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Guiding Vector Fields for Robot Motion Control

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Guiding Vector Fields for Robot Motion Control

Weijia Yao

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The research described in this dissertation has been carried out at the Faculty of Science and Engineering, University of Groningen, the Netherlands.

disc

The research reported in this dissertation is part of the research program of the Dutch Institute of Systems and Control (DISC). The author has successfully completed the education program of the Graduate School DISC.

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Guiding Vector Fields for Robot Motion Control

PhD thesis

to obtain the degree of PhD at the
 University of Groningen
 on the authority of the
 Rector Magnificus Prof. C. Wijmenga
 and in accordance with the decision by the College of Deans.

This thesis will be defended in public on

Friday 8 October 2021 at 12.45 hours

by

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born on 14 November, 1992
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To my family

献给我的家人

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Weijia Yao
Groningen, the Netherlands
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ACRONYMS

VF-PF	Vector-Field Guided Path-Following
VF-CAPF	Vector-Field Guided Path-Following with Collision-Avoidance
UAV	Unmanned Aerial Vehicle
LOS	Line-of-sight
NLGL	Nonlinear Guidance Law
LQR	Linear Quadratic Regulator
ISS	Input-to-state Stable/Stability
GAS	Globally Asymptotically Stable / Global Asymptotic Stability
DOA	Domain of Attraction