{-1.5}$
917.8	$0.025^{+0.001}_{-0.006}$	0.0076 ± 0.0007	142^{+36}_{-26}	-3.2 ± 0.06	$0.16^{+0.1}_{-0.09}$	1.7 ± 0.2
	0.0095 ± 0.0002	$0.01^{+0.002}_{-0.0007}$	142^{+20}_{-50}	-1 ± 0.02	$0.51^{+0.2}_{-0.1}$	$2^{+1.5}_{-0.9}$

2.5 Discussion

We fitted the rms and time lag spectra of the lower kHz QPO of the NS LMXB 4U 1636 – 53 at different QPO frequencies with a physical model for the variability. Our model assumes that soft photons from the NS surface only, are up-scattered inside a hot and homogeneous corona that is modeled as a spherically symmetric shell that extends to a certain distance, L , from the NS surface. In our simplified model the accretion disc is technically assumed to lie outside of the corona. Furthermore, our corona is not associated with the boundary layer, although the latter might in reality be included in what we call the Comptonising medium. Our model takes into account the fact that after an average number of scatterings, a fraction of the hard photons, η , that have not yet escaped the corona will return to the NS surface, forming a feedback loop, and will cause a delayed heating of the NS up to a higher observed temperature, T_s . Our fits yield two statistically acceptable solutions, one with a cooler (cold-seed) and another with a hotter (hot-seed) NS surface. Through our fits, we can place a lower limit to the size of the corona, L , between 1-8 km, in the sense that this is the required corona size in order to explain the lower kHz QPO properties based on inverse Compton scattering. Our fits indicate that the fraction of the NS luminosity that is due to feedback from the corona, f_η , is more than 50% in all model cases. We also find that the oscillations of the NS temperature lag the oscillations of the corona temperature by $150 \mu\text{s}$ on average, due to the finite travel time of the feedback photons from the corona back to the NS surface. Thus, the soft lags, as explained by our model, are due to the finite escape time in the corona of photons illuminating the neutron star surface. The oscillating thermodynamic properties of the corona, namely the external heating rate, $\dot{H}_{ext}$, and corona temperature, T_e , exhibit a maximum variability when the lower kHz QPO frequency is at 700 Hz, suggesting a resonance between the source of soft photons (in our case the NS surface) and the corona. To address the cold- and hot-seed model degeneracy that we encountered, we note that in the cold-seed case the inferred values of the photon power-law index, Γ , tend to be more compatible with the measured values of Zhang et al. (2017). Furthermore, the inferred time delay between the corona and NS temperature oscillations are in good agreement with the photon diffusion time in the corona, deduced by random walk arguments, only in the cold-seed model case. In general, the corona appears to be the more active among the two components, corona and NS, and our results suggest that the corona is primarily responsible for the radiative properties of the lower kHz QPOs in LMXBs.

2.5.1 Insight about the properties of the corona

We find a mean (over the different frequency intervals) electron density of $\bar{n}_e \sim 10^{25} \text{ m}^{-3}$ for both the cold- and hot-seed model case, and an average electron temperature, kT_e , of around 4 keV and 6.8 keV for the cold- and hot-seed case, respectively. The resulting Debye length, λ_D , is $\sim 8 \times 10^{-3} \text{ m}$. The size of the corona that we find here is significantly larger than this, i.e. $\lambda_D \ll L$, and therefore the corona can be treated as nearly charge neutral, which allows us to assume that the number density of ions, n_i , is equal to n_e . We also estimate a Coulomb mean free path of about $7 \times 10^{-2} \text{ m}$ for the electron-ion scattering and so the average electron temperature, T_e , can be taken as equal to the average ion temperature, T_i , assuming thermal equilibrium has been achieved through ion-electron collisions. We must note that since thermalisation between ions and electrons happens at a rate $\frac{m_e}{m_i}$ times slower than the scattering rate, the "thermal equilibration length" will be $\sim 123 \text{ m}$, which is still small compared to the km wide corona. Finally, an estimate of the outward radiation pressure in the corona, $P_{rad} = 4\sigma T_e^4/3c$, based on our estimated electron temperature, yields a value of $\sim 10^{16} \text{ dynes cm}^{-2}$, which agrees, to an order of magnitude, with the pressure due to gravity, $P_g = n_e m_p g L$, assuming a typical surface gravity of around $g = 10^{12} - 10^{13} \text{ m s}^{-2}$ for the NS in 4U 1636 – 53. This balance between gravity and radiation pressure emerges without us forcing any condition for hydrostatic equilibrium and thus supports the idea that a spherical corona can indeed be naturally sustained.

2.5.2 On the corona geometry and seed photon source type

As discussed in section 2.2, the current version of our model assumes that the NS surface is the source of seed photons for Comptonisation and no extra component is added. However, Zhang et al. (2017) fitted the spectra of NS 4U 1636 – 53, during the occurrence of the kHz QPOs, using the model *nthcomp* of *XSPEC* with a disc blackbody seed source and a simple blackbody on top of that. The timing data that are used in this work come from Ribeiro et al. (2017) and Ribeiro et al. (2019), the first of which used the best-fitting spectral parameters of Zhang et al. (2017). Thus any comparison between our best-fitting values of kT_e , kT_s and τ_T , or their dependence on QPO frequency, to the results of Ribeiro et al. (2017) must take into account the different spectral approaches. In fact, it is physically more accurate for our model to use a factor of $\frac{1}{2}$ instead of $\frac{1}{3}$ in equation (2.3), because the former is correct when the seed source intensity is a delta-like function at the center of the corona, while the latter is correct when the source intensity is distributed sinusoidally from the center of the corona, as was suggested by Sunyaev & Titarchuk (1980). However, since our variability model is constructed on top of an equilibrium solution, we need this equilibrium solution to resemble the observed energy

spectrum as much as possible, thus the use of $\frac{1}{3}$. In principle, one should assume that the seed photons could come both from the surface of the NS and part of the inner accretion disc, assuming the corona is sufficiently large.

Another crucial factor when modelling thermal Comptonisation semi-analytically, is the corona geometry. The corona geometry is regulated through the spatial dependence of the scattering optical depth τ (or τ_T). The simplest possible geometry is a spherical shell and that is what we assume in this work. However, the geometry of the corona of LMXBs is believed to be more complex and for that reason it is very common to use Monte Carlo methods to model thermal Comptonisation, since there is freedom in choosing the desired geometry. In timing analyses, such as the one presented in Ribeiro et al. (2019), it would be nearly impossible to use Monte Carlo methods to study the timing properties of the QPOs, so semi-analytical models such as the one presented in Lee & Miller (1998), Lee, Misra & Taam (2001), Kumar & Misra (2014) and this work are extremely useful.

Finally, in Figure 2.9 we plot the feedback parameter, η , retrieved for both our cold- and hot-seed model through f_η , against the measured size, L , of the corona alongside their MCMC uncertainties. The same quantities were plotted by Kumar & Misra (2016b) for the system 4U 1608–52 in the right hand panel of Figure 8 in their paper. Kumar & Misra (2016b), however, did not fit the energy dependent rms and time lags at different frequencies but matched the data of de Avellar et al. (2013) for the soft time lag between two energy bands with the predictions of a cold- and hot-seed model. In other words, at each lower kHz QPO frequency Kumar & Misra (2016b) used only one data point for the time lag due to the lack of more data, hence the large uncertainties in their work. Our detailed fits result in significantly smaller uncertainties for η and L . Thus, a comparison of our data to Monte Carlo simulations, such as those presented in Kumar & Misra (2016a), that predicted the feedback, η , given a specific size, L , and an assumed corona geometry, can shed more light on what the most realistic geometry of the corona of NS LMXBs is.

As it is apparent from Figures 2.1 and 2.2 show that our model does not exhibit a break, and a subsequent drop, of the rms amplitude above a certain energy, $E_{break} \sim 12$ keV, as reported in Ribeiro et al. (2019). Mathematically, in order for the rms amplitude to decrease at a certain energy we need to add to our continuum model an extra component, whose flux would exhibit a sufficient increase at around 12 keV. If the corona is small and optically thick, as we find in this work, the hard X-rays that leave the corona will irradiate the accretion disc; This will increase the disc temperature and produce a reprocessed component that peaks at an energy that depends on the composition of the part of the disc that is irradiated and the irradiating flux itself. Thus, a decrease

of the fractional rms as a function of energy is to be expected if we included a reflected component that was less variable in our model.

Furthermore, since the flux that irradiates the part of the disc that lies outside of the corona is already varying, the reprocessed component will also undergo oscillations. Finally, the inner parts of the disc will be irradiated faster than the outer parts, due to the finite light travel time, which in combination with the already varying incident flux will create a complex time lag as a function of energy. The initial version of the model that we present here can not yet capture all the physical processes that happen around the NS. Although out of the scope of this paper, the next step is a model in which a combination of reprocessed emission from the disc and Comptonisation in the corona can be implemented to improve fitting of the timing properties of kHz QPOs, since reverberation alone can not provide a satisfactory explanation to the energy dependence of the time lags as has been shown by Cackett (2016) for the NS LMXB 4U 1608 – 52.

2.5.3 The size of the corona and the rms amplitude of the lower and upper kHz QPOs

In some models that provide an explanation of the dynamical origin of both the lower and upper kHz QPO, a beat between the NS spin frequency and some other frequency (Alpar & Shaham 1985), e.g., the Keplerian frequency at the marginally stable circular orbit (MSCO), or at the sonic radius of the accretion flow (Miller, Lamb & Psaltis 1998; see also Alpar & Shaham 1985). These models propose that the lower kHz QPO is produced close to the NS surface, and is linked to mass accretion rate instabilities caused by inhomogeneities in the accretion disc. On the other hand, for the upper kHz QPO the flux oscillations could also originate on the NS surface (Miller, Lamb & Psaltis 1998), or they could originate further away and be caused either by occultations of self-luminous disc inhomogeneities or by Doppler beaming of the same inhomogeneities when their radial distance or orbital inclination do not allow occultations (Stella & Vietri 1998). Gilfanov, Revnivtsev & Molkov (2003), proposed that the lower kHz QPO originates in a boundary layer close to the NS surface, and it is associated with Comptonisation, and de Avellar et al. (2013) proposed that, regardless of the dynamical mechanism that explains the kHz QPO frequencies, the radiative mechanism that governs the timing properties of the lower and upper kHz QPOs should be different. Recently, Ribeiro et al. (2017) showed that the frequency dependence of the fractional rms amplitude of the upper kHz QPO, although in general different from that of the lower kHz QPO, exhibits the same behaviour as that of the lower kHz QPO around certain QPO frequencies. The latter finding indicates that when the lower and upper kHz QPOs are (separately) observed within a certain

Figure 2.9 – The feedback parameter, η , retrieved from our model from the fits to the rms and time lag energy spectra of the lower kHz QPO in 4U 1636 – 53, plotted against the best-fitting value of the size L . The cold- (blue squares) and hot-seed model (red circles) form two separate regimes.

QPO frequency range, their variability could be caused by the same radiative mechanism.

From the findings of Gilfanov, Revnivtsev & Molkov (2003), de Avellar et al. (2013) and Ribeiro et al. (2017), we assume that the flux that oscillates at the upper kHz QPO frequency is produced in the disc, the flux that oscillates at the lower kHz QPO frequency is produced close to the NS surface. Furthermore, depending on its size, a spherically symmetric corona can cover both the NS surface and the inner parts of the accretion disc. In this scenario, the balance between the NS radius, a , the corona size, L , and the radius of the MSCO, R_{ms} , determines whether the fractional rms amplitude of the upper kHz QPO is affected by Comptonisation, and thus whether it would resemble the behaviour of the fractional rms amplitude of the lower kHz QPO. Wang et al. (2017) fitted the spectrum of 4U 1636 – 53, with different reflection models, and found an inner disc radius, $R_{in} = (5.1 - 10.3)R_g$, in the intermediate state, during which the lower kHz QPO is typically observed (Zhang et al. 2017) ($R_g = GM/c^2$, with M the mass of the neutron star and G and c the gravitational constant and the speed of light, respectively). Assuming a minimum NS mass of $1.55M_\odot$, as suggested by Bulik, Kluźniak & Zhang (2000) based on the maximum kHz QPO frequencies observed in 4U 1636 – 53, yields a minimum inner radius of $\sim 12 - 23$ km.

Assuming a minimum NS radius of ~ 11 km for 4U 1636 – 53 (Stiele, Yu & Kong 2016) which is smaller than the radius of the MSCO, a width of the corona of ~ 8 km at 770 Hz (see Figure 2.3, for the cold-seed source case) would imply that the inner parts of the disc would be covered by the corona at specific frequencies. Based on these arguments, it is very interesting that a hump in the energy averaged rms amplitude of the upper kHz QPO is apparent at 770 Hz (Ribeiro et al. 2019), while the energy averaged rms amplitude of the lower kHz QPO peaks at about the same frequency. Although the energy averaged rms amplitude of the upper kHz QPO exhibits a different dependence upon QPO frequency to that of the lower kHz QPO, Ribeiro et al. (2019) showed that around 770 Hz the energy averaged rms amplitude of the upper kHz QPO resembles that of the lower. If the upper kHz QPO originates in the accretion disc, as its frequency increases and the orbiting material, that causes the QPO, moves inwards there will be a critical frequency of the upper kHz QPO (e.g. 770 Hz) where if the corona is extended enough the orbiting material will cross inside the corona and the upper kHz QPO will be affected by the properties of the corona and thus its radiative mechanism might be the same as that of the lower kHz QPO. We note that the crossing of the orbiting material inside the corona would happen when the lower kHz QPO has a frequency of around 515 Hz given an estimate of the constant separation between the lower and upper kHz QPO in 4U 1636 – 53 of about 255 Hz, as reported in Zhang et al. (1997).

A more accurate estimation of the critical upper kHz frequency, where the orbiting material crosses inside the corona is non-trivial and would require detailed general relativistic simulations. In particular, the high NS spin of ~ 582 Hz reported for 4U 1636–53 (Strohmayer & Markwardt 2002) does not allow us to apply corrections to the radius of the MSCO and to the Keplerian frequency of the accreting matter, that are of first order to the dimensionless angular momentum $j \equiv cJ/GM^2$ (Hartle & Thorne 1968; Miller, Lamb & Cook 1998 and Miller, Lamb & Psaltis 1998), where J is the orbital angular momentum of the NS. However interesting, such an analysis falls outside the scope of this work.

2.5.4 Future prospects with eXTP and NICER

The enhanced X-ray Timing and Polarimetry mission (eXTP; Zhang et al. 2019), and the Neutron-star Interior Composition ExploreR (NICER; Gendreau, Arzoumanian & Okajima 2012), are two very promising missions that can help validate some of the results presented here. NICER is currently active on the International Space Station while eXTP is expected to be launched in the mid 2020s. The improved timing and spectral capabilities of both eXTP and NICER will allow phase resolved spectroscopy of LMXBs (see in't Zand et al. 2019 for an extensive discussion on the science with eXTP). The variability and time delay between the time dependent temperatures T_e and T_s can be measured and compared to the predictions of the model presented here (e.g Figure 2.5). Furthermore, measuring NS radii with NICER through pulse profile modeling (Özel et al. 2016), combined with knowledge about the size of the corona, can help us understand whether it is correct to use the accretion disc as a source of seed photons in the case of a spherical corona.

2.6 Conclusion

We developed a numerical model that predicts the energy dependence of the rms amplitude and time lags of QPOs in NS LMXBs. Using observations of the lower kHz QPOs in the NS LMXB 4U 1636 – 53 we propose that the dependence of the variability of this QPO upon energy and QPO frequency is due to the surrounding Comptonising medium (corona) whose temperature exhibits the same dependence upon QPO frequency. We demonstrated that this model allows one to fit the timing properties of QPOs and get constraints on the thermodynamic properties and spatial extent of the corona. More importantly, our model makes predictions that can be tested with current and future instruments and thus can shed light on whether Comptonisation is the mechanism responsible for the radiative properties of the lower kHz QPOs in NS LMXBs.

Acknowledgements

E.M.R acknowledges the support from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brazil). D.A acknowledges support from the Royal society. O.B. is grateful for the support of a University of Southampton Diamond Jubilee International Visiting Fellowship. This work is part of the research programme Athena with project number 184.034.002, which is (partly) financed by the Dutch Research Council (NWO). This research has made use of data obtained from the High Energy Astrophysics Science Archive Research Center, provided by NASA's Goddard Space Flight Center.

Appendix

2.A Numerical Schemes

2.A.1 Steady state solution

In this section we compute the SSS, $n_{\gamma 0} = n_{\gamma} [\frac{\partial}{\partial t} = 0]$ of equation (2.1). After the change of variables $n_{\gamma} = N n_c$ and $E = x k T_e$ equation (2.1), in steady state, becomes:

$$\frac{d^2 N}{dx^2} = -\frac{dN}{dx} + N \left(\frac{2}{x^2} - \frac{2}{x} + \frac{c_2}{x^2} \right) - \frac{1}{e^{x \frac{T_e}{T_s}} - 1}, \quad (2.8)$$

where $c_2 = \frac{m_e c^2}{k T_e N_{esc}(x)}$ and $n_c = \frac{2\pi m_e t_c k T_e 0.3 a^2}{h^3 [(a+L)^3 - a^3]}$. Equation (2.8), which is dimensionless after the change of variables, is solved as a boundary value problem, using a finite difference scheme. The derivatives in equation (2.8) were replaced by second-order accurate central differences formulas. The discretised equation on an arbitrary mesh is:

$$\begin{aligned} N^{j-1} \underbrace{\left(\frac{1}{\delta x^2} - \frac{1}{2\delta x} \right)}_L + N^j \underbrace{\left(-\frac{2}{\delta x^2} - \frac{2}{x_j^2} + \frac{2}{x_j} - \frac{c_2}{x_j^2} \right)}_{D(x_j)} \\ + N^{j+1} \underbrace{\left(\frac{1}{\delta x^2} + \frac{1}{2\delta x} \right)}_U = - \underbrace{\frac{1}{e^{x_j \frac{T_e}{T_s}} - 1}}_{C(x_j)}, \end{aligned} \quad (2.9)$$

where $j = 1, 2, \dots, M - 1$, with M the number of mesh points and δx the mesh size, i.e. step size of the method. (Note that although we use δ for the step size, it does not represent any kind of perturbation. The condition applied at the boundaries is naturally $N^0 = N^M = 0$. The solution of equation (2.8) comes from solving a linear system that is formed using equation (2.9). In particular, if we write equation (2.9) for every consecutive triad of points on the mesh, we can form an $(M - 1) \times (M + 1)$ linear system. In matrix form this equation will be $\mathbf{A} \times \mathbf{N} = \mathbf{C}$. Taking into account the boundary conditions, the first and last columns of $\mathbf{A}$ can be incorporated in $\mathbf{C}$ so now the new coefficient matrix $\mathbf{T}$ of the system is square $(M - 1) \times (M - 1)$. Matrix $\mathbf{T}$ is tri-diagonal with L,D,U (appearing in 2.9) being its sub-diagonal, diagonal and super-diagonal elements respectively. We solved this system using Gaussian elimination with partial pivoting using the routine *dgtsv* from the LAPACK library (Anderson et al. 1999) as implemented in *scipy*'s package *linalg* (Jones et al. 2001–).

2.A.2 Solution of the linearized equation

Initially, we derive the formulas that describe the fractional amplitudes of the perturbations $\delta \dot{H}_{ext}$, δT_e and δT_s and discuss how they are connected to δn_γ . Following the reasoning and formulation of Kumar & Misra (2014), and using the first law of thermodynamics, we can write:

$$\frac{3}{2}k \frac{\partial T_e}{\partial t} = \dot{H}_{ext} - \langle \Delta \dot{E} \rangle, \quad (2.10)$$

where $\langle \Delta \dot{E} \rangle$ can be viewed as the work rate produced per particle in an ideal gas, here electrons in the medium, $\dot{H}_{ext}$ is the heat offered to the system per unit time per particle, and $\frac{3}{2}k(\partial T_e/\partial t)$ is the change in the internal energy per particle per unit time. The term $\langle \Delta \dot{E} \rangle$ for the Comptonization process is identified as the Compton cooling rate per electron and is given by:

$$\langle \Delta \dot{E} \rangle = \int_{E_{min}}^{E_{max}} (4kT_e - E) \frac{E}{m_e c} n_\gamma \sigma_T dE, \quad (2.11)$$

where E_{min} and E_{max} are arbitrary limits. The bolometric luminosity of the NS surface, is $4\pi a^2 \sigma T_s^4$, where σ is the Stefan Boltzman constant. According to Lee, Misra & Taam (2001) and Kumar & Misra (2014) it is valid to assume that a fraction η of the output Comptonized photons return to the NS, increasing its temperature and thus its luminosity. Based on this effect we can model the luminosity of the NS as a superposition of a luminosity of a black body

component with an inherent temperature $T_{s,inh}$, and a fraction, η , of the output spectrum. In that sense we can write:

$$4\pi a^2 \sigma T_s^4 = 4\pi a^2 \sigma T_{s,inh}^4 + \eta V_c \int \frac{n_\gamma}{t_c N_{esc}(E)} E dE \quad , \quad (2.12)$$

where $V_c = (4/3)\pi[(a + L)^3 - a^3]$ is the volume of the medium. Hereafter η will be referred to as the feedback parameter; it is obvious from equation (2.12) that there is an upper limit η_{max} for η that is obtained if we apply $T_{s,inh} = 0$. Assuming that the inherent NS temperature, $T_{s,inh}$, does not undergo oscillations then δT_s is the result of the oscillating photon density, δn_γ , that affects the observed temperature, T_s , through feedback.

The amplitudes δT_e , δT_s can be obtained by linearizing equations (2.10) and (2.12) after substituting the perturbed terms. The derived formulas for the amplitudes, after the proper change of variables, are:

$$\delta T_e = \frac{\dot{H}_{ext0} \delta \dot{H}_{ext} + \frac{n_c \sigma_{KN}(x)(kT_{e0})^3}{m_e c} (\int x^2 N \delta n_\gamma dx - 4 \int x N \delta n_\gamma dx)}{-\frac{3}{2} i \omega k T_{e0} + \frac{4(kT_{e0})^3 n_c \sigma_{KN}(x)}{m_e c} \int x N dx} \quad , \quad (2.13)$$

and

$$\delta T_s = \frac{k^4 \eta V_c n_c (kT_{e0})^2}{16\pi a^2 \sigma (kT_{s0})^4 t_c} \int \frac{x N \delta n_\gamma}{N_{esc}(x)} dx \quad . \quad (2.14)$$

In 2.13, $\dot{H}_{ext0}$ is the, assumed constant, external heating rate when the corona is at thermal equilibrium. In order to calculate this term we must consider the equilibrium solution ($\frac{\partial T_e}{\partial t} = 0$), of equation (2.10), which leads to:

$$\dot{H}_{ext0} = \frac{(kT_{e0})^3 n_c \sigma_{KN}(x)}{m_e c} \left(4 \int x N dx - \int x^2 N dx \right) \quad . \quad (2.15)$$

With equation (2.15), we complete the set of equations that are used to construct the model. Finally, we must note that the integrals that appear in all of the aforementioned formulas must be computed numerically and the integration limits are arbitrary in general.

Equation (2.7), which already is in dimensionless form, was discretized using second-order accurate central difference formulas, to become:

$$\begin{aligned}
 & \underbrace{\delta n_{\gamma}^{j-1} \left(-\frac{1}{\delta x^2} + \frac{1}{2\delta x} + \frac{dN}{dx} \right) \bigg|_{x_j} \frac{1}{N\delta x}}_{L(x_j)} + \underbrace{\delta n_{\gamma}^j \left(\frac{2}{\delta x^2} + \frac{1}{N} \frac{1}{e^{x_j \frac{T_e}{T_s}} - 1} - \frac{ic_5}{x_j^2} \right)}_{D(x_j)} \\
 & + \underbrace{\delta n_{\gamma}^{j+1} \left(-\frac{1}{\delta x_j^2} - \frac{1}{2\delta x} - \frac{dN}{dx} \right) \bigg|_{x_j} \frac{1}{N\delta x}}_{U(x_j)} = \\
 & \underbrace{\delta T_e \left(-\frac{2}{x_j^2} + \frac{1}{N} \frac{d^2 N}{dx^2} \right) \bigg|_{x_j} + \delta T_s \frac{1}{N} \frac{T_{e0}}{T_{s0}} \frac{x_j}{\left(e^{x_j \frac{T_{e0}}{T_{s0}}} + e^{-x_j \frac{T_{e0}}{T_{s0}}} - 2 \right)}}_{C(x_j)}, \tag{2.16}
 \end{aligned}$$

Equation (2.16) represents a linear system of equations, where the left hand side can be used to construct a tri-diagonal coefficient matrix with $L(x_j)$, $D(x_j)$ and $U(x_j)$ the sub diagonal, diagonal and super diagonal elements respectively. For simplicity we write equation (2.16) as:

$$\begin{aligned}
 \delta n_{\gamma}^{j-1} L(x_j) + \delta n_{\gamma}^j D(x_j) + \delta n_{\gamma}^{j+1} U(x_j) &= C_1(x_j) \delta T_e + \\
 & C_2(x_j) \delta T_s, \tag{2.17}
 \end{aligned}$$

where $C_1(x_j)$ and $C_2(x_j)$ can be found by comparing equation (2.17) to (2.16). Assuming M mesh points, just as in equation (2.9) so that $j = 1, 2, \dots, M$, the left hand side of equation (2.17) can be represented by a product of a tri-diagonal array of coefficients, with L, D and U being its sub diagonal, diagonal and super diagonal elements respectively, and a column array of the unknown values of δn_{γ} on the grid.

On the right hand side of equation (2.17) the terms δT_s and δT_e are integral functions of $\delta n_{\gamma}(x)$ thus we can write for δT_e :

$$\delta T_e = k_0(x) + k_1(x) \int Q_1(x) \delta n_{\gamma} dx + k_2(x) \int Q_2(x) \delta n_{\gamma} dx, \tag{2.18}$$

where k_1, k_2, k_3 , Q_1 and Q_2 can be found by comparing equation (2.13) to (2.18). For δT_s we can write:

$$\delta T_s = p_1 \int Q_3(x) \delta n_{\gamma} dx, \tag{2.19}$$

where p_1 and Q_3 can be found by comparing equation (2.14) to (2.19). We can now use the composite 3/4 Simpson rule in order to replace the integrals

in equations (2.18) and (2.19) with analytic expressions. The definite integral of a random function F whose analytic form is unknown, using second order Lagrange polynomials, can be written as

$$\int_a^b F(x)dx = \frac{\delta x}{3} \sum_{j=1}^{M/2} [F(x_{2j-2}) + 4F(x_{2j-1}) + F(x_{2j})] . \quad (2.20)$$

In our case, the integrals in equations (2.18) and (2.19), that we want to approximate, have the general form $I_i = \int Q_i(x)\delta n_\gamma(x)dx$ where i is an index that denotes a different function. According to equation (2.20) our integrals I_i can be written as a weighted sum of the values of $\delta n_\gamma(x_j)$ on the grid as:

$$\begin{aligned} I_i = \frac{\delta x}{3} & \left[Q_i(x_0)\delta n_\gamma(x_0) + Q_i(x_M)\delta n_\gamma(x_M) \right] + \\ & \frac{4\delta x}{3} \sum_{\substack{j=1 \\ \text{mod } j/2 \neq 0}}^{M-1} Q_i(x_j)\delta n_\gamma(x_j) \\ & + \frac{2\delta x}{3} \sum_{\substack{j=1 \\ \text{mod } j/2 = 0}}^{M-2} Q_i(x_j)\delta n_\gamma(x_j) . \end{aligned} \quad (2.21)$$

Using equations (2.18), (2.19) and (2.21) the amplitudes δT_e and δT_s can be substituted to the right hand side of equation (2.17), as the sum of the values of $\delta n_\gamma(x_j)$ on the entire grid. The final form of the coefficient array, corresponding to the solution of the system of equations formed by equation (2.17), will not be tri-diagonal anymore. The advantage of this mathematical technique is the fact that no initial assumptions are needed for δT_e and δT_s , and also there is no need of an iterative scheme that requires the aforementioned amplitudes to converge in order to obtain a final solution.

A variable corona for GRS 1915+105

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Submitted 04 October 2020, Accepted 16 March 2021

Published in Monthly Notices of the Royal Astronomical Society, volume 503, pages 5522-5533, 2021.

Abstract

Most models of the low frequency quasi periodic oscillations (QPOs) in low-mass X-ray binaries (LMXBs) explain the dynamical properties of those QPOs. On the other hand, in recent years reverberation models that assume a lamp-post geometry have been successful in explaining the energy-dependent time lags of the broad-band noise component in stellar mass black-holes and active galactic nuclei. We have recently shown that Comptonisation can explain the spectral-timing properties of the kilohertz (kHz) QPOs observed in neutron star (NS) LMXBs. It is therefore worth exploring whether the same family of models would be as successful in explaining the low-frequency QPOs. In this work, we use a Comptonisation model to study the frequency dependence of the phase lags of the type-C QPO in the BH LMXB GRS 1915+105. The phase lags of the QPO in GRS 1915+105 make a transition from hard to soft at a QPO frequency of around 1.8 Hz. Our model shows that at high QPO frequencies a large corona of $\sim 100\text{--}150 R_g$ covers most of the accretion disc and makes it 100% feedback dominated, thus producing soft lags. As the observed QPO frequency decreases, the corona gradually shrinks down to around $3\text{--}17 R_g$, and at 1.8 Hz feedback onto the disc becomes inefficient leading to hard lags. We discuss how changes in the accretion geometry affect the timing properties of the type-C QPO.

Keywords: numerical methods — X-rays binaries — black holes

3.1 Introduction

Low frequency (LF) Quasi-Periodic Oscillations (QPOs) in stellar-mass black hole binaries (BHB) have been known for many years (see reviews by Motta 2016 and Ingram & Motta 2020). These QPOs are distinct peaks in the Power Density Spectra (PDS) of X-ray light curves of these sources. Based on the strength of the underlying broad-band variability, centroid frequency, ν , and quality factor, $Q = \nu/\Delta\nu$, where $\Delta\nu$ is the full width at half maximum around the centroid frequency, LF QPOs are divided into three types, type-A, -B and -C (Casella et al. 2004). While all of these types share similar centroid frequencies, type-C are the most frequent QPOs, with high Q -factors. Although type-C QPOs usually appear in the range of a few mHz to 10 Hz, they have also been detected at frequencies as high as 30 Hz (Revnivtsev, Trudolyubov & Borozdin 2000). Models that explain the dynamical origin of QPOs in BHBs were proposed about two decades ago when Stella & Vietri (1998) introduced the idea of relativistic Lense-Thirring precession (LTP) as the underlying mechanism. Follow-up theoretical work by Stella, Vietri & Morsink (1999), Psaltis & Norman (2000), Fragile, Mathews & Wilson (2001), Schnittman, Homan & Miller (2006) and Ingram, Done & Fragile (2009) connected the LF QPOs in BHBs to the LTP frequency. Since then, there have been a plethora of studies that used timing and spectral features to add support to LTP as the origin of LF QPOs (Motta et al. 2014a; Motta et al. 2014b; Ingram et al. 2016 and

Ingram et al. 2017). Other models, implementing different physical mechanisms were being developed almost at the same time; some of those are the Two-Component Advection Flow model (TCAF, Molteni, Ryu & Chakrabarti 1996; Chakrabarti et al. 2008) and the Accretion-ejection instability model (Tagger & Pellat 1999). For recent reviews of observations and theory of LF QPOs, we refer the reader to Done, Gierliński & Kubota (2007), Motta (2016) and Ingram & Motta (2020).

The picture of the geometry of accretion onto a BH presented in Ingram, Done & Fragile (2009) consists of two main components. The first component is the classical optically thick geometrically thin accretion disc (Shakura & Sunyaev 1973), which is thought to be truncated at a radius r_t from the BH, typically larger than the inner-most stable circular orbit (ISCO), r_{ISCO} , depending on the spectral state (Ichimaru 1977; Esin, McClintock & Narayan 1997; Poutanen, Krolik & Ryde 1997; Gierliński, Done & Page 2008;). The second component is a geometrically thick hot flow that is assumed to reside between the truncated disc and the ISCO, and to be misaligned with respect to the accretion disc plane. General relativistic hydrodynamical simulations suggest that, under certain conditions, such thick inner flows can occur (Fragile & Anninos 2005; Fragile et al. 2007). The core idea of Ingram, Done & Fragile (2009) is that the type-C QPO is produced by such torus-like inner flow which precesses at the LTP frequency. The outer radius of the torus, r_o , is smaller or equal to the truncation radius of the accretion disc, while the inner radius is set by the limit at which the surface density of the torus becomes sufficiently low, due to the high tilt angle of its inner parts (see Ingram, Done & Fragile 2009 for a detailed description). The idea of a precessing torus, rather than a precessing disc, is partly motivated by the fact that the energy spectrum that is modulated at the frequency of the type-C QPO (covariance or rms spectrum), is well described by a Comptonised spectrum (Sobolewska & Życki 2006).

The general spectral behaviour of BHBs is usually described by a transition from a hard, less luminous, to a soft luminous state, imprinted as a q-shaped trajectory on the Hardness-Intensity Diagram (HID; Belloni et al. 2005 and Homan & Belloni 2005), with regimes of intermediate hardness and luminosity. The frequency of the type-C QPO increases as the source moves from harder to softer states. Depending on the model, the change in hardness in the HID can be interpreted as a change of the outer radius of a hot inner flow (Ingram, Done & Fragile 2009) or, for models with an extended Comptonising medium (Kazanas, Hua & Titarchuk 1997), as a change in properties of this Comptonising medium such as size, optical depth and temperature. Hereafter, we will refer to this extended Comptonising medium as the corona (Thorne & Price 1975; Sunyaev & Truemper 1979).

The timing properties of the type-C QPO, namely the fractional rms amplitude in a broad energy band and phase lag between a hard and a soft band, have also been shown to depend upon QPO frequency and source inclination. In particular, Motta et al. (2015) suggested that the rms amplitude of the type-C QPO as a function of QPO frequency is systematically higher for high than for low-inclination sources. Similarly, van den Eijnden et al. (2017) provided evidence that above a certain QPO frequency in low-inclination sources the phase lags of the type-C QPOs are positive (hard lags), meaning that high-energy photons arrive at the observer after the low-energy photons, and these hard lags increase with QPO frequency, whereas for high-inclination sources the phase lags are negative (soft lags) and decrease with QPO frequency. The aforementioned results favour a geometric origin for the type-C QPOs, which was further supported by more quantitative results, such as the modulation at half the QPO frequency of the centroid energy of the iron emission line in H1743–322 (Ingram et al. 2016). Many of the observational findings mentioned above still remain unexplained.

Over the years, considerable work has been done to study the dependence of the phase lags between two broad energy bands not only upon QPO frequency, as we mentioned in the previous paragraph, but also upon the frequency of the broad-band noise. At low Fourier frequencies of the broad-band noise, the lags are hard (Miyamoto et al. 1988; Kotov, Churazov & Gilfanov 2001), and thought to be produced by fluctuations of the mass accretion rate that propagate from the outer to the inner part of the accretion disc (Lyubarskii 1997; Arévalo & Uttley 2006; Ingram & van der Klis 2013), and are imprinted on the Comptonised and un-Comptonised emission that is used to calculate these lags. At higher Fourier frequencies of the broad-band noise the magnitude of the lags decreases, and the lags usually become soft, providing strong evidence for reverberation (Uttley et al. 2011; De Marco et al. 2015). Strong reverberation signatures were also recently reported in the broad-band noise component of MAXI J1820 + 070 by Kara et al. (2019), who studied the soft phase lags of the broad-band noise both in the energy and frequency domain. These authors suggest that a compact corona with a height of about $5 R_g$, where $R_g = GM/c^2$ is the gravitational radius, and G , M and c are the gravitational constant, BH mass and speed of light, respectively, illuminates an accretion disc that is truncated at $\sim 2 R_g$, thus producing the measured soft lags. As the corona contracts, the soft lags become shorter due to the shorter light travel-time. Further attention to reverberation was given, since an accurate reverberation mapping model was developed by Ingram et al. (2019) that is able to put constraints on the mass of the BH, as was shown by Mastroserio, Ingram & van der Klis (2019).

Undoubtedly, reverberation is an essential physical mechanism for modeling

spectra of BHBs at certain states. However, the treatment of the corona as a point-like source at a fixed height above the accretion disc in reverberation models limits the prospects of a self-consistent explanation of the observed phenomenology. In fact, several previous studies (Miller et al. 2010; Legg et al. 2012; Mizumoto et al. 2018; Mizumoto et al. 2019) have presented the timing analyses of Active Galactic Nuclei (AGNs) where, in general, the phenomenology is similar and requires the presence of an extended corona that partially covers the accretion disc. Moreover, the soft lags of the LF QPOs in stellar mass BHBs, on which we focus from now on, are usually much larger than the lags of the broad-band noise component (Wijnands, Homan & van der Klis 1999), and therefore a compact corona or hot flow with a small scale height is unable to produce such large soft lags. Specifically for stellar mass BHBs, Mizumoto et al. (2016) showed that in the case of GRS 1915+105 the lack of a variation of the iron line at different timescales further supports the idea of an extended corona. In addition, De Marco & Ponti (2016) showed that in order to explain the large soft lags of H 1743–322 during the hard states, when the type-C QPO is present, with reverberation, one has to assume an extended corona as an illuminating source, with a height of a few hundred R_g . We note also that the broad-band noise component in NS and BHC sources consists of a combination of QPO-like components (Nowak 2000; Belloni, Psaltis & van der Klis 2002) and therefore it is natural to study the rms and lag spectra of these QPO-like components to understand the properties of the broadband noise.

Although the soft lags of LF QPOs have been extensively studied in the literature, little work has been done to explain quantitatively their energy or frequency dependence in the context of a radiative mechanism. Nobili et al. (2000) suggested a Comptonisation model that consists of a two-component corona, which produces both hard and soft lags through Compton up- and down-scattering, respectively, in these two different components. Their model successfully explains both the magnitude and the change of sign of the lags as a function of QPO frequency in GRS 1915+105.

Recently, Karpouzas et al. (2020) presented a model that explains the observed soft lags of the lower kHz QPO in neutron star (NS) LMXB, as a delayed heating of the seed photon source by photons previously up-scattered in the corona. This effect is referred to as feedback. In the case of NS LMXBs, it has been shown through Monte Carlo simulations (Kumar & Misra 2016a) that feedback should play an important role even among different corona geometries, and that feedback depends on the size of the corona. Although the geometry of the corona and seed photon source are very different between NS and BH LMXBs, one can in principle test how well feedback of up-scattered photons in an extended corona can explain the soft lags observed in BH LMXBs. In fact, based on the model of Karpouzas et al. (2020), García et al. (2021) showed that

a two-component extended corona explains the energy dependence of the time lags and rms amplitude of the type-B QPO in MAXI J1348–630, and so in this work we extend our analysis for type-C QPOs. A good candidate to perform such a test is GRS 1915+105 (Castro-Tirado, Brandt & Lund 1992; Castro-Tirado et al. 1994). GRS 1915+105 is one of the most observed, and thus best studied, sources in the Rossi X-ray timing explorer (Bradt, Rothschild & Swank 1993, RXTE) archive. The compact object in GRS 1915+105, is a BH with a mass $M_{BH} = 12.4^{+2}_{-1.8} M_{\odot}$, (Reid et al. 2014) and spin estimated, with spectral techniques, to be between $a^* = 0.68$ and $a^* = 0.99$ (Šrámková et al. 2015).

GRS 1915 + 105 is a peculiar source, in the sense that it never showed a full Q-shaped cycle on the HID. For a dedicated review on GRS 1915+105, we refer the reader to Fender, Belloni & Gallo (2004). Its spectral behaviour was separated in 14 classes based on the work of Belloni et al. (2000) (see also Klein-Wolt et al. 2002 and Huppenkothen et al. 2017). Timing studies of GRS 1915+105, which are mainly what we are interested in, date back to Chen, Swank & Taam (1997) and Morgan, Remillard & Greiner (1997) and since then, several others focused on the spectral-timing properties of LF QPOs in this source (Markwardt, Swank & Taam 1999; Vignarca et al. 2003; Rodriguez et al. 2004). One of the most notable timing properties related to the type-C QPO phase lag between two broad bands of GRS 1915+105 is possibly the switch from hard lags, at QPO frequencies below ~ 2 Hz (Reig et al. 2000; Pahari et al. 2013; Zhang et al. 2020), to zero at around 2 Hz, and then to soft lags at QPO frequencies above that limit. The slope of the lag-energy spectrum follows the same behaviour, changing from positive below 1.8 Hz to zero at around 1.8 Hz, and then negative above 1.8 Hz. Alongside with the phase lags, the rms amplitude, both energy- and frequency-dependent, of the type-C QPO also shows a particular dependence upon QPO frequency (Zhang et al. 2020).

Recent work from Dutta & Chakrabarti (2016) and Chatterjee, Chakrabarti & Ghosh (2017), implementing the TCAF model (Chakrabarti et al. 2008), linked the QPO frequency to the size of the Centrifugal pressure supported Boundary Layer (CENBOL), which serves as the corona in the TCAF model. These studies suggested that changes in the size of the CENBOL can explain the change in sign of the time lag. More recently, Dutta, Pal & Chakrabarti (2018) applied the TCAF model to data of GRS 1915+105, and showed how a gradual change in the size of the CENBOL can explain the time evolution of the type-C QPO frequency, above and below the 1.8 Hz limit. Dutta, Pal & Chakrabarti (2018) concluded that, as the QPO frequency decreases from over 5 Hz to below 1.8 Hz, the CENBOL increases in size from around $135 R_g$ to over $500 R_g$, and then it decreases again to about $100 R_g$, when the QPO

frequency increases beyond 5 Hz.

Here, we use the Comptonisation model of Karpouzas et al. (2020) to explain the dependence of both the rms amplitude and phase lags upon QPO frequency and energy. In sections 2 and 3 we, respectively, introduce the model and explain its application to the data of Zhang et al. (2020). In section 4 we show our results. Finally, in section 5 we discuss our results and provide a new view to how the corona evolves over time, and how the latter affects the timing behaviour of GRS 1915+105.

3.2 Comptonisation model

In this work we use the model presented by Karpouzas et al. (2020), which is an adaptation of the Comptonisation model proposed by Lee & Miller (1998); Lee, Misra & Taam (2001); and Kumar & Misra (2014). The main idea behind these models is that any QPO can be described as an oscillation of the time averaged spectrum, $n_{\gamma 0}$, that is coupled to oscillations of other properties of the system, such as the corona temperature, kT_e , seed-photon source temperature, kT_s , and the external heating provided to a finite-sized corona, $\dot{H}_{ext}$. No assumption is made about the origin of the QPO frequency in the model. As far as the model is concerned, the QPO is an oscillation of the X-ray flux available for Compton up-scattering in the corona. As a consequence, the properties of the corona and seed-photon source can react to the QPO and oscillate coherently. Another possibility is that the QPO itself is produced by some instability in T_e and $\dot{H}_{ext}$ or some oscillatory mode in the corona (see Ingram, Done & Fragile 2009; Fragile, Straub & Blaes 2016; Fragile 2020).

The model of Karpouzas et al. (2020), assumes that the seed photon source is a blackbody and that the Comptonising corona is a spherically symmetric homogeneous shell with optical depth τ and thickness L around that blackbody. The model also takes into account feedback of up-scattered photons onto the blackbody. When applying the model in practice, first we calculate $n_{\gamma 0}$ by solving the Kompaneets equation (Kompaneets 1957) in steady-state. Next, we linearise the Kompaneets equation assuming that kT_e , kT_s and $\dot{H}_{ext}$ undergo small oscillations at exactly the QPO frequency, and solve for the complex amplitude, δn_{γ} , of the averaged spectrum, $n_{\gamma 0}$. We note that the oscillation of the seed-photon source temperature, kT_s , is attributed to feedback photons. Feedback, within the model, is regulated by the feedback fraction, f_{η} , that is defined as the fraction of the luminosity of the seed source that is solely due to feedback. The complex amplitude, δn_{γ} , holds information about the energy-dependent amplitude and phase lags at the QPO frequency, ν_{QPO} , that was used to perform the linearisation. We note here that only $n_{\gamma 0}$ and δn_{γ} are functions of photon energy, while kT_e , kT_s , L , τ and f_{η} are considered constant

for a specific QPO frequency. In Karpouzas et al. (2020) we showed that the model can fit the energy-dependent time lags and fractional rms amplitude of the kHz QPOs of the NS LMXB, 4U 1636 – 53. In this paper we make a first attempt to quantitatively explain the timing properties of the type-C QPO in the BH LMXB GRS 1915+105 using this same model. Our approach introduces two caveats. Firstly, the source of seed photons that we use is a blackbody instead of a disc-blackbody. Secondly, the photon-loss term in the Kompaneets equation assumes an equal optical depth seen by all seed-photons, which is a crude approximation in the case of a disc whose spatial extent is comparable to the size of the corona. We will expand on these issues in the Discussion section.

3.3 Data Analysis and Modeling

Zhang et al. (2020) presented an extensive timing analysis of 620 observations of GRS 1915+105 showing type-C QPOs, after analysing all of the RXTE archival data (1996–2012). The QPO frequencies ranged from 0.4 Hz to 6.3 Hz. Zhang et al. (2020) measured the phase lags of all type-C QPOs between two broad bands, 2–5.7 keV and 5.7–15 keV. They also measured the lags between multiple narrow energy bands, which we hereafter refer to as lag-energy spectra. In the paper of Zhang et al. (2020), lag-energy spectra at nine selected QPO frequencies were shown for reference. Similarly, Zhang et al. (2020) measured the fractional rms amplitude both in the full PCA band (2–60 keV), and in separate bands. The latter will be referred to as the rms-energy spectrum. The primary focus of Zhang et al. (2020) was to study the frequency dependence of the phase lags between the two broad bands mentioned above, which they called the lag-frequency spectrum. The authors fitted a broken line to the lag-frequency spectrum and placed a sufficiently precise limit of 1.8 ± 0.1 Hz on the QPO frequency at which the measured phase lags change from hard to soft. Their best-fitting broken-line to the lag-frequency spectrum also exhibited a statistically significant difference in slope, with a value of -0.21 ± 0.02 below 1.8 Hz and -0.1 ± 0.01 above 1.8 Hz, indicating a potential change in the physical mechanism that produces the lags.

Zhang et al. (2020) showed that the frequency dependence of the rms amplitude in the full PCA band, which we will refer to as the rms-frequency spectrum, also changes behaviour around the same QPO frequency, switching from increasing with QPO frequency for QPO frequencies below 1.8 Hz, to decreasing for QPO frequencies above 1.8 Hz. Although the model of Karpouzas et al. (2020) can predict both energy and frequency dependence of the fractional rms amplitude and phase lags, in this work we first study the frequency-dependent fractional rms amplitude in the full band, and phase lags between the two broad bands

used in Zhang et al. (2020). The latter approach allows us to analyze all 620 QPO frequencies, while avoiding the computationally expensive process of fitting the rms- and lag-energy spectra at all QPO frequencies. By studying the frequency dependence, we obtain an initial estimate of how the model parameters depend upon QPO frequency. Then we proceed with fitting the rms- and lag-energy spectra presented by Zhang et al. (2020), at representative QPO frequencies, and compare with our initial estimate for completeness. The challenge of the frequency-dependent analysis, with which we start, is that we only have two measurements at our disposal, namely the fractional rms amplitude and phase lag for each set of the 7 parameters of the model, kT_e , kT_s , τ , L , f_η , $\delta\dot{H}_{ext}$ that are needed to produce a pair of simulated fractional rms amplitude and phase lag at a certain QPO frequency, ν_{QPO} . Thus, one can not perform a fitting in the classical sense. In the following, we describe how we can overcome the aforementioned limitation by matching the predicted, from the model, frequency dependence of the fractional rms amplitude and phase lags to the measured ones. Hereafter, we will refer to the term fractional rms amplitude simply as rms amplitude.

As shown by Karpouzas et al. (2020), for a given set of the 7 parameters mentioned above, the model simultaneously predicts the rms amplitude in any band and phase lag between any two bands. However, the amplitude of the external heating rate, $\delta\dot{H}_{ext}$, only affects the rms amplitude, acting as a normalisation, while the phase lags are not dependent on this parameter. Therefore, a good approach is to only simulate the lag-frequency spectrum at first, and then re-scale the simulated rms-frequency spectrum, that was anyway produced alongside the lag-frequency spectrum, by adjusting $\delta\dot{H}_{ext}$ in order to match the measured rms amplitude values. In the following sections we discuss the modeling of the phase lags first, and then we study the resulting rms amplitude.

3.3.1 Modeling of the lag-frequency spectrum

To explain the change of sign of the phase lag around 1.8 Hz and its dependence upon QPO frequency, we designed a simple computational experiment using the model of Karpouzas et al. (2020). In particular, we generated 10^6 random combinations of the model parameters uniformly for kT_e in the range 3–100 keV, kT_s in the range 0.1–3 keV, L in the range 2–500 R_g , f_η in the range 0–1 and ν_{QPO} in the range 0.4–6.3 Hz. We then calculated τ using kT_e and a random uniformly distributed value of Γ between 1.5 and 3, to cover a wide enough range of the power-law index. To calculate τ , for every selected value of kT_e and Γ , we used the formula:

$$\tau = \sqrt{2.25 + \frac{3}{\frac{kT_e}{m_e c^2} [(\Gamma + 0.5)^2 - 2.25]}}, \quad (3.1)$$

where m_e is the electron mass. We set the external heating rate, $\delta\dot{H}_{ext}$, to a constant arbitrary value of 1%, since it only affects the measured rms amplitude and, as has been shown by Karpouzas et al. (2020), it has a dependence upon QPO frequency, which we do not know a priori. Later, when we compare with the measured rms amplitude, we can re-scale $\delta\dot{H}_{ext}$ to match the data, and reveal its QPO frequency dependence and actual value.

For each randomly generated QPO frequency, there is a corresponding randomly generated combination of the five model parameters kT_s , kT_e , L , f_η and τ , that when given as input to the model generates a model estimate of the phase lag between the bands 2–5.7 keV and 5.7–15 keV, at that QPO frequency. The energy bands were chosen such that we maintain consistency with Zhang et al. (2020). If one plots all the values of the model estimated phase lags as a function of the randomly generated QPO frequency, as we have done with gray circles in Figure 3.1, no obvious correlation between the two is observed. However, the data suggests that there exists a significant correlation between phase lag and QPO frequency. To accommodate for the observed correlation, we created two sub-samples from our total simulated sample of phase lags, one with pairs of simulated phase lags and QPO frequencies for which the phase lags were positive, at QPO frequencies below 1.8 Hz and clustered around the best-fitting broken line of Zhang et al. (2020) below 1.8 Hz, and another with pairs of simulated phase lags and QPO frequencies for which the phase lags were negative, at QPO frequencies above 1.8 Hz and clustered around the best-fitting broken line above 1.8 Hz. As a condition for the clustering around the best-fitting broken line, we demanded that the predicted phase lag from the model can deviate from the best-fitting broken line as much as one standard deviation of the data. The aforementioned computational experiment is illustrated in Figure 3.1, alongside with the original data and best-fitting broken line of Zhang et al. (2020).

For each pair of simulated phase lag and QPO frequency, that we restricted alongside the best-fitting broken line in Figure 3.1, there is also a pair of the five remaining model parameters kT_s , kT_e , L , f_η and τ . Our goal was to check whether the aforementioned model parameters, that were used to generate the clustered simulated points around the broken line, have any correlation with QPO frequency that separates them from the total simulated population which is uniformly sampled. We will discuss the dependence of the model parameters upon QPO frequency in the Results section of this paper, where they are presented, but for now we will discuss the other by-product of our

Figure 3.1 – The measured lag-frequency spectrum of GRS 1915+105 (black points with error bars) alongside with all of our simulations. Grey circles represent all of the simulated pairs of phase lags and QPO frequencies. Red represents the simulated pairs that cluster within $1\text{-}\sigma$ of the best-fitting broken line (blue dashed line) to the measured phase lags. The break, where the phase lags switch from hard to soft, is denoted by the cyan dotted line at ~ 1.8 Hz. The white circles that track the best-fitting broken line show individual observations, at selected QPO frequencies, for which the energy dependence of the phase lags was measured (see section 3.3.3).

model, the rms amplitude, which at this point is already calculated but not compared with the measured rms amplitude.

3.3.2 The rms-frequency spectrum

As we mentioned above, our model can predict the rms amplitude at any QPO frequency, simultaneously with the phase lag, for any combination of the model parameters. However, we remind the reader that to compare with measured rms amplitude we need to take into account the amplitude of the external heating rate, $\delta\dot{H}_{ext}$. Up to this point, we have clustered the simulated pairs of phase lags and frequencies alongside the best-fitting broken line (Figure 3.1). In Figure 3.2, we plot the simulated rms-frequency spectrum, i.e. the pairs of simulated rms amplitude and frequency, only for the points for which the corresponding simulated phase lags are clustered around the broken-line of Figure 3.1. On top of that, we plot the rms amplitude measurements of Zhang et al. (2020) alongside with their uncertainties. In Figure 3.2, we re-scaled the simulated rms-frequency spectrum by multiplying all of the values by a factor of 11, which is equivalent to assuming an external heating rate amplitude of $\delta\dot{H}_{ext} = 11\%$ instead of 1% that we used before, equal at all QPO frequencies. This re-scaling is totally arbitrary and only serves as a way to visually compare the simulated rms-frequency spectrum to the measured values of Zhang et al. (2020).

One can immediately distinguish two clusters of simulated points in Figure 3.2. These clusters reflect the degeneracy caused by the lack of prior knowledge of the spectral parameters of the source in these observations (Karpouzas et al. 2020), kT_e , τ and kT_s . To detect and separate the clusters, we used the DBSCAN algorithm (Ester et al. 1996). One of the clusters (blue rectangle in Figure 3.2) corresponds to very low values of the rms amplitude ($< 1\%$) and does not populate the higher QPO frequencies (> 3 Hz) very well, even when arbitrarily re-scaled. The other cluster (red rectangle in Figure 3.2) appears at higher rms amplitude but does not fully populate the lower QPO frequencies (< 2 Hz) sufficiently well. Interestingly, by forcing the simulated lag-frequency spectrum to follow the broken line in Figure 3.1, the corresponding simulated rms-frequency spectrum for the upper cluster (red rectangle in Figure 3.2), has a significant negative correlation, exactly above the 1.8 Hz level, as observed in the measured rms amplitude.

Since $\delta\dot{H}_{ext}$ is QPO frequency-dependent, in general, the simulated rms-frequency spectrum can not perfectly match the data yet, for any arbitrary re-scaling factor that is constant at each frequency. However, if one matches the measured rms-frequency spectrum with the simulated one, the expected frequency dependence of $\delta\dot{H}_{ext}$ can be extracted. To perform the aforementioned matching, first we divided the simulated frequency range in 15 bins. In each bin we calculated the mode of the distribution of the simulated rms amplitude values and used that as the expected value of the simulated rms amplitude in that

Figure 3.2 – The measured rms-frequency spectrum of GRS 1915+105 (black points with errorbars) alongside with the simulated rms-frequency pairs that follow the broken line of Figure 3.1 (contours). The contours were produced by smoothing the two-dimensional space of the simulated points with a Gaussian kernel. The rms amplitude is re-scaled by a factor of 11% to better compare with the data. The red and blue rectangles showcase the separation of the two prominent clusters in the simulations.

bin. We also calculated the $1 - \sigma$ confidence interval around the mode, and used that value as the uncertainty of the simulated rms amplitude in that bin. After doing that, we have an rms-frequency relation which we can match to the measured rms-frequency relation. To do the latter, we simply found a normalisation, different at each QPO frequency that, when multiplied with the simulated rms amplitude, re-produces the measured rms amplitude at each QPO frequency. The value of the normalisation factor at each QPO frequency represents the required external heating rate fractional amplitude, $|\delta\dot{H}_{ext}|$, that best fits the data. We plot the result of this matching in the top panel of Figure 3.3, while the lower panel shows the values of the normalisation factor required at every QPO frequency.

We must note that we discarded the lower cluster (blue rectangle in Figure 3.2) of the simulated rms-frequency spectrum because it would require an extreme variability of the external heating rate ($\delta\dot{H}_{ext} \sim 100\%$), to agree with the measured rms amplitude. In Figure 3.2, the contour levels correspond to a smoothing of the simulated rms-frequency pairs in the two-dimensional space using a Gaussian kernel. Seemingly, the contours in Figure 3.2 show no simulated rms-frequency pairs below 1 Hz. This happens only because the model, under the priors assumed, predicts too few points below 1 Hz, compared to the ones above that frequency, that they appear insignificant after the smoothing process, and thus not visible. This effect will be discussed below.

To study the physical parameters of the model and their dependence upon QPO frequency, which is discussed in the next section, we separated the simulated QPO frequency domain in two regimes, one below 1.8 Hz and the other above 1.8 Hz. For the regime above 1.8 Hz, we used all the simulated models that belonged to the upper cluster of the rms-frequency spectrum (red rectangle of Figure 3.2). For the regime below 1.8 Hz, we used all the available simulated models due to the fact that the simulations in this regime were more scarce. The issue of scarce model sampling below 1.8 Hz will be addressed in Section 3.5.3.

3.3.3 MCMC fitting to the rms- and lag-energy spectra

Zhang et al. (2020) presented rms- and lag-energy spectra of GRS 1915+105 for selected type-C QPO frequencies in the complete range 0.4–6.3 Hz. Those selected frequencies are plotted as white circles in Figure 3.1, and can be viewed as snapshots of the timing properties of GRS 1915+105 at a given QPO frequency. They showed that, on average, the initially negative slope of the lag-energy spectra systematically increases following a linear trend, while the QPO frequency decreases down to ~ 2 Hz. Below 2 Hz, the slope of the lag-energy spectrum, on average, crosses the zero level (switching to hard lags)

Figure 3.3 – Matching of the simulated rms-frequency spectrum to the measured one for the type-C QPOs in GRS 1915+105. The top panel shows the data (black error-bars) alongside with the full simulated values of the rms amplitude (contours) which have been averaged in selected frequency bins (white circles). The blue error-bars indicate the $1\text{-}\sigma$ interval around the mode of each distribution of simulated rms amplitude in a particular frequency bin. The red dashed line in the lower panel is the required external heating rate amplitude, at each QPO frequency bin, that minimises the residuals between the white circles (top panel) and the average measured rms (back errors).

and continues to increase following a significantly different linear trend than the one formed above the 2 Hz limit. Up to this point, our analysis has omitted any information about the energy dependence of the rms amplitude and phase lags provided by the model. Nevertheless, we arrived at an initial estimate of how the physical parameters depend upon QPO frequency. The dependence of the physical parameters upon QPO frequency are shown as black circles in Figure 3.4.

Using this initial estimate as a starting point, we performed an individual Markov-chain Monte Carlo (MCMC) fitting to each rms- and lag-energy spectrum selected by Zhang et al. (2020). For the MCMC fitting and analysis we used the affine-invariant ensemble sampler (Foreman-Mackey et al. 2013) and followed the same approach as in Karpouzas et al. (2020). We must note that in the case of the rms-energy spectrum, for reasons that we discuss in Section 3.5.3, we neglected the rms amplitude measurements above 25 keV. The best-fitting model, alongside with the data of Zhang et al. (2020), at each QPO frequency is plotted separately for the frequencies above 1.8 Hz in Figure 3.5 and for the frequencies below 1.8 Hz in Figure 3.6. As expected, the MCMC fitting gives a separate estimate of the dependence of the physical parameters upon QPO frequency. In the next sections we will discuss these results and how they compare to the analysis that does not take the energy dependence of the rms amplitude and phase lags into account.

3.4 Results

In section 3.3.1, we discussed the clustering of the simulated phase lags alongside the best-fitting broken-line of Zhang et al. (2020). This clustering provides an initial estimate of the frequency dependence of the model parameters, thus revealing what, in the context of the model, is the physical mechanism that drives the lag-frequency relation. In Figure 3.4 we plot five of the model parameters, namely the corona size, L , feedback fraction, f_η , electron temperature of the corona, kT_e , optical depth, τ , and seed photon source temperature, kT_s , as a function of the QPO frequency plus the photon power-law index, Γ , which is not an independent parameter but was used to generate the values of τ for the simulation (see section 3.3.1).

To estimate the frequency dependence of the parameters, we divided the simulated frequency domain of the cases that clustered alongside the broken line in 15 bins. In each frequency bin we studied the distribution of each one of the five parameters and used the mode of the distribution as a best estimate, and the $1 - \sigma$ confidence interval around that mode as the uncertainty of that estimate. These parameters, and their corresponding uncertainties at each QPO frequency are summarized in Table 3.1. In Figure 3.4 the black circles

represent the values of the parameters as a function of QPO frequency derived from the clustering alongside the best-fitting broken-line (Figure 3.1). Hereafter, for simplicity, we will refer to the results derived from the clustering around the best-fitting broken-line process as the frequency-only-approach. The red circles in the same Figure represent the best-fitting parameters derived from the MCMC fitting to the individual rms- and lag-energy spectra of Zhang et al. (2020) at selected QPO frequencies (see Karpouzas et al. 2020 for details on the parameter estimation using the MCMC method). We will refer to the process of MCMC fitting to the rms- and lag-energy spectra as the energy-dependent-approach. The physical parameters derived from the energy-dependent-approach are summarized separately in Table 3.2.

Based on the frequency-only-approach we find that the corona size decreases systematically from $115 R_g$ at around 6 Hz down to about $7.2 R_g$ at 2 Hz, and then increases again to almost $160 R_g$ at 0.9 Hz. Alongside with the decreasing corona size, in Figure 3.4 we plot the evolution of the disc inner radius assuming that the type-C QPO is the manifestation of the LTP frequency around a BH with mass $12.4M_\odot$ and spin, a^* , between 0.68 and 0.99. At the same time, the feedback fraction, f_η , stays at a maximum value of ~ 1 , which means that the disc is mainly heated up by up-scattered photons, for QPO frequencies above 2 Hz and then drops abruptly to zero at frequencies below 2 Hz, where the measured lags also change sign. For the rest of the parameters, kT_e , τ and kT_s , we did not find significant correlations with QPO frequency, except in the case of the corona temperature which exhibits, on average, lower values at frequencies below 2 Hz than above 2 Hz. More specifically, the corona temperature, kT_e , remains above 70 keV, on average, at frequencies above 2 Hz, while at 1.7 Hz it becomes ~ 60 keV and decreases down to about 11 keV at 1.2 Hz. The amplitude of the external heating rate, $|\delta\dot{H}_{ext}|$, which is derived through comparison with the measured rms-frequency spectrum, decreases from around 10% at 5.9 Hz, to 20% at 2 Hz. Below 2 Hz, $|\delta\dot{H}_{ext}|$ becomes extremely high ($>100\%$), a fact that will be discussed later in section 3.5.3.

Figure 3.4 – Dependence of the physical parameters upon the type-C QPO frequency in GRS 1915+105. In each panel the black circles show the parameter estimates based on the total QPO frequency sample (frequency-only-approach) while the red circles show the parameter estimates from the MCMC fitting applied to individual selected frequencies (energy-dependent-approach). The red shaded area, in the upper left panel shows the range of the predicted evolution of the disc inner radius assuming that the type-C QPO frequency is produced by LTP and assuming a range for the masses and spins of the BH in GRS 1915+105. The dotted cyan vertical lines, in all panels, denote the 1.8 Hz limit.

Table 3.1 – Physical parameters of the model at different frequencies of the type-C QPO in GRS 1915 + 105. The frequencies in the first column represent the center of the selected bins in which the distribution of each parameter was analysed. The amplitude of the external heating rate (last column) is deduced by comparison of the simulated rms-frequency spectra to the measured one (see Figure 3.3). The uncertainties of the model parameters were calculated as described in Figure 3.3 and are omitted when only upper limits for the parameters could be constrained.

ν_{qpo} [Hz]	L [R_g]	f_η	kT_e [keV]	kT_s [keV]	τ	Γ	$ \delta\dot{H}_{ext} $ [%]
0.9	157^{+56}_{-113}	0	15^{+24}_{-0}	$0.4^{+1.2}_{-0}$	$2.3^{+2.0}_{-0}$	1.9 ± 0.3	256.2
1.2	185^{+38}_{-75}	0	11^{+49}_{-0}	$0.3^{+1.1}_{-0}$	1.7^{+2}_{-0}	$1.5^{+0.5}_{-0}$	310.6
1.5	231^{+0}_{-64}	0	16^{+44}_{-9}	$0.2^{+1.6}_{-0}$	$1.2^{+2.2}_{-0}$	$1.5^{+0.7}_{-0}$	432.2
1.7	7.6^{+84}_{-0}	$0.1^{+0.5}$	57^{+29}_{-34}	$0.2^{+2.0}_{-0}$	$1.0^{+1.6}_{-0}$	$1.5^{+0.7}_{-0}$	36.5
2.0	7.2^{+34}_{-0}	$1_{-0.2}$	93^{+5}_{-29}	$0.3^{+1.2}_{-0.1}$	$1.0^{+0.7}_{-0.0}$	$1.6^{+0.5}_{-0.2}$	21.1
2.3	7^{+43}_{-0}	$1_{-0.2}$	76^{+22}_{-31}	$0.3^{+1.2}_{-0.1}$	$1.0^{+1.4}_{-0.0}$	$1.7^{+0.4}_{-0.2}$	19.8
2.6	30^{+34}_{-23}	$1_{-0.2}$	79^{+19}_{-28}	$0.4^{+1.1}_{-0.3}$	$1.0^{+0.8}_{-0.0}$	$1.9^{+0.3}_{-0.4}$	19.1
2.8	44 ± 36	$1_{-0.2}$	93^{+5}_{-25}	$0.6^{+1.0}_{-0.4}$	1.5 ± 0.6	$1.8^{+0.4}_{-0.3}$	18.8
3.1	58 ± 38	$1_{-0.2}$	100_{-26}	$0.5^{+1.2}_{-0.3}$	1.5 ± 0.6	$1.7^{+0.5}_{-0.2}$	18.5
3.4	75 ± 40	$1_{-0.2}$	76^{+22}_{-27}	$0.7^{+1.1}_{-0.5}$	$1.1^{+0.9}_{-0}$	$1.7^{+0.4}_{-0.2}$	18.3
3.7	79^{+39}_{-52}	$1_{-0.2}$	100_{-27}	$0.9^{+0.9}_{-0.6}$	$1.0^{+0.8}_{-0}$	$2^{+0.3}_{-0.4}$	17.8
3.9	96 ± 42	$1_{-0.2}$	74^{+23}_{-29}	$0.6^{+1.3}_{-0.3}$	$1^{+1.4}_{-0}$	$1.7^{+0.4}_{-0.3}$	17.1
4.2	107 ± 42	$1_{-0.2}$	100_{-29}	$1^{+1}_{-0.7}$	$1^{+1.4}_{-0}$	$1.7^{+0.5}_{-0.2}$	16.2
4.5	89^{+44}_{-73}	$1_{-0.2}$	91^{+6}_{-24}	$0.9^{+1.2}_{-0.5}$	$0.9^{+1.3}_{-0.0}$	1.9 ± 0.3	15.0
4.8	101^{+44}_{-59}	$1_{-0.2}$	100_{-24}	$1^{+1.1}_{-0.7}$	1.1^{+1}_{-0}	$1.8^{+0.4}_{-0.3}$	13.6
5.0	116 ± 44	$1_{-0.2}$	91^{+6}_{-24}	1.4 ± 0.9	$1.0^{+1.4}_{-0}$	$1.6^{+0.6}_{-0.1}$	12.2
5.3	115^{+45}_{-60}	$1_{-0.2}$	79^{+18}_{-30}	$0.8^{+1.2}_{-0.5}$	$1^{+0.8}_{-0}$	$1.8^{+0.4}_{-0.3}$	11
5.6	117^{+44}_{-59}	$1_{-0.2}$	73^{+24}_{-30}	1.3 ± 0.9	$1^{+0.8}_{-0}$	2 ± 0.3	10.2
5.9	116^{+45}_{-74}	$1_{-0.2}$	79^{+18}_{-30}	$2.9^{+0}_{-0.9}$	$1.1^{+0.9}_{-0}$	$1.8^{+0.4}_{-0.3}$	10.1

In the energy-dependent-approach, we fitted the rms- and lag-energy spectra of the QPO at the individual frequencies selected by Zhang et al. (2020). This allows us to better constrain the physical parameters of the model. This approach is equivalent to taking a snapshot of what the parameters would be at the time when the QPO frequency had a particular value. Based on the results of the energy-dependent-approach, we find that the corona size stays above $100 R_g$ when the QPO frequency is above 2.5 Hz, while at around 2 Hz it becomes $8.7 R_g$ with a lower $1 - \sigma$ uncertainty of $6 R_g$, indicating that around that frequency the corona size, within the uncertainties, can be as low as the lowest value in our simulations, which was $2 R_g$. However, we note here that, based on the MCMC, the best-fitting corona size between 2 Hz and 3.5 Hz does not seem to agree with the smooth decrease in size that we derive from the frequency-only approach. This happens because the observations between 2 Hz and 3.5 Hz, for which we had energy-dependent measurements (white circles in Figure 3.1), happened to be selected such that the phase lags were very low, compared to other observations in the same frequency range, which naturally leads to larger fitted corona sizes, L , since that parameter is mostly affected by the magnitude of the phase lag. The feedback fraction, f_η , decreases with QPO frequency smoothly from almost 100%, at 5 Hz to almost zero at 0.5 Hz. The corona temperature, although badly constrained, is high (>40 keV) at QPO frequencies above 2 Hz, and it becomes significantly lower (<13 keV) at QPO frequencies below 2 Hz. The optical depth of the corona shows a systematic increase from 0.7 at 5 Hz to 6.2 at 0.5 Hz. The seed-photon source temperature also shows a decrease, on average, from values above 1 keV at QPO frequencies above 2 Hz, to values below 0.3 keV at QPO frequencies below 2 Hz. Finally, the amplitude of the external heating rate increases from 17% at 5 Hz to $\sim 40\%$ at 2 Hz, and then it becomes higher than 100% at QPO frequencies below 2 Hz.

The last column of Table 3.2 summarizes the χ^2 of the fits to the rms- and lag-energy spectra at each QPO frequency. For the calculation of χ^2 we combined both the rms amplitude and phase lags, using the same approach as in Karpouzas et al. (2020). The high χ^2 values are, in most cases, because of the bad fit of the model to the rms-energy spectrum at energies higher than 20 keV. Although, as we mentioned before, the measured rms in energy bands above 25 keV was not used in the MCMC fitting, the rms amplitude, predicted by the model, fails to flatten out at high energies. The non-flattening of the model rms amplitude at high energies was already noticed in the case of NS LMXBs (Karpouzas et al. 2020), and attributed to a lack of a non-oscillating flux component in the model, such as reflection, that would contribute to a reduction of the rms amplitude at higher energies. This explanation is also valid in the case of GRS 1915+105, where reflection signatures appear in the

energy spectrum when the type-C QPOs are observed (Misra et al. 2020).

Table 3.2 – Physical parameters of the model as a function of the type-C QPO frequency in GRS 1915+105. The best-fitting model parameters are derived via MCMC fitting to the energy-dependent rms amplitude and phase lags at each frequency, while the corresponding uncertainties were taken as the $1-\sigma$ confidence interval around the mode of the MCMC posterior distributions for each model parameter. The values of Γ in the seventh column are derived using the best-fitting temperature, kT_e , and optical depth, τ , while the uncertainties of Γ are computed through propagation, using the uncertainties of the same parameters.

ν_{qpo} [Hz]	L [R_g]	f_η	kT_e [keV]	kT_s [keV]	τ	Γ	$ \delta\dot{H}_{ext} [\%]$	$\chi^2/d.o.f$
0.466	1348_{-56}^{+45}	0.01 ± 0.004	$7.1_{-0.3}^{+0.2}$	0.2 ± 0.05	$6.2_{-0.3}^{+0.3}$	2.0 ± 0.1	995_{-4}^{+2}	880.9/5
0.865	479_{-81}^{+55}	0.3 ± 0.1	$7.7_{-0.8}^{+1.0}$	0.2 ± 0.04	$4.6_{-0.6}^{+0.4}$	2.3 ± 0.2	926_{-600}^{+62}	482.4/5
1.334	208_{-42}^{+38}	0.4 ± 0.1	12 ± 2.4	0.1 ± 0.05	3.6 ± 0.7	2.3 ± 0.4	826_{-218}^{+127}	54.6/5
1.651	161_{-29}^{+25}	0.4 ± 0.1	13_{-2}^{+3}	0.1 ± 0.03	4.0 ± 0.7	2.1 ± 0.3	719 ± 187	153.9/5
2.077	9_{-6}^{+8}	0.7 ± 0.1	111_{-43}^{+51}	1.3 ± 0.3	$2.3_{-1.1}^{+1.3}$	$1.3_{-0.3}^{+0.2}$	39_{-10}^{+7}	24.3/5
2.538	138_{-28}^{+31}	0.9 ± 0.1	86_{-40}^{+52}	1.3 ± 0.2	$1.0_{-0.4}^{+0.6}$	$2.1_{-0.8}^{+0.6}$	21 ± 3	7.0/5
3.137	136_{-34}^{+31}	0.9 ± 0.1	76_{-45}^{+42}	$1.2_{-0.3}^{+0.4}$	1.1 ± 0.5	2.1 ± 0.8	21 ± 5	17.6/5
4.097	96_{-14}^{+10}	0.8 ± 0.04	$40.3_{-14.2}^{+18.1}$	1.0 ± 0.1	$1.2_{-0.3}^{+0.4}$	$2.7_{-0.8}^{+0.6}$	$17.3_{-2.1}^{+1.8}$	11.2/5
4.979	130_{-22}^{+15}	0.99 ± 0.01	69_{-16}^{+27}	$2.9_{-0.2}^{+0.1}$	0.7 ± 0.2	2.7 ± 0.6	17_{-2}^{+1}	42.4/5

3.5 Discussion

We propose a novel explanation for the change of the time lags of the type-C QPO in GRS 1915+105 from soft, at high QPO frequencies, to hard, at low QPO frequencies. We applied a Comptonisation model that incorporates feedback of the hard corona photons onto the soft photon source to a large data set of timing measurements of this source with the RXTE satellite. We used the rms amplitude and phase lag measurements of Zhang et al. (2020) combined with the model of Karpouzas et al. (2020) and conclude that a corona with time-varying size, accompanied by a switch in feedback efficiency, is able to explain the data. In particular, we find that, as the QPO frequency decreases from 5 Hz to 2 Hz, the size of the corona decreases from around $120 R_g$ to a few R_g . During this contraction, the accretion disc is covered by the corona, the feedback of Compton up-scattered photons onto the disc is very efficient ($\sim 100\%$), and most of the disc flux is due to feedback photons. As the QPO frequency decreases, so does the feedback efficiency because the corona covers less and less area of the disc, as its size decreases. At the same time, assuming that the truncation radius of the disc is anti-correlated to the QPO frequency, the inner radius of the disc increases and at some critical frequency (~ 1.8 Hz) the disc is no longer covered by the corona. From that point on, the efficiency of feedback decreases as the disc inner radius moves further out and the corona size continues to decrease. In Figure 3.7, we present a schematic representation for the evolution of the size of the corona in four phases as the type-C QPO frequency decreases from 5 Hz down to 1 Hz. Our fits are systematically worse when the QPO frequency is below 1.8 Hz than when it is above that value. Our results indicate that when the QPO frequency is below 1.8 Hz the size of the corona increases to values much larger than $130 R_g$, the feedback efficiency decreases to values $< 50\%$ and down to zero, and also the power of the source of external heating for the corona oscillates with an amplitude larger than 100%. The aforementioned paradigm of a decreasing corona and feedback efficiency fits well with all available timing measurements of the type-C QPO in the frequency range of 1.8–5 Hz, and explains both the decrease in phase lags between two energy bands, and the smooth decrease in the slope of the lag-energy spectrum. The change in the behaviour of the lags at 1.8 Hz and below indicates either a possible switch in the physical mechanism that produces the lags or a change in the geometry of the Comptonising region that the model does not take into account.

3.5.1 Comparison to other models

Nobili et al. (2000) proposed a model that was able to explain the lag-frequency spectrum of GRS 1915+105. The difference between their approach and ours

Figure 3.5 – Best-fitting energy-dependent models to the rms- and lag-energy spectra at selected type-C QPO frequencies above 1.8 Hz in GRS 1915+105. In the left and right panels the rms- and lag-energy spectra are plotted, respectively, alongside with the best fitting models. For the different QPO frequencies, the data-points and model-lines are plotted with different styles colors. In the bottom sub-panels of each side we plot the residuals between the data and model normalised by the uncertainties in the data.

Figure 3.6 – Same as in Figure 3.5, for the type-C QPO frequencies in GRS 1915+105 at QPO frequencies below 1.8 Hz.

Figure 3.7 – Schematic representation of the evolution of the corona size in GRS 1915+105 at four typical QPO frequencies. In panel a, at high QPO frequency, a spherically symmetric corona (blue layered circle) covers the inner accretion disc (thick horizontal line) and feedback is very efficient. In panel b the QPO frequency is lower and, assuming it reflects the Lense-Thirring frequency at the inner edge of the disc, the inner-disc radius increases; the size of the corona has decreased enough so that it is only marginally covering the inner disc, while feedback efficiency has decreased. In panel c, where the QPO frequency is at around 2 Hz, the inner-disc radius is higher, the corona has reached its minimum size and is now inside the disc. Finally, in panel d the corona size increases again as the QPO frequency decreases, and the inner edge of the disc increases further.

is in the mechanism that produces the soft lags. In the model of Nobili et al. (2000), the soft lags are caused by Compton down-scattering of soft disc photons that have previously suffered saturated Comptonisation in an inner optically thick component of the corona. In the model of Nobili et al. (2000) the Compton down-scattering happens in an outer cooler component of the corona.

In the model of Karpouzas et al. (2020), the soft lags are interpreted as a delayed response of the seed photon source to feedback photons that impinge back on the disc after being Compton up-scattered in the corona. The differences in the physical assumptions between the two models are reflected on the correlation between the inferred corona size and disc inner radius. In the model of Nobili et al. (2000), the disc inner radius is positively correlated to the size of the corona, whereas in our model the two quantities are anticorrelated down to the limit of ~ 2 Hz and then positively correlated for lower QPO frequencies. Our model, however, explains not only the lag-frequency spectrum, but also both the rms- and lag-energy spectra at different QPO frequencies.

Dutta, Pal & Chakrabarti (2018) based on the TCAF model, provide an explanation opposite to ours when it comes to the evolution of the Comptonising cloud at different type-C QPO frequencies. More specifically, although the size of our corona is of the same order of magnitude as the size of the CENBOL in Dutta, Pal & Chakrabarti (2018), in their interpretation there is no systematic or rapid decrease in the CENBOL size at around 1.8 Hz. Since a transition of a type-C QPO frequency around 1.8 Hz typically happens in matter of a few days (Dutta, Pal & Chakrabarti 2018), the predicted decrease in the size of the corona should happen on the same time scale. Regardless of whether the decrease in corona size is systematic or very rapid, our fits to both the lag-energy and lag-frequency spectra agree that the size should be significantly lower around 1.8 Hz. If our explanation is correct, a reduction in the size of the corona, while the temperature remains the same or decreases, would lead to a reduction in the hard X-ray intensity at low QPO frequencies. Stiele & Kong (2018) and Bhargava et al. (2019) provide evidence of positive correlation between QPO frequency and X-ray flux. Furthermore, if matter from the corona is not advected onto the black hole then, given appropriate conditions in the magnetic field, matter could be directed towards the jet, in which case enhanced radio emission should be present at low QPO frequencies. In support to our expectations, Yan et al. (2013), found that increased radio activity is present when the frequency of the type-C QPO is low.

Although at first glance our explanation appears to be in conflict with the model of Ingram, Done & Fragile (2009), this is not the case. Ingram, Done & Fragile (2009) assumed that the inner flow around an accreting black hole

consists of a geometrically thick torus located inside the truncation radius of a non-precessing geometrically thin disc. As the torus precesses, it illuminates different parts of the disc causing the modulation of the X-ray flux that produces the LF QPO. Ingram et al. (2016) noted, however, that the same pattern would result if the torus was fixed and it was the disc the one that precessed at the LT frequency (Schnittman, Homan & Miller 2006; Tsang & Butsky 2013). As Ingram et al. (2016) explained, their choice of one geometry over the other was based on the fact that the rms spectrum of the QPO is hard and hence the emission at the QPO frequency could not come from the disc.

As we showed here, this argument is not applicable, and a scenario with a precessing disc and a fixed corona is possible. Indeed, in our model the rms spectrum of the QPO is a consequence of inverse Compton scattering of soft disc photons in the corona (the torus in the scenario of Ingram, Done & Fragile 2009), such that the high rms amplitude values of the QPO at high energies simply reflect the variability of the soft disc emission at the LT frequency that is scattered in the corona. This, plus the feedback from the corona to the disc, naturally explain both the variability of the iron line discussed in Ingram et al. (2016) and the rms spectrum of the QPO.

Our interpretation of a corona with a different size at different QPO frequencies, agrees with recent evidence presented by Kara et al. (2019) of a contracting corona in the BH transient MAXI J1820+070. Kara et al. (2019) studied the broad-band noise component through a reverberation model that assumes a point-like corona above the accretion disc, and presented evidence that the height of the corona changes with time. Here, we used the LF QPOs and applied a Comptonisation model that assumes an extended corona, and provided evidence that the size of this corona changes with QPO frequency. Since the QPO frequency changes with time, the corona size also changes with time. The results of Kara et al. (2019) and ours could be due to a similar mechanism that acts throughout the different spectral states and depending on the exact state, and its spectral-timing properties can be detected by using different analysis and modelling techniques.

3.5.2 On the possible connection between the corona and the jet

Persistent and transient BH LMXBs exhibit significant radio emission (Fender 2001). The radio activity is related to the existence of a jet (Band & Grindlay 1986; Levinson & Blandford 1996; Georganopoulos, Aharonian & Kirk 2002; Reig, Kylafis & Giannios 2003), that can produce hard X-rays through Compton up-scattering. Jet models, such as the ones presented in Reig, Kylafis & Giannios (2003) and Kylafis & Reig (2018), successfully explain the hard lags as a function of broad-band frequency in the context of Comptonisation of disc

seed-photons in the jet. However, to the best of our knowledge, the soft lags of the type-C QPOs, and particularly the transition from hard to soft lags, has yet to be explained by jet models.

The radio emission in GRS 1915+105 is occasionally anti-correlated to the hard X-ray flux (Trudolyubov 2001). The fact that the radio emission of GRS 1915+105 is stronger at lower type-C QPO frequencies (Yan et al. 2013), combined with the fact that the lags are hard at low QPO frequencies and, as we showed here, Comptonisation in a uniform corona provides systematically worse fits to the data, naturally leads to the following scenario: At high QPO frequencies an extended corona with efficient feedback onto the disc can explain both the power-law component in the energy spectrum and the lags and rms-amplitude of the type-C QPO. As the size of the corona decreases, feedback onto the disc and Comptonisation in the corona are no longer the dominant components that drive the timing-spectral properties, and so the jet takes over as the medium that produces the power-law and the hard lags. This scenario, if validated, can explain all the spectral and timing properties at once, as well as the correlation between X-ray and radio luminosity.

3.5.3 Model Caveats

We modelled the rms-frequency, lag-frequency, rms-energy and lag-energy spectra of the type-C QPO in GRS 1915+105, using the Comptonisation model of Karpouzas et al. (2020). As with every model, there are certain caveats that we address here. Firstly, the model tends to produce less simulated phase-lag and rms-amplitude values at QPO frequencies lower than 1.8 Hz (Figures 3.1 and 3.2). This happens because in the model the energy-dependent rms-amplitude and lags do not change as smoothly for low values of the feedback fraction, f_η , as for high values ($f_\eta > 0.3$). This behaviour is inherent to the model and depends on the assumptions upon which it is built. To produce hard lags, the model requires less feedback, since in the model increasing the feedback means increasing the time-delay of the soft component with respect to the hard one, and so regimes of hard lags will tend to be poorly sampled. A detailed study of how the model depends upon each physical parameter will be presented in an upcoming publication (García et al. in prep.).

Secondly, the model of Karpouzas et al. (2020) uses a simple blackbody as the seed-photon source for Comptonisation. Despite this simplification, it is remarkable how good the model works when applied to the data. While a multi-colour disc-blackbody would be a more realistic assumption, this would not only add two additional free parameters (source inclination and disc inner radius), but it would complicate the definition of both the optical depth, which should then have an extra spatial dependence, and of the feedback, since the

feedback efficiency should then depend on the inner disc radius. Addressing the aforementioned problems will be part of a future work, especially since source inclination is suspected to play a role in the switch of the time lags behaviour as a function of QPO frequency (van den Eijnden et al. 2017).

Finally, the model yields a seemingly worse fit to the data of the rms-energy spectra above 25 keV, and also in general at QPO frequencies below 1.8 Hz. The first issue was also addressed in Karpouzas et al. (2020) and is likely due to the lack of a reflection component in our model. The second issue, which becomes even more apparent from the very large values of the required external heating rate amplitude ($|\delta\dot{H}_{ext}| > 100\%$), can be due to either a break-down of the assumption of the model at QPO frequencies below 1.8 Hz, or to an actual switch in the physical mechanism, as mentioned in the previous sub-section. To answer as many of these questions as possible and retain applicability to large data-sets, a more detailed modeling of the disc-corona-jet geometry and interaction in a semi-analytical framework is needed.

3.6 Conclusions

We applied our newly developed Comptonisation model, initially designed to explain the lower kHz QPO in NS LMXBs, to the type-C QPO of the BH LMXB GRS 1915+105. We used a large data-set of timing measurements of this source obtained using archival data from the RXTE satellite. We showed that a spherically symmetric and uniform corona of a finite size is able to explain these timing properties, under the condition that the size of the corona decreases as a function of QPO frequency, down to the critical frequency of 1.8 Hz, where the lags turn from soft to hard. Furthermore, the switch of the lags from soft to hard is moderated by the feedback efficiency of up-scattered photons onto the accretion disc that lies inside the corona. For the first time, we were able to fit the energy dependent rms amplitude and time lags of type-C QPOs, which supports the idea that Comptonisation in the corona should be an essential component of every spectral and timing model.

Acknowledgements

D.A acknowledges support from the Royal society. This work is part of the research programme Athena with project number 184.034.002, which is (partly) financed by the Dutch Research Council (NWO). LZ acknowledges support from the Royal Society Newton Funds. TMB acknowledges financial contribution from the agreement ASI-INAF n. 2017-14-H.0. Y.Z. acknowledges the support from China Scholarship Council (CSC 201906100030). This research has made use of data obtained from the High Energy Astrophysics Science

Archive Research Center, provided by NASA's Goddard Space Flight Center.

The effect of Comptonisation in a corona with variable size.

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Under preparation to be submitted to the Monthly Notices of the Royal Astronomical Society

Abstract

The energy spectra of black hole (BH) low-mass X-ray binaries (LMXBs) typically show a high energy tail, associated to Comptonisation of seed photons from an accretion disc, that are fed into a corona of hot electrons. Although the exact geometry of the corona and interaction mechanism between the disc and corona are still under debate, a growing number of studies suggest that the physical size of the corona may be changing throughout the state transitions that BH LMXBs go through. In this work we have developed a new framework, based upon an existing thermal Comptonisation model, that assumes a spherically symmetric corona with variable size that can include only part of the accretion disc, depending on that size. Assuming a part of the disc is being left out of the corona, we show that, the emerging spectrum can be modelled as a sum of two terms that depend both upon the size of the corona and source inclination. We discuss how adding disc inclination and corona size as free parameters affect the shape of the energy spectra of BH LMXBs, and show that significant discrepancies for different values of those parameters occur at energies below 1 keV mainly, if one uses the full expression for the disc spectrum instead of the approximation often used in the literature. With NICER, and possibly other future instruments capable of measuring fluxes below 1 keV, more detailed and realistic modelling of the energy spectra is required and so this work is an effort towards that direction.

Keywords: numerical methods — X-rays binaries — black holes

4.1 Introduction

It is believed that the high energy tail of the spectra of black hole (BH) low-mass X-ray binaries (LMXBs) is mainly produced by Comptonisation in mildly relativistic plasmas (Gierliński et al. 1997; Zdziarski et al. 1998; Wardziński et al. 2002; Zdziarski & Gierliński 2004; Done, Gierliński & Kubota 2007; Burke, Gilfanov & Sunyaev 2017), which we refer to as coronae. At the same time, the low energy part of the spectra is attributed to an optically thick and geometrically thin accretion disc (Shakura & Sunyaev 1973) that can be well modelled by a multi-color blackbody model (Mitsuda et al. 1984). Low-energy (soft) seed photons from the disc can enter into the hot corona and, after multiple scatterings with the free electrons, escape at a later time with higher energies.

Different geometries for the corona have been proposed (Reig, Kylafis & Giannios 2003; Giannios 2005; Haardt & Maraschi 1991, 1993; Galeev, Rosner & Vaiana 1979; Haardt, Maraschi & Ghisellini 1994; Markoff, Nowak & Wilms 2005; Miller, Homan & Miniutti 2006; Miller et al. 2006), and the topic is still under great debate. The existence of cyclical state transitions in BH LMXBs, depicted in the Hardness Intensity Diagram (Homan et al. 2001; Belloni et al. 2005; Homan & Belloni 2005; HID), implies that the geometry of the corona

could also dynamically change. In fact, recent spectral-timing studies (Kara et al. 2019; You et al. 2021; Karpouzas et al. 2021) have shown that the size of the corona in some BH LMXBs can either increase or decrease depending on spectral state. Although the aforementioned studies have assumed different geometries for the corona, we will focus on the one presented by Karpouzas et al. (2021). By fitting a variable Comptonisation model (Karpouzas et al. 2020) to the lag- and rms-spectra of the type-C quasi-periodic oscillations (QPOs) in the BH LMXB GRS 1915+105, these authors showed that as the QPO centroid frequency changes from ~ 6 Hz to ~ 2 Hz, the corona size decreases from $\sim 100R_g$ to $\sim 10R_g$, where R_g is the Schwarzschild radius $R_g = \frac{2GM}{c^2}$, M is the mass of the compact object, G the gravitational constant and c the speed of light. When the QPO centroid frequency decreases further below 2 Hz, the corona size increases again to $> 100R_g$. The results of Karpouzas et al. (2021) indicate that as the source transitions from the soft to the hard state, the corona initially contracts and after a critical type-C QPO frequency has been reached (~ 2 Hz), the corona expands again. The suggestion that the corona size might change continuously is the main motivation for the current work.

A simple and effective way to model thermal Comptonisation in BH LMXBs, is by assuming that the corona is a sphere that surrounds the accretion disc. Models that were built upon the assumption of spherical symmetry of the corona date back to Sunyaev & Titarchuk (1980). These authors provided an analytic solution to the Kompaneets equation (Kompaneets 1957) that describes Comptonisation in an infinite spherical corona, but treated the corona as finite by adding a photon loss term in the Kompaneets equation, which was calculated by solving the photon diffusion equation. Lightman & Zdziarski (1987) and Zdziarski, Johnson & Magdziarz (1996) treated the same problem numerically, also including relativistic corrections to the Kompaneets equation (Cooper 1971), and their approach was later implemented into the model *nthcomp* (Życki, Done & Smith 1999) that, to date, is among the primary tools for analyzing observed spectra in the spectral analysis suite *Xspec* (Arnaud 1996). Other models that treat Comptonisation exist within *Xspec*, such as *comptt* (Hua & Titarchuk 1995), *compps* (Poutanen & Svensson 1996), *simpl* (Steiner et al. 2009), and *thcomp* (Zdziarski et al. 2020).

Thermal Comptonisation alone is not enough to completely describe all the features in the energy spectra of BH LMXBs. Usually, in the soft state an additional soft component is required, which is assumed to be the accretion disc and is modelled as a multi-color disc blackbody (Mitsuda et al. 1984) that is implemented in *Xspec* as the *diskbb* model (Mitsuda et al. 1984; Makishima et al. 1986). To reproduce the high energy part of the spectrum, *nthcomp* assumes that all of the accretion disc can be covered by a spherical corona and so all photons emitted by the disc can effectively undergo the same av-

erage number of scatterings. Furthermore, *nthcomp* uses *diskbb* as a seed for Comptonisation, making the assumption that the disc is truncated at a radial distance that is significantly larger than the characteristic radius R_* , where R_* is roughly equal to R_g . In short, the *nthcomp* model of *Xspec* assumes that the disc is truncated at a distance R , where $R \gg R_*$, and that the spherical corona includes all the photons emitted by the disc. Accretion disc models that do not adopt the assumption $R \gg R_*$ already exist in the literature (Zimmerman et al. 2005), but are not widely used when modelling Comptonisation. A problem that arises from the current use of *nthcomp* is that, if an additional soft component is required on top of the Comptonised one to fit the spectra sufficiently well, some part of the accretion disc will be added twice since it will be considered both as seed for Comptonisation and also as directly observed. To correct for the latter effect, in this work we have used the outer boundary (radius) of a finite spherical corona and only included seed photons that come from the disc that extends from an inner physical radius, to the outer boundary of that corona. The remaining accretion disc that lies outside of that spherical corona has been modelled separately as an additive component. In section 4.2, we develop the numerical framework based on the approach of Lightman & Zdziarski (1987), which allows a variable size for the corona and thus partial coverage of the accretion disc. In section 4.3 we present and discuss simulations with our spectral model for different combinations of the additional physical parameters introduced by our method. Finally, in appendix 4.A we discuss the derivation of a disc blackbody model in the context of a variable corona size, and also compare our model to the *diskbb* model of *Xspec*.

4.2 Modelling Comptonisation in a finite-size spherical corona

4.2.1 Review of the Comptonization model

Comptonisation in the spectra of BH LMXBs, can be modelled by solving the Kompaneets equation (Kompaneets 1957) in steady state. To be consistent with current models used in the literature for fitting observed energy-spectra such as *nthcomp* (Życki, Done & Smith 1999), we follow Lightman & Zdziarski (1987) and write the steady state Kompaneets equation as:

$$\frac{c}{L} \tau_T \frac{d}{dx} \left[\omega(x) x^4 \left(\Theta \frac{du}{dx} + u \right) \right] + \dot{n}_s - \dot{n}_{esc} = 0, \quad (4.1)$$

and

$$\omega(x) = (1 + 4.6x + 1.1x^2)^{-1}, \quad (4.2)$$

where c is the speed of light, L is the radius of an assumed spherical corona, τ_T is the Thomson optical depth, $x = h\nu/m_e c^2$ and $\Theta = kT_e/m_e c^2$, where T_e is the temperature of the corona and ν , h , m_e , k are the frequency, Planck constant, electron mass and Boltzman constant, respectively. The assumption of a spherical corona with constant electron number density, n_e , allows us to write $\tau_T = n_e \sigma_T L$, where σ_T is the Thomson cross section. The function $\omega(x)$ is an empirical correction for relativistic effects (Cooper 1971), and is valid up to $x \sim 1$, where $u = n/x^2$ and n is the photon occupation number (number density inside the medium, in units of $\text{m}^{-3} \text{keV}^{-1}$).

As is obvious in equation (4.1) we include a photon seed term, $\dot{n}_s$, and a photon loss term, $\dot{n}_{esc}$. The treatment of the seed photon source is one of the main points of the present work. For the photon escape rate, following the reasoning of Lightman & Zdziarski (1987), we have used the formula:

$$\dot{n}_{esc} = \frac{c\tau_T}{L} n(x) \left[\tau_T + \frac{1}{3} \tau_T \tau_{KN} f(x) \right]^{-1}, \quad (4.3)$$

where

$$f(x) = \begin{cases} 1, & x \leq 0.1, \\ (1-x)/0.9, & 0.1 < x < 1, \\ 0, & x \geq 1, \end{cases} \quad (4.4)$$

and given the Klein-Nishina cross section for electron-photon scatterings, σ_{KN} ,

$$\tau_{KN}(x) = \frac{\tau_T \sigma_{KN}(x)}{\sigma_T}. \quad (4.5)$$

In equation (4.3) the photon escape rate is basically the photon number density inside the medium, $n(x)$, divided by the average escape time of a photon, $t_{esc} = t_c N_{esc}$, where $N_{esc} = \tau_T + (\tau_T \tau_{KN} f(x))/3$ is the average number of scatterings until a photon escapes a corona of finite size L , assuming that the seed source intensity decreases in intensity with distance from the center (Sunyaev & Titarchuk 1980). The function $f(x)$ was constructed by Lightman & Zdziarski (1987) to achieve a good agreement between the solution of equation (4.1) and Monte Carlo simulations in both the relativistic and non-relativistic regime.

4.2.2 The seed photon source in a variable size corona

To allow the corona size to vary, which will in turn allow us to study the effect of small coronae, one has to find a proper expression for the seed photon source that represents the accretion disc. Although equation (4.1) can be written in a dimensionless form so that the corona size, L , appears explicitly only in the normalization, the photon escape term, $\dot{n}_{esc}$, depends on optical depth, τ_T , which is a function of size and electron number density, and so L will implicitly affect the shape of the spectrum. Traditionally, Comptonisation models, such as *nthcomp*, use the full accretion disc as seed, thus assuming that the corona is always large enough to include every soft photon emitted by the disc. However, since an additional Shakura-Sunyaev disc component is sometimes required on top of Comptonisation to explain the soft part of the spectrum, an inconsistency occurs within the spherical model since all the available disc photons have already been used as seed for Comptonisation and so an additional "naked" disc is not justified. In the *nthcomp* model, which we use as a reference here, τ_T is not treated as a free parameter, since in the context of the model it is geometry dependent, but is calculated through the power law photon index, Γ , which depends upon both τ_T and T_e . A value for Γ is chosen to fit the high-energy tail of the spectrum and, depending on T_e , the model internally calculates the correct τ_T and solves equation 4.1.

To model the seed source, we take the flux from the surface of an optically thick accretion disc as was given by Shakura & Sunyaev (1973). By integrating the flux as a function of radial distance in the disc relative to the central object using the temperature gradient, one can estimate the seed photon rate in a Comptonizing corona when part of an accretion disc resides inside that corona. The inner distance at which matter can be accreted is denoted by R_* , while R is the distance from the center of a coordinate system centered at the compact object. As we have mentioned, $R_* \sim R_g$. By integrating inside the finite medium from $R = R_{in}$ to $R = R_{out} = L$, the seed photons injected per unit time and volume at a normalised energy x from an accretion disc that extends from $r_{in} = R_{in}/R_*$ to $\beta = L/R_*$ can be written as:

$$\dot{n}_s(x) = \frac{8\pi^2 m_e^2 c^2 x^2 R_*^2}{h^3 V_{tot}} \int_{r_{in}}^{\beta} \frac{r dr}{e^{x/\Theta_s(r)} - 1}, \quad (4.6)$$

where the integration is with respect to $r = R/R_*$ and $\Theta_s(r)$ is the temperature gradient of the disc based on Shakura & Sunyaev (1973), h is the Planck constant, m_e the electron rest mass, c the speed of light and V_{tot} the total volume of the corona. In Appendix 4.A we discuss the derivation of equation 4.6, and present a comparison between equation (4.6) and the current disc black-body model (Mitsuda et al. 1984) as implemented in *Xspec* (Arnaud 1996). In

particular, the model of Mitsuda et al. (1984) uses an approximation for $\Theta_s(r)$ that is valid only in the limit $R \gg R_\star$. We note that the temperature at the inner-most disc radius, T_{in} , is no longer a parameter of the model, contrary to usual spectral models, and instead we use a critical temperature, $T_\star$, which is a theoretical quantity (equation 4.18 in Appendix 4.A) that works as an anchor for our model in order to calculate $\Theta_s(r)$. The theoretical temperature $T_\star$ appears as a consequence of using the full radial profile of the disc temperature, instead of the approximation for $R \gg R_\star$ (see Appendix 4.A for details). It is obvious that, by using β as an upper limit for the integration in equation (4.6), we introduce L as a free parameter in the model again, in the sense that we can now control the size of the corona by choosing how much of the disc lies inside that corona. The integral in equation (4.6) is calculated numerically using the composite Simpson's rule.

After a change of variables such that $y = \log x$, and $n = n_c N$ to allow for a finer resolution of a numerical solution at low energies, equation (4.1) becomes:

$$\frac{d^2 N}{dy} - \ln 10 \frac{dN}{dy} + c_2(y) \frac{dN}{dy} + c_1(y) N + \frac{10^{2y}}{\omega(y)} \int_{r_{in}}^{\beta} \frac{r dr}{e^{10y/\Theta_s(r)} - 1} = 0 \quad , \quad (4.7)$$

where :

$$c_2(y) = \frac{10^y \ln 10}{\Theta} + \frac{1}{\omega(y)} \frac{d\omega}{dy}, \quad (4.8)$$

$$c_1(y) = \frac{10^y \ln 10}{\Theta \omega(y)} \frac{d\omega}{dy} + \frac{10^y 2 (\ln 10)^2}{\Theta} - 2 (\ln 10)^2 - \frac{2 \ln 10}{\omega(y)} \frac{d\omega}{dy} - \frac{(\ln 10)^2}{\Theta \omega(y) \left[\tau_T + \frac{1}{3} \tau_T \tau_{KN}(y) f(y) \right]}, \quad (4.9)$$

and

$$n_c = \frac{8\pi^2 m_e^2 c R_\star^2 L (\ln 10)^2}{\Theta h^3 \tau_T V_{tot}}. \quad (4.10)$$

Equation (4.7) is solved using a finite differences scheme. The derivatives in equation (4.7) are approximated with second-order-accurate central difference formulas. After discretization, the linear system is solved via Gaussian

elimination with partial pivoting, using the subroutine *dgtsv* of the package *LAPACK* (Anderson et al. 1999).

When the assumed spherical corona is sufficiently small, a significant part of the disc may remain exposed outside of that corona, and will be detected in soft X-rays. The contribution of the part of the disc that remains uncovered by the corona to the low energy part of the spectrum will depend upon disc inclination, simply because the telescope will receive more photons from a face-on disc (disc plane perpendicular to observers line of sight) than from an edge-on disc (disc plane parallel to observers line of sight). On the contrary, disc inclination does not play a role for the X-ray flux coming from the corona, because even one scattering of disc photons in the corona by hot electrons will isotropise the emission and render inclination irrelevant. The geometry of the problem is illustrated in figure 4.1.

It is obvious that the two spectral components, (a) the Comptonised flux from the spherical corona that includes part of the accretion disc, and (b) the direct emission from the part of the disc that is not covered by the corona, can be treated as independent and will have different normalisations, thus one has to be careful when adding them. Solving equation (4.7) gives the number density of photons inside the medium, which is $n(x) = n_c N(x)$ at steady state, but that is not the flux that escapes from the outer boundary of the spherical corona. The photon number density that is emitted per unit surface, time and energy, from the corona can be written as:

$$\dot{N}_{emit}^s(x) = cP_{esc}n_cN(x). \quad (4.11)$$

Multiplying by the escape probability $P_{esc} = 1/N_{esc}$, we take into account the fact that a photon reaching the outer boundary surface of the corona within one mean free path is not guaranteed to escape due to a potential back-scattering inside the corona, which will delay (but not prevent) its escape. With respect to an observer at distance D , the surface of the corona will emit a Comptonized component which is equal to a total number $\dot{N}_{obs}^s(x) = 4\pi L^2 \dot{N}_{emit}^s(x) / 4\pi D^2 = 4\pi L^2 c P_{esc} n_c N(x) / 4\pi D^2$ of photons. Subsequently, the total number of photons that an observer would receive per unit time, area and energy, given P_{esc} and equation (4.10), is:

$$\dot{N}_{obs}^s = \frac{8\pi^2 m_e^2 c^2 R_\star^2 (\ln 10)^2}{h^3 D^2 \Theta (\tau_T^2 + \frac{1}{3} \tau_T^2 \tau_{KN})} N(x). \quad (4.12)$$

For the part of the accretion disc that lies outside of the corona (outer boundary of the corona is at $\beta = L/R_\star$), the total amount of photons that an observer would receive is:

Figure 4.1 – Comptonisation geometry. The outer sphere is the corona and the red dashed line is the accretion disc plane at an angle $(\frac{\pi}{2} - i)$, where i is the inclination of the source with respect to the observer. The part of the disc that lies inside the corona is denoted by the light red filled area. The rest of the disc (dark red area) is left outside of the corona. The compact object, with its characteristic inner radius R_* , is represented by the gray sphere. The disc is truncated at an inner radius r_{in} shown as a gap between the disc and the compact object. The dark red arrow denotes direct emission from the disc, while the light red arrow denotes the Comptonised emission after consecutive scatterings in the corona.

$$\dot{N}_{obs}^d = \frac{4\pi m_e^2 c^2 \cos i R_\star^2}{h^3 D^2} x^2 \int_\beta^\infty \frac{r dr}{e^{x/\Theta_s(r)} - 1}, \quad (4.13)$$

where in reality the upper limit is not infinity but the physical size of the disc. The combined spectrum is the sum $\dot{N}_{obs}^s + \dot{N}_{obs}^d$, which we write as

$$\dot{N}_{obs}^{tot} = N_{norm} \left[\frac{2\pi(\ln 10)^2}{\Theta(\tau_T^2 + \frac{1}{3}\tau_T^2\tau_{KN})} N(x) + \cos i x^2 \int_\beta^\infty \frac{r dr}{e^{x/\Theta_s(r)} - 1} \right], \quad (4.14)$$

where

$$N_{norm} = \frac{4\pi m_e^2 c^2 R_\star^2}{h^3 D^2}, \quad (4.15)$$

is the common normalisation. Equations (4.14) and (4.15) summarise the model for the spectrum of an accretion disc when part of that disc lies inside a Comptonizing region. The free parameters are the temperature of the medium, Θ , the characteristic temperature, $T_\star$, required to calculate the gradient $\Theta_s(r)$, the Thomson optical depth, τ_T , the inner (or truncation) radius of the disc, r_{in} , the outer limit of the corona, β , the inclination of the disc, i , and the normalization N_{norm} .

4.3 Results and discussion

The new spectral model that we have proposed through equation 4.14 has two peculiarities that need to be explored. The first one is in the Comptonisation component that corresponds to the term with $N(x)$ in equation 4.14, which includes a new parameter, β , that defines the radius of the spherical corona. The second peculiarity is that the term related to the disc component that lies outside of the corona contains the cosine of the inclination of the accretion disc. To study the effect of the β parameter on the Comptonisation spectra, we have calculated $N(x)$ by solving equation 4.7 for different values of β in the range $3-\infty$, where we have used ∞ to denote that β is sufficiently large to include the whole accretion disc.

While studying the effect of β , on the energy spectra, we have fixed the corona temperature, optical depth, critical temperature and normalised disc inner radius to the values $kT_e = 35$ keV, $\tau = 2.1$, $kT_\star = 1$ keV and $r_{in} = 1$, respectively, which are typical for the case of GRS 1915+105 in the soft intermediate state. We note again that $kT_\star$ is not the temperature at the inner disc radius, but

Figure 4.2 – The Comptonised spectrum for different values of β . The inner radius of the disc used as seed was taken at $r_{in} = 1$. The spectral parameters were chosen to be $\Gamma = 2$ (which gives $\tau_T = 2.1$), $kT_e = 35$ keV and $kT_\star = 1$ keV.

one can assume that it implicitly depends on spectral state, because its precise value depends upon mass accretion rate. The value $kT_\star = 1$ keV is physically reasonable for a typical $10M_\odot$ black hole accreting at a rate of $\sim 0.5\dot{M}_{Edd}$ during the soft intermediate state. The Comptonised spectra derived for different values of β are shown in Figure 4.2. We have chosen values of β between 3 and 100 which, depending on the exact ISCO radius, correspond to physical sizes, L , between a few tens to a few hundreds R_g , respectively, which is in agreement with constraints of the corona size for GRS 1915+105 presented in Karpouzas et al. (2021). The overall decrease in flux between the different cases is expected since the corona will cover a smaller part of the disc as β becomes smaller. Furthermore, the sharp flux drop at low energies for decreasing values of β is due to the exclusion of the softer (outer) parts of the disc. Figure 4.2 shows that at energies above 1 keV, the spectra for the different values of β are very similar albeit a different normalisation. However, below 1 keV the

change in the shape of the Comptonised component, due to different β values, can not be accounted for by a re-normalisation. Thus, the danger of always fitting a model that assumes $\beta \sim \infty$ mainly appears in the low energy part of the spectrum that is currently used to constrain the hydrogen column density in the interstellar medium, N_H , through photoelectric absorption models.

The second peculiarity of our model that needs to be discussed is the dependence upon source inclination, which appears through the second term of the sum of equation 4.14. As we have mentioned already, it is reasonable to expect that if a part of the X-ray emitting accretion disc is not covered by the spherical corona, the flux received from that part of the disc should depend upon viewing angle. As one would expect, the effect of inclination should also appear mostly at lower energies and should be stronger or weaker depending on the combination of r_{in} and β . To study the effect of inclination, we have simulated energy spectra where we now take into account both Comptonisation from the corona and direct emission from the disc that lies outside of that corona. For the aforementioned simulations we have used equation 4.14, and assumed the same spectral parameters as before, namely $kT_e = 35$ keV, $\tau = 2.1$, $kT_\star = 1$ keV. First, we kept the normalised disc inner radius fixed at $r_{in} = 1$ and examined two cases for the normalised corona size of $\beta = 15$ and $\beta = 100$. In each case with different β we varied the disc inclination between 0° and 90° and plotted the resulting spectra together on the top left (for $\beta = 15$) and right (for $\beta = 100$) panels of Figure 4.3. Finally, we studied the same case as above and only changed the normalised disc inner radius from $r_{in} = 1$ to $r_{in} = 10$; the resulting spectra for the same range of disc inclinations are plotted in the lower panels of Figure 4.3.

In Figure 4.3, the simulated energy spectra have been plotted only up to 1 keV in energy since above that limit inclination changes the shape of the spectra by less than 1%. As expected, when the corona normalised size is big ($\beta = 100$), the contribution of direct emission from the disc is important only at very low X-ray energies, regardless of how small or big the normalised disc inner radius is (top and bottom right panels in Figure 4.3). On the other hand, when β is comparable to r_{in} , the direct emission from the disc becomes significant at higher energies and thus inclination has a more prominent effect in the shape of the spectra at energies up to ~ 1 keV. The latter case is shown in the left top and bottom panels of Figure 4.3. In the bottom left panel of Figure 4.3, where the disc inner radius is large ($r_{in} = 10$) and the corona is small ($\beta = 15$), the change in the shape of the spectrum due to inclination is profound for energies up to 0.75 keV, while for a disc with low inner radius ($r_{in} = 1$) and same critical temperature, $kT_\star$, the effect of different inclinations on the shape of the spectrum is noticeable mainly below 0.4 keV. In the examples discussed above we have shown that for reasonable values of the spectral parameters of

Figure 4.3 – The total energy spectrum as a combination of a Comptonised component from a finite corona and the accretion disc that remains outside of that corona, for different values of disc inclination between $i = 0^\circ$ (yellow solid lines) and $i = 90^\circ$ (blue dash-dotted line). In each panel, we plot simulated spectra in the energy range $10^{-3} - 1$ keV. For all spectra, we have assumed that $kT_e = 35$ keV, $kT_\star = 1$ keV, $\Gamma = 2$. (which gives $\tau \sim 2.2$). In the top two panels we have assumed that the normalised inner radius of the disc, r_{in} , is unity (the lowest possible value), and have varied the normalised size of the corona between $\beta = 15$ (top left panel) and $\beta = 100$ (top right panel). In the bottom panels we show the same case as the panels above, but for a bigger normalised disc inner radius, $r_{in} = 10$.

a BH LMXB, disc inclination will affect the shape of the energy spectra down to energies that are already observable, e.g. with missions like the NICER.

4.4 Conclusions

We have developed a new numerical framework to model components in the energy spectra of BH LMXBs. We have used an existing thermal Comptonisation model that assumes a spherically symmetric corona. By introducing a new expression for the seed photon term, we have allowed the spherical corona to cover only part of the seed photon source, which is the accretion disc. By doing the latter we have shown that the contribution of the part of the disc that remains outside of the corona to the total flux depends upon source inclination. Different from the current spectral models, our approach introduces both source inclination and the radius of the spherical corona as free parameters, contrary to the current spectral models, and can thus lead to constraints on those parameters that other models can not provide. Finally, our simulations show that the effects of source inclination and small corona sizes on the energy spectra are most prominent at energies below 1 keV, and can only be detectable with NICER. The growing evidence for the geometrical evolution of the corona during state transitions is motivating enough to start making models that account for such evolution, and so the model that we have presented here aims to build upon that idea.

Acknowledgements

D.A acknowledges support from the Royal society. This work is part of the research programme Athena with project number 184.034.002, which is (partly) financed by the Dutch Research Council (NWO). The authors would like to thank A. Zdziarski, P. Zycki and J.Garcia for the helpful discussions and comments on the manuscript.

Appendix

4.A Modelling of the seed source term

For an optically thick disc, every infinitesimal annulus $dA = 2\pi R dR$ can be assumed to emit as separate blackbodies of temperature $T(R)$, and so following Mitsuda et al. (1984) the rate at which photons of energy E are injected into a spherical corona can be written as an integral between an inner radius, R_{in} , and outer radius, R_{out} , as:

$$\dot{n}_E(E) = 2 * \frac{4\pi^2 E^2}{h^3 c^2 V_{tot}} \int_{R_{in}}^{R_{out}} \frac{R dR}{e^{E/kT(R)} - 1}, \quad (4.16)$$

where h is the Planck constant, c the speed of light, k the Boltzman constant and V_{tot} the total volume of the corona. The factor 2 in equation (4.16) accounts for the fact that both sides of the disc inject the same number of photons into the corona. The independent variable, R , in the integration represents the distance from the center of the compact object. Equation (4.16) implies that each annulus dR of the disc contributes to the total flux as a single blackbody component of temperature $T(R)$, which is given by:

$$\frac{T(R)}{T_\star} = \left(\frac{R_\star}{R}\right)^{3/4} \left[1 - \left(\frac{R_\star}{R}\right)^{1/2}\right]^{1/4}. \quad (4.17)$$

Equation (4.17) (Shakura & Sunyaev 1973) is the temperature gradient, where $R_\star$ indicates a critical limit near the compact object and $T_\star$ is a characteristic temperature given by

$$T_{\star} = \left(\frac{3GM\dot{M}}{8\pi R_{\star}^3 \sigma} \right)^{1/4}, \quad (4.18)$$

where G is the Gravitational constant, M the mass of the compact object, $\dot{M}$ the mass accretion rate and σ the Stefan-Boltzman constant. Note that the equation $T(R) = T_{\star}$ has no solution, so $T(R) < T_{\star} \forall R$. It is therefore wrong to say that $T_{\star}$ is the temperature at the inner radius $R_{\star}$. Furthermore, the approximation $R \gg R_{\star}$ is often used, (e.g. Mitsuda et al. 1984), leading to equation (4.17) taking the simple form

$$\frac{T}{T_{\star}} = \left(\frac{R}{R_{\star}} \right)^{-3/4}. \quad (4.19)$$

After substituting the temperature as a function of R from equation (4.19) into equation (4.16), the integration can be done with respect to temperature and we can write:

$$\dot{n}_T(E) = \frac{4}{3} \frac{8\pi^2 E^2 R_{\star}^2}{h^3 c^2 V_{tot}} \int_{T_{out}}^{T_{in}} \frac{\left(\frac{T}{T_{\star}} \right)^{-11/3} \frac{dT}{T_{\star}}}{e^{E/kT} - 1}. \quad (4.20)$$

For simplicity, let us change to the dimensionless variables $t = T/T_{\star}$ and $r = R/R_{\star}$ so that equations (4.16) and (4.20) become respectively:

$$\dot{n}_r = \frac{8\pi^2 E^2 R_{\star}^2}{h^3 c^2 V_{tot}} \int_{r_{in}}^{\beta} \frac{r dr}{e^{E/kT(r)} - 1}, \quad (4.21)$$

where $\beta = R_{out}/R_{\star}$, and

$$\dot{n}_t = \frac{4}{3} \frac{8\pi^2 E^2 R_{\star}^2}{h^3 c^2 V_{tot}} \int_{t_{out}}^{t_{in}} \frac{t^{-\frac{11}{3}} dt}{e^{E/tkT_{\star}} - 1} \quad (4.22)$$

Note that due to the approximation of equation 4.19 we can replace $T_{\star}$ and $R_{\star}$ with T_{in} and R_{in} respectively and thus rewrite equation (4.22) as

$$\dot{n}_{t,spec} = \frac{4}{3} \frac{8\pi^2 E^2 R_{in}^2}{h^3 c^2 V_{tot}} \int_{t_{out}}^1 \frac{t^{-\frac{11}{3}} dt}{e^{E/tkT_{in}} - 1}, \quad (4.23)$$

where now $t = T/T_{in}$. The subscript *Xspec* in equation (4.23) is used to denote that this equation is the one implemented by the *diskbb* model in *Xspec* and is the same as in Mitsuda et al. (1984), apart from the normalisation factor.

The lower limit of the integration is set to zero ($t_{out} = 0$), which implies that the seed photons provided by the disk come from a practically infinite distance from the center of accretion. The upper limit of the integration in equation (4.23) is always unity ($t_{in} = 1$) indicating that we are only taking into account emission that extends down to R_{in} at most.

On one hand, in equation (4.22) the upper limit of integration is variable, indicating how far away from R_* we are integrating (the further away we are from R_* the lower T_{in} will be with respect to T_* and so the upper limit will be closer to zero). On the other hand, the integral in equation 4.23 has a static upper limit set to unity, but the denominator of the exponential term as well as the normalisation factor are connected to T_{in} and R_{in} respectively, and not to the static values T_* and R_* . In that sense, we are trading the simplicity provided by just changing the integration upper limit (equation 4.22) to the ease of using a static upper limit and only one parameter (T_{in}). With the help of equation (4.19), equation (4.23) can be written as

$$\dot{n}_{t,spec} = \frac{4}{3} \frac{8\pi^2 E^2 R_*^2}{h^3 c^2 V_{tot}} \int_{t_{out}}^1 \left(\frac{T_{in}}{T_*} \right)^{-\frac{8}{3}} \frac{t^{-\frac{11}{3}} dt}{e^{E/tkT_{in}} - 1}, \quad (4.24)$$

where now the normalisation factor is the same as in equation (4.22). Using equation (4.23) with $R_{in} \sim R_*$, or equally equation (4.24) with $T_{in} \sim T_*$, violates the assumption $R \gg R_*$. Thus, the use of the *XSPEC* version of *diskbb* should be limited to cases where the disc does not extend down to the ISCO.

We can easily test how big r_{in} should be in order for equations (4.21) and (4.24) to converge. In other words, we can test at what distance from R_* would equation (4.24) be valid. To do this test, we compute the spectrum given by equation (4.21) with $\beta = \infty$ and r_{in} taking increasing values between $r_{in} = 1$ and $r_{in} = 10^{13}$. For each value of r_{in} , we calculate T_{in} directly from equation (4.19) and we then compute the corresponding spectrum given by equation (4.24). Finally, for every r_{in} we compute the integrals of equations (4.21) and (4.24) over all energies and calculate the fractional deviation between them using equation (4.21) as reference. We repeated the aforementioned test for a range of characteristics temperatures, kT_* , between 0.3 keV and 1.5 keV in order to generalize our result. The results of the test are summarised in Figure 4.4. In the aforementioned figure one can see that even at distances of $10 R_*$ ($r_{in} = 10$), the fractional deviation is almost 10%. On the other hand, for $r_{in} > 10^3$ equations (4.21) and (4.24) agree to a 1% level. We only show the case of $kT_* = 0.3$ keV, since that parameter has a negligible effect.

Figure 4.4 – The difference between the disc blackbody model that uses the assumption $R \gg R_*$, through equation (4.24), and the one without that assumption, defined by equation (4.21), as a function of disc inner radius, r_{in} . The difference is shown in units of fractional flux deviation in the y-axis. We have used a characteristic temperature, $kT_* = 0.3 \text{ keV}$, since changing that parameter yields a negligible difference.

The X-ray corona becomes the radio jet in the microquasar GRS 1915+105

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To be submitted in Nature.

Abstract

GRS 1915+105 was the first stellar-mass black-hole in our Galaxy to display a superluminal radio jet, similar to those observed in active galactic nuclei with a super-massive black hole at the centre. It has been proposed that the radio emission in GRS 1915+105 is fed by instabilities in the accretion disc by which the inner parts of the accretion flow is ejected in the jet. Here we show that there is a significant correlation between: (i) the radio flux, coming from the jet, and the flux of the iron emission line, coming from the disc and, (ii) the temperature of the corona that produces the high-energy part of the X-ray spectrum via inverse Compton scattering and the amplitude of a high-frequency variability component coming from the innermost part of the accretion flow. At the same time, the radio flux and the flux of the iron line are strongly anti-correlated with the temperature of the X-ray corona and the amplitude of the high-frequency variability component. These correlations persist over ~ 10 years, despite the highly variable X-ray and radio properties of the source in that period. Our findings provide, for the first time, incontrovertible evidence that the energy that powers this black-hole system can be directed either to the X-ray corona or the jet. When this energy is used to power the corona, raising its temperature, there is less energy left to fuel the jet and the radio flux drops, and vice versa. These facts, plus the modelling of the variability in this source lead to the conclusion that in GRS 1915+105 the X-ray corona morphs into the jet.

5.1 Introduction

The X-ray spectrum of black-hole binaries can be decomposed into three main radiation components: (i) A thermal component that dominates the emission at low (soft) energies (Shakura & Sunyaev 1973), below ~ 5 keV, due to an accretion disc through which mass flows from the binary companion to the black hole. (ii) A power-law like component that dominates the spectrum at energies above ~ 5 – 10 keV, caused by inverse-Compton scattering of the disc photons in a corona of highly-energetic electrons (Sunyaev & Titarchuk 1980). If the energy distribution of the electrons is Maxwellian, this component features a high-energy cut off at an energy that depends upon the electron temperature of the corona, kT_e . (iii) A broad emission line at ~ 6.5 – 7 keV due to iron, produced when photons from the corona reflect off the disc, with the line profile being set by special- and general-relativistic effects (Fabian 1989; Fabian et al. 2009). The X-ray emission of accreting black-hole binaries exhibits high-amplitude variability from tens of milliseconds to decades (Greiner, Morgan & Remillard 1996; Belloni et al. 2000). Depending on the relative importance of these spectral components and the strength of the variability, accreting black holes display two main states (Méndez & van der Klis 1997): In the low-hard state the X-ray spectrum (~ 1 – 20 keV range) is dominated by emission from

the corona, and the Fourier power spectrum shows variability of up to 50% of the average luminosity over a broad range of time scales plus narrow quasi-periodic oscillations, QPOs. In the high-soft state the emission is dominated by the accretion disc, and the variability drops to less than 5% of the average luminosity. Black-hole binaries in the low-hard state emit in radio with a spectrum that is consistent with self-absorbed synchrotron radiation from an optically thick and compact jet (Fender et al. 1997).

During the transition from the low-hard to the high-soft state, some black-hole binaries show radio emission from individual, spatially-resolved, plasma clouds that are ejected in a jet at speeds close to the speed of light. The radio spectrum of these discrete ejections is consistent with synchrotron emission from optically-thin material (Mirabel & Rodríguez 1994). GRS 1915+105 harbours a $12^{+2.0}_{-1.8}$ solar-mass black hole (Reid et al. 2014) and is very variable both in X-rays and radio wavelengths (Belloni et al. 2000; Pooley & Fender 1997). In X-rays, the emission switches from times in which a bright accretion disc with a temperature $kT_{\text{bb}} \approx 2$ keV and small inner-radius dominates the spectrum, to times in which the corona dominates the spectrum, the disc is relatively cool, $kT_{\text{bb}} \approx 0.5\text{--}1$ keV and is inferred to have a large inner radius (Belloni et al. 1997). In several observations, mostly those with a cool disc, in addition to a band-limited noise component, a narrow and strong QPO, called type-C QPO (Casella, Belloni & Stella 2005), appears in the Fourier power spectrum at frequencies between ~ 0.4 Hz and 6.5 Hz (Zhang et al. 2020). The frequency of the QPO increases as the temperature of the disc increases and the spectrum of the source softens. Besides this QPO (and harmonics and sub-harmonics of the fundamental frequency), the power spectrum of GRS 1915+105 sometimes shows a broad variability component at $\sim 60\text{--}80$ Hz (Trudolyubov 2001) that we will call the high-frequency bump. The high-frequency bump appears when the spectrum of the source is dominated by the corona, but observations in which the corona dominates the emission do not always show this high-frequency bump. At the same time, observations of GRS 1915+105 in which the spectrum is dominated by the corona are sometimes, but not always, accompanied by high radio fluxes (Muno, Morgan & Remillard 1999; Fender et al. 1999).

5.2 Data analysis

We studied a large dataset of 1800+ X-ray observations of GRS 1915+105 obtained with the Rossi X-ray Timing Explorer (RXTE) between 1996 and 2012, combined with almost daily observations of the source at 15 GHz with the Ryle telescope. Our final sample consists of 410 observations for which we have simultaneous data both in X-rays and radio. For each of these observations

we have a measurement of (i) the radio flux density at 15 GHz, (ii) the X-ray hardness ratio calculated as the ratio of the intensity in the 13–60 keV to the 2–7 keV band, (iii) the frequency of the fundamental component of the type-C QPO, (iv) the phase lag at the QPO frequency for photons in the 5.7–15 keV band with respect to those in the 2–5.7 keV band, (v) the 2–60 keV fractional rms amplitude of the high-frequency bump, and (vi) the best-fitting parameters to the X-ray energy spectra of the source. These vastly different types of data, consisting of X-ray and radio fluxes and spectral and timing properties of a single source, come from wavelengths that are more than eight orders of magnitude apart and sample time scales that are more than eleven orders of magnitude different, from ten milliseconds to a decade.

5.2.1 Power spectra

We examined all the RXTE archival observations of GRS1915+105 from 1996 to 2012 obtained with the Proportional Counter Array (PCA). The observations that we used for our analysis belong to the class (Belloni et al. 2000) χ , state C, equivalent to the hard state in other (transient) black-hole sources. For each observation we computed the Fourier power spectrum in the full energy band (absolute PCA channel 0–249) every 128 s with a time resolution of 1/512 s, corresponding to a Nyquist frequency of 256 Hz. We averaged all the 128-s power spectra within an observation, subtracted the contribution due to Poisson noise (Zhang et al. 1995) and normalised (Belloni & Hasinger 1990) these averaged power spectra to units of fractional rms squared per Hz. (We ignored the background count rate for this calculation since it was always negligible compared to the source count rate.) We subsequently applied a logarithmic frequency rebin to the data such that the size of a bin increases by $\exp(1/100)$ with respect to the size of the previous one, and we used XSPEC version 12.9 to fit the resulting power spectra with a sum of Lorentzian functions (Nowak 2000) that represent the broad-band noise component and a number of QPOs (Zhang et al. 2020). As in previous studies (Trudolyubov 2001) we included a Gaussian component centred at zero frequency in the model to fit a high-frequency bump at 60–80 Hz in the power spectra. We show two power spectra with the best-fitting model in Figure 5.1.

For the rest of the analysis we selected only observations where at least one narrow QPO peak is present on top of the broad-band noise component in the power spectra, which is typical for the type-C QPOs (Casella, Belloni & Stella 2005). Based on the fitting results we only retained features that were detected at a significance greater than 3σ and had a Q factor, defined as the ratio of the QPO frequency to its width, of 2 or more. We further checked the spectrogram of each observation, which shows visually the Fourier power spectrum as it varies with time, and excluded ~ 130 observations in which the

Figure 5.1 – X-ray Fourier power spectra of two observations of GRS 1915+105 (data with errors) fitted with a model consisting of several Lorentzians (solid lines with the individual components indicated with dotted lines) for the QPO (plus harmonic and sub-harmonic components) and a zero-centred Gaussian for the high-frequency bump. In both power spectra the strong QPO fundamental component is at ~ 2 Hz; the high-frequency bump peaking at ~ 70 Hz is significantly detected, with an rms amplitude of $8.0 \pm 0.3\%$, in the power spectrum plotted in black (ObsID 30703-01-34-00 of September 21 1998), and is significantly absent, with a 95% confidence upper limit of the rms amplitude of 1%, in the power spectrum plotted in blue (ObsID 30402-01-09-01 of April 10 1998).

QPO frequency changed significantly within a full RXTE observation. Our final sample contains a total of 620 observations.

5.2.2 Lag spectra

Following the method described in Vaughan & Nowak (1997) and Nowak et al. (1999) we produced a frequency-dependent phase-lag spectrum (lag-frequency spectrum) between the 2–5.7 and 5.7–15 keV energy bands for each observation. Since our sample includes observations during the PCA calibration epochs 3–5, to account for changes in the PCA channel-to-energy gain factor we selected the closest absolute channels that matched these energy bands, but the exact boundaries of each band still differ slightly between epochs. To calculate the phase lags of the QPO we averaged the lag-frequency spectra around the centroid frequency of the QPO, $\nu_0 \pm FWHM/2$, where $FWHM$ is the full-width at half-maximum of the Lorentzian that we used to fit the QPO profile. In principle, the phase lags in the range of frequencies of the QPO can be affected by the lags of the underlying broad-band noise component. However, in GRS 1915+105 the rms amplitude of the QPO is much higher than that of the broad-band noise and the phase lags at the frequency of the QPO are dominated by the QPO itself, with the contribution of the noise component being negligible (van den Eijnden, Ingram & Uttley 2016). In this work, a positive (negative) lag means that the hard photons lag (lead) the soft photons. No correction for the dead-time-driven cross-talk (van der Klis et al. 1987) was done because this effect was found to be negligible. (See Zhang et al. (2020) for other details of the timing analysis.) In Figure 5.2 we show nine representative power and lag-frequency spectra covering the range of frequencies of the QPO in GRS 1915+105.

5.2.3 Energy Spectra

We used the RXTE/PCA* Standard 2 data to extract energy spectra separately for each observation in our sample. We corrected the energy spectra for dead time and used the FTOOLS PCABACKEST and PCARSP in HEADAS v.6.27 to, respectively, extract background spectra and produce response files for each observation. We fitted the energy spectra of all observations jointly with the model VPHABS*(DISKBB+GAUSS+NTHCOMP). The component VPHABS accounts for the interstellar absorption along the line of sight to the source; we used the abundance and cross-section tables given by Wilms, Allen & McCray (2000) and Verner et al. (1996), respectively, with the column density of hy-

* We also fitted the RXTE/HEXTE (High Energy X-ray Timing Experiment) data of those observations in which the instrument was operational; the results were consistent with the ones of the RXTE/PCA, but since data of this instrument were not available for all observations, we did not use the RXTE/HEXTE data for the rest of the analysis.

Figure 5.2 – Power-density (upper half of each panel) and phase-lag (lower half of each panel) spectra of nine observations of GRS 1915+105 with RXTE. Each panel corresponds to an observation in which the QPO was at the frequency given, together with the ObsID, in the legend of the panel. We calculated the power spectra using the photons from the full PCA band, and the phase-lag spectrum using photons in the 5.7–15 keV band with respect to the photons in the 2–5.7 keV band. The lags are given in units of $\text{rad}/2\pi$ and indicate whether, at a given frequency, the variability of the low-energy photons lag (negative lags) or lead (positive lags) that of the high-energy photons. The black vertical lines indicate the range of frequencies that we used to measure the lags of the QPO.

drogen, N_{H} , linked across observations and free to vary. Because N_{H} in the direction of GRS 1915+105 is quite high (Miller et al. 2013), $N_{\text{H}} \sim 6 \times 10^{22}$ atoms cm^{-2} , we also let the abundance of iron in the interstellar material free to vary, since neutral iron produces an absorption edge at $E \approx 7.1$ keV that was apparent in the fit residuals. The component DISKBB (Mitsuda et al. 1984) represents the emission from an optically thick and geometrically thin accretion disc, and has parameters kT_{bb} and N_{dbb} that are, respectively, the temperature at the inner disc radius and the normalisation of the component defined as the ratio of the inner disc radius to the distance to the source squared multiplied by the cosine of the inclination of the disc with respect to the line of sight. The component NTHCOMP (Zdziarski, Johnson & Magdziarz 1996) represents the emission due to inverse-Compton scattering from the corona. The parameters of this component are the power-law index, Γ , the electron temperature of the corona, kT_{e} , the temperature of the source of soft photons that enter into, and are up scattered in, the corona, kT_{seed} , and a normalisation that gives the flux density at 1 keV. We assumed that the source of seed soft photons is the accretion disc, and linked the temperature kT_{seed} to the temperature of the DISKBB component for each observation separately during the fits. In this model, the optical depth of the corona, τ , assumed to be homogeneous, is a function (Sunyaev & Titarchuk 1980) of Γ and kT_{e} . Finally, the component GAUSS represents a broad iron emission line at 6.5–7 keV due to reflection of corona photons off the accretion disc (Fabian 1989; Fabian et al. 2009). While a full reflection model (García et al. 2014; Dauser et al. 2014) would be more appropriate to describe this effect, the PCA instrument does not have enough spectral resolution to distinguish between the profile of a line in a full reflection model and a Gaussian component. The parameters of the GAUSS component are the energy, width and total photon flux of the line, E_{g} , σ , and N_{g} , respectively. Finally, we calculated the total observed flux of the source and the intrinsic (i.e., not affected by interstellar absorption) flux of the full model and of all the individual model components in the 2–25 keV energy range. In Figure 5.3 we show the energy spectra, with the best-fitting model, of the same two observations in Figure 5.1.

5.2.4 Hardness ratio

For each RXTE observation of GRS 1915+105 we computed a hardness ratio value defined as the ratio of the background-subtracted count rate of the source in the 13–60 keV band to that in the 2–7 keV band. As with the lag-frequency spectra, we selected the closest absolute channels that matched these energy bands in each PCA gain epoch. Before calculating the ratios, we corrected each observed count rate for instrumental dead time and normalised them by the count rate in the same band from the Crab nebula to account for possible

Figure 5.3 – X-ray energy spectra (data points with errors) of the same two observations of GRS 1915+105 shown in Figure 5.1, fitted with a model consisting of the emission from the accretion disc, the corona and a broad iron line. The dashed lines represent the best-fitting model of the corona, while the solid lines through the data correspond to the full model. In the spectrum plotted in black (ObsID 30703-01-34-00 of September 21 1998) the temperature of the corona is $kT_e = 10.9 \pm 0.8$ keV, while in the one plotted in blue (ObsID 30402-01-09-01 of April 10 1998) $kT_e = 6.3 \pm 0.5$ keV. The difference in temperature between the two spectra is apparent by the fact that, at high energies, the flux drops quicker in the spectrum plotted in blue than in the one plotted in black. The bottom panel shows the residuals, $\Delta\chi = (\text{data} - \text{model})/\text{error}$, of the best-fitting model.

changes of the effective area of the instrument with time.

5.2.5 Radio fluxes

For the flux-density data of GRS 1915+105 in radio we used measurements (Pooley & Fender 1997) from the Ryle Telescope at 15 GHz with the four mobile antennas set in a compact array configuration within 100 m of the nearest fixed antenna yielding a resolution of ~ 30 arcsec at that frequency. The flux-density scale was calibrated with observations of the nearby quasars 3C 48 or 3C 286. The observations consist mostly of 32-s samples with an rms noise of 6 mJy that decreases as the square root of the integration time; flux-density values below about 1 mJy may be unreliable. See Pooley & Fender (1997) for other details of the analysis of the radio data. Finally, we cross-correlated all the X-ray and radio data based on the date of the observations, which left us with a sample of 410 observations with simultaneous radio flux densities at 15 GHz and X-ray energy, power-density and lag-frequency spectra and hardness ratios. Having described the observations and analysis we used, it is worth noting that the measurements presented in this paper come from very different types of data and totally independent analysis techniques: The hardness ratio, iron-line flux and kT_e come from X-ray light curves and time-averaged energy spectra, the frequency and lags of the QPO and the rms amplitude of the high-frequency bump come from Fourier power spectra of high-time resolution data, and the radio flux was measured independently in a totally different frequency band than the X-ray data.

5.3 Results and Discussion

In Figure 5.4 we plot the X-ray hardness ratio as a function of the frequency of the QPO for these 410 observations of GRS 1915+105. The x and y axes of the four panels in this Figure are the same, while the colours of the points in each panel represent, respectively, the simultaneous 15-GHz radio-flux measurements from the jet (Fig. 5.4a), the electron temperature of the X-ray corona (Fig. 5.4b), the flux of the iron emission line in the X-ray spectrum (Fig. 5.4c), and the fractional rms amplitude of the high-frequency bump (Fig. 5.4d). In all four panels blue (red) indicates high (low) values of the quantity represented by the colours. The QPO frequency generally increases as the hardness ratio decreases and the source spectrum softens, consistent with a decreasing inner radius of the accretion disc that leads to an increase of both the disc flux (Shakura & Sunyaev 1973) and QPO frequency (Stella & Vietri 1998; Ingram, Done & Fragile 2009). The relation, however, is significantly broader than the spread expected from the errors in each quantity (the errors are smaller than the size of the points). The colours of the points in the four panels of this Fig-

ure show that the radio flux, the flux of the iron line, the temperature of the corona and the fractional rms amplitude of the high-frequency bump depend upon both QPO frequency and hardness ratio. This double dependence is the reason for the striking match between the two panels on the left and the two on the right, and the almost perfect mirror match between the two top and the two bottom panels. In other words, the breadth of the hardness-ratio/QPO-frequency relation is consistently set by either of the four quantities, such that if we plotted the data in a diagram with the QPO frequency along the x axis, the hardness ratio along the y axis and any of these four quantities along the z axis, the points would lie on a 2-dimensional surface in 3D.

The two panels of Figure 5.5 show the flux of the iron line as a function of the X-ray flux in the 2–25-keV band for all observations of GRS 1915+105. In the left panel the colour of the points represents the temperature of the corona, while in the right panel the colours represent the phase lags between the 2–5.7 keV and 5.7–15 keV bands at the frequency of the QPO. While the flux of the iron line increases as the total flux of the source increases, when the temperature of the corona is low, the radio flux is high and the lags at the QPO frequency are positive the correlation (Neilsen & Lee 2009) is steeper than when the temperature is high, the radio flux is low and the lags are negative.

5.3.1 Fits to the power spectra

In Figure 5.1 we plot two representative Fourier power spectra of GRS 1915+105 to show both the type-C QPO and the high-frequency bump. The power spectrum plotted in black corresponds to the observation from September 21 1998 (ObsID 30703-01-34-00), while the power spectrum plotted in blue corresponds to the observation from April 10 1998 (ObsID 30402-01-09-01). Both power spectra were calculated using photons from the full PCA band, nominally 2–60 keV. The QPO is the narrow feature that appears in both power spectra at ~ 2 Hz, with a second harmonic and subharmonic at, respectively, ~ 4 Hz and ~ 1 Hz. In the case of GRS 1915+105 it is relatively straightforward to distinguish the QPO fundamental from the subharmonic and harmonic components because the fundamental is always the strongest peak in the power spectrum. Not only that, the QPO fundamental, subharmonic and harmonic components show very different rms amplitude and lags dependence when plotted vs. their own frequency (Zhang et al. 2020). The model includes also two broad Lorentzians centred, respectively, at 0 Hz and 2.5 Hz to fit the broad-band noise component in the power spectrum. The high-frequency bump is the broad feature extending up to ~ 100 Hz in the black power spectrum (Belloni et al. 1999; Revnivtsev, Gilfanov & Churazov 2000; Nowak 2000; Belloni, Psaltis & van der Klis 2002; Pottschmidt et al. 2003; Motta et al. 2014b,a).

Figure 5.4 – Hardness ratio vs. QPO frequency of GRS 1915+105. Each point corresponds to one of the 410 observations of GRS 1915+105. The hardness ratio is the ratio of the observed intensity from 13–60 keV to 2–7 keV in units of the Crab nebula. Average errors of the QPO frequency and hardness ratio are, respectively, ± 0.05 Hz and ± 0.001 Crab units, and are both smaller than the size of the points. The colour of the points in each panel represent: **a**, The 15-GHz radio-flux with average error of $\pm(1-5)$ mJy. **b**, The electron temperature of the X-ray corona with average error of ± 0.75 keV. **c**, The flux of the iron emission line with average error of $\pm 0.7 \times 10^{-10}$ erg cm $^{-2}$ s $^{-1}$. **d**, The fractional rms amplitude of the high-frequency bump with average error of $\pm 0.7\%$. In panel **d** the circles indicate measured values and the triangles upper limits. (All errors and upper limits in this work represent 1- σ and 95% confidence intervals, respectively.)

We fitted this bump with a zero-centred Gaussian and let the width and normalisation free to vary during the fits. It is apparent that the QPO at 2 Hz and its second harmonic and subharmonic are present in both power spectra, but the high-frequency bump is only present in the observation of September

Figure 5.5 – Flux of the iron line vs. total flux in the 2–25 keV range for GRS 1915+105. Each point corresponds to one of the 410 observations of GRS 1915+105. The colour scale represents the temperature of the corona (upper panel) and the phase lags of the QPO in radians (lower panel). Negative (positive) lags indicate that, at the QPO frequency, the soft photons in the band 2–5.7 keV arrive after (before) the hard ones in the band 5.7–15 keV. The average error of the plotted quantities is shown at the bottom right of the plots. The lines indicate the two separate correlations.

1998 and significantly absent in the one of April 1998. We computed the rms amplitude of the high-frequency bump by integrating the power of the best-fitting Gaussian from 0 Hz to ∞ . The rms amplitude of the high-frequency bump in the observation of September 1998 is $8.0 \pm 0.3\%$ of the total luminosity of the source in the 2–60 keV band, whereas the 95% confidence upper limit of the rms amplitude in the observation of April 1998 is only 1% of the total luminosity. Since the high-frequency bump was not significantly detected in the observation of April 1998, to calculate the upper limit we fixed the width of the Gaussian to the value we obtained from the fit to the observation of September 1998. We followed the same procedure to compute the rms amplitude or, when appropriate, the upper limits of the high-frequency bump in the rest of the observations. In the observations in which the high-frequency bump is significantly detected the width of this feature ranges from ~ 30 Hz to ~ 100 Hz. In those observations in which the high-frequency bump was not significantly detected, to compute the upper limits we fixed the width of the Gaussian to 70 Hz, which is the average value that we obtained from the fits of the observations in which the high-frequency bump was significantly detected. We report a detailed analysis of the properties of the high-frequency bump in a separate paper (Zhang et al. 2021).

5.3.2 Fits to the energy spectra

In Figure 5.3 we show the energy spectra of the same two observations for which we show the power spectra in Figure 5.1. The spectrum plotted in black corresponds to the observation from September 21 1998 (ObsID 30703-01-34-00), while the spectrum plotted in blue corresponds to the observation from April 10 1998 (ObsID 30402-01-09-01). This Figure shows that the flux of the source was lower and, at the same time, the spectrum was harder (extending to higher energies) in September 1998 than in April 1998. This difference is reflected in the values of the best-fitting temperature of the corona in these two observations, which are $kT_e = 10.9 \pm 0.8$ keV and $kT_e = 6.3 \pm 0.5$ in the observations of September 1998 and April 1998, respectively. From the comparison of this Figure and Figure 5.1 it is apparent that the high-frequency bump is present when the temperature of the corona is high. Indeed, when we fit all the observations we find that the rms amplitude of the high-frequency bump is higher when the temperature of the corona is higher, regardless of the values of the hardness ratio and QPO frequency in each observation (see Figure 5.4).

5.3.3 Energy-dependent rms amplitude and phase lags of the QPO: The corona becomes the jet

In Figure 5.2 we plot the power- and lag-frequency spectra of nine observations of GRS 1915+105, representative of the range of frequencies covered by the type-C QPO. The phase lags of the QPO change from positive when the frequency of the QPO is lower than ~ 2 Hz to negative when the frequency of the QPO is higher than ~ 2 Hz. (See Zhang et al. 2020 for details of this result and a discussion of the implication of this behaviour). At the same time, as the QPO frequency increases the slope of the lag spectrum decreases (Karpouzas et al. 2021), going from positive when the frequency increases from ~ 0.5 Hz to ~ 2 Hz, to 0 when the frequency is ~ 2 Hz, and finally to negative when the QPO frequency increases further from ~ 2 Hz to ~ 6 Hz.

The energy-dependent rms amplitude and phase lags of the QPO in GRS 1915+105 are well explained by a time-dependent model (Karpouzas et al. 2020) in which photons from the accretion disc are inverse Compton scattered in a corona of characteristic size L , and a fraction of those photons subsequently impinges back onto the disc before reaching the observer. The time-averaged version of this variability model is equivalent to the NTHCOMP model in XSPEC. In the current version of the model we assume that the corona is spherically symmetric and has constant optical depth and electron temperature. Most likely the geometry of the corona is more complex than that, and the optical depth and electron temperature change as a function of the distance to the black hole. In a non-spherical corona the rms amplitude and lags of the QPO may change with the inclination of the binary system with respect to the line of sight (Motta et al. 2015; van den Eijnden et al. 2017). We note, however, that both the rms amplitude and lags of the QPO in this and other sources change in a systematic way with QPO frequency, which in turn changes over time scales of minutes to hours, much faster than potential changes of the binary inclination that we expected to happen over periods of a few tens of days to a few thousand years (Wijers & Pringle 1999). This evinces that, although inclination may play a role, the thermodynamical properties of the corona have a much larger effect than inclination in setting the rms amplitude and lags of the QPO.

Fits with this time-dependent model to the data of GRS 1915+105 show (Karpouzas et al. 2021; García et al. 2021) that, as the QPO frequency decreases from ~ 6 Hz to ~ 2 Hz, the feedback fraction is close to 100% and remains more or less constant. At the same time the size* of the corona decreases from

* In the model we solve the Kompaneets equation (Kompaneets 1957) for a spherical corona; since the actual geometry of the corona is likely different, the values given here should be considered as a characteristic size of the corona.

Figure 5.6 – Size of the corona as a function of QPO frequency for GRS 1915+105. Each point corresponds to one of the 410 observations of GRS 1915+105. The size of the corona comes from the fits of a variable-Comptonisation model (Karpouzas et al. 2020, 2021) to the rms and lag spectra of the QPO (García et al. 2021). The colour of the points indicates the feedback fraction, the fraction of photons from the corona that returns to the disc. The black solid line is the weighted average of the points. The grey dashed line shows the inner radius of the accretion disc in km assuming that the QPO frequency is the Lense-Thirring precession frequency for a 12 solar-mass black hole.

$L \approx 1000\text{--}2000$ km ($\sim 60\text{--}120 R_g$ for a 12 solar-mass black hole) to $L \approx 100$ km ($\sim 6\text{--}10 R_g$), the lags are negative and their magnitude decreases. At a QPO frequency of ~ 2 Hz the corona size is minimum, the feedback fraction drops abruptly to zero and the lags become zero. If the QPO frequency is produced by Lense-Thirring precession (Stella & Vietri 1998; Ingram, Done & Fragile 2009) of the inner edge of the accretion disc, at a QPO frequency of 2 Hz the size of the corona is more or less equal to the inner disc radius (dashed line in Figure 5.6). When the QPO frequency decreases below ~ 2 Hz, the corona size increases again up to $L \approx 2 \times 10^4$ km ($\sim 1200 R_g$) at the lowest QPO frequency, the feedback fraction remains constant close to zero and the lags become positive and increase with decreasing QPO frequency. Although the size of the corona increases again when the QPO frequency is below ~ 2 Hz, the fact that the feedback fraction is close to zero whereas when the QPO frequency is above ~ 2 Hz the feedback fraction is high, implies that the corona does not cover the inner parts of the accretion disc. This in turn demonstrates that the geometry of the corona has changed (see Fig. 5.7).

Figure 5.8 shows the time evolution of the QPO frequency superimposed on 10 years of the radio flux measurements of GRS 1915+105 with the Ryle telescope. To produce this Figure we reanalysed all the RXTE observations in the period of the radio monitoring campaign, and added 357 observations with QPOs that were not in the sample of 410 observations discussed so far. These 357 observations were not included in the original analysis (Zhang et al. 2020) because the QPO frequency drifts by a few tenths of Hz in the observation period, which could bias the measurement of the rms and lags of the QPO. But since the frequency drift is only slightly larger than the error of the average QPO frequency in each observation, we decided to use these data for this Figure because this allows us to have a denser coverage of the time evolution of the QPO frequency during the radio monitoring campaign. Figure 5.8 shows that whenever the QPO frequency is below 2 Hz, the source displays a strong radio flare* and, vice versa, whenever there is a radio flare the QPO frequency goes below 2 Hz. This behaviour repeats consistently for all the radio flares in these observations; there is no radio flare in which the QPO frequency does not cross below 2 Hz, and there is no case of a QPO frequency below 2 Hz without a radio flare. Given the data, we estimated that the probability of having a QPO frequency below 2 Hz and a radio flare at the same time if the two phenomena were uncorrelated is less than 2×10^{-10} .

* While it is apparent to the eye from the Figure, it is difficult to quantify what a strong radio flare is. Here we call a flare strong if its flux density exceeds ~ 50 mJy for more than ~ 10 days and the flare lasts more than ~ 100 days.

Figure 5.7 – Schematic picture of the corona turning into the jet in GRS 1915+105. When the QPO frequency is in the 2–6 Hz range the lags are negative, the corona is hot and extended, and covers the inner parts of the disc (panel **a**). The magnitude of the lags and the size of the corona decrease and the inner disc radius increases as the QPO frequency decreases. When the QPO frequency is ~ 2 Hz the lags are zero and the size of the corona is equal to the inner radius of the accretion disc (panel **b**). As the frequency of the QPO decreases further below ~ 2 Hz the lags turn positive and the corona becomes the jet (panels **c** and **d**)

Figure 5.8 – Time evolution of QPO frequency and radio flux for GRS 1915+105, full data. The points connected by a line show the time evolution of the frequency of the QPO in the X-ray power spectrum of GRS 1915+105. The light-blue curve (smoothed with a Gaussian kernel) shows the simultaneous measurements of the radio flux density at 15 GHz (y axis rescaled). A strong radio flare appears whenever the lags of the QPO turn from soft to hard (blue points), corresponding to the QPO frequency crossing from above to below ~ 2 Hz. The horizontal band shows the range of QPO frequencies over which the transition occurs. The probability of having a positive QPO lag always and only during a radio flare if the two phenomena were uncorrelated is less than 2×10^{-10} .

Taken together, the results shown in Figures 5.6 and 5.8 and the sudden change of the feedback fraction (Karpouzas et al. 2021; García et al. 2021) from ~ 1 to ~ 0 when the QPO frequency is, respectively, above or below 2 Hz are consistent with the scenario described earlier, in which the energy that is initially stored in the X-ray corona is gradually released into the radio jet and, quite possibly, the X-ray corona itself becomes the jet. The morphing of the extended corona into the jet is strengthened by the relation between the actual luminosity of the corona and that expected if the emission is due to inverse-Compton scattering of seed photons from the accretion disc. In Extended Figure 5.9 we show the ratio of the expected to the observed corona flux vs. the observed flux of the corona. We calculated the expected flux as the product of the bolometric disc flux times e^y , where y is the Compton parameter. (This is the expression for the case that the optical depth is $\tau > 1$.) Since the Compton y parameter gives the number of scatterings times the relative energy that a photon gains per scattering, if the seed-photon source is the disc, the expected and observed corona fluxes should be roughly the same. This Figure shows that the observed and expected fluxes are the same (within a factor 3) only when the lags of the QPO are negative (yellow, orange and red points), i.e., when the QPO frequency is higher than ~ 2 Hz and the disc is enshrouded by the corona. When the lags are positive (light blue and blue points) and the corona morphed into the jet, if the seed photons were from the disc the expected flux of the corona is much lower than the one observed. In this phase a different source must be providing the seed photons for Comptonisation. Since in this phase the jet is active, the most likely source of seed photons is synchrotron emission (Veledina 2018) from the same electrons that produce the jet. Indeed, if we calculate the expected corona flux by multiplying the observed synchrotron flux coming from the jet extrapolated to the X-ray band times e^y , the expected corona flux is much higher when the lags of the QPO are positive than when they are negative.

These results provide a unique clue to understand the nature of the corona and the jet in this object, and what powers these components. Inverse Compton scattering cools down the corona by transferring energy from the electrons to the soft disc photons. The temperature of the corona in black-hole binaries, however, increases during periods in which the photon flux of the disc drops and Compton cooling is less effective. This means that a source of power balances the inverse Compton cooling and sets the temperature of the corona. Our findings show that in GRS 1915+105 the energy provided by this mechanism is split to either power the jet or heat the corona. The thermal energy stored in a spherical corona of optical depth τ and size L around a black hole of mass

M is $E_{th} \approx 3.4 \times 10^{26} \tau \left(\frac{kT_e}{1 \text{ keV}} \right) \left(\frac{M}{M_\odot} \right)^2 \left(\frac{L}{R_g} \right)^2$ erg, where $R_g = GM/c^2$ is

Figure 5.9 – Ratio of the expected to the observed corona flux vs. observed corona flux. The expected corona flux is calculated as the bolometric disc flux times e^y , where y is the Compton parameter. Each point corresponds to one of the 410 observations of GRS 1915+105. The average error of the plotted quantities is shown at the upper right of the plot. The colour scale represents the phase lags, in units of $(\text{rad}/2\pi)$, of the QPO (see Fig. 5.5 for an explanation). The triangles indicate upper limits of the ratio of the expected to the observed corona flux. The expected and observed fluxes are roughly the same (within a factor of ~ 3) when the lags are negative, the corona is extended and hot, and it covers the inner parts of the accretion disc (colours towards red), whereas the expected flux drops significantly when the lags are positive, the geometry of the corona changes and its temperature drops as it turns into the jet (colours towards blue).

the gravitational radius of the black hole and G and c are, respectively, the gravitational constant and the speed of light (Merloni & Fabian 2001).

As discussed here, the change of sign of the lags is explained if a fraction of the disc photons that are Comptonised in the corona impinges back onto the disc before reaching the observer (Karpouzas et al. 2020). When this feedback fraction is low, time delays due to Comptonisation dominate and the lags are positive; when this fraction is high, reprocessed disc photons reach the observer later than those from the corona and the lags become negative. Taking the corona sizes from fits with this model (Karpouzas et al. 2021), $L \approx 10-1200 R_g$, plus $\tau \approx 1-6$ and $kT_e \approx 5-40$ keV from the spectral fits, we find that $E_{th} \approx 10^{31}-10^{35}$ erg. If this energy is released over the time scale of the high-frequency bump*, the thermal luminosity is 2 to 5 orders of magnitude lower than the observed luminosity of the corona in GRS 1915+105. The alternative is that the corona is powered by magnetic energy (Merloni & Fabian 2001; Malzac, Merloni & Fabian 2004), e.g., shear energy due to differential rotation of the magnetic-field lines that thread the accretion disc. This magnetic energy would also be responsible for the synchrotron radio emission and the jet ejection mechanism in black-hole binaries.

We propose that in GRS 1915+105 the corona turns into the jet and that both are, at different times, the same physical component. Based on the results shown here and fits with the model of the lags, the process proceeds as follows: (1) When the QPO frequency is ~ 6 Hz the corona is relatively large and enshrouds the inner parts of the accretion disc (Fig. 5.7a); as seen from the corona, the disc covers half of the sky and hence there is a high probability that photons from the corona return to the disc leading to a high feedback fraction and negative QPO lags. As the QPO frequency decreases from ~ 6 Hz to ~ 2 Hz, the inner edge of the disc moves outwards and the magnitude of the lags and the size of the corona decrease. In this phase the magnetic field that threads the disc and powers the corona is relatively disorganised, the magnetic energy is dissipated stochastically leading to a high corona temperature. (2) At a QPO frequency† of ~ 2 Hz the size of the corona becomes equal to the truncation radius of the accretion disc, the feedback fraction decreases abruptly and the QPO lags become zero. From this point on the corona moves inside the accretion disc (Fig. 5.7b), the magnetic field starts to become coherent on longer spatial scales, the stochastic energy dissipation decreases and the temperature of the corona drops. At the same time the magnetic-field lines

* Note that this is the shortest variability time scale in the data; using any other time scale to estimate the thermal luminosity would make the discrepancy even bigger.

† The transition occurs when the lags to the QPO change from negative to positive; although the exact value of the QPO frequency (Zhang et al. 2020) at which this transition occurs is between 1.8 and 2 Hz, here we will always write 2 Hz for simplicity.

start to channel material from the corona into the direction perpendicular to the accretion disc, and low-level radio emission from the jet starts to appear. (3) As the QPO frequency continues decreasing below ~ 2 Hz the inner disc radius moves further out and the size of the corona increases again but, given that the lags are positive, this time the feedback fraction remains low. From this it follows that the corona does not cover the inner parts of the disc, which can only be the case if the geometry of the corona changes such that it becomes more prominent in the direction perpendicular to the disc (Fig. 5.7c); as seen from the corona, the disc now covers a much smaller area of the sky. As the magnetic-field lines become more spatially coherent (Meier 2005) there is less stochastic energy dissipation, the corona temperature drops further and the material from the corona that was channeled off the originally extended corona is expelled away from the accretion-disc plane and becomes the radio jet (Fig. 5.7d).

Our multi-wavelength correlations match the proposal that, during the initial parts of an outburst, the X-ray corona of the black-hole binary MAXI J1820+070 contracts (Kara et al. 2019) and then re-expands (Wang et al. 2021). Here we find that in the case of GRS 1915+105 the contracting corona becomes the radio jet (Vadawale et al. 2003; Rodriguez, Corbel & Tomsick 2003; Fender, Belloni & Gallo 2004) and that, at least part of the time, the corona and the jet are actually one and the same physical component (Levinson & Blandford 1996; Giannios, Kylafis & Psaltis 2004; Markoff, Nowak & Wilms 2005).

The appearance of radio flares when the QPO frequency is below 2 Hz reinforces our interpretation. Figure 5.10 shows that in the periods that the frequency of the QPO moves more or less stochastically between ~ 2 Hz and ~ 8 Hz (red circles) the radio flux (light blue curve) is low. Occasionally, the QPO frequency evolves in a more systematic way: it starts to decrease, crosses the value of 2 Hz and moves up again; at the same time the radio flux increases sharply and a radio flare lasting a few tens of days is observed. Figure 5.10 shows this behaviour over a period of about 500 days in which the source shows two radio flares. Extended Data Figure 5.8 shows that the same behaviour repeats consistently over a period of 10 years and about a dozen radio flares.

The relation between the iron-line flux and the total flux is consistent with the above scenario. The flux of the iron line depends more strongly upon X-ray flux when the temperature of the corona is low, that is when the corona has turned into the jet (Neilsen & Lee (2009)), than when the corona is more extended and covers the inner parts of the accretion disc. This points to a lamp-post geometry (Miniutti & Fabian 2004) in which the corona, which is now the jet,

Figure 5.10 – Time evolution of QPO frequency and radio flux for GRS 1915+105. The points connected by a line show the time evolution of the frequency of the QPO in the X-ray power spectrum of GRS 1915+105. The light-blue curve (smoothed with a Gaussian kernel) shows the simultaneous measurements of the radio flux density at 15 GHz (y axis rescaled). A strong radio flare appears whenever the lags of the QPO turn from soft to hard (blue points), corresponding to the QPO frequency crossing from above to below ~ 2 Hz. The horizontal band shows the range of QPO frequencies over which the transition occurs. Here we show the first two radio flares in our data, but the same behaviour occurs consistently for all radio flares during our observations.

illuminates the disc anisotropically (Kylafis, Reig & Papadakis 2020; Reig & Kylafis 2021), which does not cover the accretion disc (see Fig. 5.5) and hence the emission reflected off the disc and the flux of the iron line are not (or mildly) attenuated by the corona (Matt, Fabian & Reynolds 1997; Petrucci et al. 2001). The behaviour we observe in GRS 1915+105 could explain the deviations from a single track in the radio/X-ray correlation of other accreting galactic black holes (Hannikainen et al. 1998; Corbel et al. 2003; Gallo, Fender & Pooley 2003; Coriat et al. 2011; Gallo, Miller & Fender 2012; Gallo, Degenaar & van den Eijnden 2018). If, as in the case of GRS 1915+105, the energy powering these systems is used to either accelerate the jet or heat the corona, different sources, or the same source at different times, will show lower radio fluxes at a given X-ray flux (or, equivalently, higher X-ray fluxes at a given radio flux), depending on how much energy is directed towards, respectively, launching the

radio jet or heating the X-ray corona.

5.4 Acknowledgements

This work is part of the research programme Athena with project number 184.034.002, which is (partly) financed by the Dutch Research Council (NWO). Y.Z. acknowledges the support from China Scholarship Council Scholarship (201906100030). TMB acknowledges financial contribution from the agreement ASI-INAF n.2017-14-H.0 and thanks the Team Meeting at the International Space Science Institute (Bern) for fruitful discussions. D.A acknowledges support from the Royal Society. O.B. and D.A. acknowledge support of a Diamond Jubilee International Visiting Fellowship by the University of Southampton. We thank G. Pooley for making the radio data available. This research has made use of data and/or software provided by the High Energy Astrophysics Science Archive Research Center (HEASARC), which is a service of the Astrophysics Science Division at NASA/GSFC. This research made use of NASA's Astrophysics Data System.

Summary and future prospects

6.1 A summary of the thesis results

In this thesis I have designed a variable Comptonisation model that explains the properties of the fast variability in neutron-star (NS) and black-hole (BH) low-mass X-ray binaries (LMXBs). As the title suggests, this model is based on the phenomenon of inverse-Compton scattering, that occurs when low-energy photons from the NS or accretion disc are up-scattered to higher energies via collisions with the high-energy free electrons that exist around the LMXB, and are forming an atmosphere that we call the corona. A key aspect of the model is feedback, in the sense that a fraction of the up-scattered photons can stochastically impinge back onto the NS or accretion disc, causing re-heating. The model focuses on explaining the timing properties of quasi-periodic oscillations (QPOs) such as, the fractional rms and time lags, but also the steady-state energy spectra regardless of any variability.

In Chapter 1, we have discussed the mathematical background for modelling and fitting the energy-dependent fractional rms and time lags in NS LMXBs, using a variable Comptonisation model. The new numerical framework that we proposed works without the need of prior or initial assumptions, and leads to numerically stable solutions across a vast range of physical parameters. In Chapter 2, we have applied our newly developed variable Comptonisation model to a BH LMXB. Although the model was originally designed to work for NS LMXBs, we were able to explain the energy-dependence of the fractional rms and time lags surprisingly well also in the case of BHs. Furthermore, in chapter 3 we have given, for the first time, a physical interpretation as to why the slope of the energy-dependent time lags in the BH LMXB GRS 1915+105 switches from negative when the QPO frequency is higher than a

critical value to positive when the QPO frequency is lower than this critical value. In particular, we have proposed that a spherically symmetric corona that is first decreasing in size (contraction) and then increasing (expansion) can explain the observed behaviour since it is naturally adjusting the feedback onto the accretion disc. In Chapter 4, motivated by our results so far, we have studied the effects of changes in the corona size on the energy spectra of BH LMXBs. To do the latter, we modified existing spectral models and introduced new physical parameters that can now be constrained by fitting the model to observations. Using simulations, we showed that the finite size of the corona mainly affects the energy spectra at energies below 1 keV. Finally, in Chapter 5 we have analysed hundreds of X-ray and radio observations of the BH GRS 1915+105 obtained over a period of ~ 10 years, and reached the conclusion that not only the corona size but also the corona geometry changes as the spectrum of this BH changes. Based on a series of quantitative findings, we have proposed that depending on the structure of the magnetic field a spatially extended corona can gradually turn into a jet and vice versa, in a process that is recurring as the data suggest.

6.2 Future prospects

Based on the work presented in this thesis, future prospects can be discussed both with regards to the model and to potential applications of such model. First and foremost, the current version of the model works under the assumption of spherical symmetry and homogeneity of the corona. In reality the corona must have a much more complex geometry and a radial dependence of temperature and optical depth. Also, the free electrons in the corona can have a bulk velocity component, especially in the regime where the jet dominates, a fact that needs to be taken into account in the equations of the model. Furthermore, a fraction of the high-energy X-rays that leave the corona of both NS and BH LMXBs after being up-scattered will necessarily illuminate the accretion disc, leading to a formation of a reflection component which will create additional time-delays and thus will affect both the energy-dependent fractional rms and time lags. Although all of the above are important physical phenomena that have been shown to affect greatly spectral-timing measurements in LMXBs, incorporating them in a semi-analytical framework, such as the one presented in this thesis, is highly non-trivial and will for sure be an interesting and challenging endeavor that will keep myself and others busy in the years to come.

With regards to future applications of the model to observations, the possibilities are only limited by the amount of data that are available. We have shown here that the model works well both for NS and BH LMXBs and that

it provides insights in the spectral and geometrical evolution of those sources. However, due to limitations in computing power and human resources and also the fact that the archive of X-ray observations is overwhelmingly large, we have not yet been able to perform a population study of LMXBs. Analysing the spectral-timing properties of many different LMXBs with our model is an obvious next step, as we want to test whether more systems exhibit a similar behaviour.

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Nederlandse Samenvatting

Compacte objecten zoals een neutronenster (afgekort NS) en een zwart gat (afgekort BH, van de Engelse benaming black hole) bestaan vaak dichtbij een gewone ster, zoals onze zon. Door de extreem sterke zwaartekracht van het compacte object wordt materie onttrokken van de gewone ster, ook wel de begeleidente ster of donorster genaamd, en wordt de materie naar het compacte object getrokken waardoor een accretieschijf rondom het compacte object wordt gevormd. Dit systeem heet een lage massa röntgendubbelster (afgekort LMXB, van de Engelse benaming low-mass X-ray binary) omdat de begeleidente ster een lage massa heeft (minder dan die van onze zon) en het systeem straalt voornamelijk röntgenstraling uit. De röntgenstraling kan enkel buiten de atmosfeer van de aarde worden gedetecteerd worden en daarom moeten satellieten met een speciaal röntgenstraling-telescoop in een baan om de aarde worden geplaatst zodat LMXBs bestudeerd kunnen worden. De röntgenstraling telescopen worden op een enkele plek gericht, namelijk daar waar de LMXB gelegen is, en leggen de fotonen die worden uitgestraald extreem snel vast. Meestal worden LMXBs een aantal uren gemonitord. Wanneer voldoende informatie is verzameld en opgeslagen door de waarnemer is het mogelijk om de 'lightcurve' van de bron te construeren, die beschrijft hoe de intensiteit van de bron verandert met de tijd, en het energiespectrum, dat ons informeert over de fysieke verschijnselen die plaatsvinden rond de LMXB.

Verondersteld wordt dat LMXBs een atmosfeer bevatten, gelijk aan die van de zon, genaamd de corona. De corona heeft een temperatuur van honderden miljoenen graden Kelvin, waardoor materie enkel in de vorm van hete plasma kan bestaan. De accretieschijf rondom het compacte object, in geval van BH LMXBs, of een combinatie van een neutronenster oppervlak en accretieschijf, in geval van NS LMXBs, stoot fotonen uit die de corona binnendringen. Zodra een foton de corona binnendringt, kan het ofwel onaangetast ontsnappen, ofwel een interactie aangaan met de corona en daarbij meer energie krijgen in een pro-

ces dat comptonisatie wordt genoemd. Om comptonisatie te bestuderen leggen we de focus op hoge energie van het energiespectrum van LMXBs, genaamd de high-energy tail, dat waarschijnlijk door dat mechanisme wordt geproduceerd. Door wiskundige modellering van comptonisatie observaties te vergelijken, kunnen we meer inzicht krijgen over de temperatuur en optische dichtheid van de corona. Verder is aangetoond dat de high-energy tail van het energiespectra van LMXBs op bepaalde momenten met 20% kan oscilleren. Uit metingen blijkt zelfs dat de 'high-energy tail', samen met de rest, van het energiespectrum van LMXB's quasi-periodiek oscilleert, en dit fenomeen staat bekend als Quasi-Periodic Oscillations (QPO's). De frequentie van QPO's kan variëren van enkele Hz, in welk geval ze laagfrequente (LF) QPO's worden genoemd, tot duizenden Hz, in welk geval ze kilo-Hertz (kHz) QPO's worden genoemd. Studies waarbij QPO's betrokken zijn, worden variabiliteits- of timingstudies genoemd. Aangezien de hoogenergetische staart wordt toegeschreven aan Comptonization in de corona, en aangezien het volledige spectrum een QPO kan vertonen, is het niet meer dan redelijk om aan te nemen dat Comptonization en QPO's op de een of andere manier met elkaar verbonden zouden moeten zijn. Om bovengenoemde reden zijn variabele Comptonisatiemodellen nodig om de waargenomen variabiliteit zelfconsistent te verklaren, in de context van Comptonisatie.

In dit proefschrift hebben we een variabel compensatiemodel gebouwd dat kan worden gebruikt om de waargenomen variabiliteit in LMXB's te modelleren. Het idee waarop ons model is gebaseerd is eerder in de literatuur voorgesteld, maar sommige wiskundige beperkingen maakten nauwkeurige vergelijkingen met waarnemingen moeilijk. In hoofdstuk 2 van dit proefschrift leggen we de natuurkunde van het variabele Comptonisatiemodel uit en bieden we een nieuw numeriek kader waarmee de modelvergelijkingen efficiënt kunnen worden opgelost. In hetzelfde hoofdstuk passen we ons model toe op de kHz QPO's in NS LMXB 4U 163653 en laten we zien dat: (a) het model de waarnemingen voldoende goed kan verklaren, (b) de waargenomen variabiliteit voornamelijk wordt aangedreven door oscillaties in de temperatuur van de corona dan oscillaties in de NS-temperatuur, en (c) de grootte van de corona die we vinden, wanneer we het model vergelijken met waarnemingen, ongeveer 10 km is, wat impliceert dat de kHz QPO's geassocieerd zijn met een zeer klein gebied rond de NS.

In hoofdstuk 3 hebben we ons nieuw ontwikkelde model toegepast op de LF QPO's in de BH LMXB GRS 1915+105. Hoewel het model dat we in hoofdstuk 2 beschrijven oorspronkelijk voor NS LMXB's was, waren de resultaten net zo goed wanneer ze werden toegepast op BH LMXB's. We ontdekten in het bijzonder dat terwijl de bron GRS 1915+105 overgaat van een zeer lichtgevende naar een minder lichtgevende toestand, de corona aanvankelijk samentrekt van

enkele duizenden km tot slechts enkele tienden van een km, en dan weer vergroot tot enkele honderden kilometers. De verandering in de grootte van de corona die we met ons model vonden door enkel variabiliteit te bestuderen, impliceert veranderingen in de fysieke parameters die Comptonisatie beschrijven, welke feitelijk worden bevestigd door onafhankelijke waarnemingen van de energiespectra van GRS 1915+105. De succesvolle toepassing van ons model op zowel LF QPO's in een BH LMXB, als ook op kHz QPO's in NS LMXB's, suggereert dat Comptonisation en QPO's met elkaar verbonden zijn.

Gemotiveerd door de resultaten als gepresenteerd in hoofdstuk 3, die een corona suggereren met variabele grootte rond BH LMXB's, hebben we in hoofdstuk 4 een nieuw Comptonisatiemodel ontwikkeld. In het nieuwe model hebben we een accretieschijf gebruikt in plaats van een NS als bron die fotonen in de corona injecteert. Ook hebben we aangenomen dat de corona sferisch symmetrisch is en slechts een deel van de accretieschijf kan bedekken, terwijl het resterende deel direct wordt blootgesteld aan de waarnemer. Door energiespectra van BH LMXB's te simuleren, hebben we aangetoond dat onze aannames enkele aanvullende fysieke parameters introduceren, zoals schijfhelling en schijfbinnenradius die kunnen worden beperkt door waarnemingen. Verder hebben we aangetoond dat, aangezien de helling van de schijf en de binnenstraal kunnen variëren doorheen bronnen en spectrale toestanden, verschillende waarden van die parameters zullen leiden tot verschillende vormen van de energiespectra, met name bij lage fotonenergieën. De resultaten als gepresenteerd in hoofdstuk 4 kunnen worden gezien als de eerste stap naar het bouwen van meer gedetailleerde spectrale modellen, die rekening houden met mogelijke veranderingen in de geometrie van de corona van BH LMXB's.

In Hoofdstuk 5 analyseren we ~ 1800 waarnemingen van de BH LMXB GRS 1915+105 die bijna zestien hebben geduurd, die vergezeld gingen van dagelijkse waarnemingen van de radioflux op 15 GHz. Hoewel de radiogegevens niet de volledige zestien jaar beschikbaar waren, konden we 410 waarnemingen kruisen waarin een QPO prominent aanwezig was, en tegelijkertijd waren radiogegevens binnen enkele dagen beschikbaar. Door die 410 waarnemingen te analyseren, komen we tot de conclusie dat naarmate de QPO-frequentie afneemt van ~ 6 Hz tot $\sim 0,4$ Hz, de radioflux systematisch toeneemt, en de impliciete coronatemperatuur neemt systematisch af. Verder weten we uit Hoofdstuk 3 dat de afhankelijkheid van feedback efficiëntie van QPO frequentie een verandering in de geometrie van de corona suggereert wanneer de QPO frequentie onder ~ 2 Hz komt. Op basis van het bovenstaande stellen we voor dat deze anticorrelatie tussen radio-intensiteit en coronatemperatuur te wijten is aan een mogelijke verandering in de structuur van het magnetische veld. Door het magnetische veld kan de corona bestaan in de vorm van een hete uitgestrekte wolk die de accretieschijf op een bepaald moment omhult,

maar op een ander moment de materie naar een uitstromende straal leidt die sterk uitzendt in radiogolflengten. De laatste stelling wordt ondersteund door aanvullende bevindingen die we presenteren, zoals de verandering in hoe de flux van de ijzer-K-lijn is gecorreleerd met de totale waargenomen röntgenflux onder en boven een QPO-frequentie van ~ 2 Hz, en ook het feit dat er alleen significante flitsen in de radio is wanneer de QPO-frequentie onder ~ 2 Hz komt.

Acknowledgements

The time has come to write the epilogue of the thesis, and acknowledge certain people that not only made this thesis possible, but were responsible for making most of my journey during the PhD enjoyable and when that was not possible, tolerable. First and foremost I want to thank my supervisors professors Mariano Méndez and Diego Altamirano for offering me the PhD position. I remember that when I was called at the Kapteyn institute for a interview, I was captivated by the offices at Kapteyn and also by the beauty of Groningen. While I was travelling back, I was hoping to be offered that position, regardless of whether my potential supervisors would be a good match for me. Little did I know about how good my relationship with my supervisors would be. The nature of my PhD project was such that it required many hours of study, working out math on paper and then writing numerical models in the computer. Both of my supervisors understood that and did not put any pressure on me, by demanding delivery of results within strict deadlines. In fact, the freedom that they gave me to tackle the problem any way I wanted, led to me being able to produce results relatively fast. Recognising my work as good, when that was the case, and making me aware of it was of vital importance since I have the tendency to discredit my own work, a fact that Mariano made me aware of. Furthermore, Mariano's and Diego's concern for my personal well-being alongside the professional one made me feel safe and calm, allowing me to continue working even when I was going through difficult personal moments. During my visits and time at Southampton, Diego made sure that I felt like home, and introduced me to some wonderful people, for which I will always be grateful. Finally, Mariano and particularly Diego have the superpower of being patient with me when I forget to do mandatory courses and other tasks required by PhD students.

Although a PhD always seemed like a natural continuation of my studies, I was not sure that I would follow one until I met and worked with certain people.

In particular, I would like to thank Professor John Hugh Seiradakis (John), Dr. Dimitrios Mislis (Dimitris) and Dr. Angelos Tsiaras (Angelos). John, who is no longer with us, accepted to supervise my undergraduate thesis back in 2012. Through his kind spirit, attention to detail and drive for excellence, he persuaded me that an academic life was worth pursuing. Through John I met Dimitris, who at the time was a post-doc at the same university. Dimitris introduced me to scientific research for the first time, in the context of transiting exoplanets, and also to programming with python. Angelos and I met early during our undergraduate studies, but started collaborating closely much later, during the end of our studies. Angelos and myself have spent many long nights at the Holomon Astronomical Station, the observatory of our university, and many hours writing python code together. Angelos also helped with the co-supervision of my master thesis, while I was working with him at the University College London (UCL), where he was doing his PhD.

Arriving to the Netherlands and being welcomed into my office at the Kapteyn institute was one of the most rewarding experiences in my life. The secretaries at Kapteyn, Martine, Christa and Ramona, were very helpful and willing to answer any questions anytime. Special thanks should also go to the computer group of Kapteyn Martin, Leon and Eite who I have been regularly bombarding with questions and emails during the last four years. Furthermore, I would like to thank my office mates Giulio, Hannah, Cecilia, and Tirna for the nice conversations during coffee breaks, office meals and also for tolerating my eating in the office. At this point I want to thank my PhD mentor, and member of the reading committee, Inga Kamp. During the past four years Inga has been available for discussions regarding the progress of my PhD and my personal struggles, and was always paying attention to my presentations or talks and always came up with interesting questions. Also, to the rest of the professors, postdocs, PhD students and staff at Kapteyn, thank you for making sure that the institute is a place where you always feel safe and comfortable to work at. Regarding internal collaborators, the list is so long and I only hope I will not forget someone. Federico García, who has been a postdoctoral researcher in our group, has single handedly made the model more user-friendly and accessible. I have spent many hours discussing and coding with Federico and I would like to thank him for his patience and attention to detail. Federico and his student Candela Bellavita, from Argentina, have worked hard lately and succeeded in making the model faster and also extending the physics. I would also like to thank members of our own X-ray group that I have met and worked with over the years. In particular Kevin Alabarta, Evandro Ribeiro, Yanan Wang, Valentina Peirano, Yuexin Zhang and Sai Na. Kevin is a good friend and a former housemate of mine and so I want to thank him particularly for the time that we spent together. To the rest of the group, although I may

not have gotten to know you very well, I always enjoyed our conversations both personally and in the group meetings. I am very thankful for all the hard-working and at the same time kind people that I have around me, and so I hope I can continue collaborating with all of you.

With regards to external collaborators, I would like to thank Omer Blaes who I met at Southampton through Diego. Omer has been a co-author in some of our papers and has also helped me understand better the results and implications of my work. Apart from a great scientist and teacher, Omer is a wonderful and fun person and I have always enjoyed being around him. Also, I would like to thank Tomaso Belloni who has also been a co-author in many papers. Tomaso was always happy to discuss topics related to either data or applications of our model, and always came up with important and helpful comments. Furthermore, I would like to thank professor Nick Kylafis and Pablo Reig for long conversations on Comptonisation and our model. In particular, Nick has criticised our model in a positive way thus pushing us to understand it better on a qualitative level. He has made us aware of potential caveats but also ways to improve the model, and so I thank him for that. Finally I would like to thank Liang Zhang whose work and results were of key importance for many of the results presented here, and also Divya Rawat for analysing X-ray data and applying our model.

Of course, this PhD would not have been possible without the support of my friends and loved ones. My mother, Maria Tsaousi, and father, Dimitris Karpouzas, have always supported me in the pursuit of excellence since I was a little child. They were proud of me but also gave me enough space to do things my way. They supported me both financially and emotionally when I moved to the Netherlands, despite the problems that we were facing. My brothers, Mike and Nick Karpouzas, and also my cousins, aunts and uncles, made sure that every time I went back to Greece I had a great time. My partner and future wife, Kyra Baurdoux, has been providing me with love, comfort and support during the last three years of the PhD. Kyra has been tolerating my endless hours of sitting in front of the computer, my being distracted with thoughts of work even when not working, my complaining about not being able to find my clothes in my own closet and the way I am eating peanut butter sandwiches. Finally, my friends George Katsianis, Manthos Karydas, Ioannis Katharopoulos and Georgios Vernardos have always been available for scientific discussions regarding theoretical and philosophical topics regarding both my work and in general. Georgios Vernardos was also working in the Kapteyn institute during the most part of my PhD, and has also been my housemate for ~ 2 years and so I want to thank him for the nice times that we had together.