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Event-Triggered Synchronization of Multiagent Systems With Partial Input Saturation

Xuegang Tan , Ming Cao , Senior Member, IEEE, Jinde Cao , Fellow, IEEE, Wenying Xu , Member, IEEE, and Yiping Luo

Abstract—This article is concerned with the distributed event/self-triggered synchronization problem for general linear multiagent systems with partial input saturation. Both the event-based and self-triggered laws are designed using the local sampled, possibly saturated, state, which ensures the bounded synchronization of the multiagent systems and exclusion of the Zeno-behavior. The continuous communication between agents is avoided under these triggering protocols. Different from the existing related works, we show the fully distributed design for multiagent systems, where the synchronization criteria, the designed input laws, and the proposed triggering protocols do not depend on any global information of the communication topology. In addition, the computational load of multiagent systems is reduced significantly.

Index Terms—Event-triggered communication, fully distributed, multiagent systems (MASs), partial input saturation, synchronization.

I. INTRODUCTION

RECENTLY, distributed cooperative control for multiagent systems (MASs) has drawn considerable attention in the control community due to its wide applications in various areas, such as robotic formation [1]–[3]; smart grids [4], [5]; multivehicle platooning [6]; and social wireless networks [7]. Under the distributed cooperative control, updating data and exchanging

information are necessary for a group of agents to accomplish some team tasks, which may lead to high computational and communication costs [8], [9]. Unfortunately, most agents in MASs are equipped with simple equipment due to budget constraints, which results in the fact that the power resources of agents and the bandwidth of communication network are inevitably constrained. Based on this fact, it is difficult for agents to work for long hours. In addition, the constraints of equipment, such as simple sensors or actuators, usually lead to the communication channel or input becoming saturated. To overcome the drawbacks mentioned above, most researchers focus on the twofold questions: The first is how to reduce the data update frequency of actuators or sensors. The second is how to design the control protocols for MASs under some saturation conditions to guarantee good system performances.

As previously mentioned, the power resource, and the communication and computation abilities of agents are limited. Hence, designing distributed cooperative algorithms with low computational load and communication frequency is very important for MASs. To solve the problems, event-based control schemes are introduced to deal with the cooperative problems for MASs, which offers the advantage of reducing the communication frequency and lowering input updates [10]–[14]. Designing distributed event-based protocol for each agent is a core task in the event-based cooperative control problem, which includes the control rules and the event-based communication laws [15]–[19]. In [11] and [12], event-based and self-triggered impulsive synchronization were investigated, respectively. In [10] and [15], the event-based control protocols were designed for a class of single-integrator MASs. To ensure the synchronization of the nonlinear MASs, a model-based event-triggered control strategy was proposed in [20]. The work [21] proposed a periodic event-based control approach for a class of linear MASs with communication delays, which shows the fact that it is more flexible in scheduling communication than periodic sampling. To avoid the continuous communication or monitoring, the self-triggered control approaches for MASs were introduced in [16]–[18].

It is worth mentioning that, in most literature relating to the cooperative algorithm design for MASs (e.g., event-based control scheme discussed in [10]–[19]), the communicated information from neighbors has usually been implicitly assumed to be accurately obtained. However, most of MASs do not satisfy such an assumption in practice, which will result in state saturation (or partial state saturation) in the process of agents executing

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some global tasks. The state saturation of agents often is induced by the physical limits of the protection equipments or devices, such as overflow phenomenon of digital controller and the round-off errors caused by the finite-word-length in numerical computation [22], [23]. The saturation effects, if not adequately handled, can seriously degrade the control performances of the closed-loop response of MASs. This naturally gives rise to fundamental difficulties on the design of the distributed cooperation control schemes for MASs. Thus, one particular topic of research interest is the saturation of the control input using sensed or communicated data [26]–[28]. Input saturation often appears when implementing a controller, and most of the existing related works focus on this topic [24], [25], [29]–[31]. A node-based and an edge-based adaptive antiwindup control protocols were designed for MASs with input saturation in [26], in which no global information of the communication topology is used in the consensus protocol. Another work [28] studied a control protocol design method for a class of leader-following nonlinear MASs with saturation input. In [30], the synchronization of heterogeneous multiagent Euler–Lagrange systems with input and actuator saturation was considered. It should be noted that, to the best of our knowledge, many research investigations have been devoted to the synchronization control of MASs under saturation input. However, few focus on the synchronization of MASs with saturated input under the event-based schemes, because the control design of MASs with input saturation and discontinuous communication is even more difficult. Designing the distributed event-based communication protocols for such MASs with input saturation deserves more attention, which is of not only high theoretical importance but also practical significance.

In addition, note that most of the existing criteria related to the distributed cooperative control for MASs rely on the global information of the communication topology including the Laplacian matrix of the graph or its nonzero eigenvalues. Such control protocols or triggering conditions involving the global information are not fully distributed (see [11], [12], [20]). As the number of nodes increases, the computation load of the MASs under these methods will increase. Different from the schemes in [11], [12], and [20], a method estimating the lower bound of the second smallest eigenvalue associated with the Laplacian matrix is introduced to solve the synchronization problem of MASs [32]–[34]. Although the strategy proposed in [32]–[34] requires that each agent obtains the total agent number, it reduces skillfully the dimension of matrix inequality in the synchronization criteria. Thus, this reduces the computational load of the MASs greatly. In [13], [26], and [35], adaptive control was considered to realize the fully distributed control for MASs. However, the process of seeking the appropriate control parameters based on the adaptive laws will increase the computation load. The other distributed approaches, as introduced in [16], realize the fully distributed control only for some special MASs such as the single-integrator model. The most important is that the related works assume that the agents execute the cooperative tasks under the ideal communication with requiring the accurate state information of agents. As a result, the proposed distributed control protocols may not be practical for such MASs under saturation due to communication. The event-triggered

synchronization problem of MASs with input saturation via a fully distributed control is interesting and challenging.

Motivated by the above observations, the objective of this article is to investigate a *fully* distributed event-based framework for a general linear MAS with partial input saturation, where the partial information of control input is subject to the saturation effects due to different physical limitations. Some new results on the event-triggered communication scheme for MASs are obtained, which can be summarized as follows:

- 1) Three control rules are designed for a class of general linear MASs where the communication among agents is subject to saturation. In addition, an event-based communication protocol and a self-triggered communication law are proposed, respectively, and both of them guarantee that the Zeno-behavior does not exhibit in the closed-loop systems.
- 2) The established criteria are suitable for a class of general linear MASs, which ensure that the agents achieve bounded synchronization.
- 3) A distributed event-based control algorithm is developed, and a simulation example is presented to demonstrate the effectiveness of the bounded synchronization conditions. Only the sampled information is utilized in the triggering communication rules, in which the continuous communication among agents is avoided. In addition, it should be emphasized that the control protocols, the triggering laws, and the criteria do not rely on the Laplacian matrix of the communication topology and its eigenvalues, which makes the control more distributed and can reduce the computation load of the MASs greatly.

The rest of this article is organized as follows. In Section II, a class of linear MASs is introduced, and two definitions are given, respectively. Section III proposes the bounded synchronization criteria. In Section IV, an example is given to illustrate the effectiveness of the proposed algorithm. Section V concludes this article.

Some notations to be used are given as follows. The sets of nonnegative integers, n -dimensional real vectors, and $(n \times m)$ -dimensional real matrices are denoted by $\mathbb{N}_+$, $\mathbb{R}^n$, and $\mathbb{R}^{n \times m}$, respectively. Let Y^T stand for the transpose of a vector or matrix Y , respectively, and $\lambda_{\max}(R)$ (resp. $\lambda_{\min}(R)$) be the maximal (resp. minimal) eigenvalue of the matrix R when its eigenvalues are real. In addition, $\text{diag}\{d_1, d_2, \dots, d_n\}$ is a diagonal matrix, whose diagonal elements are $d_1, d_2, \dots, d_n$. Let $\|\cdot\|$ denote the norm of a vector or matrix, and $\begin{bmatrix} A & C \\ * & B \end{bmatrix}$ is the short-handed notation for the symmetric matrix $\begin{bmatrix} A & C \\ C^T & B \end{bmatrix}$. Lastly, I stands for the identity matrix with appropriate dimensions.

II. PROBLEM FORMULATION AND PRELIMINARIES

We consider that the communication topology of the MASs is an undirected connection graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ denotes the vertex set corresponding to the N agents, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{A} = [a_{ij}]_{N \times N}$ is the adjacency matrix of $\mathcal{G}$. For any agent $i \in \mathcal{V}$, agent $j \in \mathcal{V}$ is called a neighbor of i if i and j can communicate each

other, and $(i, j) \in \mathcal{E}$ is the edge between the two agents with weight a_{ij} . Let $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ denote the neighbor set of agent i . Suppose that there is no self edges (i, i) in $\mathcal{G}$, i.e., $i \notin \mathcal{N}_i$ and $(i, i) \notin \mathcal{E}$. In addition, because the graph $\mathcal{G}$ is undirected, $a_{ij} = a_{ji} > 0$ if and only if $(i, j) \in \mathcal{E}$; otherwise, $a_{ij} = 0$. The Laplacian matrix $L = [l_{ij}]_{N \times N}$ of the graph $\mathcal{G}$ is then $L = D - \mathcal{A}$, where $D = \text{diag}\{d_1, d_2, \dots, d_N\}$ is the degree matrix with $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. Obviously, $l_{ij} = -a_{ij}$ if $i \neq j$; otherwise, $l_{ij} = \sum_{j \in \mathcal{N}_i} a_{ij}$.

Consider a linear MAS with N agents, whose dynamics can be described as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i = 1, 2, \dots, N \quad (1)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$. $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, respectively, are the state and the input of agent i . The control input $u_i(t)$ is subject to the sampling state saturation, and takes the form

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij} [K_{i1} \text{Sat}_\Delta(\text{Sat}_\Delta(x_j(t_k^j)) - x_i(t_k^i)) - K_{i2} x_i(t)] \quad (2)$$

where $K_{i1}, K_{i2} \in \mathbb{R}^{m \times n}$, $t \in [t_k^i, t_{k+1}^i)$, $\forall k \in \mathbb{N}_+$, and it assumes that each agent i can obtain the accurate real-time state of itself.

For any $x \in \mathbb{R}^n$, $\text{Sat}_\Delta(x) = [\text{Sat}_\Delta(x_{i1}), \text{Sat}_\Delta(x_{i2}), \dots, \text{Sat}_\Delta(x_{in})]^T$ denotes the saturation function induced by the physical devices of the agents or the communication between agents, and $\text{Sat}_\Delta(x_{ij}) = \text{sign}(x_{ij}) \min\{|x_{ij}|, \Delta_j\}$ with a saturation threshold $\Delta_j > 0$ for all $j \in \{1, 2, \dots, n\}$.

Remark 1: In some practical cases, transmitted or sampled information may suffer from saturation due to various physical limitation [36], [37]. In this article, the control input (2) includes two parts: 1) Agents' own states that can be accurately measured and 2) the relative information between agent i and its neighbors that is subject to the saturation effects. Such a phenomenon is referred to as *partial input saturation*. It is worthwhile to mention that such saturation effects, if not adequately handled, can seriously deteriorate the control performance of MASs.

For any agent i , its dynamics can be rewritten as

$$\dot{x}_i(t) = \tilde{A}x_i(t) + B\tilde{u}_i(t), t \in [t_k^i, t_{k+1}^i) \quad \forall k \in \mathbb{N}_+ \quad (3)$$

where there exists a matrix $K \in \mathbb{R}^{m \times n}$ which satisfies $K = \sum_{j \in \mathcal{N}_i} a_{ij} K_{i2}$ for any $i \in \mathcal{V}$, such that $\tilde{A} = A - BK$; $\tilde{u}_i(t) = K_{i1} q_i(t_k)$, and $q_i(t_k) = \sum_{j \in \mathcal{N}_i} a_{ij} \text{Sat}_\Delta(\text{Sat}_\Delta(x_j(t_k^j)) - x_i(t_k^i))$. It is easy to check that there is a positive constant $\tilde{\Delta}$, such that $\|q_i(t_k)\|^2 \leq \tilde{\Delta}$. In addition, $\{t_k^i : k \in \mathbb{N}_+\}$ denotes the event-triggered time sequences of agent i , which is decided by the triggering condition to be given later. For analysis purposes, merge all the time sequences in the chronological order, i.e., $\{t_k\} = \bigcup_{i=1}^N \{t_k^i : k \in \mathbb{N}_+\}$, where t_k satisfies $t_0 < t_1 < \dots < t_k < t_{k+1} < \dots$, $\lim_{k \rightarrow \infty} t_k = +\infty$. This property will be proved later.

In addition, for convenience of presentation, two definitions are given below.

Definition 1: [38] $D^+ f(x) = \limsup_{\Delta \rightarrow 0^+} \frac{f(x+\Delta) - f(x)}{\Delta}$ if $f(x)$ is differentiable at its right side.

Definition 2: [39] For any agents $i, j \in \mathcal{V}$ of MASs (1) with initial conditions $x_i(t_0), x_j(t_0) \in \mathbb{R}^n$, the *bounded synchronization* of the MASs is achieved with an error bound $\varepsilon > 0$, if there exist a constant $\delta = \delta(t_0) > 0$ and a set $\mathcal{S}$, such that the state error $e_{ij}(t) = x_j(t) - x_i(t)$ satisfying $\|e_{ij}(t_0)\| \leq \delta$ converges to the set $\mathcal{S} = \{e_{ij}(t) \in \mathbb{R}^n | \|e_{ij}(t)\| \leq \varepsilon\}$ as t goes to infinity.

III. SYNCHRONIZATION ANALYSIS

The event/self-triggered synchronization analysis for MASs (1) with partial input saturation under the fully distributed control approach will be given in this section. And a novel fully distributed control framework is proposed to optimize the control performance of the MASs.

A. Event-Based Communication Scheme

We first give the event-triggered communication rule for agent i as below

$$t_{k+1}^i = \inf \left\{ t : t > t_k^i, \sigma_i \|q_i(t_k)\|^2 + e^{-\lambda_i(t-t_0)} + \tau \leq \|e_i(t)\|^2 \right\} \quad (4)$$

where $\sigma_i > 0, \lambda_i \geq 0, \tau > 0$, $e_i(t) = x_i(t_k^i) - x_i(t)$, and $q_i(t_k)$ is given as in (3).

The discussion of excluding the Zeno-behavior for the designed triggering law is first given as follows.

Theorem 1: Under the event triggering rule (4), the Zeno-behavior does not appear in the closed-loop system (1).

Proof: Let $V_i(t) = e_i^T(t) e_i(t)$, $t \in [t_k^i, t_{k+1}^i) \forall k \in \mathbb{N}_+, i = 1, 2, \dots, N$. From (3) and $\tilde{u}_i(t) = K_{i1} q_i(t_k)$, we have

$$\begin{aligned} D^+ V_i(t) &= e_i^T(t) (\tilde{A}x_i(t) + BK_{i1} q_i(t_k)) \\ &\quad + (\tilde{A}x_i(t) + BK_{i1} q_i(t_k))^T e_i(t) \\ &= -e_i^T(t) (\tilde{A}^T + \tilde{A}) e_i(t) \\ &\quad + e_i^T(t) [\tilde{A}x_i(t_k^i) + BK_{i1} q_i(t_k)] \\ &\quad + [\tilde{A}x_i(t_k^i) + BK_{i1} q_i(t_k)]^T e_i(t) \\ &\leq e_i^T(t) [I_n - (\tilde{A}^T + \tilde{A})] e_i(t) + \|\Omega_i(t_k)\|^2 \\ &\leq \gamma e_i^T(t) e_i(t) + \|\Omega_i(t_k)\|^2 \end{aligned}$$

where $\gamma = \|I_n - (\tilde{A}^T + \tilde{A})\|$, $\Omega_i(t_k) = \tilde{A}x_i(t_k^i) + BK_{i1} q_i(t_k)$. Because of the fact that $V_i(t_k^i) = 0$, one has

$$V_i(t) \leq \frac{1}{\gamma} \|\Omega_i(t_k)\|^2 (e^{\gamma(t-t_k^i)} - 1).$$

According to the triggering condition (4), if the next event occurs at t_{k+1}^i , the following inequality holds

$$0 < e^{t_{k+1}^i - t_0} + \tau \leq \frac{1}{\gamma} \|\Omega_i(t_k)\|^2 (e^{\gamma(t_{k+1}^i - t_k^i)} - 1).$$

Should Zeno-behavior take place, there would exist a positive constant T , such that $\lim_{k \rightarrow +\infty} t_k^i = T < +\infty$, i.e., $t_{k+1}^i - t_k^i \rightarrow 0$ as $k \rightarrow +\infty$. As a result, $0 < e^{t_{k+1}^i - t_0} + \tau \leq 0$ which

contradicts $e^{t_{k+1}^i - t_0} > 0$ and $\tau \geq 0$. This implies that the Zeno-behavior can be successfully excluded in the closed-loop systems. The proof is complete. ■

Remark 2: Theorem 1 guarantees the following property of the sequence $\{t_k\}_{k=1}^\infty$: $t_0 < t_1 < \dots < t_k < t_{k+1} < \dots$, $\lim_{k \rightarrow \infty} t_k = +\infty$. In the following theorems, we will discuss the event-based synchronization of the system (1) with partial input saturation.

Theorem 2: Consider the system (1) with the control law (2) and the triggering rule (4). If there are constants $\alpha_{ij} > 0$, matrices $K, K_{i1} \in \mathbb{R}^{m \times n}$, and positive definite matrices $P_{ij}, Q_{ij}, R_{ij} \in \mathbb{R}^{n \times n}$ satisfying $P_{ij} = P_{ji}$, $Q_{ij} = Q_{ji}$, $R_{ij} = R_{ji}$, respectively, such that

$$\begin{bmatrix} \Theta_{ij} & \Xi_{ij} & -Q_{ij} & -P_{ij}BK_{i1} \\ -R_{ij} & R_{ij} & 0 & \\ * & -\frac{1}{2}\lambda_i I & 0 & \\ * & * & -\eta_i I & \end{bmatrix} \leq 0 \quad (5)$$

where $\Theta_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK) + \alpha_{ij}I] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$, $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$, and the parameters λ_i, σ_i satisfy the triggering rule (4), then the system (1) realizes the bounded synchronization, i.e., for any $i, j \in \mathcal{V}$, the error $e_{ij}(t) = x_j(t) - x_i(t)$ converges to the set

$$\mathcal{S} = \left\{ e_{ij}(t) \in \mathbb{R}^n \left\| e_{ij}(t) \right\| \leq \sqrt{\frac{\Gamma}{a_{ij}\lambda_{\max}(P_{ij})}} \right\} \quad (6)$$

where $\Gamma = \sum_{l=1}^N \sum_{s=1}^N a_{ls} [\frac{\lambda_{\max}(P_{ls})}{\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta})]$ and $\tilde{\Delta}, \tau$ are positive constants.

Proof: Let $V(t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) P_{ij} e_{ij}(t) + e^{\lambda_i(t-t_0)}]$, where $\lambda_i > 0$ and P_{ij} are positive definite matrices satisfying $P_{ij} = P_{ji}$. For any $t \in [t_k, t_{k+1}) \forall k \in \mathbb{N}_+$, we have

$$\begin{aligned} D^+V(t) &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) P_{ij} \dot{e}_{ij}(t) \\ &\quad + \dot{e}_{ij}^T(t) P_{ij} e_{ij}(t) - \lambda_i e^{\lambda_i(t-t_0)}]. \end{aligned} \quad (7)$$

From (3), we have

$$\begin{aligned} D^+V(t) &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) (\tilde{A}^T P_{ij} + P_{ij} \tilde{A}) e_{ij}(t) \\ &\quad + e_{ij}^T(t) P_{ij} B \tilde{u}_j(t) + \tilde{u}_j^T(t) B^T P_{ij} e_{ij}(t) \\ &\quad - e_{ij}^T(t) P_{ij} B \tilde{u}_i(t) + \tilde{u}_i^T(t) B^T P_{ij} e_{ij}(t) \\ &\quad - \lambda_i e^{\lambda_i(t-t_0)}]. \end{aligned} \quad (8)$$

From $e_{ij}(t) = x_j(t) - x_i(t)$, one has $e_{ij}(t) = e_j(t) - e_i(t) + \hat{e}_{ij}(t_k^{ij})$, where $\hat{e}_{ij}(t_k^{ij}) = e_j(t_k^j) - e_i(t_k^i)$. Hence, by using the free-weighting matrix technique, there are positive definite matrices Q_{ij}, R_{ij} , such that

$$\begin{aligned} 0 &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} [-e_{ij}^T(t) Q_{ij} e_{ij}(t) + e_{ij}^T(t) Q_{ij} e_j(t) \\ &\quad - e_{ij}^T(t) Q_{ij} e_i(t) + e_{ij}^T(t) Q_{ij} \hat{e}_{ij}(t_k^{ij})] \end{aligned} \quad (9)$$

$$\begin{aligned} 0 &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} [\hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_{ij}(t) - \hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_j(t) \\ &\quad + \hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_i(t) - \hat{e}_{ij}^T(t_k^{ij}) R_{ij} \hat{e}_{ij}(t_k^{ij})]. \end{aligned} \quad (10)$$

Since $a_{ij} = a_{ji}$, $P_{ij} = P_{ji}$, $Q_{ij} = Q_{ji}$, $R_{ij} = R_{ji} \quad \forall i, j \in \mathcal{V}$, it holds that

$$\begin{aligned} &\sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) Q_{ij} e_j(t) - \hat{e}_{ij}^T(t_k^{ij}) Q_{ij} e_j(t)] \\ &= - \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) Q_{ij} e_i(t) - \hat{e}_{ij}^T(t_k^{ij}) Q_{ij} e_i(t)], \quad (11) \\ &\sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) P_{ij} B \tilde{u}_j(t) + \tilde{u}_j^T(t) B^T P_{ij} e_{ij}(t)] \\ &= - \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) P_{ij} B \tilde{u}_i(t) + \tilde{u}_i^T(t) B^T P_{ij} e_{ij}(t)]. \end{aligned} \quad (12)$$

Thus, according to $\tilde{u}_i(t) = K_{i1} q_i(\hat{t}_k)$, and (8)–(12), one has

$$\begin{aligned} D^+V(t) &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left\{ e_{ij}^T(t) \left[\frac{1}{2} (\tilde{A}^T P_{ij} + P_{ij} \tilde{A}) - Q_{ij} \right] e_{ij}(t) \right. \\ &\quad - e_{ij}^T(t) P_{ij} B K_{i1} q_i(\hat{t}_k) - q_i^T(\hat{t}_k) K_{i1}^T B^T P_{ij} e_{ij}(t) \\ &\quad - 2e_{ij}^T(t) Q_{ij} e_i(t) + e_{ij}^T(t) Q_{ij} \hat{e}_{ij}(t_k^{ij}) \\ &\quad + 2\hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_i(t) + \hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_{ij}(t) \\ &\quad \left. - \hat{e}_{ij}^T(t_k^{ij}) R_{ij} \hat{e}_{ij}(t_k^{ij}) - \frac{1}{2} \lambda_i e^{\lambda_i(t-t_0)} \right\}. \end{aligned} \quad (13)$$

We know that $-e^{-\lambda_i(t-t_0)} \leq \sigma_i \|q_i(\hat{t}_k)\|^2 + \tau - \|e_i(t)\|^2 \quad \forall t \in [t_k, t_{k+1}) \quad \forall k \in \mathbb{N}_+$. In addition, there exists a positive constant $\tilde{\Delta}$, such that $\|q_i(\hat{t}_k)\|^2 \leq \tilde{\Delta}$ due to $q_i(\hat{t}_k) = \sum_{j \in \mathcal{N}_i} a_{ij} (\text{Sat}_\Delta(\text{Sat}_\Delta(x_j(t_k^j)) - x_i(t_k^i)))$. Thus, there is a constant $\theta_i > 0$, such that $\theta_i(\tilde{\Delta} - \|q_i(\hat{t}_k)\|^2) \geq 0$. One has

$$\begin{aligned} D^+V(t) &\leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left\{ e_{ij}^T(t) \left[\frac{1}{2} (\tilde{A}^T P_{ij} + P_{ij} \tilde{A}) - Q_{ij} \right] e_{ij}(t) \right. \\ &\quad - e_{ij}^T(t) P_{ij} B K_{i1} q_i(\hat{t}_k) - q_i^T(\hat{t}_k) K_{i1}^T B^T P_{ij} e_{ij}(t) \\ &\quad - 2e_{ij}^T(t) Q_{ij} e_i(t) + e_{ij}^T(t) Q_{ij} \hat{e}_{ij}(t_k^{ij}) \\ &\quad + 2\hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_i(t) + \hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_{ij}(t) \\ &\quad + \frac{\lambda_i}{2} (\sigma_i \|q_i(\hat{t}_k)\|^2 + \tau - \|e_i(t)\|^2) \\ &\quad \left. - \hat{e}_{ij}^T(t_k^{ij}) R_{ij} \hat{e}_{ij}(t_k^{ij}) + \theta_i(\tilde{\Delta} - \|q_i(\hat{t}_k)\|^2) \right\} \end{aligned}$$

for any $t \in [t_k, t_{k+1}) \forall k \in \mathbb{N}_+$. Letting $\xi_{ij}(t) = [e_{ij}^T(t), \hat{e}_{ij}^T(t_k), e_i^T(t), q_i^T(t_k)]^T$, one obtains

$$D^+V(t) \leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[\xi_{ij}^T(t) \Omega_{ij} \xi_{ij}(t) + \frac{\lambda_i \tau}{2} + \theta_i \tilde{\Delta} \right]$$

where

$$\Omega_{ij} = \begin{bmatrix} \hat{\Theta}_{ij} & \Xi_{ij} & -Q_{ij} & -P_{ij}BK_{i1} \\ * & -R_{ij} & R_{ij} & 0 \\ * & * & -\frac{1}{2}\lambda_i I & 0 \\ * & * & * & -\eta_i I \end{bmatrix}$$

$\hat{\Theta}_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK)] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, and $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$. From (5), we know that

$$D^+V(t) \leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[\frac{\lambda_i \tau}{2} + \theta_i \tilde{\Delta} - \frac{\alpha_{ij}}{2\lambda_{\max}(P_{ij})} e_{ij}^T(t) P_{ij} e_{ij}(t) \right]. \quad (14)$$

Let $\tilde{V}(t) = \sum_{i=1}^N \sum_{j=1}^N a_{ij} V_{ij}(t)$, where

$$V_{ij}(t) = \frac{1}{2} e_{ij}^T(t) P_{ij} e_{ij}(t) + \left(\frac{1}{2} + \beta_{ij} \right) e^{-\lambda_i(t-t_0)} - \frac{\lambda_{\max}(P_{ij})}{\alpha_{ij}} \left(\frac{\lambda_i \tau}{2} + \theta_i \tilde{\Delta} \right) \quad (15)$$

$\beta_{ij} = \frac{\alpha_{ij}}{2(\lambda_i \lambda_{\max}(P_{ij}) - \alpha_{ij})}$ and $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$.

One obtains

$$D^+ \tilde{V}(t) \leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[-\frac{\alpha_{ij}}{\lambda_{\max}(P_{ij})} V_{ij}(t) \right]. \quad (16)$$

Thus, for any $t \in [t_k, t_{k+1}) \forall k \in \mathbb{N}_+$, we have

$$\tilde{V}(t) \leq \tilde{V}(t_k) e^{-\alpha(t-t_k)} \quad (17)$$

where

$$\alpha = \min \left\{ \frac{\alpha_{11}}{\lambda_{\max}(P_{11})}, \frac{\alpha_{12}}{\lambda_{\max}(P_{12})}, \dots, \frac{\alpha_{ij}}{\lambda_{\max}(P_{ij})}, \dots, \frac{\alpha_{NN}}{\lambda_{\max}(P_{NN})} \right\}.$$

We know that $\tilde{V}(t)$ is continuous for any $t \geq t_0$. By induction, it follows that

$$\tilde{V}(t) \leq \tilde{V}(t_0) e^{-\alpha(t-t_0)}. \quad (18)$$

That is

$$\begin{aligned} \|e_{ij}(t)\|^2 &\leq \frac{2\tilde{V}(t_0)}{a_{ij}\lambda_{\min}(P_{ij})} e^{-\alpha(t-t_0)} \\ &+ \sum_{l=1}^N \sum_{s=1}^N a_{ls} \left[\frac{\lambda_{\max}(P_{ls})}{a_{ij}\lambda_{\min}(P_{ij})\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta}) - \frac{1 + 2\beta_{ls}}{a_{ij}\lambda_{\min}(P_{ij})} e^{-\lambda_l(t-t_0)} \right]. \end{aligned} \quad (19)$$

As $t \rightarrow +\infty$, the error $\|e_{ij}(t)\|$ converges to the set in (6) with the convergence rate $\min\{\frac{\alpha}{2}, \frac{\lambda_i}{2}\}$, $i = 1, 2, \dots, N$, i.e., the MAS

(1) with the controller (3) achieves the bounded synchronization based on the event-triggered communication law (4). The proof is complete. ■

Remark 3: Because of saturation, each agent's information on its neighbors is imprecise. As a result, the MAS (1) under input (2) reaches bounded synchronization, but not asymptotic synchronization or exponential synchronization.

Remark 4: Theorem 2 realizes the fully distributed control for MASs without involving the global graph information, in which the computation load of this algorithm can be further reduced. Although the global graph information is not utilized in the theorem, it relies on the sum of the weights of an agent and its neighbors. This means that the method may not be invalid when the graph is changing, for example, the nodes are added/deleted. The controller maybe need to redesign. In the following, an improved input rule $u_i(t)$ is designed to be

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij} [K_{i1} \text{Sat}_{\Delta}(\text{Sat}_{\Delta}(x_j(t_k^j)) - x_i(t_k^i)) - K_{i2} x_i(t)] \quad (20)$$

where $K_{i2} = \frac{K}{\sum_{j \in \mathcal{N}_i} a_{ij}}$, $K_{i1}, K_{i2}, K \in \mathbb{R}^{m \times n}$, and $t \in [t_k^i, t_{k+1}^i) \forall k \in \mathbb{N}_+$. In this input, suppose that the agent i can obtain its real time state. Thus, the dynamics of the MASs can be rewritten as

$$\dot{x}_i(t) = \tilde{A}x_i(t) + B\tilde{u}_i(t), t \in [t_k^i, t_{k+1}^i) \forall k \in \mathbb{N}_+ \quad (21)$$

where $\tilde{A} = A - BK$, $\tilde{u}_i(t) = K_{i1} q_i(t_k^i)$, and $q_i(t_k^i) = \sum_{j \in \mathcal{N}_i} a_{ij} \text{Sat}_{\Delta}(\text{Sat}_{\Delta}(x_j(t_k^j)) - x_i(t_k^i))$.

Theorem 3: Consider the system (1) with the control law (20) and the triggering rule (4). If there are constants $\alpha_{ij} > 0$, matrices $K, K_{i1} \in \mathbb{R}^{m \times n}$, and positive definite matrices $P_{ij}, Q_{ij}, R_{ij} \in \mathbb{R}^{n \times n}$ satisfying $P_{ij} = P_{ji}, Q_{ij} = Q_{ji}, R_{ij} = R_{ji}$, respectively, such that

$$\begin{bmatrix} \Theta_{ij} & \Xi_{ij} & -Q_{ij} & -P_{ij}BK_{i1} \\ * & -R_{ij} & R_{ij} & 0 \\ * & * & -\frac{1}{2}\lambda_i I & 0 \\ * & * & * & -\eta_i I \end{bmatrix} \leq 0 \quad (22)$$

where $\Theta_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK) + \alpha_{ij}I] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$, $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$, and the parameters λ_i, σ_i satisfy the triggering rule (4), then the system (1) realizes the bounded synchronization, i.e., for any $i, j \in \mathcal{V}$, the error $e_{ij}(t) = x_j(t) - x_i(t)$ converges to the set

$$\mathcal{S} = \left\{ e_{ij}(t) \in \mathbb{R}^n \mid \|e_{ij}(t)\| \leq \sqrt{\frac{\Gamma}{a_{ij}\lambda_{\max}(P_{ij})}} \right\} \quad (23)$$

where $\Gamma = \sum_{l=1}^N \sum_{s=1}^N a_{ls} [\frac{\lambda_{\max}(P_{ls})}{\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta})]$ and $\tilde{\Delta}, \tau$ are positive constants. ■

Proof: The proof is similar with Theorem 2, which is omitted here.

Remark 5: The input (20) shows that the agent's controller does not only depend on the sampling information of its neighbors but also rely on the real-time state of itself. This means that the agent has to monitor the state of itself, and update its controller in real time. The performance in reducing resource

consumption of this control scheme can be further improved. The control law (20) can be redesigned as

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij} [K_{i1} \text{Sat}_\Delta(\text{Sat}_\Delta(x_j(t_k^j)) - x_i(t_k^i)) - K_{i2} x_i(t_k^i)] \quad (24)$$

where $K_{i2} = \frac{K}{\sum_{j \in \mathcal{N}_i} a_{ij}}$, $K_{i1}, K_{i2}, K \in \mathbb{R}^{m \times n}$, and $t \in [t_k^i, t_{k+1}^i), \forall k \in \mathbb{N}_+$. This design scheme can avoid the real-time updating of controller.

The dynamics of the MASs can be rewritten as

$$\dot{x}_i(t) = \tilde{A} x_i(t) + B \tilde{u}_i(t), t \in [t_k^i, t_{k+1}^i) \quad \forall k \in \mathbb{N}_+ \quad (25)$$

where $\tilde{A} = A - BK$, $\tilde{u}_i(t) = K_{i1} q_i(\hat{t}_k) - K e_i(t)$, and $q_i(\hat{t}_k) = \sum_{j \in \mathcal{N}_i} a_{ij} \text{Sat}_\Delta(\text{Sat}_\Delta(x_j(t_k^j)) - x_i(t_k^i))$.

Under (4) and (24), the following bounded synchronization result can be obtained.

Theorem 4: Consider the system (1) with the control law (24) and the triggering rule (4). If there are constants $\alpha_{ij} > 0$, matrices $K, K_{i1} \in \mathbb{R}^{m \times n}$, and positive definite matrices $P_{ij}, Q_{ij}, R_{ij} \in \mathbb{R}^{n \times n}$ satisfying $P_{ij} = P_{ji}, Q_{ij} = Q_{ji}, R_{ij} = R_{ji}$, respectively, such that

$$\begin{bmatrix} \Theta_{ij} & \Xi_{ij} & P_{ij}BK - Q_{ij} & -P_{ij}BK_{i1} \\ * & -R_{ij} & R_{ij} & 0 \\ * & * & -\frac{1}{2}\lambda_i I & 0 \\ * & * & * & -\eta_i I \end{bmatrix} \leq 0 \quad (26)$$

where $\Theta_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK) + \alpha_{ij}I] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$, $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$, and the parameters λ_i, σ_i satisfy the triggering rule (4), then the system (1) realizes the bounded synchronization, i.e., for any $i, j \in \mathcal{V}$, the error $e_{ij}(t) = x_j(t) - x_i(t)$ converges to the set

$$\mathcal{S} = \left\{ e_{ij}(t) \in \mathbb{R}^n \mid \|e_{ij}(t)\| \leq \sqrt{\frac{\Gamma}{a_{ij} \lambda_{\max}(P_{ij})}} \right\} \quad (27)$$

where $\Gamma = \sum_{l=1}^N \sum_{s=1}^N a_{ls} \left[\frac{\lambda_{\max}(P_{ls})}{\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta}) \right]$ and $\tilde{\Delta}, \tau$ are positive constants.

Proof: Construct the function $V(t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) P_{ij} e_{ij}(t) + e^{-\lambda_i(t-t_0)}]$, where $\lambda_i > 0$ and P_{ij} are positive definite matrices satisfying $P_{ij} = P_{ji}$. Similar with (7)–(12), one has

$$\begin{aligned} D^+ V(t) &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left\{ e_{ij}^T(t) \left[\frac{1}{2} (\tilde{A}^T P_{ij} + P_{ij} \tilde{A}) - Q_{ij} \right] e_{ij}(t) \right. \\ &\quad - e_{ij}^T(t) P_{ij} B K_{i1} q_i(\hat{t}_k) - q_i^T(\hat{t}_k) K_{i1}^T B^T P_{ij} e_{ij}(t) \\ &\quad + e_{ij}^T(t) P_{ij} B K e_i(t) + e_i^T(t) K^T B^T P_{ij} e_{ij}(t) \\ &\quad - 2e_{ij}^T(t) Q_{ij} e_i(t) + e_{ij}^T(t) Q_{ij} \hat{e}_{ij}(t_k^{ij}) \\ &\quad + 2\hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_i(t) + \hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_{ij}(t) \\ &\quad \left. - \hat{e}_{ij}^T(t_k^{ij}) R_{ij} \hat{e}_{ij}(t_k^{ij}) - \frac{1}{2} \lambda_i e^{\lambda_i(t-t_0)} \right\} \quad (28) \end{aligned}$$

using $\tilde{u}_i(t) = K_{i1} q_i(\hat{t}_k) - K e_i(t)$.

Because of saturation and the triggering scheme (4), we know that there exist positive constants $\tilde{\Delta}$ and θ_i , such that $\theta_i(\tilde{\Delta} - \|q_i(\hat{t}_k)\|^2) \geq 0$ and $-e^{-\lambda_i(t-t_0)} \leq \sigma_i \|q_i(\hat{t}_k)\|^2 + \tau - \|e_i(t)\|^2 \quad \forall t \in [t_k^i, t_{k+1}^i) \quad \forall k \in \mathbb{N}_+$. Thus

$$\begin{aligned} D^+ V(t) &\leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left\{ e_{ij}^T(t) \left[\frac{1}{2} (\tilde{A}^T P_{ij} + P_{ij} \tilde{A}) - Q_{ij} \right] e_{ij}(t) \right. \\ &\quad - e_{ij}^T(t) P_{ij} B K_{i1} q_i(\hat{t}_k) - q_i^T(\hat{t}_k) K_{i1}^T B^T P_{ij} e_{ij}(t) \\ &\quad + e_{ij}^T(t) P_{ij} B K e_i(t) + e_i^T(t) K^T B^T P_{ij} e_{ij}(t) \\ &\quad - 2e_{ij}^T(t) Q_{ij} e_i(t) + e_{ij}^T(t) Q_{ij} \hat{e}_{ij}(t_k^{ij}) \\ &\quad + 2\hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_i(t) + \hat{e}_{ij}^T(t_k^{ij}) R_{ij} e_{ij}(t) \\ &\quad - \hat{e}_{ij}^T(t_k^{ij}) R_{ij} \hat{e}_{ij}(t_k^{ij}) + \theta_i(\tilde{\Delta} - \|q_i(\hat{t}_k)\|^2) \\ &\quad \left. + \frac{\lambda_i}{2} (\sigma_i \|q_i(\hat{t}_k)\|^2 + \tau - \|e_i(t)\|^2) \right\}. \quad (29) \end{aligned}$$

Letting $\xi_{ij}(t) = [e_{ij}^T(t), \hat{e}_{ij}^T(t_k^{ij}), e_i^T(t), q_i^T(\hat{t}_k)]^T$, one obtains

$$D^+ V(t) \leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[\xi_{ij}^T(t) \Omega_{ij} \xi_{ij}(t) + \frac{\lambda_i \tau}{2} + \theta_i \tilde{\Delta} \right]$$

where

$$\Omega_{ij} = \begin{bmatrix} \hat{\Theta}_{ij} & \Xi_{ij} & P_{ij}BK - Q_{ij} & -P_{ij}BK_{i1} \\ * & -R_{ij} & R_{ij} & 0 \\ * & * & -\frac{1}{2}\lambda_i I & 0 \\ * & * & * & -\eta_i I \end{bmatrix}$$

$\hat{\Theta}_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK)] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, and $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$. From (26), we know that

$$\begin{aligned} D^+ V(t) &\leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[\frac{\lambda_i \tau}{2} + \theta_i \tilde{\Delta} - \frac{\alpha_{ij}}{2\lambda_{\max}(P_{ij})} e_{ij}^T(t) P_{ij} e_{ij}(t) \right]. \end{aligned}$$

Similar to (15)–(18), one obtains that

$$\begin{aligned} \|e_{ij}(t)\|^2 &\leq \frac{2\tilde{V}(t_0)}{a_{ij} \lambda_{\min}(P_{ij})} e^{-\alpha(t-t_0)} \\ &\quad + \sum_{l=1}^N \sum_{s=1}^N a_{ls} \left[\frac{\lambda_{\max}(P_{ls})}{a_{ij} \lambda_{\min}(P_{ij}) \alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta}) \right. \\ &\quad \left. - \frac{1 + 2\beta_{ls}}{a_{ij} \lambda_{\min}(P_{ij})} e^{-\lambda_l(t-t_0)} \right]. \end{aligned}$$

As $t \rightarrow +\infty$, the error $\|e_{ij}(t)\|$ converges to the set in (27) with the convergence rate $\min\{\frac{\alpha}{2}, \frac{\lambda_i}{2}\}$, $i = 1, 2, \dots, N$, i.e., the MAS (1) achieves the bounded synchronization under the controller (24) and the triggering law (4). This completes the proof. ■

B. Self-Triggered Communication Scheme

In this subsection, we design a self-triggered communication scheme to realize the fully distributed synchronization of MAS (1). To avoid the case of monitoring the real-time state of the

plant, a self-triggered communication rule for agent i is designed as

$$t_{k+1}^i = \inf \left\{ t : t > t_k^i, \sigma_i \|q_i(\hat{t}_k)\|^2 + \tau + e^{-\lambda_i(t-t_0)} - \frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t-t_k^i)} - 1) \leq 0 \right\} \quad (30)$$

where $\gamma = \|I_n + A^T + A\|$, $\Omega_i(\hat{t}_k) = (A - BK)x_i(t_k^i) + BK_{i1}q_i(\hat{t}_k)$.

Remark 6: For each agent, only the sampled information from its neighbors is utilized in the two triggering conditions, which has the advantage in term of avoiding continuous communication among agents. However, the triggering error $e_i(t)$ depends on the real-time state of agent i according to the event-based rule (4). This means that the agent has to monitor its state in real time, which consumes resources. The new triggering condition (30) overcomes this drawback because the triggering error $\|e_i(t)\|^2$ of agent i in (4) is estimated by $\frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t-t_k^i)} - 1)$, in which only the sampled information at the instants of the triggered events are utilized. Similar to Theorem 1, it is easy to check that $\|e_i(t)\|^2 \leq \frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t-t_k^i)} - 1)$. The proof is omitted here.

Theorem 5: The Zeno-behavior can be excluded in the closed-loop system (1) with partial saturation input (25) if the agents of system (1) communicate with each other under the self-triggered rule (30).

Proof: If the next event occurs at t_{k+1}^i , depending on the self-triggered condition (30), the following equation holds

$$\begin{aligned} & \frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t_{k+1}^i - t_k^i)} - 1) \\ &= \sigma_i \|q_i(\hat{t}_k)\|^2 + \tau + e^{-\lambda_i(t_{k+1}^i - t_0)}. \end{aligned}$$

Hence

$$e^{-\lambda_i(t_{k+1}^i - t_0)} + \tau \leq \frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t_{k+1}^i - t_k^i)} - 1).$$

If Zeno-behavior cannot be avoided, then there exist a positive constant T , such that $\lim_{k \rightarrow +\infty} t_k^i = T < +\infty$, i.e., $t_{k+1}^i - t_k^i \rightarrow 0$ as $k \rightarrow +\infty$. As a result, $e^{-\lambda_i(T-t_0)} + \tau \leq 0$ which contradicts $e^{-\lambda_i(T-t_0)} + \tau > 0$. This means that the constant $T^* := \inf_{k \in \mathbb{N}_+} \{t_{k+1}^i - t_k^i\} > 0$ for any $i \in \mathcal{V}$. This implies that the Zeno-behavior can be successfully excluded in the closed-loop systems. The proof is complete. ■

Theorem 6: Consider the system (1) with the control law (24) and the triggering rule (30). If there are constants $\alpha_{ij} > 0$, matrices $K, K_{i1} \in \mathbb{R}^{m \times n}$, and positive definite matrices $P_{ij}, Q_{ij}, R_{ij} \in \mathbb{R}^{n \times n}$ satisfying $P_{ij} = P_{ji}, Q_{ij} = Q_{ji}, R_{ij} = R_{ji}$, respectively, such that

$$\begin{bmatrix} \Theta_{ij} & \Xi_{ij} & P_{ij}BK - Q_{ij} - P_{ij}BK_{i1} \\ -R_{ij} & R_{ij} & 0 \\ * & -\frac{1}{2}\lambda_i I & 0 \\ * & * & -\eta_i I \end{bmatrix} \leq 0 \quad (31)$$

where $\Theta_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK) + \alpha_{ij}I] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$, $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$, and the parameters λ_i, σ_i satisfy the triggering rule (30), then the system (1) realizes bounded synchronization,

i.e., for any $i, j \in \mathcal{V}$, the error $e_{ij}(t) = x_j(t) - x_i(t)$ converges to the set

$$\mathcal{S} = \left\{ e_{ij}(t) \in \mathbb{R}^n \mid \|e_{ij}(t)\| \leq \sqrt{\frac{\Gamma}{a_{ij}\lambda_{\max}(P_{ij})}} \right\} \quad (32)$$

where $\Gamma = \sum_{l=1}^N \sum_{s=1}^N a_{ls} \left[\frac{\lambda_{\max}(P_{ls})}{\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta}) \right]$, and $\tilde{\Delta}, \tau$ are positive constants.

Proof: Construct the function $V(t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} [e_{ij}^T(t) P_{ij} e_{ij}(t) + e^{-\lambda_i(t-t_0)}]$, where $\lambda_i > 0$ and P_{ij} are positive definite matrices satisfying $P_{ij} = P_{ji}$. Similar with (7)–(12), one knows that (28) also holds here.

Moreover, because of the bounded property of $q_i(\hat{t}_k)$, there exist positive constants $\tilde{\Delta}$ and θ_i , such that $\theta_i(\tilde{\Delta} - \|q_i(\hat{t}_k)\|^2) \geq 0$. In addition, according to the triggering condition (30) and $e_i^T(t) e_i(t) \leq \frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t-t_k^i)} - 1)$, for any $t \in [t_k^i, t_{k+1}^i) \forall k \in \mathbb{N}_+$, one has

$$\begin{aligned} -e^{-\lambda_i(t-t_0)} &\leq \sigma_i \|q_i(\hat{t}_k)\|^2 + \tau - \frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t-t_k^i)} - 1) \\ &\leq \sigma_i \|q_i(\hat{t}_k)\|^2 + \tau - \|e_i(t)\|^2 \end{aligned}$$

where the inequality also holds in the interval $[t_{\tilde{k}}, t_{\tilde{k}+1})$ since $[t_{\tilde{k}}, t_{\tilde{k}+1}) \subseteq [t_k^i, t_{k+1}^i) \forall \tilde{k}, k \in \mathbb{N}_+$. Thus, similar with (29), one has

$$D^+ V(t) \leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[\xi_{ij}^T(t) \Omega_{ij} \xi_{ij}(t) + \theta_i \tilde{\Delta} + \frac{\lambda_i}{2} \tau \right]$$

where

$$\xi_{ij}(t) = [e_{ij}^T(t), \hat{e}_{ij}^T(t_k^i), e_i^T(t), q_i^T(\hat{t}_k)]^T,$$

$$\Omega_{ij} = \begin{bmatrix} \hat{\Theta}_{ij} & \Xi_{ij} & P_{ij}BK - Q_{ij} - P_{ij}BK_{i1} \\ -R_{ij} & R_{ij} & 0 \\ * & -\frac{1}{2}\lambda_i I & 0 \\ * & * & -\eta_i I \end{bmatrix}$$

$\hat{\Theta}_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK)] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i$, and $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$. From (31), we know that

$$\begin{aligned} D^+ V(t) &\leq \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[\frac{\lambda_i \tau}{2} + \theta_i \tilde{\Delta} - \frac{\alpha_{ij}}{2\lambda_{\max}(P_{ij})} e_{ij}^T(t) P_{ij} e_{ij}(t) \right]. \end{aligned}$$

Similar to (15)–(18), one obtains that

$$\begin{aligned} \|e_{ij}(t)\|^2 &\leq \frac{2\tilde{V}(t_0)}{a_{ij}\lambda_{\min}(P_{ij})} e^{-\alpha(t-t_0)} \\ &\quad + \sum_{l=1}^N \sum_{s=1}^N a_{ls} \left[\frac{\lambda_{\max}(P_{ls})}{a_{ij}\lambda_{\min}(P_{ij})\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta}) \right. \\ &\quad \left. - \frac{1 + 2\beta_{ls}}{a_{ij}\lambda_{\min}(P_{ij})} e^{-\lambda_l(t-t_0)} \right]. \end{aligned}$$

As $t \rightarrow +\infty$, the error $\|e_{ij}(t)\|$ converges to the set in (32) with the convergence rate $\min\{\frac{\alpha}{2}, \frac{\lambda_i}{2}\}$, $i = 1, 2, \dots, N$, i.e., the

MAS (1) with the controller (24) achieves the bounded synchronization under the self-triggered law (30). This completes the proof. ■

Remark 7: Note that the synchronization performance is affected by the triggering condition and the saturation bound based on $\Gamma = \sum_{l=1}^N \sum_{s=1}^N a_{ls} [\frac{\lambda_{\max}(P_{ls})}{\alpha_{ls}} (\lambda_l \tau + 2\theta_l \tilde{\Delta})]$. For any given parameters of the triggering condition and the saturation bound, some proper parameters must be chosen to optimize the bound. So an optimization problem can be formulated as follows:

minimize Γ

$$\text{s.t.} \quad \begin{bmatrix} \Theta_{ij} & \Xi_{ij} & P_{ij}BK - Q_{ij} & -P_{ij}BK_{i1} \\ -R_{ij} & R_{ij} & 0 & 0 \\ * & -\frac{1}{2}\lambda_i I & 0 & 0 \\ * & * & -\eta_i I & 0 \end{bmatrix} \leq 0$$

where $\Theta_{ij} = \frac{1}{2}[(A - BK)^T P_{ij} + P_{ij}(A - BK) + \alpha_{ij}I] - Q_{ij}$, $\Xi_{ij} = \frac{1}{2}(R_{ij} + Q_{ij})$, $\eta_i = \theta_i - \frac{1}{2}\lambda_i \sigma_i > 0$, and $\alpha_{ij} \neq \lambda_i \lambda_{\max}(P_{ij})$, $i, j = 1, 2, \dots, N$.

Remark 8: Compared with the existing methods handling the coordination control of MASs with input saturation such as [23]–[25], our results may seem to be conservative in terms of synchronization performance, where the state error of MASs only converges into a bounded set. However, our method has the advantage in avoiding the continuous communication among agents and the real-time updating of controllers since only the sampled relative information is utilized in the event-based communication rule and the controller of MASs. In addition, unlike the existing criteria such as those reported in [11], [12], and [18], the results in this article do not rely on the Laplacian matrix information of $\mathcal{G}$ or its number of nodes. This means that the method has the advantages of reducing the algorithmic complexity and computation load. Theorems 3–6 show that the designed event-based strategies will still be valid and do not need to be redesigned if the network graph remains connected property after changing (e.g., switching connected topology and added/removed nodes).

Before the end of this section, the synchronization algorithm for MASs under the distributed event-based communication strategy is given as shown in Algorithm 1.

IV. NUMERICAL SIMULATION

In this section, a numerical simulation is presented to illustrate the effectiveness of the proposed fully distributed event-based control strategy for a class of linear MASs with partial input saturation.

In this simulation, the cases of adding and deleting nodes are considered, respectively. Suppose that the original network consists of six agents where the communication topology is shown in Fig. 1(b). The communication topology with the deleted nodes and added nodes are shown in Fig. 1(a) and (c), respectively. Node 6 in Fig. 1(a) denotes the deleted agent, and agent 7 is added into the network in Fig. 1(c).

Consider the dynamic of the agent i as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i = 1, 2, \dots, 7 \quad (33)$$

Fig. 1. Communication topology of agent networks.

Algorithm 1: Fully Distributed Control for MASs Under the Event-Based Communication Strategy.

Require:

- 1: Initialize the parameters of the triggering conditions (4) or (30): σ_i , λ_i , τ , and γ ;
- 2: Let the first triggering instant $t_0^i = t_0$ of agent i ;
- Initialize the state of agents $x_i(t_0)$, $i = 1, 2, \dots, N$;

Ensure:

- 3: **for** t **do** t_0 to t_{end}
 - 4: **for** i **do** 1 to N
 - 5: Update $\|q_i(\hat{t}_k)\|^2$ and $\|e_i(t)\|^2$ (or $\frac{\|\Omega_i(\hat{t}_k)\|^2}{\gamma} (e^{\gamma(t-t_k^i)} - 1)$);
 - 6: **if** Condition (4) (or (30)) holds **then**
 - 7: $t_{k+1}^i \leftarrow t$;
 - 8: Update the sampling information: $x_i(t_{k+1}^i)$;
 - 9: **end if**
 - 10: Update the state $x_i(t)$ based on control input $u_i(t)$.
 - 11: **end for**
 - 12: **end for**
-

where

$$A = \begin{bmatrix} 0.03 & 1.5 & 0 \\ -1.5 & 0.03 & 0 \\ -1.0 & 0 & -0.3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$x_i(t) \in \mathbb{R}^3$ is the state of agent i with the initial state $x_i(t_0)$ randomly taking its value in $[-3, 3]^3$; input $u_i(t) \in \mathbb{R}^3$ is given as (24) with

$$K_{i1} = \begin{bmatrix} 1.2094 & -0.1164 & 0.0982 \\ 0.1812 & 1.2206 & 0.0466 \\ 0.1612 & -0.0810 & 1.1922 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.0234 & 0 & 0 \\ 0 & 0.0234 & 0 \\ 0 & 0 & 0.0234 \end{bmatrix}.$$

The bound of the input saturation is $\Delta = [0.75, 0.90, 1.20]$. The parameters of triggering condition (4) are $\sigma_i = 0.008$, $\lambda_i = 2.3$, $\tau = 0.07$. By solving (26), one obtains $\theta_i = 2.5092$, $\alpha_{ij} = 0.6000$, and

$$P_{ij} = \begin{bmatrix} 0.1880 & -0.0638 & 0.1593 \\ -0.0638 & 0.2274 & -0.1628 \\ 0.1593 & -0.1628 & 0.3251 \end{bmatrix}$$

$$Q_{ij} = \begin{bmatrix} 0.9836 & -0.0036 & 0.0041 \\ -0.0036 & 0.9898 & -0.0202 \\ 0.0041 & -0.0202 & 0.9782 \end{bmatrix}$$

Fig. 2. Synchronization simulation of the MASs (33) for the case of deleting agent.

Fig. 3. Synchronization simulation of the MASs (33) for the case of adding agent.

$$R_{ij} = \begin{bmatrix} 1.1205 & 0.0002 & 0.0003 \\ 0.0002 & 1.1192 & 0.0023 \\ 0.0003 & 0.0023 & 1.1218 \end{bmatrix}$$

for any $i, j \in \{1, 2, \dots, 7\}$. Similarly, the other theorems can be verified, which is omitted here.

The simulations of Theorem 4 are shown in Figs. 2 and 3. The synchronization of the MASs (33) with deleting node is depicted by Fig. 2. Here, Fig. 2(a) and (b) shows the evolution of the state $x_i(t)$ and the norm of the state error $\|x_i(t) - \bar{x}(t)\|$

of agent i ($i = 1, 2, \dots, 6$), which shows the fact that the MASs (33) achieves the bounded synchronization. The input of the agents is shown in Fig. 2(c). The fact of excluding the Zeno-behavior in the MASs (33) is verified in Fig. 2(d).

Similarly, the simulations of the MASs (33) with one deleted node is shown in Fig. 3, where Fig. 3(a) and (b) shows the evolution of the state and the norm of the state error of agents. Fig. 3(c) shows the input of the agents. The triggering instants of agents are shown in Fig. 3(d). The simulations show that the MASs (33) with added or deleted nodes achieve the bounded

synchronization, where the saturated inputs due to communication depend on the designed event triggering rule (4). Only the neighbor information of an agent is used in the designed controller and the event-triggered rules. In addition, the synchronization criteria do not rely on the global graph information either. This means our method realizes the fully distributed control for MASs, which has the advantages in terms of reducing the complexity and computation load of the synchronization algorithm.

V. CONCLUSION

In this article, we have designed a fully distributed event-triggered protocol for MAS with communication induced input saturation to solve the synchronization problem. Several bounded synchronization criteria without involving the global graph information have been obtained by using the event-triggered techniques and system stability theory. The proposed fully distributed cooperative schemes have the advantages in terms of reducing the computational load and saving the resource, which has great potential in practical applications. In addition, it is proved that the Zeno-behavior does not appear in the closed-loop systems. An event-triggered algorithm implemented in a distributed way has been designed. Finally, an example has been given to verify the effectiveness of the obtained results. However, the presented synchronization criteria hold only when the communication typology is undirected, and the synchronization of the MASs is bounded. It remains a challenge to establish the less conservative results by fully distributed event-triggered protocol, such as how to design the fully distributed event-triggered communication law for MASs with directed communication graph, how to realize the bounded synchronization with tighter bound, and how to establish the asymptotical synchronization and exponential synchronization for MASs.

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