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Observational signatures of ultracompact objects and bouncing universe in nonlocal gravity

Peng, Jun

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Observational signatures of ultracompact objects and bouncing universe in nonlocal gravity

Jun Peng

**university of
 groningen**

**faculty of science
and engineering**

Jun Peng

Van Swinderen Institute PhD series 2021

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Observational signatures of ultracompact objects and bouncing universe in nonlocal gravity

PhD thesis

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Rector Magnificus Prof. C. Wijmenga
and in accordance with
the decision by the College of Deans.

This thesis will be defended in public on
Thursday 7th October 2021 at 14.30 hrs.

by

Jun Peng

born on 20 January 1993
in Liaoning, China

Supervisors

Prof. A. Mazumdar

Prof. E. Pallante

Assessment committee

Prof. R. Brandenberger

Prof. V. P. Frolov

Prof. D. Roest

Abstract

Nonlocal gravity is an elegant proposal for resolving the unitary of higher derivative gravity as the modification of Einstein's General Relativity (GR) in the ultraviolet (UV) regime. In this thesis, we study some applications of nonlocal gravity in astrophysics and cosmology, specifically including ultracompact objects (UCOs) and bouncing universe.

UCO model is mainly proposed for resolving black hole (BH) singularity and information-loss paradox due to Hawking radiation associated with horizon. The UCOs are horizonless and own reflectivity of particles or waves compared with BHs which are purely absorptive. Therefore new phenomena would arise due to reflectivity. In this thesis, we study the observational signatures of UCOs in both optical aspects such as shadows and photon rings, and acoustic properties such as ringdown signal and QNMs. First, we analyze the optical appearance of an asymmetric thin-shell wormhole (ATW) illuminated by a thin accretion disk. We show additional photon rings and a potential lensing band from the ATW space-time with respect to the black hole case. Although we take the ATW as a toy model, our analysis may provide an optically observational signature to distinguish UCOs from black holes. Then we towards to study the gravitational wave (GW) echoes when experiencing nonlocal effect by generalizing the gravitational perturbation equation to a nonlocal version of Schrodinger-like equation. We find that nonlocality amplifies echoes due to the less damping of transmission of waves. Our analysis will provide smoking-gun signatures on how the nonlocality affects the GW echoes.

Bouncing universe is coming up for resolving cosmological singularity. In the context of GR, inevitable exotic matter is required to drive a bouncing scenario, which typically results in ghost or gradient instability. Bouncing universe can also be obtained in higher derivative gravity, which is also plagued by ghost instability in the gravitational sector. In this thesis, we obtain homogeneous and isotropic non-singular bouncing universes without adding matter in nonlocal gravity. We show that at the linearized level around the bounce, it is possible to constrain the nonlocal form factor such that there is only a scalar propagating degree of freedom. The scalar mode can be made free from perturbative ghost instabilities, and has oscillatory and bounded evolution across the bounce. We also show that it is possible to realize an anisotropic bouncing solution driven by the proper matters along with a cosmological constant. We show that the anisotropy grows slower than in GR during the contraction phase, and is bounded across the bounce. And it is possible to simultaneously satisfy positivity of energy density and, at least in the late time de Sitter phase, avoid the introduction of propagating ghost/tachyonic modes with respect to local theory.

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Publications

The present PhD thesis is based on the following collaborative works:

- P1** Luca Buoninfante, Anupam Mazumdar, **Jun Peng**
Nonlocality amplifies echoes
Phys.Rev.D 100, no. 10, 104059 (2019)
arXiv:1906.03624
- P2** Sravan Kumar, Shubham Maheshwari, Anupam Mazumdar, **Jun Peng**
Stable, nonsingular bouncing universe with only a scalar mode
Phys.Rev.D 102, no. 2, 024080 (2020)
arXiv:2005.01762
- P3** **Jun Peng**, Minyong Guo, Xing-Hui Feng
Influence of Quantum Correction on the Black Hole Shadows, Photon Rings and Lensing Rings
Chin.Phys.C 45, no. 8, 085103 (2021)
arXiv:2008.00657
- P4** **Jun Peng**, Minyong Guo, Xing-Hui Feng
Observational Signature and Additional Photon Rings of Asymmetric Thin-shell Wormhole
Submitted, 2021
arXiv:2102.05488
- P5** Sravan Kumar, Shubham Maheshwari, Anupam Mazumdar, **Jun Peng**
An anisotropic bouncing universe in nonlocal gravity
JCAP 07, no. 025 (2021)
arXiv:2103.13980

Acronyms and conventions

List of Acronyms

ATW asymmetric thin-shell wormhole

BH black hole

BKL Belinskii-Khalatnikov-Lifshitz

CDM cold dark matter

EOM equation of motion

FLRW Friedmann-Lemaître-Robertson-Walker

GR general relativity

GW gravitational wave

IDG infinite derivative gravity

NEC null energy condition

QFT quantum field theory

QNM quasi-normal mode

SM standard model

SVT scalar-vector-tensor

UCO ultracompact object

UV ultraviolet

Conventions

Throughout the entire thesis, we use the most plus metric signature $(-, +, +, +)$.

Latin indices i, j, k and so on, generally run over the three spatial coordinate labels, usually taken as 1, 2, 3.

Greek indices μ, ν, ρ, σ etc., generally run over the four spacetime coordinate labels 1, 2, 3, 0, where 0 stands for the time coordinate.

$\square$ is the D'Alembert operator, $\square = g^{\mu\nu} \nabla_\mu \nabla_\nu$.

We are working in Natural units, i.e. $c = \hbar = 1$. In general relativity, it's convenient to define a reduced Planck mass

$$M_p := \sqrt{\frac{\hbar c}{8\pi G}} \simeq 2.4 \times 10^{18} \text{GeV}/c^2 \simeq 4.3 \times 10^{-9} \text{kg},$$

where $G = 6.67 \times 10^{-8} \text{g}^{-1} \text{cm}^3 \text{s}^{-2}$ is the Newton constant, so $M_p = 1/\sqrt{8\pi G}$ in Natural units.

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Chapter 1

Introduction

Space, time, and the universe are ancient topics in human history. In 1687, Isaac Newton proposed that the force governing the motion of the celestial body is the same as the gravity felt by objects on Earth. This is the famous “law of universal gravitation”, which tells us that any two massive bodies attract each other. The gravity is proportional to the product of the masses of the two bodies and anti-proportional to the distance of the two bodies, i.e.

$$F = \frac{GM_1M_2}{r^2}, \tag{1.1}$$

where G is the Newtonian constant.

Newton’s gravity is extremely successful in classical physics. However, some crucial problems are difficult to explain. For one thing, Newton’s law of universal gravitation does not explain how gravity propagates, as it seems that if there is a massive object, a nearby massive object is immediately affected by gravity, a concept known as "action at a distance". Secondly, classical mechanics is not compatible with the electromagnetic theory. The theory of electromagnetism doesn’t follow the Galilean relativity principle. The speed of light is constant and independent of observers.

To address the limitations of classical mechanics, Albert Einstein proposed

Special Relativity (SR) in 1905. Einstein developed the work of Lorentz and Poincaré on Newton's view of space and time, presenting the principle of special relativity and the invariance of the speed of light. By changing the properties of time and space, Einstein succeeded in unifying Newton's mechanics and Maxwell's electrodynamics within the framework of his SR. But the SR is not perfect. On the one hand, the SR is only applicable to inertial frames, and once in the accelerating reference frame, SR will no longer be suitable. On the other hand, SR can not describe gravity. After a series of *gedanken experiments*, Einstein proposed the principle of general covariance and the equivalence principle:

- the laws of physics have the same form in any reference frame,
- in a local acceleration reference frame, the inertial force can be equivalent to gravity.

According to the principle of equivalence, Einstein realized that the gravitational force between objects is originated from the curvature of spacetime. With the help of Riemann geometry, Einstein came up with his famous General Relativity (GR) in 1915. GR shows that gravity is actually a warping effect of spacetime. Its basic idea can be summarised in a sentence by Wheeler

Matter tells spacetime how to curve, spacetime tells matter how to move.

The mathematical description of GR is the famous Einstein equation,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (1.2)$$

where $G_{\mu\nu}$ is the Einstein tensor, a purely geometric quantity, $T_{\mu\nu}$ is the energy-momentum tensor of matters. The constant κ is associated with the gravitational constant by $\kappa^2 = 8\pi G/c^4$ in SI units. So in Natural units $\kappa = \sqrt{8\pi G} = 1/M_p$, where M_p is the reduced Planck mass. Einstein equation describes how spacetime is curved by matters. Another important equation is the so-called geodesic equation

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0, \quad (1.3)$$

which describes the motion of a particle in curved spacetime.

GR plays a key role in astrophysics and cosmology. In relativistic astrophysics, GR is an indispensable tool in the research of black holes (BH), pulsars, quasars, and stellar evolution [1–4]. In cosmology, it is the basic theoretical framework to study the evolution of the universe and the formation of large-scale structures [5–7].

As a relativistic theory of gravity, GR is significant in strong relativistic situation, i.e. strong gravitational regime. In special relativity, the relativistic effect can be characterized by the ratio of the moving velocity of a particle with the speed of light v/c . When the moving velocity of one particle approaches the speed of light, the relativistic effect will be explicit. For a spherical object with mass M and radius R , the escape velocity of a body from the surface is $\sqrt{2GM/R}$. As we already discussed, the more comparable to the speed of light for the velocity $\sqrt{2GM/R}$, the more manifest the relativistic effect will be. Thus in GR, we can characterize the relativistic effect or the strength of the gravitational field by a dimensionless quantity

$$\Phi \equiv \frac{GM}{Rc^2}. \quad (1.4)$$

Here we reserve the constants G and c in SI units to show the meaning of Φ explicitly. When $\Phi \ll 1$, GR can be approximated to linearized GR, the relativistic modification is small. In fact, Φ is precisely the (minus) Newtonian gravitational potential. For Earth, $\Phi \approx 10^{-9}$, this relativistic effect must be considered in high precision global positioning system (GPS). For Sun, $\Phi \approx 10^{-6}$, this relativistic effect is observable in practice. The classical three tests for GR in history: Mercury perihelion precession, deflection of light and time delay, were done in the solar system.

So far, GR has been tested with high accuracy in the regime $\Phi \ll 1$ of weak gravitational fields. The precise test in strong-field regime comparably lacks until two recent famous events came up. One is the gravitational wave (GW) emitting from binary BHs or neutron stars merging detected by LIGO-Virgo [8, 9]. Another is the photo image of the black hole at the center of the galaxy M87 (M87*) released by the Event Horizon Telescope (EHT) [10–12]. For a neutron star with one solar mass and ten kilometers radius, $\Phi \approx 0.1$, the gravitational

field is very strong. For a BH, $\Phi \approx 0.5$, the gravitational field is so strong that light can't escape. As for cosmology, the standard Λ CDM (cosmological constant and cold dark matter) model in the framework of GR can describe the evolution of our universe very well [6].

Though GR is exceptionally successful even in strong-field regime, it is well known that GR suffers from ultraviolet (UV) divergences. At the classical level, this is reflected in the black hole and cosmology singularities which is unacceptable since the spacetime curvature and matter density go to infinity. Significantly, the singularity theorems, proposed by Roger Penrose and Stephen Hawking [13, 14], guarantee that the singularities are inevitable in the frame of GR. Another fatal issue is that, at the quantum level, one can see that the GR is non-renormalizable by doing a power counting to understand [15].

When we want to quantize gravity from a perturbative viewpoint like quantum field theory (QFT) which describes standard model (SM) including strong, weak and electromagnetic interactions, we must study the gravitational dynamics. The first step is constructing the action of GR. Just as scalar and electromagnetic fields, we may expect the gravitational action to be an integral over a Lagrangian L_g which depends on the metric and its first derivatives

$$S \sim \int d^4x \sqrt{-g} L_g[g_{ab}, \partial_c g_{ab}], \quad (1.5)$$

where L_g indicates the Lagrangian of gravity. Since L_g contains only up to the first derivatives of the metric, we will obtain field equations that are second order in the derivatives of the metric, which is precisely what we will expect for a dynamical system. For the theory of gravity, the above scheme is faced with difficulties. The first derivative of metric can't form a tensor, so it is impossible for us to construct a general covariant scalar L_g only by the metric and its first derivative.

The solution to this problem is to construct a general covariant action

$$S \sim \int d^4x \sqrt{-g} L_g[g, \partial g, \partial^2 g]. \quad (1.6)$$

That is to say the Lagrangian depends not only on the metric and its first derivative, but also on its second derivative, but only linearly on the second derivative of the metric. In this way, we can get rid of the second derivative term of the Lagrangian through the boundary term. Then a second derivative equation of motion with respect to the metric is obtained. Thanks to the linearity of Riemann tensor R_{abcd} on the second derivative of metric, we can use it to construct such a scalar, i.e. the Ricci scalar R which gives the famous Einstein-Hilbert action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R \quad (1.7)$$

for GR.

From the viewpoint of perturbative quantum gravity, we consider a linear perturbation of metric around Minkowski background, i.e.

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad (1.8)$$

where $h_{\mu\nu} \ll 1$. The expansion of Einstein-Hilbert action under this perturbation takes the following form

$$S \sim \int d^4x [\partial h \partial h + \kappa h \partial h \partial h + \kappa^2 h^2 \partial h \partial h + \dots]. \quad (1.9)$$

The first term $\partial h \partial h$ is the kinetic term which governs how the graviton propagates, just like $\partial\phi\partial\phi$ for a scalar field. The cubic and higher terms in h determines the interaction of the graviton with itself. Because of the presence of $\sqrt{-g}$ and the inverse of $g_{\mu\nu}$, the interaction terms are infinite, while interactions in SM are finite. Another major difference is that while SM is renormalizable, GR is non-renormalizable, which can be seen by dimensional analysis [15]. Varying the kinetic term with respect to $h_{\mu\nu}$ gives the perturbation equation

$$\square h_{\mu\nu} = 0 \quad (1.10)$$

in transverse and traceless gauge, where $\square$ is the D'Alembert operator in Minkowski spacetime, $\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$. This indicates that the graviton is a massless spin-2

particle in GR.

Now if we loosen the equivalence principle, only reserve the principle of general covariance, in other words we allow higher derivative or non-minimal coupling existence in the equation of motion describing gravity, the action determining gravitational dynamics will be abundant. As illustrated above, GR should be modified in the UV regime. Among numerous modified theories, higher derivative gravities whose action includes higher curvature invariants have attracted a lot of attention. It is believed that higher curvature interactions naturally arise in a UV complete quantum gravity. In fact, string theory as one of the most promising candidates of quantum gravity ¹, generically gives infinitely subleading higher curvature terms correcting the Einstein-Hilbert action [22, 23].

A simplest higher derivative gravity is $f(R)$ gravity whose action is an arbitrary function of Ricci scalar R [24]. $f(R)$ gravity has been extensively considered in the context of cosmology. It may provide new explanations for dark matter, dark energy or inflation beyond the standard Λ CDM model in GR [24–27]. We take the simplest R^2 gravity as an example, i.e.

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \alpha R^2], \quad (1.11)$$

where α is the coupling constant, which has the dimension $[\alpha] = [M]^{-2}$. To analyze the physical modes, we meanwhile implement the perturbation (1.8). The perturbation equations around Minkowski background are

$$(\square - m_s^2)h = 0, \quad \square h_{\mu\nu} = 0, \quad (1.12)$$

where h is the trace of $h_{\mu\nu}$, and $m_s^2 = 1/6\alpha$. So besides the usual massless spin-2 graviton, we have an extra massive spin-0 scalar mode. It has been shown that $f(R)$ is equivalent to scalar-tensor theory [24]. Though $f(R)$ gravity makes some progress, especially in cosmology, it is still nonrenormalizable. However when

¹It should be pointed out that the non-renormalizability of GR is based on perturbative QFT. Some non-perturbative schemes to quantize gravity include loop quantum gravity [16–18], causal set theory [19, 20] and casual dynamical triangulations [21].

considering all quadratic curvature terms ²[28]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + aR^2 + bR^{\mu\nu}R_{\mu\nu} + cR^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}], \quad (1.13)$$

where a, b, c are the coupling constants, which have the dimension $[M]^{-2}$ similarly. It has been proved that the theory is renormalizable from a viewpoint of perturbation around Minkowski spacetime [28], but at the cost of introducing a massive spin-2 graviton with the negative kinetic term, known as Weyl ghost that leads to Ostrogradski instabilities by making the Hamiltonian unbounded from below [29], while at the quantum level it breaks the unitarity of scattering process. Specifically, for action (1.13), the perturbation equations around Minkowski background are [28]

$$(\square - m_s^2)h = 0, \quad \square(\square - m^2)h_{\mu\nu} = 0, \quad (1.14)$$

where m_s, m are defined in terms of coupling constants a, b, c . h is the trace of $h_{\mu\nu}$, so it is a massive spin-0 scalar mode of mass m_s . The general solution of the fourth-order equation for $h_{\mu\nu}$ is a linear superposition of $h_{\mu\nu}^m$ (m stands for massless) and $h_{\mu\nu}^M$ (M stands for massive) which satisfy [28]

$$\square h_{\mu\nu}^m = 0, \quad -(\square - m^2)h_{\mu\nu}^M = 0, \quad (1.15)$$

respectively. So we have an usual massless spin-2 graviton $h_{\mu\nu}^m$ as in GR. However we have an additional massive spin-2 graviton $h_{\mu\nu}^M$ of mass m whose kinetic term has wrong sign, which is a manifestation of its ghost-like behavior.

So one may seek a modified theory of gravity without incurring pathologies, i.e., the incompatibility of renormalization and unitarity, which is extremely non-trivial. A turning point to resolve such problems is considering giving up the key principle of standard QFT, i.e. locality. In local theory, the Lagrangians are

²The quadratic term of Riemann tensor $R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ can be cancelled out by Gauss-Bonnet theorem, i.e. the combination $R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ is a pure topological term in four dimension, having no contribution to equation of motion.

constructed by polynomials of fields and derivatives of fields

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi), \quad (1.16)$$

where the order n of derivatives is finite. A nonlocal theory is constructed by introducing non-polynomial differential operators, such as [19, 20, 30–34]

$$\mathcal{L} = \mathcal{L}\left(\phi, \partial\phi, \dots, \partial^n\phi, \frac{1}{\square}\phi, \ln(\square/M_s^2)\phi, e^{\square/M_s^2}\phi, \dots\right), \quad (1.17)$$

which contains infinite order covariant derivatives. A new energy scale of non-locality M_s , beyond which new physics should be manifest, is introduced in logarithm and exponential operators to make the arguments dimensionless ($[\square] = [M]^2$).

In nature, infinite covariant derivatives in the field equations may require an infinite amount of initial data to solve the field equation such that the locality is violated. Authors in [35, 36] found that it is possible to realize a well-defined initial value problem for some specific choice of nonlocal operator, though differential equations are infinite order. In the context of nonlocal theory, many studies have been made in the relation between renormalization and unitarity [37–39]. For example, in 1997, E. T. Tomboulis has shown that the four-dimensional gauge theories and gravitational theories with non-polynomial actions, in particular in the form of an exponential, are perturbatively super-renormalizable and no unphysical poles are introduced in the propagators [38]. In string theory, which is intrinsically nonlocal, it was shown that the point-like singularity could be regularized [34, 40]. All these facts inspired people to investigate the nonlocal theory of gravity. In 2005, authors in [41] argued that the absence of ghosts in the modified propagator with asymptotic freedom could be realized in nonlocal quadratic scalar curvature gravity model which is proposed initially to resolve cosmology singularities. In 2012, authors in [42] generalized the study to the most general nonlocal curvature-quadratic gravitational action and showed that such theory with asymptotic freedom in the UV regime could be made free from ghosts around Minkowski background. Similarly, the most general nonlocal

curvature-quadratic gravitational action can also be constructed around de Sitter (dS) and anti-de Sitter (AdS) background without introducing ghost degree of freedom [43, 44]. In order to avoid introducing extra degrees of freedom, it is natural to choose the non-polynomial differential operators as *analytic* functions, such as $e^\square$. The nonlocal gravity whose action is constructed in terms of analytic differential operators is called *infinite derivative gravity* (IDG) [42–44], with action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\square)R^{\mu\nu\rho\sigma} \right], \quad (1.18)$$

where $\mathcal{F}_i(\square)$ ($i = 1, 2, 3$) are *analytic* functions of D'Alembertian operators $\square = g^{\mu\nu}\nabla_\mu\nabla_\nu$ and contain infinite orders of derivatives. Furthermore $\mathcal{F}_i(\square)$ are set to be dimensionless using the energy scale of nonlocality M_s , i.e.

$$\mathcal{F}_i(\square) = \sum_{n=0}^{\infty} f_{i,n} \left(\frac{\square}{M_s^2} \right)^n, \quad (1.19)$$

where both $\square/M_s^2$ and the coefficients $f_{i,n}$ are dimensionless. M_s is the new physical scale at which higher derivative terms become important. The direct bound on $M_s > 0.004\text{eV}$ arises from constraints of Newtonian potential at low energies [45], while high energy constraints come from gravitational waves and non-Gaussianity [46]. Through choosing appropriate form factors $\mathcal{F}_i(\square)$, we can derive the propagator [42, 47]

$$\Pi_{\text{IDG}} = e^{\gamma(-\frac{k^2}{M_s^2})} \Pi_{\text{GR}}. \quad (1.20)$$

In this case the propagator is modified by exponential of an entire functions $\gamma(-k^2/M_s^2)$, which do not introduce any new dynamical degrees of freedom. Moreover, at the quantum level, several attempts have been made towards the renormalization of IDG [48, 49], which give us a hint that the UV behavior could be improved in IDG. Moreover, the authors in [49–51] have shown that the IDG could be super-renormalizable by choosing the peculiar nonlocal operator which behaves polynomially in UV. The nonlocal theories with non-analytic differential

operator, such as $\frac{1}{\square}, \ln(\square)$, are studied in [20, 30–33] and we will not focus on this direction.

Moreover the IDG with action (1.18) gives a regularized gravitational potential [42, 49, 52]

$$\Phi = -\frac{GM}{r} \operatorname{erf}\left(\frac{M_s r}{2}\right), \quad (1.21)$$

where $\operatorname{erf}(\frac{M_s r}{2})$ is an Error function. When $r \rightarrow \infty$, Φ approaches Newtonian potential, whereas $r \rightarrow 0$, Φ approaches a finite value. In this sense, IDG can resolve point-like singularity. It was shown that IDG not only can resolve the point-like singularity in static [42, 45, 53–55], but also the ring singularity in a rotating spacetime metric [56], and with the presence of torsion as well [57]. Moreover, exact non-singular spherically symmetric solutions have been found in dynamical scenario [58–60]. In the linear regime, promising results have been realized, which implies that IDG could be a very good candidate for resolving classical singularities. However to discuss more on the classical singularities, i.e. BH singularity and cosmology singularity, one has to working in the full non-linear regime. In [52], the full field equation for the most general IDG action (1.18) is obtained. Nevertheless solving the full field equation is extremely challenging. So far, exact non-singular bouncing solutions were found, around spatially flat, homogeneous and isotropic backgrounds [41, 42, 61–64]. In contrast, the solutions of non-linear spherically symmetric metric are still lacking.

The detection GW and BH photos not only verify GR in the strong-field regime but also are convincing evidence of the existence of BH. As predicted by GR, any compact object with a mass larger than a few solar masses must collapse to a BH. BH physics is an important research field in classical and quantum gravity. Though enough pieces of evidence verify the existence of BH, there are some unsolved questions in BH physics. The first is the BH singularity mentioned above. This problem seems not to be very urgent. Since according to cosmic censorship conjecture [13], the singularity must be shaded by an event horizon. While the existence of horizon results in another issue, information-loss paradox [65], which is related to breakdown of unitary due to Hawking radiation. To address these problems, various horizonless ultracompact objects (UCOs) were

proposed [66, 67]. The formation of UCOs attributes to either correction to GR or beyond standard model fields coupled to gravity [68–73]. In fact, it was shown that, in a dynamical collapse, there is no formation of horizon and singularities in IDG [59]. In this sense, IDG could also be seen as a good candidate for resolving horizon problems in BH physics. It has been found that the Schwarzschild metric obtained in GR can only be valid in the local regime, i.e. $r \gg 1/M_s$ [53] and the size of the nonlocal regime is always larger than the Schwarzschild radius [53, 74, 75]. Thus the horizon problem could potentially be resolved in IDG. Moreover, the astrophysical context of UCO model is constructed in IDG recently, which is called *nonlocal star* [74, 75].

The main works of this thesis include two aspects: the optically and acoustically observational signatures of UCOs and bouncing universe in nonlocal gravity.

- The BH photos are based on gravitational lensing. The trajectory of light ray in curved spacetime is determined by a null geodesic. When the gravity is so strong that light ray forms a closed orbit called light ring (or photon sphere in spherical spacetime), we will observe a shadow of BH. When the emission source is near BH, we can observe more interesting "photon rings" structure [76]. Both shadow and photon ring of BH are associated with light ring, and the light ring is only determined by the geometry of spacetime. The ringdown signal of GW emitted from binary BHs merging is characterized by the quasi-normal modes (QNMs)³ of BH [77, 78]. The QNMs are the fingerprint of BH [79, 80]. According to the GR uniqueness and no-hair theorem [81–83], the QNMs spectrum of neutral stationary BHs depends uniquely on two parameters, i.e., their mass and angular momentum. So we can test GR and BH in strong gravitational regime through its optical properties such as shadows and photon rings, and acoustic properties such as ringdown signal, QNMs. The crucial distinction between UCO and BH is that the existence of horizon of BH guarantee it is absolutely absorptive. UCOs can be characterized by their compactness which can be characterized by the inverse of their effective radius and reflectivity which

³See Appendix B for physical discussion on QNMs.

is associated with the reflection of particles or waves on their effective surface [66, 67]. An UCO as a BH mimicker can stimulate all observational properties of a BH with arbitrary accuracy, only if its compactness is high enough. However, additional phenomenons would arise due to reflectivity in both optical and acoustic aspects. Therefore we take the asymmetric thin-shell wormhole (ATW) as a toy model to capture the optically observational signatures of UCOs. Furthermore, we take toy model, i.e. nonlocal scalar field in double delta potential, to capture the nonlocal effect on the acoustically observational signatures of UCOs.

- Standard Λ CDM cosmological model in GR explains the evolution of our universe very well except for some problems in the very early time, such as big bang singularity and inflation [84, 85]. Bouncing universe aspires to provide an alternative scenario to solve these problems [86, 87]. In GR, we have no bouncing solutions unless adding exotic matters which violating null energy condition (NEC) [86–89]. The violation of NEC typically leads to ghost or gradient instability [90, 91]. Another problem with bouncing cosmologies is the growth of vector (metric and matter) modes with time during the contraction phase, thereby invalidating perturbation theory [92]. In some bouncing models [93], and even in power-law inflation [94], scalar modes can also grow. In the context of higher derivative modified gravity, quadratic scalar curvature ($R + R^2$) is an interesting model to consider when hunting for non-singular solutions [95, 96]. After adding a cosmological constant to $R + R^2$ theory, one can indeed obtain bouncing solutions, but which nevertheless are plagued by the presence of negative radiation energy density $\rho < 0$. Extending the action by adding an arbitrary number of higher derivative terms typically leads to ghost degrees of freedom around, for instance, flat space and (A)dS. On the other hand, an infinite covariant derivative, nonlocal extension can potentially resolve the problem of ghosts, by choosing a suitable function that captures all order derivative terms in the action. In this nonlocal quadratic scalar curvature model, bouncing solutions were found with $\rho > 0$, around spatially flat, homogeneous and

isotropic backgrounds [41, 42, 61–64]. The stability of scalar perturbations around these solutions in the presence of radiation was studied in [61, 97]. One can deduce that the presence of radiation implies growth of vector perturbations during the contraction phase [98]. Therefore one of the objectives of this thesis is finding a healthy, non-singular bouncing scenario with bounded amplitudes of perturbations around spatially flat, homogeneous and isotropic background.

However, according to Misner’s initial chaos, there is no prior reason to assume any spacetime symmetries (such as FLRW) for the universe to begin with [99, 100]. Misner proposed a mixmaster universe where FLRW can emerge at late times from an initial extremely anisotropic, chaotic oscillatory state. Mixmaster phenomena with bounce and cyclic cosmologies have been widely studied in several Bianchi spacetimes in GR with energy condition-violating fluids [101–110]. In the Bianchi-I class, in particular, bouncing cosmology has been studied in frameworks inspired from string theory [111–113]. A generic feature of Bianchi-I anisotropic bounce in GR is that the effective energy density from spacetime anisotropies grows during the contraction phase as a^{-6} (a is the average scale factor), faster than matter, radiation and spatial curvature, leading to a chaotic mixmaster phenomenon [100, 101, 114] and finally resulting in Belinskii-Khalatnikov-Lifshitz (BKL) instability [115–118]. However, see [119–123] for examples where an Ekpyrotic scenario can control the growth of anisotropies. For Bianchi-IX, it was also noted that quadratic curvature gravity and low energy string theory alleviate the chaotic behavior at initial times [103, 124]. In a recent study, it was shown that the vacuum field equations of nonlocal gravity containing a GR term, nonlocal quadratic in Ricci scalar, Ricci tensor and Weyl tensor terms can potentially avoid the BKL type singularity [125]. Therefore another aim of this thesis is to find a healthy anisotropic non-singular bouncing universe, which could ameliorate the challenges mentioned above.

This thesis is organised as follows:

In Chapter 2, I first give a brief introduction about the concept and phenomenology of ultracompact objects (UCOs) as BH mimickers. Then I give a brief introduction about gravitational perturbations around non-maximally symmetric backgrounds, such as black hole perturbation. BH perturbation is foundation of investigating the BH QNMs and GW ringdown. Cosmological perturbation will be illustrated in the next Chapter.

In Chapter 3, we consider a specific UCO model, asymmetric thin-shell wormhole (ATW), and study its optically observational appearance illuminated by a thin accretion disk. Because of the asymmetry of effective potential of radial null geodesic, a photon may be bounced back when it falls into another spacetime through the throat of wormhole. We show that this reflection mechanism by ATW makes an essential distinction in the observational appearance compared to BH which is purely absorptive.

In Chapter 4, I first give a brief introduction of nonlocal gravity, and then provide smoking-gun signatures of nonlocal interactions while studying reflection and transmission of waves bouncing through two Dirac delta potentials. These signatures can be potentially detectable in the context of GW ringdown, where two Dirac delta potentials can mimic the two potential barriers at the surface and at the photon sphere of an UCO, or, at the two photon spheres of a wormhole, experiencing nonlocal interactions. We discuss the influence of nonlocal interaction on the damping time scale of GW echoes.

In Chapter 5, I first give a brief overview of the bouncing universe model, especially I describe the problems existed in the frame of GR. Furthermore, since there is no prior reason to assume a homogeneous and isotropic universe to begin with, we study the anisotropy and shows it goes as a^{-6} , which indicating an instability due to the growth of anisotropy in the contraction phase, i.e. BKL instability, will disrupt a bounce. Next I introduce the cosmology perturbations, which is the foundation of investigating the formation of the galaxies and galaxy clusters and analyzing the stability of bouncing solutions.

In Chapter 6, we study bouncing universe in nonlocal gravity. We show that

non-singular bounce solution exists in the absence of matter. At the linearized level around the bounce, it is possible to constrain the nonlocal form factor such that there is only a scalar propagating degree of freedom, and no vector or tensor modes. The scalar mode can be made free from perturbative ghost instabilities, and has oscillatory and bounded evolution across the bounce. We also generalize our discussion to anisotropic universe by adding matter fields.

I conclude and make some outlooks in Chapter 7.

Chapter 2

Ultracompact objects: models and phenomenology

In this Chapter, we will give a short overview of the concepts and essential pieces of knowledge regarding the UCO models, which are useful to introduce the later Chapter 4 and Chapter 5 of the thesis. In Section 2.1, we will give a brief introduction of the concepts of the UCOs. And we provide a more concrete illustration of the common phenomenology that UCOs preserved. One is the photon sphere and shadow. Another is the QNMs and GW echos. With respect to the BH case, we show the differences in these two aspects from the UCOs. In Section 2.2, we derive the perturbation equations for the black hole case in the frame of GR, which is the first step for studying the QNMs and GW ringdown.

2.1 Ultracompact objects

2.1.1 Models of ultracompact objects

As a class of fundamental solutions to GR, BHs play a significant role in astrophysics and theoretical physics. The direct detection of GW from binary BHs merging and the first BH picture of M87* provide strong evidence for the exis-

tence of BHs [10–12]. These also show that GR is correct even in very strong gravitational regime. As predicted by uniqueness and "no-hair" theorems within GR [81–83], any compact object with mass larger than a few solar masses must collapse to a BH. Although the existence of BH is widely accepted by gravity community, there are several open issues in BH physics:

- Information-loss paradox [65] which is related to breakdown of unitary during Hawking’s radiation;
- Black hole singularity where GR breaks down.

These two problems should be hopefully resolved within an ultraviolet complete quantum gravity theory.

Nevertheless the statement “BHs exist in our Universe” is not absolutely definite. A BH owns its name due to the fact that nothing – not even light – can escape from the region enclosed by its horizon which is the real defining quantity of a BH. In fact, one can design ultracompact objects (UCOs) without a classical BH horizon, which mimic all observational properties of a BH with arbitrary accuracy. The formation of UCOs attributes to either correction to GR or beyond standard model fields coupled to gravity. UCOs may be classified in terms of their [66, 67]

- compactness, i.e. the inverse of their effective radius r_0 in units of the total mass M . It is customary to define a “closeness” parameter ϵ such as $r_0 = r_+(1 + \epsilon)$, where r_+ is the location of the horizon of the corresponding BH with the same mass and spin. Therefore, the BH limit corresponds to $\epsilon \rightarrow 0$;
- reflectivity $\mathcal{R}$ at their effective surface. This quantity is generically complex and frequency-dependent and can also depend on other object’s parameters such as the spin ¹. Note that the perfect absorption limit $\mathcal{R} \rightarrow 0$ is corresponding to a classical BH horizon.

¹The reflectivity of waves is defined in (2.12).

Figure 2.1: The classification of compact objects (COs) in parameter space [66, 67]. An object could be called compact when it features an innermost stable circular orbit (ISCO), $r_0 < 6M$ ($\epsilon < 2$). A compact object which features a photon sphere is called ultracompact object, $r_0 < 3M$ ($\epsilon < 1/2$). An ultracompact object whose radius $r_0 \leq 2.038M$ ($\epsilon < 0.019$) has a "clean" photon sphere, and therefore is called the clean photon sphere object (ClePhO).

The quantity of proposed UCO models is huge and ever-growing. Famous examples of UCOs are “gravitational-vacuum stars” or gravastars [68, 69]. These are configurations supported by a negative pressure, which might arise as an hydrodynamical description of one-loop QFT effects in curved spacetime, so they do not necessarily require exotic new physics. A more ambitious model of UCO

aim to replace BH completely is the fuzzball in string theory, which provides a quantum description of the horizon [70, 71]. Another known UCO model is wormhole [72, 73], which can be constructed by cut-and-paste method (thin-shell wormhole) , or can be obtained by adding exotic matter fields in GR, or may arise from modified gravity theories.

Although the formation mechanisms of UCOs vary from each other vastly, the UCOs share a common phenomenology. And we will give a more concrete description in the next two sections.

2.1.2 Photon spheres and shadows

For simplicity, we start with a four-dimensional spherically symmetric object and assume that its exterior is described by vacuum GR. Birkhoff's theorem guarantees that any vacuum spherically symmetric spacetime is described by the Schwarzschild geometry

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad f(r) = 1 - \frac{2M}{r}, \quad (2.1)$$

and M is the total mass of the spacetime.

The null geodesic equation in this geometry can be obtained with the help of two conserved quantities, the energy $E = f\dot{t}$ and angular momentum $L = r^2\dot{\phi}$, where a dot denotes a derivative with respect to proper time. Then the radial equation of motion is

$$\dot{r}^2 + V_{eff} = \frac{1}{b^2}, \quad V_{eff} = \frac{f}{r^2}, \quad (2.2)$$

where $b = \frac{L}{E}$ is called the impact parameter. In the Schwarzschild geometry, the effective potential V_{eff} has one peak located at $r = 3M$ (determined by $V'_{eff}(r) = 0$) where a circular null geodesic is possible. This radius defines a surface called the photon sphere, or on an equatorial plane, a light ring. The photon sphere has a number of interesting properties, and is useful to understand certain features of compact objects. Due to Eq.(2.2), the critical impact parameter which give

Figure 2.2: The top plot is the effective potential of radial null geodesic in Schwarzschild BH. The bottom plot is the photon trajectories (photons are coming from far right) in Schwarzschild BH in the Euclidean polar coordinates (r, ϕ) with different impact parameters $b > b_c$ (black), $b = b_c$ (red), $b < b_c$ (blue). The black hole is shown as a solid disk.

circular null geodesic is given by [76]

$$b_c = \frac{1}{\sqrt{V_{\text{eff}}(3M)}} = 3\sqrt{3}M. \quad (2.3)$$

We can see explicitly from Fig.2.2 that when impact parameter $b > b_c$, the photons move toward the black hole from infinity approaching one closest point, and then move away from the black hole back to infinity. When impact parameter $b < b_c$, the photons always fall into the black hole. When $b = b_c$, the photons will revolve around the black hole at radius $r = 3M$, i.e. the photon sphere. Thus we have a dark area called shadow characterized by the critical impact parameter $b_c = 3\sqrt{3}M$. Naturally the necessary condition for an UCO as a BH mimicker is that it features the same shadow. In other words, the radius of UCOs can't shade the photon sphere which corresponds to compactness parameter $\epsilon < 1/2$. It should be pointed out that neutron stars or boson stars do not belong to UCOs, since in general they are lowly compact.

The most prominent distinction between UCO and BH is that UCO as regular body, the metric function f is well behaved in its interior, never changes sign nor diverges. Thus the effective potential for radial null geodesic exhibits an additional potential barrier inside the photon sphere, which is originated from a partial reflective surface, a centrifugal barrier at the center, or by the geometry. See Figure.2.3 for the effective potential well of gravastar as an example. So photons with impact parameter $b < b_c$ are possibly bounced back by the interior potential barrier. This reflectivity of photons for UCOs provides important optically observational signatures to distinguish from BHs.

Figure 2.3: The effective potential of radial null geodesic in a gravastar.

2.1.3 Quasi-normal modes, ringdown and echos

Gravitational wave emitted from a compact binary coalescence can be divided into three stages: inspiral, merge and ringdown. The ringdown is the final stage when the remnant relaxes to an equilibrium solution and is characterized by its complex eigen frequencies, the so-called quasi-normal modes (QNMs) which describe the response of the compact object to a perturbation [79, 80].

For a static UCO whose exterior is described by vacuum GR. After separation of the variables, perturbations are governed by a Schrodinger-like equation² [80]

$$\frac{d^2\Psi}{dr_*^2} + [\omega^2 - V]\Psi = 0, \quad (2.4)$$

where r_* is the tortoise coordinate defined by $dr_* = dr/f$, i.e.

$$r_* = r + 2M \log\left(\frac{r}{2M} - 1\right). \quad (2.5)$$

For matter field perturbation with spin- s , the effective potential is [80]

$$V = f \left(\frac{l(l+1)}{r^2} + (1-s^2)\frac{2M}{r^3} \right), \quad (2.6)$$

where l is the so called multiple number coming from the angular separation. For axial gravitational perturbation, the effective potential is the so called Regge-Wheeler potential [126]

$$V_{\text{axial}} = f \left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3} \right). \quad (2.7)$$

For polar gravitational perturbation, the effective potential is the so called Zerilli potential [127]

$$V_{\text{polar}} = 2f \left(\frac{\lambda^2(\lambda+1)r^3 + 3\lambda^2Mr^2 + 9M^2(\lambda r + M)}{r^3(\lambda r + 3M)^2} \right), \quad (2.8)$$

where $\lambda = (l-1)(l+2)/2$. The effective potential of axial gravitational perturba-

²See next section for derivation of the perturbation equation.

tion has similar profile to those of matter field perturbations, and can be viewed as spin-2 tensor field perturbation. They own a peak approximately at the photon sphere $r \approx 3M$, which is the unstable circular orbit of photons around the compact object. In the UCO case, the absence of the event horizon at $r_* \rightarrow -\infty$ implies the existence of a potential well between the UCO surface $2M(1+\epsilon)$ and the photon sphere $3M$. The potential well can support long-lived semi-trapped modes which are responsible not only for a completely different QNM spectrum with respect to the BH case, but also for the gravitational wave echoes.

Specifically, QNMs are complex eigenvalues of perturbation equation (2.4) with specific boundary conditions. For an isolate system, the perturbation is purely outgoing at infinity,

$$\Psi \sim e^{i\omega r_*}, \quad \text{as } r_* \rightarrow +\infty. \quad (2.9)$$

In the BH case, the event horizon would require that the perturbation is purely ingoing,

$$\Psi \sim e^{-i\omega r_*}, \quad \text{as } r_* \rightarrow -\infty. \quad (2.10)$$

In the UCO case, the absence of horizon and the existence of partial reflective surface make the choosing of boundary condition at the radius r_0 of the UCO totally different from the BH case. The solution is generally a superposition of ingoing and outgoing waves,

$$\Psi \sim C_{in} e^{-i\omega r_*} + C_{out} e^{i\omega r_*}, \quad \text{as } r_* \rightarrow r_*^0, \quad (2.11)$$

where $r_*^0 \equiv r_*(r_0)$ and $r_0 = 2M(1+\epsilon)$ is the radius of the UCO. The surface reflectivity of the UCO is then defined as [67]

$$\mathcal{R} = \frac{C_{out}}{C_{in}} e^{2i\omega r_*^0}. \quad (2.12)$$

A perfectly reflecting UCO has $|\mathcal{R}|^2 = 1$, whereas a totally absorbing compact object has $|\mathcal{R}|^2 = 0$ as in the BH case.

For a BH, a pulse crossing the photon sphere, a fraction of which travels to outside observers gives the ringdown signal, which is to a very good level described

Figure 2.4: The effective potential for axial gravitational perturbation ($l = 2$) of a Schwarzschild BH (top panel) and an UCO with $\epsilon = 10^{-6}$ (bottom panel) [66, 67]. The effective potentials have a peak approximately at the photon sphere, $r \approx 3M$. In the UCO case, as we already mentioned in the last subsection, the effective potential features a well between the surface of the UCO and the photon sphere.

by the lowest QNMs. The other fraction travels into the horizon and dies off, so it has no effect for outside observers. For a UCO, the pulse traveling inwards is reflected either at its surface or at its center. The reflected pulse crossing the

photon sphere, a fraction of which transmits to outside observers and the other fraction is reflected back to the object. Thus this pulse is semi-trapped between the object and the photon sphere. The repeated reflections and transmissions gives rise to a series of echoes of ever-decreasing amplitude, with characteristic echo delay time [66]

$$\tau_{echo} \sim 4M |\log \epsilon|. \quad (2.13)$$

From the detection of the ringdown and echo it is possible to infer the QNMs of the remnant and distinguish UCOs from BHs. Due to the GR uniqueness theorem, the QNM spectrum of Kerr BHs depends uniquely on two parameters, i.e., their mass and angular momentum. While the QNM spectrum of UCOs depend also on the properties of themselves, in particular their compactness and reflectivity.

2.2 Perturbations and master wave equations

A stationary isolated black hole is an extremely simple object which can be described by only a few parameters such as mass, angular momentum, and charge due to no-hair theory. But in practice, a real black hole is always dynamic, such as interacting with surroundings, even evaporating due to Hawking radiation. When we want to study about stability, gravitational wave, tidal effect, and Hawking evaporation of black holes, we have to start from an analysis of their perturbations.

2.2.1 Matter fields perturbation

In general, perturbations of a black hole spacetime can be performed in two ways: by adding fields (scalar, electromagnetism, Dirac fields etc.) to the black hole spacetime or by perturbing the black hole metric (the gravitational field) itself. In the linear approximation, i.e., when a field does not backreact on the background, the first type of perturbation is reduced to the propagation of fields in the background of a black hole, which is, in many cases, a general covariant equation of motion of the corresponding field.

On the other hand, the perturbation analysis of rotational black holes is very tedious, and has no universal method. I will give a brief review of perturbation analysis of general spherically symmetric black hole in n dimensions, whose metric takes the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n-2}^2, \quad (2.14)$$

where $d\Omega_{n-2}^2 = \gamma_{ij}d\theta^i d\theta^j$ is the metric of $(n-2)$ dimensional sphere $\mathbb{S}^{n-2}$.

First we take a massless scalar field as an example to derive the master perturbation equation. The covariant equation of motion for massless scalar field in curved spacetime is given by Klein-Gordon equation

$$\square\phi = 0. \quad (2.15)$$

According to the spherical symmetry of background spacetime, we can decompose the scalar wave function as

$$\phi(t, r, \theta, \phi) = e^{-i\omega t} Y_l(\theta) R(r)/r, \quad (2.16)$$

where $Y_l(\theta) = Y_{l0}(\theta, \phi)$ are the spherical harmonics with $m = 0$. After introducing the tortoise coordinate r_* which can map the horizon to negative infinity,

$$dr_* = \frac{dr}{f}. \quad (2.17)$$

By submitting (2.16) into (2.15), we can put the radial wave equation for $R(r)$ into a Schrodinger-like form

$$\frac{d^2 R}{dr_*^2} + [\omega^2 - V(r)]R = \omega^2 R \quad (2.18)$$

with potential [79]

$$V(r) = f \frac{l(l+n-3)}{r^2} + f f' \frac{n-2}{2r} + f^2 \frac{(n-2)(n-4)}{4r^2}, \quad (2.19)$$

where l is the so called multipole number. Note that the potential depends only

on l and not on m due to the spherical symmetry of the background. Though the covariant form of the equation of motion is quite different for fields with different spin in curved backgrounds, the effective potentials of master Schrodinger-like wave equations have very similar form. For $D = 4$ Schwarzschild black hole, they take the form as (2.6), in which $s = 0$ for scalar field, $s = 1/2$ for spinor field and $s = 1$ for electromagnetic field.

2.2.2 Gravational perturbation

Now we focus on the gravitational perturbations of spherically symmetric black holes in the framework of general relativity in details. A gravitational perturbation is described by a small deviation from the background metric, i.e.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}. \quad (2.20)$$

For convenience, we denote $h_{\mu\nu} = \delta g_{\mu\nu}$. From this variation we can easily compute the variation of the affine connections

$$\delta \Gamma_{\mu\nu}^{\rho} = \frac{1}{2}(\bar{\nabla}_{\mu} h_{\nu}^{\rho} + \bar{\nabla}_{\nu} h_{\mu}^{\rho} - \bar{\nabla}^{\rho} h_{\mu\nu}). \quad (2.21)$$

Next we can get the variation of the Riemann tensor

$$\delta R^{\rho}{}_{\sigma\mu\nu} = \bar{\nabla}_{\mu} \delta \Gamma_{\nu\sigma}^{\rho} - \bar{\nabla}_{\nu} \delta \Gamma_{\mu\sigma}^{\rho}. \quad (2.22)$$

The vacuum Einstein equation is

$$R_{\mu\nu} = 0. \quad (2.23)$$

Thus the perturbed equation is

$$\delta R_{\mu\nu} = 0. \quad (2.24)$$

Scalar-Vector-Tensor (SVT) decomposition of metric perturbations

In general the perturbation tensor $h_{\mu\nu}$ has $n(n+1)/2$ components. The main technical challenge for analysing gravitational perturbations is the separation of variables in a set of coupled equations with respect to the components of $h_{\mu\nu}$. Thanks to the $SO(n-1)$ rotational symmetry of background spacetime (2.14), the separation of variables is possible with the help of the following tensorial decomposition of $h_{\mu\nu}$ [128, 128, 129].

According to the foliation of background spacetime (2.14), we can split the metric perturbation $h_{\mu\nu}$ as

$$h_{ab}, \quad h_{ai}, \quad h_{ij}, \quad (2.25)$$

where indices a, b represent the time coordinate t and radial coordinate r , and indices i, j represent coordinates of $\mathbb{S}^{n-2}$. Furthermore, the h_{ai} can be decomposed as

$$h_{ai} = D_i \Psi_a + \hat{h}_{ai}, \quad D^i \hat{h}_{ai} = 0, \quad (2.26)$$

where D is the covariant derivative with respect to the metric δ_{ij} on $\mathbb{S}^{n-2}$. The h_{ij} can be decomposed as

$$h_{ij} = \hat{h}_{ij} + D_i \hat{\Psi}_j + D_j \hat{\Psi}_i + \left(D_i D_j - \frac{1}{n-2} \delta_{ij} \Delta \right) \Psi + \Phi \delta_{ij}, \quad (2.27)$$

$$D^i \hat{\Psi}_i = 0, \quad D^i \hat{h}_{ij} = 0, \quad \delta^{ij} \hat{h}_{ij} = 0, \quad (2.28)$$

where $\Delta = \delta^{ij} D_i D_j$ is the Laplacian for $\mathbb{S}^{n-2}$. Hatted quantities are divergenceless or transverse. Eventually we can classify all variables into three groups according to their transformation law under the $SO(n-1)$ rotational symmetry of $\mathbb{S}^{n-2}$:

- scalar perturbations $h_{ab}, \Psi_a, \Psi, \Phi$
- vector perturbations $\hat{h}_{ai}, \hat{\Psi}_i$
- tensor perturbation $\hat{h}_{ij}$

It is convenient to expand the scalar perturbations by scalar spherical harmonics Y as

$$h_{tt} = f(r)H_0(t, r)Y, \quad h_{tr} = H_1(t, r)Y, \quad h_{rr} = f^{-1}(r)H_2(t, r)Y, \quad (2.29)$$

$$\Psi_t = \mathcal{H}_0(t, r)Y, \quad \Psi_r = \mathcal{H}_1(t, r)Y, \quad \Psi = r^2\mathcal{G}(t, r)Y, \quad \Phi = r^2K(t, r)Y. \quad (2.30)$$

Now the scalar perturbations are determined by the expansion coefficients

$$H_0, \quad H_1, \quad H_2, \quad \mathcal{H}_0, \quad \mathcal{H}_1, \quad \mathcal{G}, \quad K. \quad (2.31)$$

Similarly, we can expand the vector perturbations by transverse vector spherical harmonics Y_i as

$$\hat{h}_{ti} = h_0(t, r)Y_i, \quad \hat{h}_{ri} = h_1(t, r)Y_i, \quad \hat{\Psi}_i = r^2h_2(t, r)Y_i, \quad (2.32)$$

Then the vector perturbations are determined by the expansion coefficients

$$h_0, \quad h_1, \quad h_2, \quad (2.33)$$

We can expand the tensor perturbation by transverse and traceless tensor spherical harmonics Y_{ij} as

$$\hat{h}_{ij} = r^2h_T(t, r)Y_{ij}. \quad (2.34)$$

Then the tensor perturbations are determined by a single variable

$$h_T. \quad (2.35)$$

For detailed introduction of scalar, vector and tensor spherical harmonics, one may refer to the appendix A in [130]. Finally, the perturbation equations split into three decoupled subsets each of which contains only variables belonging to one of the above three types of perturbation. Note that it is important not to confuse the scalar type of gravitational perturbations (sometimes simply called "scalar perturbations") with perturbation of adding the test scalar field, and the vector type of gravitational perturbations (sometimes simply called "vector

perturbations") with perturbation of adding the test vector field.

Gauge freedom

Note that the perturbed Einstein equation (2.2.2) are invariant under the gauge transformations

$$x^\mu \rightarrow x^\mu + \xi^\mu, \quad \delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu. \quad (2.36)$$

In order to extract the dynamics of the physical degrees of freedom from the perturbed equations, the gauge freedom must be eliminated through fixing gauge or constructing gauge invariant variables. The gauge invariant variables can be constructed from the harmonic expansion coefficients (2.31), (2.33), (2.35) (the tensor perturbation h_T is already gauge invariant).

Usually, it is convenient to fix a gauge to eliminate gauge freedom. Conventionally, we choose the Regge-Wheeler gauge [126]

$$\Psi_a = 0, \quad \Psi = 0, \quad \hat{\Psi}_i = 0. \quad (2.37)$$

In this gauge, the SVT decomposition of metric perturbations becomes

$$h_{ai} = \hat{h}_{ai}, \quad D^i \hat{h}_{ai} = 0, \quad (2.38)$$

$$h_{ij} = \hat{h}_{ij} + \Phi \gamma_{ij}, \quad D^i \hat{h}_{ij} = 0, \quad \gamma^{ij} \hat{h}_{ij} = 0. \quad (2.39)$$

Now the residual physical degrees of freedom are

- scalar perturbations h_{ab}, Φ
- vector perturbations $\hat{h}_{ai}$
- tensor perturbation $\hat{h}_{ij}$

Correspondingly, in this gauge the harmonic expansion coefficients

$$\mathcal{H}_0 = \mathcal{H}_1 = \mathcal{G} = h_2 = 0. \quad (2.40)$$

We are left with

- scalar perturbations H_0, H_1, H_2, K
- vector perturbations h_0, h_1
- tensor perturbation h_T

2.2.3 Regge-Wheeler and Zerilli equations

In four dimension, $\hat{h}_{ij}$ has three independent components. However due to SVT decomposition (See (2.29)), $\hat{h}_{ij}$ is transverse and traceless, i.e.

$$D^i \hat{h}_{ij} = 0 = \hat{h}_i^i, \quad (2.41)$$

which means we can impose three constraint equations. So the tensor perturbations are not dynamical and can be gauged off in $D = 4$. The scalar type of gravitational perturbation is also called "even/polar/electric" perturbation which is responsible for deformation of the black hole horizon. The vector type of gravitational perturbation is also called "odd/axial/magnetic" perturbation which is connected with small rotations of the black hole. In the following, we present the explicit master perturbation equations for $D = 4$ Schwarzschild black hole.

odd/axial/magnetic perturbation

The odd perturbation metric in Regge-Wheeler gauge takes the form [126]

$$h_{\mu\nu}^{odd} = e^{i\omega t} \begin{pmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ Sym & Sym & 0 & 0 \end{pmatrix} \sin \theta \partial_\theta P_l(\cos \theta). \quad (2.42)$$

Note that $h_0(r), h_1(r)$ are the vector type of perturbations as explained in (2.32). Since the radial equations are independent of the index m in spherical harmonics $Y_{\ell m}$, we have set $m = 0$ here and in the following. We can define a master variable

$$\Psi_- = \left(1 - \frac{2M}{r}\right) \frac{h_1}{r}. \quad (2.43)$$

By substituting the odd perturbation metric into the perturbation equation Eq.(2.2.2), it is easy to obtain the Regge-Wheeler equation in the tortoise coordinate

$$\frac{d^2 \Psi_-}{dr_*^2} + [\omega^2 - V_-] \Psi_- = 0 \quad (2.44)$$

with the Regge-Wheeler potential V_- given by Eq.(2.7). The variable h_0 can be solved using Ψ_- . We can see from Eq.(2.6) that the vector type of gravitational perturbation is equivalent to adding the test spin-2 field.

even/polar/electric perturbation

The even perturbation metric in Regge-Wheeler gauge takes the form [127]

$$h_{\mu\nu}^{even} = e^{i\omega t} \begin{pmatrix} H_0(1 - 2M/r) & H_1 & 0 & 0 \\ H_1 & H_2(1 - 2M/r)^{-1} & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} P_l(\cos \theta) \quad (2.45)$$

where H_0, H_1, H_2, K are the scalar type of perturbations as illustrated in (2.31). We can define a master variable

$$\Psi_+ = u(r) \frac{H_1}{i\omega} + v(r) K. \quad (2.46)$$

It needs few steps to obtain the Zerilli equation in the tortoise coordinate

$$\frac{d^2 \Psi_+}{dr_*^2} + [\omega^2 - V_+] \Psi_+ = 0 \quad (2.47)$$

with the Zerilli potential V_+ given by Eq.(2.8). The other variables can be solved using Ψ_+ .

2.3 Discussion

As BH's mimicker and substitute, UCOs' exterior geometry of spacetime can be identical to the ones of corresponding BHs up to their surface. Thus it is important to distinguish UCOs from BHs in both theoretical and observational aspects. The release of the direct image of the core of the M87 galaxy by the EHT collaboration makes it possible to probe GR and BH with shadows. Since the exterior spacetime may be identical, an UCO and a BH potentially cast the same shadows. However when they are illuminated by the accretion disk around them, there will be "photon rings" due to photons that orbit the UCO or BH. The reflectivity of photons on the surface of UCOs gives more abundant signatures of "photon rings" with respect to the case of BHs. This is the subject in next chapter.

Photon ring tests of gravity is somewhat limited, because it is based on the geodesic motion of photons in curved spacetime, and therefore it only involves the geometry rather than the dynamical properties of the gravity in contrast to the observation of GW. Theoretical analysis shows that the ringdown phase of binary UCOs coalescence features echoes effect due to reflectivity of wave on the surface. The ringdown signal is characterized by QNMs which are complex eigenfrequencies of perturbation equations under specific boundary conditions. A remarkable feature of the spectrum of BH QNMs in GR is that it respects the no-hair and unique theorem, i.e. the spectrum is completely determined by the mass, charge and spin of the BH, whatever the type of perturbation. Another important feature of QNMs in GR is the isospectrality which says the gravitational perturbations of the axial and the polar modes for most known BHs share an identical spectrum, even though their master equations are completely different from each other [131]. All these features will break down for UCOs, so we can distinguish them by comparing the QNM spectra in principle.

Chapter 3

Photon rings of asymmetric thin-shell wormhole

Recently, a distinct shadow mechanism was proposed by Wang et al. from the asymmetric thin-shell wormhole (ATW) in the paper [132]. On the other hand, Gralla et al.'s work represented a nice description of photon rings in the presence of an accretion disk around a black hole [76]. In this Chapter, we are inspired to thoroughly investigate the observational appearance of accretion disk around the ATW. In Section 3.1, we first give detailed illustrations about the concepts of photon rings and lensing rings proposed by Gralla et al. in the paper [76]. Moreover, through studying the observational appearance of the accretion disk around a black hole, we can achieve a further understanding of the photon rings and the lensing rings from the observational point of view. In Section 3.2, we take a ATW as a toy model to capture the optical properties of UCOs. We generalized the same procedure introduced in the previous section to see the observational appearance of the ATW in the presence of the accretion disk. This Chapter is mainly based on my papers **P3** and **P4**.

3.1 Photon rings of black hole

The first black hole image of M87* released by the Event Horizon Telescope plays a significant role in the frontier of general relativity [10]. It provides a direct and powerful observational information for general relativity in the strong gravitational regime. As a result the research of black hole shadow becomes very popular thereafter [76, 133–155].

As we know shadow is a kind of darker area which is formed when an opaque body imprisons the transmit of light. By definition, the shape of the shadow would depend on the location and intensity of the light source. The same is true for black hole shadows. For the standard usage of the term "shadow", the light source is thought to be distributed throughout the entire space desultorily, including from behind the observer. In this situation, the shadow radius is determined by the critical impact factor which gives the photon sphere. But if a particular light source (like accretion disk) near the black hole is so bright that the natural light sources can be ignored, the appearance of a black hole cannot described by the standard "shadow". Recently, Gralla et al. presented an elegant description of observational appearance of black hole when considering emission disk around black hole [76]. To avoid ambiguity, the standard "shadow" is suggested to be called the "critical curve" [76]. In this section, we give a brief review of Gralla et al's analysis about black hole shadows, lensing rings and photon rings.

3.1.1 Light bending: diret, lensed and photon ring

The orbit of one particle traveling in curved spacetime is described by geodesic equations which can be expressed as the Euler-Lagrange equation

$$\frac{d}{d\tau} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} \right) = \frac{\partial \mathcal{L}}{\partial x^\mu}, \quad (3.1)$$

where τ is the proper time, $\dot{x}^\mu$ is the four-velocity of particle and $\mathcal{L}$ is the Lagrangian. For the spherically symmetric line element

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3.2)$$

the Lagrangian is

$$\mathcal{L} = \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \frac{1}{2}\left(-f\dot{t}^2 + \frac{\dot{r}^2}{f} + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right). \quad (3.3)$$

For static and spherically symmetric metric, we can always restrict the particle moving on the equatorial plane, i.e. $\theta = \frac{\pi}{2}$ and $\dot{\theta} = 0$. In addition, the Lagrangian $\mathcal{L}$ doesn't involve coordinates t and ϕ explicitly, so we have two conserved quantities

$$E = -\frac{\partial\mathcal{L}}{\partial\dot{t}} = f\dot{t}, \quad L = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = r^2\dot{\phi} \quad (3.4)$$

as the respective energy and angular momentum of one particle travelling around the black hole. For null geodesic $\mathcal{L} = 0$, then we can obtain the orbit equation

$$\left(\frac{dr}{d\phi}\right)^2 + V_{eff} = 0 \quad (3.5)$$

with the effective potential given by

$$V_{eff} = -r^4\left(\frac{1}{b^2} - \frac{f(r)}{r^2}\right), \quad (3.6)$$

where $b = \frac{L}{E}$ is called the impact parameter. The circular orbit corresponds to

$$V_{eff} = 0, \quad V'_{eff} = 0. \quad (3.7)$$

Because of the spherical symmetry, all circular orbits form a closed surface which is the so called photon sphere whose radius is determined by

$$\left(\frac{f(r_{ph})}{r_{ph}^2}\right)' = 0 \quad (3.8)$$

and the corresponding impact parameter b_c gives the standard shadow radius

$$b_c = \frac{r_{ph}}{\sqrt{f(r_{ph})}}. \quad (3.9)$$

Gralla et al's proposal is based on the classification of null geodesic in black

Figure 3.1: Behavior of photons in Schwarzschild black hole as a function of impact parameter b . The top plot is the total number of orbits $n = \phi/2\pi$. The colors correspond to $n < 3/4$ (black), $3/4 < n < 5/4$ (yellow), and $n > 5/4$ (red), defined as the direct, lensed, and photon ring trajectories, respectively. The bottom plot is a selection of associated photon trajectories in the Euclidean polar coordinates (r, ϕ) . The spacing in impact parameter is $1/10$, $1/100$, and $1/1000$ in the direct (black), lensed (yellow), and photon ring (red) bands, respectively. The black hole is shown as a solid disk.

holes. Hence, let's start to investigate the trajectory of a light ray traveling around the black hole. It is convenient to make a transformation $u = 1/r$. The orbit equation now becomes

$$\left(\frac{du}{d\phi}\right)^2 = G(u), \quad (3.10)$$

where

$$G(u) = \frac{1}{b^2} + 2Mu^3 - u^2 \quad (3.11)$$

for Schwarzschild black hole. When impact parameter $b > b_c$, the light ray move toward the black hole from infinity approaching one closest point, and then move away from the black hole back to infinity. When impact parameter $b < b_c$, the light ray always falls into the black hole. When $b = b_c$, the light ray will revolve around the black hole at radius r_{ph} , i.e. the photon sphere.

For $b > b_c$, the turning point corresponds to the minimally positive real root of $G(u) = 0$, which we will denote by u_m . According to (3.10), the total change of azimuthal angle ϕ for certain trajectory with impact parameter b can be calculated by

$$\phi = 2 \int_0^{u_m} \frac{du}{\sqrt{G(u)}}, \quad b > b_c. \quad (3.12)$$

For $b < b_c$, we only focus on the trajectory outside the horizon, so the total change of azimuthal angle ϕ is obtained by

$$\phi = \int_0^{u_0} \frac{du}{\sqrt{G(u)}}, \quad b < b_c, \quad (3.13)$$

where $u_0 = 1/r_0$.

In order to discuss the observational appearance of emission originating near a black hole, The authors in [76] divide trajectories into direct, lensed and photon ring ones. Now we give a brief introduction. One can define the total number of orbits $n = \frac{\phi}{2\pi}$ which is obviously a function of impact parameter b . We denote the solution of

$$n(b) = \frac{2m-1}{4}, \quad m = 1, 2, 3, \dots \quad (3.14)$$

by $b_m^\pm$. Note that $b_m^- < b_c$ and $b_m^+ > b_c$. Then we can classify all trajectories as follows:

- direct: $\frac{1}{4} < n < \frac{3}{4} \Rightarrow b \in (b_1^-, b_2^-) \cup (b_2^+, \infty)$
- lensed: $\frac{3}{4} < n < \frac{5}{4} \Rightarrow b \in (b_2^-, b_3^-) \cup (b_3^+, b_2^+)$
- photon ring: $n > \frac{5}{4} \Rightarrow b \in (b_3^-, b_3^+)$

The physical picture of this classification is clear from the trajectory plots in Fig.3.1. Assuming light rays emit from north pole direction (far right of the trajectory plots), trajectories whose number of orbits $1/4 < n < 3/4$ will intersect the equatorial plane only once. Trajectories whose number of orbits $3/4 < n < 5/4$ will intersect the equatorial plane twice. Trajectories whose number of orbits $n > 5/4$ will intersect the equatorial plane at least 3 times.

3.1.2 Observational appearance of thin disk emission

Usually the term "shadow" describes the appearance of a black hole illuminated from all directions, so the shadow radius is given by critical impact parameter b_c . In practice, the emission is always accumulated in certain finite region near the black hole such as accretion disk. Armed with the previous preparations, now we are ready to consider a concrete accretion disk around the Schwarzschild black hole on the equatorial plane. There are many models in accretion disk theory, however, we focus on the optically and geometrically thin disks for simplicity.

A static observer is assumed to locate at the north pole, and the lights emitted from the accretion disk is considered isotropic in the rest frame of the static observer. In view of the spherical symmetry of the spacetime, we also suppose the emitted specific intensity only depends on the radial coordinate, denoted by $I_\nu^{\text{em}}(r)$ with emission frequency ν in a static frame. An observer in infinity will receive the specific intensity $I_{\nu'}^{\text{obs}}$ with redshifted frequency $\nu' = \sqrt{f}\nu$. Considering I_ν/ν^3 is conserved along a ray, i.e.

$$\frac{I_{\nu'}^{\text{obs}}}{\nu'^3} = \frac{I_\nu^{\text{em}}}{\nu^3}, \quad (3.15)$$

we have the observed specific intensity

$$I_{\nu'}^{\text{obs}} = f^{3/2}(r) I_{\nu}^{\text{em}}(r). \quad (3.16)$$

So the total observed intensity is an integral over all frequencies

$$I^{\text{obs}} = \int I_{\nu'}^{\text{obs}} d\nu' = \int f^2 I_{\nu}^{\text{em}} d\nu = f^2(r) I^{\text{em}}(r), \quad (3.17)$$

where $I^{\text{em}} = \int I_{\nu}^{\text{em}} d\nu$ is the total emitted intensity from the accretion disk.

In addition, the intensity of the lights emitted from the accretion disk is so big that other sources in the environment can be ignored. If a light ray from the observer intersects with the emission disk, it means the intersecting point as a light source will contribute brightness to the observer. As discussed in section 3.1.1, a light ray whose number of orbits $n > 1/4$ will intersect with the disk on the front side. If n goes larger than $3/4$, the light ray will bend around the black hole, intersecting with the disk for the second time on the back side. Further, when $n > 5/4$, the light ray will intersect with the disk for the third time on the front side again, and so on. Hence, the observed intensity is a sum of the intensities from each intersection,

$$I^{\text{obs}}(b) = \sum_m f^2 I^{\text{em}}|_{r=r_m(b)}, \quad (3.18)$$

where $r_m(b)$ is the so called transfer function which denotes the radial position of the m -th intersection with the emission disk. What needs to be emphasized is that we do not consider the absorption and reflection of light by accretion disks and the loss of light intensity in the environment in our process which is just an ideal model.

We denote the solution of orbit equation (3.10) by $u(\phi, b)$. We will focus on

Figure 3.2: The first three transfer functions in Schwarzschild black hole. They represent the radial coordinate of the first (black), second (yellow), and third (red) intersections with the emission disk.

the first three transfer functions which can be obtained by

$$\begin{aligned} r_1(b) &= \frac{1}{u(\frac{\pi}{2}, b)}, & b \in (b_1^-, \infty), \\ r_2(b) &= \frac{1}{u(\frac{3\pi}{2}, b)}, & b \in (b_2^-, b_2^+), \\ r_3(b) &= \frac{1}{u(\frac{5\pi}{2}, b)}, & b \in (b_3^-, b_3^+), \end{aligned} \quad (3.19)$$

and plot the results in Fig.3.2. As illustrated in [76], the first transfer function gives the "direct image" of the disk which is essentially just the redshift of source profile. The second transfer function gives a highly demagnified image of the back side of the disk, referred to "lensing ring". The third transfer function gives an extremely demagnified image of the front side of the disk, referred to "photon ring". The images resulted from further transfer functions are so demagnified that they can be neglected. The demagnified scale is determined by the slope of transfer function, dr/db , called the demagnification factor.

Having obtained the transfer function, we can consider concrete emission profile. Firstly, we consider the emission is sharply peaked at the innermost stable

circular orbit, and it ends abruptly at $r = r_{isco}$, such as

$$I_1^{\text{em}}(r) = \begin{cases} I^0 e^{-(r-r_{isco})}, & r > r_{isco}, \\ 0, & r < r_{isco}. \end{cases} \quad (3.20)$$

Secondly, we consider the emission is sharply peaked at the photon sphere, and it ends abruptly at $r = r_c$ while decays fast to zero at $r = r_{isco}$, such as

$$I_2^{\text{em}}(r) = \begin{cases} I^0 \frac{2 - \tanh(r-r_{ph})}{2} e^{-(r-r_{ph})}, & r > r_{ph}, \\ 0, & r < r_{ph}. \end{cases} \quad (3.21)$$

Finally, we consider a emission decaying gradually from the horizon to the ISCO, such as

$$I_3^{\text{em}}(r) = \begin{cases} I^0 \frac{-\tanh(r-4.5)+1}{-\tanh(r_0-4.5)+1}, & r > r_0, \\ 0, & r < r_0. \end{cases} \quad (3.22)$$

We can get the observed intensities $I^{\text{obs}}(b)$ according to (3.18), and plot the results in Fig.3.3. Although Fig. 3.3 shows only a few highly idealized cases of thin disk emission near a Schwarzschild black hole (viewed face on), it illustrates two key points that we believe will hold quite generally for optically thin disk emission: (1) The emission is dominated by the direct emission, with the lensing ring emission providing only a small contribution to the total flux and the photon ring providing a negligible contribution in all cases. (2) Although the photon ring always occurs near $b \sim b_c$ and the lensing ring always occurs somewhat outside this radius, the size of the dark central area is very much dependent on the emission model. Although the black holes in the right column of Fig. 3.3 are the same size, the dark central areas are very different in size.

3.2 Photon rings of asymmetric thin-shell wormhole

As discussed in Chapter 2, with the help of the image of M87* we are still not enough to assert that the supermassive object in the center of M87* must be a

Figure 3.3: Observational appearance of a geometrically and optically thin disk with different emission profiles near Schwarzschild black hole. The left column are the plots of various emission profiles $I^{\text{em}}(r)$. The middle column are the corresponding observed intensities I^{obs} as a function of impact parameter b . The right column are the density plots of observed intensities $I^{\text{obs}}(b)$.

black hole. This is because black holes are not the only ones that have photon spheres which have the ability to shade the horizon. It has been found some ultra compact objects (UCOs) also own photon spheres [156, 157]. Furthermore, some of them are found to mimic the optical appearances of black holes including shadows [158–160]. Thus it is crucial to distinguish black holes from UCOs whose external spacetime geometries are similar or even identical to those of black holes up to the vicinity of horizon, such as gravastar, wormhole and fuzzball and so on. In fact one may distinguish black holes from some UCOs in their acoustic properties, such as echo effect [66, 161–165]. Interestingly, in Ref.[166] it was

performed a data analysis and claimed to be found statistical evidence for late time echoes in some of the LIGO events [167, 168]. However, such a result is still under debate [169] and more progress is still needed. Another observational difference between black holes and wormholes is explored in [170, 171]. Roughly speaking, the essential distinction between black holes and UCOs is that the event horizon of black holes is a one-way membrane while UCOs have no horizons. In this sense, black holes can be regarded as absolute black objects with zero reflection, while UCOs have considerable reflection.

Recently the shadow of an asymmetric thin-shell wormhole (ATW) has been studied in [132]. The authors showed that due to reflection of photons by the wormhole there exist a novel shadow which is different from that of black hole in certain parameters space. Then more examples of asymmetric thin-shell wormhole are studied in [172–174], of which the authors named their results as double shadows [174]. It's worth noting that double shadows should be called double critical curves or photon spheres in the context of our article. On the other hand, the novel shadow proposed in [132] does refer to the dark area, however, its edge is still related to the photon sphere when the throat of the ATW is inside the photon sphere, but the photon sphere here is the one which is located in the other side opposite the observers other than the usual one in the observers' side for an ATW spacetime. So far there's no discussion on the observational appearance of emission disk in ATW spacetime although no matter novel shadows or double shadows are improper terminologies in practical observation. In this section, we give a completed picture of observational appearance of accretion disk around an ATW, which can be regarded as a toy model to capture the optical properties of UCOs.

3.2.1 Null geodesic

In this subsection, we give a brief review of the asymmetric thin-shell wormhole model presented in [132]. We consider a thin-shell wormhole using cut-and-paste method, that is, two distinct spacetimes with different parameters are glued by

a thin shell

$$ds_i^2 = -f_i(r_i)dt_i^2 + \frac{dr_i^2}{f_i(r_i)} + r_i^2 d\Omega^2, \quad (3.23)$$

where $i = 1, 2$, and by focusing on the Schwarzschild case we have

$$f_i(r_i) = 1 - \frac{2M_i}{r_i}, \quad r \geq R, \quad (3.24)$$

where M_i are the mass parameters, and R is the position of thin-shell, i.e. the radius of throat. Thus we have

$$R > \max\{2M_1, 2M_2\}, \quad (3.25)$$

Figure 3.4: Asymmetric thin-shell wormhole as a black hole mimicker. We assume that observers are located at spacetime $\mathcal{M}_1$. So the spacetime geometry viewed by observers is identical to that of black hole up to the thin-shell.

In each spacetime, the radial null geodesic is

$$\left(\frac{dr_i}{d\tau}\right)^2 + V_{i,eff} = \frac{1}{b_i^2} \quad (3.26)$$

with the effective potential given by

$$V_{i,eff} = \frac{f_i(r_i)}{r_i^2}, \quad (3.27)$$

where $b_i = \frac{L_i}{E_i}$ is called the impact parameter.

Without loss of generality, we suppose that observers are located in spacetime $\mathcal{M}_1$ and set $M_1 = 1$ and $M_2 = k$. The impact parameters in two spacetimes are connected with each other by

$$\frac{b_1}{b_2} = \sqrt{\frac{R-2k}{R-2}} \equiv Z. \quad (3.28)$$

If there exists photon sphere in $\mathcal{M}_1$, it is hardly to distinguish a wormhole from a black hole based on direct optical observation, because the horizon is shaded by the photon sphere. So we are interested in this case, i.e. $R < 3$. In other words, we want to distinguish a wormhole from a black hole through more abundant information even if they look like each other.

The more abundant information indeed exists. Considering the ingoing null geodesics with $b_1 < b_1^c = 3\sqrt{3}$ in spacetime $\mathcal{M}_1$, certain geodesics would turn back passing through the throat with a necessary condition

$$b_2 = \frac{b_1}{Z} > b_2^c = 3\sqrt{3}k. \quad (3.29)$$

To summarize, the ingoing null geodesic in spacetime $\mathcal{M}_1$ whose impact parameter satisfies

$$3\sqrt{3}kZ < b_1 < 3\sqrt{3}, \quad (3.30)$$

would drop into spacetime $\mathcal{M}_2$ and then turn back to spacetime $\mathcal{M}_1$ passing through the throat. This condition can be satisfied given

$$1 < k < \frac{R}{2} \leq \frac{3}{2}. \quad (3.31)$$

We show a plot of effective potential in Fig.3.5. It can be obviously seen from this plot that the photon in spacetime $\mathcal{M}_1$ with impact parameter lying in the

Figure 3.5: Plot of effective potential of radial null geodesic in asymmetric thin-shell wormhole. We have set $M_1 = 1, M_2 = 1.2, R = 2.6$, so $b_1^c = 3\sqrt{3} \approx 5.19615$ and $Zb_2^c = 3.6$ here and in the following. Note that the ticks in spacetime $\mathcal{M}_2$ is scaled by Z^2 because the impact parameters in two spacetimes are connected with each other by (3.28)

range $Zb_2^c < b_1 < b_1^c$ can turn back when it reaches the turning point in spacetime $\mathcal{M}_2$. We will see that this reflection mechanism by wormhole make a essentially distinction in the observational appearance between wormhole and black hole in the following.

3.2.2 Trajectory of photon and deflection angle

To have a complete understanding of the observational appearance of accretion disk around asymmetric thin-shell wormhole through the shadow, photon rings and lensing rings, we need first to investigate the trajectory and deflection angle of a light ray traveling in the wormhole. It is convenient to make a coordinate transformation $u_i = 1/r_i$. The trajectory of photon is determined by orbit equa-

Figure 3.6: Photon trajectories in the Euclidean polar coordinates (r, ϕ) with impact parameters lying in the range $Zb_2^c < b_1 < b_1^c$. The photons are coming from far right in spacetime $\mathcal{M}_1$. The red solid lines stand for trajectories in spacetime $\mathcal{M}_1$, and the blue dashed lines stand for trajectories in spacetime $\mathcal{M}_2$.

tion

$$\left(\frac{du_i}{d\phi}\right)^2 = G_i(u_i), \quad (3.32)$$

where

$$G_i(u_i) = \frac{1}{b_i^2} + 2M_i u_i^3 - u_i^2. \quad (3.33)$$

All trajectories can be divided into three classes. When $b_1 > b_1^c$, photon in $\mathcal{M}_1$ from infinity approach one closest point outside the throat, and then move back to infinity in $\mathcal{M}_1$. When $Zb_2^c < b_1 < b_1^c$, photon in $\mathcal{M}_1$ from infinity drop into $\mathcal{M}_2$ and then turn back passing through the throat to infinity in $\mathcal{M}_1$. When $b_1 < Zb_2^c$, photon in $\mathcal{M}_1$ from infinity drop into $\mathcal{M}_2$ and move to infinity in $\mathcal{M}_2$.

For $b_1 > b_1^c$, the turning point in spacetime $\mathcal{M}_1$ corresponds to the minimally positive real root of $G_1(u_1) = 0$, which we will denote by u_1^{min} . According to (3.32), the total change of azimuthal angle ϕ for certain trajectory with impact parameter b_1 can be calculated by

$$\phi_1(b_1) = 2 \int_0^{u_1^{min}} \frac{du_1}{\sqrt{G_1(u_1)}}, \quad b_1 > b_1^c. \quad (3.34)$$

For

$$Zb_2^c < b_1 < b_1^c, \quad (3.35)$$

we firstly focus on the trajectory in spacetime $\mathcal{M}_1$ outside the throat. The total change of azimuthal angle ϕ in spacetime $\mathcal{M}_1$ is obtained by

$$\phi_1(b_1) = 2 \int_0^{1/R} \frac{du_1}{\sqrt{G_1(u_1)}}, \quad b_1 < b_1^c, \quad (3.36)$$

The turning point in spacetime $\mathcal{M}_2$ corresponds to the maximally positive real root of $G_2(u_2) = 0$, which we will denote by u_2^{max} . According to (3.32), the total change of azimuthal angle ϕ for certain trajectory with impact parameter b_2 in spacetime $\mathcal{M}_2$ can be calculated by

$$\phi_2(b_2) = 2 \int_{u_2^{max}}^{1/R} \frac{du_2}{\sqrt{G_2(u_2)}}, \quad b_2 > b_2^c. \quad (3.37)$$

It is convenient to define three orbit numbers

$$n_1(b_1) = \frac{\phi_1(b_1)}{2\pi}, \quad n_2(b_1) = \frac{\phi_1(b_1) + \phi_2(b_1/Z)}{2\pi}, \quad n_3(b_1) = \frac{2\phi_1(b_1) + \phi_2(b_1/Z)}{2\pi} \quad (3.38)$$

for later use.

3.2.3 Transfer functions and photon rings

As Gralla et al's proposal [76], we focus on the optically and geometrically thin accretion disks for simplicity. Both static observer and accretion disk are assumed to locate at spacetime $\mathcal{M}_1$. The static observer is assumed to locate at the north pole, and the accretion disk is on the equatorial plane.

According to ray-tracing method, if a light ray from the observer intersects with the emission disk, it means the intersecting point as a light source will contribute brightness to the observer. We first consider that a light ray completely travels in spacetime $\mathcal{M}_1$. As the black hole case, a light ray whose orbit number $n_1 > 1/4$ will intersect with the disk on the front side. If n_1 goes larger than $3/4$, the light ray will bend around the wormhole, intersecting with the disk for the second time on the back side. Further, when $n_1 > 5/4$, the light ray will intersect with the disk for the third time on the front side again, and so on. Hence, the observed intensity is a sum of the intensities from each intersection,

$$I^{\text{obs}}(b_1) = \sum_m f_1^2 I^{\text{em}}|_{r=r_m(b_1)}, \quad (3.39)$$

where $r_m(b_1)$ is the so called transfer function which denotes the radial position of the m -th intersection with the emission disk. Here and in the following we omit the subscript "1" of the radial coordinate r_1 in spacetime $\mathcal{M}_1$ for convenience.

As discussed in above sections, for asymmetric thin shell wormhole photon can turn back passing through the throat in proper parameters space, so we would have addition transfer functions. According to the definitions of orbit numbers, when $n_2 < 3/4$ and $n_3 > 3/4$, the reflected outgoing trajectories in spacetime $\mathcal{M}_1$ will intersect with the disk on the back side. When $n_2 < 5/4$ and $n_3 > 5/4$, the reflected outgoing trajectories in spacetime $\mathcal{M}_1$ will intersect with the disk on the

Figure 3.7: The plots of orbit numbers n_1, n_2, n_3 , and the ranges of impact parameter for first three transfer functions.

front side. Thus the range of impact parameter for new second transfer functions are determined by $n_2 < 3/4$ and $n_3 > 3/4$ and the range of impact parameter for new third transfer functions are determined by $n_2 < 5/4$ and $n_3 > 5/4$ as

Figure 3.8: The plots of first three transfer functions. The black, orange and red ones correspond respectively to the first, second and third transfer functions.

illustrated in the middle plot of Fig.3.8. We give all transfer functions in the right plot of Fig.3.8.

As illustrated in [76], the first transfer function gives the "direct image" of the disk which is essentially just the redshift of source profile. The second transfer function gives a highly demagnified image of the back side of the disk, referred to "lensing ring". The third transfer function gives an extremely demagnified image of the front side of the disk, referred to "photon ring". The images resulted from further transfer functions are so demagnified that they can be neglected. The demagnified scale is determined by the slope of transfer function, dr/db_1 , called the demagnification factor. We can see from the right plot of Fig.3.8 that the new third transfer function near Zb_2^c has a high slope like the usual third transfer function near b_1^c . Another new third transfer function in the left of b_1^c has a smaller slope than the usual third transfer function near b_1^c , but bigger slope than the usual second transfer function in the right of b_1^c . The new second transfer function has a modest slope like the usual first transfer function, so we had better call the resulted image as 'lensing band'.

Now we take two typical emission models which are plotted in Fig.3.9 as examples proposed by Gralla et al. [76] to make physical pictures more clear. The observed intensity can be obtained according to (3.39). We give corresponding

Figure 3.9: The left emission profile (model I) is sharply peaked and abruptly end at the innermost stable circular orbit ($6M_1$). The right emission profile (model II) decaying gradually from the photon sphere ($3M_1$) to the innermost stable circular orbit ($6M_1$).

plots and densityplots of the observed intensities in Fig.3.10 for emission model I and Fig.3.11 for emission model II. We can see that in both models we have two additional photon rings resulted from corresponding two new third transfer functions for asymmetric thin shell wormhole. The new photon ring near critical curve Zb_2^c is highly demagnified like the photon ring near critical curve b_1^c . Another new photon ring which is located at the inside of critical curve b_1^c has a considerable size, but is smaller than the lensing ring which is located at outside of critical curve b_1^c . However in emission model I, the new second transfer function make no contribution to observed intensity, since this transfer function is out of the domain of emission model. While in emission model II, we have an additional sizable lensing band between critical curves Zb_2^c and b_1^c resulted from the new second transfer function in this emission model.

3.3 Discussion

In this chapter, we studied the trajectories of photons and their deflection angles in asymmetric thin-shell wormhole connecting two distinct Schwarzschild spacetimes by the throat. Typically, we placed observers in the spacetime $\mathcal{M}_1$ of which the mass parameter is smaller than that of the other side. Our interest is mainly about the photon rings in the observers' sky, we focused on the case that the

Figure 3.10: Observed intensity and densityplot of emission model I. The top row corresponds to black hole and the bottom row corresponds to wormhole. The left collum are the observed intensities. The middle collum are the densityplots of observed intensities and the right collum are local densityplots.

Figure 3.11: Observed intensity and densityplot of emission model II. The top row corresponds to black hole and the bottom row corresponds to wormhole. The left collum are the observed intensities. The middle collum are the densityplots of observed intensities and the right collum are local density plots.

throat of the ATW is inside the photon sphere of $\mathcal{M}_1$. After giving the formulas of deflection angles in each side, we constructed new orbit numbers counting the total deflection angles and showed the completed trajectories of photons which go through the throat twice, that is, ingoing photons of $\mathcal{M}_1$ pass through the throat and turn back after they reach the turning points in spacetime $\mathcal{M}_2$, then they go through the throat again. Then, considering optically and geometrically thin accretion disks around the wormhole, we gave the transfer functions and obtained the observed intensity and density plot in the sky of observers based on some emission models.

From our calculations, we found the ATW and corresponding Schwarzschild spacetimes differ markedly in the second and third transfer functions. In particular, the second transfer function is no longer a monotone function, a new segment appears before the monotone increasing part which also exists in corresponding Schwarzschild black hole spacetimes. As for the third transfer function, more structures were found in ATW. In addition to a new monotone increasing part, the usual one splits into two branches. The underlying reasons for these new characteristics is that photons with certain impact parameters could turn back after they go through the throat while these photons in corresponding Schwarzschild black hole spacetimes would fall into the event horizon and never come back since the event horizon is a one-way membrane.

As a result, two additional photon rings are found for ATW spacetime, one of which is highly demagnified near Zb_2^c (the critical curve in opposite spacetime viewed by observers' side) like the usual photon ring near b_1^c (the critical curve in observers' spacetime) and the other one located at inside of critical curve b_1^c is much brighter and has a considerable size, even though it's smaller than the lensing ring which is located at outside of critical curve b_1^c . Besides, we also found an additional lensing band when the emission profile overlap the domain of the new second transfer function. Though the ATW is a constructed model, these additional photon rings or lensing rings (or bands) should be exclusive structures for ultra compact objects because of reflectivity ¹. Our analysis as a toy model

¹In fact, the relevant study of gravastar as another UCO example has been implemented in [175].

may provide an optically observational evidence to distinguish UCOs from black holes.

Chapter 4

Echo amplification in nonlocal gravity

The nonlocal interactions have been widely studied since the days of Yukawa [176], Pais and Uhlenbeck [177]. It has been studied widely in the context of quantum field theory in order to ameliorate the ultraviolet (UV) (or, in other words, short-distance) behavior of loop integrals and scattering amplitudes [38, 178–180]. One of the striking features of nonlocality is that it weakens the interaction, and as a direct consequence it can regularize singularities through smearing out a point-like Dirac delta distribution [34, 40–42]. The simplest nonlocal interaction can be captured by infinite derivative field theories, where the kinetic operator is generalized by means exponentials of *entire* functions, which do not introduce any new dynamical degrees of freedom, and modifies the UV behavior of the theory. The latter criteria guarantees perturbative unitarity and classical stability which, in general, are absent in any finite higher derivative theory, see [38, 41, 42, 181].

In this Chapter, we provide a smoking-gun signature for nonlocal interaction in gravitational wave echoes. In Section 4.1, we give a brief introduction to the infinite derivative gravity. In Section 4.2, we towards to study the nonlocal interaction in gravitational wave echoes by assuming a simple model of reflection and transmission of waves bouncing through two Dirac delta potentials, which

can be viewed as the cavity of the effective potentials of UCOs. The context of this Chapter is mainly based on my publication **P1**.

4.1 Nonlocal gravity

In this Section, I give a brief introduction of IDG, including the construction of action, the structures of linearized field equation and proporgator around Minkowski background, in which we will see how IDG can be free of ghost. Especially, we illustrate how the gravitational potential can be regularized in IDG, when considering the Newtonian limit around Minkowski background.

4.1.1 The general quadratic curvature action

The most general form of the covariant quadratic curvature action is given by [42]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + R_{\mu_1 \nu_1 \lambda_1 \sigma_1} \mathcal{O}_{\mu_2 \nu_2 \lambda_2 \sigma_2}^{\mu_1 \nu_1 \lambda_1 \sigma_1} R^{\mu_2 \nu_2 \lambda_2 \sigma_2} \right], \quad (4.1)$$

where the differential operator $\mathcal{O}$ only contains the covariant derivatives and metric tensor $g_{\mu\nu}$ to preserve covariance, $R_{\dots}$ can be Ricci scalar or any Riemann tensor and Ricci tensor with upper or lower indices. In principle, the most general quadratic action is captured by 14 arbitrary functions, the F_i 's. However, it can be reduced to 3 arbitrary functions after applying the Bianchi identities around the maximal symmetric background, i.e. Minkowski background, dS, and AdS background[42–44], which simplifies the most general action (4.1) to

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R + R \mathcal{F}_1(\square) R + R_{\mu\nu} \mathcal{F}_2(\square) R^{\mu\nu} + R_{\mu\nu\rho\sigma} \mathcal{F}_3(\square) R^{\mu\nu\rho\sigma} \right\}. \quad (4.2)$$

The form factors $\mathcal{F}_i(\square)$ are arbitrary functions of d'Alembertian $\square$, which can be expressed as the power series

$$\mathcal{F}_i(\square) = \sum_{n=0}^{\infty} f_{i,n} \square^n. \quad (4.3)$$

Here $\mathcal{F}_i(\Box)$ are dimensionless, i.e. dimensionless $\Box_s = \Box/M_s^2$ and dimensionless coefficients $f_{i,n}$, where M_s is the new physical scale. Note that the *nonlocal gravity* is occurred once $\mathcal{F}_i(\Box)$ is a non-polynomial of $\Box$, for example $\frac{1}{\Box}, \ln(\Box), e^\Box$, which means that the $\mathcal{F}_i(\Box)$ contains infinite-order covariant derivative covariant. And we set $M_p = 1$.

In order to illustrate the issue of ghost instabilities and singularity, we need to study a slight fluctuation around the Minkowski background, i.e.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (4.4)$$

One method of obtaining the linearized field equation is based on varying the quartic gravitational action up to the second-order of $h_{\mu\nu}$ ($\mathcal{O}(h^2)$). Hence the second-order variation of the action (4.2) reads as

$$\begin{aligned} \delta^2 S = \int d^4x \Big\{ & \frac{M_p^2}{2} \delta^{(2)}(\sqrt{-g}R) + \sqrt{-g}[\delta R \mathcal{F}_1(\bar{\Box})\delta R + \delta R_{\mu\nu} \mathcal{F}_2(\bar{\Box})\delta R^{\mu\nu} \\ & + \delta R_{\mu\nu\rho\sigma} \mathcal{F}_3(\bar{\Box})\delta R^{\mu\nu\rho\sigma}] \Big\}. \end{aligned} \quad (4.5)$$

Note that in action (4.5), the $\bar{R}^{(2)}$ terms are absent due to the background value of the Ricci scalar, Riemann tensor, and Ricci tensor is zero in terms of the Minkowski metric. See Appendix A for the linear and second-order perturbation for $R_{\mu\nu\rho\sigma}, R_{\mu\nu}, R$ and $\sqrt{-g}$. Hence we can finally obtain

$$\begin{aligned} \delta^2 S = - \int \Big[& \frac{1}{2} h_{\mu\nu} a(\Box) h^{\mu\nu} + h_\mu^\sigma b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + h c(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} h d(\Box) \Box h \\ & + h^{\lambda\sigma} \frac{f(\Box)}{\Box} \partial_\lambda \partial_\sigma \partial_\mu \partial_\nu h^{\mu\nu} \Big], \end{aligned} \quad (4.6)$$

where

$$\begin{aligned}
 a(\Box) &= 1 + 2\mathcal{F}_2(\Box)\Box + 8\mathcal{F}_3(\Box)\Box, \\
 b(\Box) &= -1 - 2\mathcal{F}_2(\Box)\Box - 8\mathcal{F}_3(\Box)\Box, \\
 c(\Box) &= 1 - 8\mathcal{F}_1(\Box)\Box - 2\mathcal{F}_2(\Box)\Box, \\
 d(\Box) &= -1 + 8\mathcal{F}_1(\Box)\Box + 2\mathcal{F}_2(\Box)\Box, \\
 f(\Box) &= 8\mathcal{F}_1(\Box)\Box + 4\mathcal{F}_2(\Box)\Box + 8\mathcal{F}_3(\Box)\Box.
 \end{aligned} \tag{4.7}$$

Obviously, the above coefficients (4.7) obey the following relations

$$\begin{aligned}
 a(\Box) + b(\Box) &= 0, \\
 c(\Box) + d(\Box) &= 0, \\
 b(\Box) + c(\Box) + f(\Box) &= 0.
 \end{aligned} \tag{4.8}$$

By varying the action (4.5) with respect to $h_{\mu\nu}$, the linearized field equations can be derived explicitly as

$$\begin{aligned}
 a(\Box)\Box h_{\mu\nu} + b(\Box)(\partial_\mu\partial_\alpha h_\nu^\alpha + \partial_\alpha\partial_\nu h_\mu^\alpha) + c(\Box)(\eta_{\mu\nu}\partial_\alpha\partial_\beta h^{\alpha\beta} + \partial_\mu\partial_\nu h) \\
 + \eta_{\mu\nu}d(\Box)\Box h + f(\Box)\Box^{-1}\partial_\beta\partial_\alpha\partial_\mu\partial_\nu h^{\alpha\beta} = -\frac{2}{M_p^2}T_{\mu\nu}.
 \end{aligned} \tag{4.9}$$

4.1.2 Propagators around Minkowski background

Now we can calculate the propagator of the quadratic curvature theory. By inverting the kinetic term of the field equation (4.9) in the momentum space, the explicit form of the propagator around Minkowski background reads [42, 47]

$$\Pi(k) = \frac{\mathcal{P}^2}{a(-k^2)k^2} + \frac{\mathcal{P}_s^0}{[a(-k^2) - 3c(-k^2)]k^2}. \tag{4.10}$$

Here $\mathcal{P}^2$ and $\mathcal{P}_s^0$ stand for the spin-2 and scalar projection operators. Since a healthy new theory needs to recover GR in the infrared limit, therefore we need

to impose the following conditions in the limit $k^2 \rightarrow 0$,

$$a(0) = c(0) = -b(0) = -d(0) = 1. \quad (4.11)$$

After considering the conditions (4.11), one can recover the propagator for GR explicitly

$$\lim_{k^2 \rightarrow 0} \Pi(k) = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi(k)_{GR}. \quad (4.12)$$

Note that the $\mathcal{P}_s^0$ term does not introduce any physical degree of freedom in GR as the two coefficients of the scalar part ($\mathcal{P}_s^0$) and the helicity-0 component of spin-2 part ($\mathcal{P}^2$) cancels precisely. Seen from the propagator of GR, it seems that there are 6 degrees of freedom, 5 from the spin-2 part, and 1 from the scalar part. While the unphysical degree of freedom can be removed by imposing the field equations and gauge invariance. Therefore only 2 physical degrees of freedom remain in GR.

$f(R)$ gravity

The action of the $f(R)$ theory is given by

$$S = \int d^4x \sqrt{-g} f(R). \quad (4.13)$$

Here $f(R)$ can be expanded around the Minkowski background, i.e. $R = 0$. Consider the second order expansion

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2} + R f_{1,0} R \right\}, \quad (4.14)$$

which is a reduction of the general form of the quadratic curvature action (4.1) by choosing the following conditions

$$\mathcal{F}_2(\square) = \mathcal{F}_3(\square) = 0, \quad \mathcal{F}_1(\square) = f_{1,0}. \quad (4.15)$$

As a direct consequence, the relevant coefficients read as

$$a(\square) = 1, \quad c(\square) = 1 + f_{1,0}\square. \quad (4.16)$$

Therefore the propagator for the R^2 theory is given by

$$\Pi_{R^2} = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 - \frac{1}{3f_{1,0}}}. \quad (4.17)$$

Compared to the GR case, a second pole appears, i.e. $k^2 = \frac{1}{3f_{1,0}}$, whose residue has a positive sign indicating that it is not a ghost. Hence with respect to GR, one more scalar degree (massive spin-0) of freedom appears due to the R^2 term correction. Note that one can also choose $f_0 > 0$ to ensure it gets rid of tachyonic instability. Although $f(R)$ theory is ghost-free, it is not renormalizable, implying it is not a good candidate for resolving the singularity problem.

Fourth Derivative Gravity

The fourth order derivative theory is give by

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2} + f_{1,0}R^2 + R_{\mu\nu}f_{2,0}R^{\mu\nu} \right\}. \quad (4.18)$$

Similarly, basing on general form (4.10) the propagator for the fourth order derivative theory can be obtained

$$\Pi_{R^2} = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 - \frac{1}{3f_{1,0} + f_{2,0}}} - \frac{\mathcal{P}^2}{k^2 - \frac{1}{-2f_{2,0}}}. \quad (4.19)$$

Hence a new massive spin-2 ghost appears, known as the Weyl ghost. And there is another massive spin-0 mode that can be shown to be absent if $3f_{1,0} + f_{2,0} = 0$ holds, which is consistent with the result obtain in [182]. Although the fourth order derivative theory is renormalizable, it plagues by the massive spin-2 ghost instability.

4.1.3 Infinite derivative gravity

So far, we give two examples in the frame of local theory (finite covariant derivatives), which indicate it is not compatible between the renormalizable and unitary in $f(R)$ theory and fourth order derivative ($R_{\mu\nu}R^{\mu\nu}$) theory. So in order to seek a new theory that could potentially resolve the singularity problem without introducing any ghost degrees of freedom, we turned our attention to the nonlocal frame.

For simplicity, let us consider a new theory that only processes the graviton as the propagating degree of freedom. For this purpose, one should impose the following condition

$$a(\square) = c(\square) \quad (4.20)$$

with the $a(\square)$ must be an entire function that does not introduce extra poles. The simplest entire function is given by

$$a(\square) = e^{-\frac{\square}{M_s^2}}. \quad (4.21)$$

Since we are working around the Minkowski background, the $F_i(\square)$ in the action can be moved to the left-hand side straightforwardly. Therefore the $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ can be replaced by the combination of the R^2 and $R_{\mu\nu}R^{\mu\nu}$ term by implementing the Euler topological invariant relation. So one can thus set $\mathcal{F}_3(\square)$ to zero. (4.20), (4.7), (4.21) implies that

$$\mathcal{F}_1(\square) = -\frac{e^{-\frac{\square}{M_s^2}} - 1}{\square}, \quad \mathcal{F}_2(\square) = 2\frac{e^{-\frac{\square}{M_s^2}} - 1}{\square}. \quad (4.22)$$

Hence the action (4.2) can be rewritten as

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R + G_{\mu\nu} \frac{e^{-\frac{\square}{M_s^2}} - 1}{\square} R^{\mu\nu} \right\}. \quad (4.23)$$

This kind of nonlocal gravity theory is known as *Infinite derivative Gravity*. In other words, a nonlocal gravity theory is called the IDG, if the differential opera-

tors of the nonlocal gravity are analytic operators, i.e. the Taylor expansion can be well-defined around $\square = 0$. Note that nonlocal gravity constructed in terms of non-analytic differential like $\frac{1}{\square}, \ln(\square)$ have been studied in other literature [20, 30–33], and we will not focus on this direction in this thesis.

Regularized gravitational potential within IDG

From the above discussion, we noticed that the infinite derivative gravity is free from ghost instabilities around Minkowski background. Now let us check whether this theory could be a good candidate for resolving the singularity problem.

First of all, we will consider the most simple case, i.e., the Newtonian approximation around the Minkowski spacetime. Hence the metric reads as

$$ds^2 = -(1 + 2\Phi(r))dt^2 + (1 - 2\Psi(r))d\vec{r}^2, \quad (4.24)$$

where $|\Phi| \ll 1, |\Psi| \ll 1$. Suppose the potential Φ and Ψ is sourced by a point source

$$T_{\mu\nu} = \rho\delta_\mu^0\delta_\nu^0 = m_g\delta^3(\vec{r})\delta_\mu^0\delta_\nu^0. \quad (4.25)$$

Submitting (4.24) and (4.25) into (4.1.1), one can get the 00 component and trace part of the field equation

$$\begin{aligned} (a - 3c)[\Delta\Phi - 2\Delta\Psi] &= \frac{m_g}{M_p^2}\delta^3(\vec{r}), \\ (c - a)\Delta\phi - 2c\Delta\Psi &= -\frac{m_g}{M_p^2}\delta^3(\vec{r}), \end{aligned} \quad (4.26)$$

where Δ is the Laplacian on three spatial metric $d\vec{r}^2$. Furthermore, considering (4.20), the linearized field equation can be simplified to be

$$2e^{-\frac{\Delta}{M_s^2}}\Delta\Phi = 2e^{-\frac{\Delta}{M_s^2}}\Delta\Psi = \frac{m_g}{M_p^2}\delta^3(\vec{r}). \quad (4.27)$$

By going to the Fourier space, one can thus obtain [42, 49, 52]

$$\Phi(r) = \frac{m_g}{2(2\pi)^3 M_p^2} \int \frac{d^3 k}{k^2} e^{i\vec{k}\cdot\vec{r}} e^{-\frac{k^2}{M_s^2}} = -\frac{m_g}{8\pi M_p^2 r} \text{erf}\left(\frac{M_s r}{2}\right). \quad (4.28)$$

As expected, the Newtonian potential in GR is regularized by the IDG.

4.2 Echoes in nonlocal gravity

As we already mentioned, the exact spherically symmetric solutions in the full non-linear regime have not been obtained in IDG. While due to the correlation between the matter field perturbation and the gravitational perturbation around the BH background as illustrated in Section 2.2, we can capture the salient feature of nonlocality by introducing a nonlocal scalar field. In other words, we can improve the Schrodinger-like wave equation by introducing infinite derivative order operators. As illustrated in Section 2.1, GW echo arises from a compact binary coalescence since the effective potential of gravitational perturbation for UCO own a cavity between the surface and the photon sphere. For simplicity, the cavity of the effective potential can be simulated by two Dirac delta potential barriers. The two Dirac deltas can be also used to model the two symmetric photon spheres of a wormhole [183]. In the case of a static spacetime metric, the double delta potential could also mimic the two potential barriers at the surface and at the photon sphere of an ultra compact object (UCO), located at $2GM(1+\epsilon)$ and $3GM$, respectively, with $\epsilon < 0.5$. By assuming a simple model of reflection and transmission of waves bouncing through two Dirac delta potentials, we will show that the transmission of waves is less damped compared to the local case due to the fact that nonlocality weakens the interaction. As a consequence, the echoes are amplified. Despite the simplicity of our model, by modifying the dynamics of propagating waves through infinite derivative order operators, we might be able to capture some salient features of nonlocality, which is otherwise extremely elusive to test.

4.2.1 Nonlocal scalar field in double delta potential

Let us consider a nonlocal massless scalar field Φ interacting with a potential V in $1+1$ dimensions with a nonlocal operator given by an exponential of an *entire* function

$$[F(\square)\square - V]\Phi = 0, \quad F(\square) = e^{-\ell_s^2 \square}; \quad (4.29)$$

we will work with the simplest kinetic operator above but other choices are also possible [38, 45, 49, 55]. In Eq.(4.29), $\ell_s = \frac{1}{M_s}$ is the fundamental length scale of nonlocality below which new physics should manifest and $\square = -\partial_t^2 + \partial_x^2$ is the flat d'Alembertian operator. Note that we adopt the metric signature; $\eta = \text{diag}(-1, 1, 1, 1)$, and use the Natural Units $\hbar = 1 = c$. By separating the time and spatial variables with the ansatz $\Phi(t, x) = e^{-i\omega t}\psi(x)$, Eq.(4.29) becomes [184]

$$\left[e^{-\ell_s^2(\partial_x^2 + \omega^2)}(\partial_x^2 + \omega^2) - V(x) \right] \psi(x) = 0, \quad (4.30)$$

which is a nonlocal version of the Schrödinger equation. For general potential barriers, $V(x)$ is assumed to be positive and to satisfy $V(x) \rightarrow 0$ as $x \rightarrow \pm\infty$, so that the general solution of $\psi(x)$ has the form

$$\psi(x) = \begin{cases} Ae^{i\omega x} + Be^{-i\omega x}, & x \rightarrow -\infty, \\ Ce^{i\omega x}, & x \rightarrow +\infty. \end{cases} \quad (4.31)$$

The reflection and transmission coefficients, $\mathcal{R}$ and $\mathcal{T}$ respectively, are defined as

$$\mathcal{R} = \frac{B}{A}, \quad \mathcal{T} = \frac{C}{A}, \quad |\mathcal{R}|^2 + |\mathcal{T}|^2 = 1. \quad (4.32)$$

For a general potential Eq.(4.30) may not be easily solvable, but there exist analytic solution in some cases. Here, we assume a symmetric double Dirac delta potentials

$$V(x) = \lambda\delta(x+a) + \lambda\delta(x-a). \quad (4.33)$$

Note that a single Dirac delta potential in nonlocal theory was first studied in [184]. The nonlocal Schrödinger equation (4.30) with the potential (4.33) can be

solved analytically by using the Lippmann-Schwinger method [185]

$$\psi(x) = \psi_0(x) - \int_{-\infty}^{\infty} dx' G(x, x') V(x') \psi(x') \quad (4.34)$$

where $G(x, x')$ is the Green function defined through the equation

$$e^{-\ell_s^2(\partial_x^2 + \omega^2)}(\partial_x^2 + \omega^2)G(x, x') = -\delta(x - x'), \quad (4.35)$$

and reads [184]

$$\begin{aligned} \mathcal{G}(z) &= \frac{i}{4\omega} [e^{i\omega z} Y(z) + e^{-i\omega z} Y(-z)] \\ \text{with } Y(z) &= 1 + \operatorname{erf}\left(i\omega\ell_s + \frac{z}{2\ell_s}\right), \end{aligned} \quad (4.36)$$

which in the limit $\ell_s \rightarrow 0$ reduces to the local Green function $\mathcal{G}_0(z) = \frac{i}{2\omega} e^{i\omega|z|}$, as expected. The Green function turns out to be symmetric, $G(x, x') = G(x', x)$, and depends only on the variable $z = x - x'$: $G(x, x') \equiv \mathcal{G}(z)$, $\mathcal{G}(z) = \mathcal{G}(-z)$. Hence, by using the Lippmann-Schwinger equation (4.34), the solution for the symmetric double delta in (4.33) can be shown to be

$$\psi(x) = e^{i\omega x} - \lambda \sum_{j,k=1}^2 \mathcal{G}(x - a_k) \Phi_{kj}^{-1} e^{i\omega a_j}, \quad (4.37)$$

where

$$\Phi_{kj} = \begin{cases} 1 + \frac{i\lambda_j}{2\omega} [1 + \operatorname{erf}(i\omega\ell_s)], & k = j; \\ \frac{i\lambda_j}{4\omega} [e^{i2\omega a} Y(2a) + e^{-i2\omega a} Y(-2a)], & k \neq j, \end{cases} \quad (4.38)$$

and we have defined $a_1 = a$, $a_2 = -a$. By assuming that the incoming wave comes from the left, we can find the reflection and transmission coefficients for the solution in Eq.(4.38) by identifying the coefficients in front of $e^{-i\omega x}$ when $x \ll a$ and of $e^{i\omega x}$ when $x \gg a$, respectively; they are given by

$$\mathcal{R} = -\frac{X}{Z}, \quad \mathcal{T} = 1 - \frac{Y}{Z} \quad (4.39)$$

where

$$\begin{aligned}
 X &= 2[-2\frac{\omega}{\lambda}i + 1 + \operatorname{erf}(i\omega\ell_s)]\cos(2\omega a) \\
 &\quad - [1 + \operatorname{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} - [1 + \operatorname{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a} \\
 Y &= 2[-2\frac{\omega}{\lambda}i + 1 + \operatorname{erf}(i\omega\ell_s)] \\
 &\quad - \{[1 + \operatorname{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} + [1 + \operatorname{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a}\}\cos(2\omega a) \\
 Z &= [-2\frac{\omega}{\lambda}i + 1 + \operatorname{erf}(i\omega\ell_s)]^2 \\
 &\quad - \frac{1}{4}\{[1 + \operatorname{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} + [1 + \operatorname{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a}\}^2 \quad (4.40)
 \end{aligned}$$

and one can easily check that $|\mathcal{R}|^2 + |\mathcal{T}|^2 = 1$.

4.2.2 Quasi-normal modes

The poles of $\mathcal{R}$ and $\mathcal{T}$ describe the QNMs of the system, which satisfy the boundary condition such that there is no incoming wave at infinity, i.e. $\psi \rightarrow e^{-i\omega x}$ as $x \rightarrow -\infty$ and $\psi \rightarrow e^{i\omega x}$ as $x \rightarrow +\infty$ [186]. In a realistic astrophysical framework, the quantities in (4.39) are crucial for understanding the effect of nonlocality in the UCOs. We can check that the poles of the reflection and transmission coefficients are given by the equations

$$\begin{aligned}
 2[-2\frac{\omega}{\lambda}i + 1 + \operatorname{erf}(i\omega\ell_s)] &= \pm \left\{ [1 + \operatorname{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} \right. \\
 &\quad \left. + [1 + \operatorname{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a} \right\}, \quad (4.41)
 \end{aligned}$$

whose roots of the type $\omega_n = \mathcal{W}_n - i\omega'_n$ are called quasi-normal frequencies (QNFs). Note that for ω with a negative imaginary part, the time dependent piece $e^{-i\omega t}$ behaves like a damped oscillation: the real part gives the energy carried by the wave and the imaginary part (inverse) gives the time scale over which the wave decays after interacting with the potential barrier. It is worthwhile mentioning that in the limit, $a = 0$, we recover the quasi-normal condition for

Figure 4.1: We have shown the behaviour of the real part of the QNFs $\mathcal{W}_n$ as a function of the negative imaginary part ω'_n for both local (black line) and nonlocal (orange line) cases; the latter was obtained by solving numerically Eq.(4.42). In the upper panel we have set $a = 1$, $\ell_s = 0.01$ and $\lambda = 10$, while in the bottom panel $a = 1$, $\ell_s = 0.01$ and $\lambda = 50$.

a single Dirac delta potential with strength 2λ , i.e. $1 - i\omega/\lambda + \text{erf}(i\omega\ell_s) = 0$ [184]; while in the limit $\ell_s = 0$, Eq.(4.41) reduces to the well known local case, $1 - i2\omega/\lambda = \pm e^{2i\omega a}$.

Generally, Eq.(4.41) cannot be solved, but we can simplify it and obtain an analytic solution by working in a specific regime which turns out to be the most interesting one from a physical point of view. Since we expect the fundamental scale of nonlocality ℓ_s to be always smaller than the distance between the surface and the photon sphere of a UCO, see [74, 162], it is sensible to require $a \gg \ell_s$. Note that we only consider a toy-model to understand the qualitative features of nonlocal effects on the propagation of waves, indeed we do not specify any

physical value for a and λ which are compatible with some astrophysical scenario. However, a quantitative analysis will require a more realistic model in which the potential in Eq.(4.30) has to take into account the presence of a horizonless curved spacetime and of a centrifugal contribution. Moreover, when studying the QNMs and the echoes produced by an initial pulse (see also below) the low frequencies are the most relevant ones as they correspond to longer damping times, therefore we can also impose the inequality $|\omega|\ell_s < a/\ell_s$. In such limits, $\text{erf}(i\omega\ell_s + a/\ell_s) \rightarrow 1$ and $\text{erf}(i\omega\ell_s - a/\ell_s) \rightarrow -1$, so that Eq.(4.41) reduces to

$$1 - i2\frac{\omega}{\lambda} + \text{erf}(i\omega\ell_s) = \pm e^{2i\omega a}, \quad (4.42)$$

which in the limit $|\omega|\ell_s \ll 1$ can be further simplified since $\text{erf}(i\omega\ell_s) \approx \frac{2}{\sqrt{\pi}}i\omega\ell_s$. Indeed, in this regime we can find an analytic expression for the QNMs in terms of the Lambert W function:

$$\omega_n = i \left[\frac{W_n(\pm \alpha a e^{\alpha a})}{2a} - \frac{\alpha}{2} \right], \quad \frac{1}{\alpha} = \frac{1}{\lambda} - \frac{\ell_s}{\sqrt{\pi}}. \quad (4.43)$$

Furthermore, we can solve numerically Eq.(4.42) and study the behaviour of the real part of the QNMs, $\text{Re}(\omega) = \mathcal{W}_n$, as a function of the negative imaginary part $-\text{Im}(\omega) = \omega'_n$. From Fig. 1, we can see that nonlocal effects make the absolute value of imaginary part of the QNFs smaller, i.e. the damping time is longer. The curve corresponding to the nonlocal case is always below the local one in the low frequency regime as shown in the Fig. 1. The closer the ring-down energy $\mathcal{W}_n$ is to the energy scale $1/\ell_s \equiv M_s$, the more relevant nonlocality is. This means the more obvious becomes the difference between the imaginary parts between local and nonlocal. Only for $\ell_s \rightarrow 0$ the two curves coincide. Moreover, when the strength of the delta potential $\lambda \gtrsim 1/\ell_s \equiv M_s$, the damping is smaller. Therefore, a crucial implication due to nonlocality is that low frequency waves survive a *longer* life-time as compared to the local case. This feature was expected since it is known that nonlocality weakens the interaction [180, 187–189]. Furthermore, when $\mathcal{W}_n\ell_s \sim \mathcal{O}(1)$, we can also notice the presence of a turning point after which the damping time decreases. This property is mathematically related to

the structure of the error function with complex argument but it is not relevant from a physical point of view since it will only describe short-lived waves with energies $\mathcal{W}_n \ell_s \gg 1$.

4.2.3 Echoes

So far we have learned that nonlocal effects weaken the interaction between wave and potential barriers, and as a consequence the time scales over which a transmitted wave decays turn out to be longer than the local case. We now wish to use the spectral features of the QNMs and study the consequences induced by non-locality on the echoes production. In order to do so we need to work in the time domain where the form of the wave can be cast as an infinite linear combination of QNMs [183, 190]:

$$\Psi(t) = \sum_{n=-\infty}^{\infty} c_n e^{-i\omega_n t}, \quad (4.44)$$

which in a more realistic scenario can represent the signal produced at the photon sphere after relaxation. Because the oscillation period of the real parts of the QNMs is $\text{Re}(\omega_{n+1} - \omega_n) \simeq \pi/2a$, which is the same also in the local case, and since $\text{Im}(\omega_n) \ll \text{Re}(\omega_n)$, the function $\Psi(t)$ is in good approximation periodic, with period $T = 4a$. This means that the primary signal is reproduced periodically, as it can be reflected by the second potential barrier (UCO's surface), thus giving rise to echoes.

This feature allows us to express the coefficients c_n as a Fourier transform of the first echo, $\Psi_{1^{\text{st}}\text{echo}}^{(0)}(t)$,

$$c_n = \frac{1}{4a} \int_0^{4a} dt \Psi_{1^{\text{st}}\text{echo}}^{(0)}(t) e^{i\omega_n t}. \quad (4.45)$$

We can choose the first echo as a Gaussian wave packet of the form

$$\Psi_{1^{\text{st}}\text{echo}}^{(0)}(t) = e^{-i\omega_0(t-t_0)} e^{-(t-t_0)^2/2\tau^2}, \quad (4.46)$$

where ω_0 is the leading frequency of the primary signal. Assuming that $\tau \ll 2a$ we can obtain an approximation for the coefficients c_n of the subsequent echoes

Figure 4.2: We have plotted the behaviour of the signal in Eq.(4.44), with coefficients (4.47), which describes the evolution of the echoes produced at a fixed spatial point, for both local (black line) and nonlocal (orange line) cases. We have chosen $t_0 = 1$, $\tau = 0.25$, and set $a = 1$, $\lambda = 50$, $\ell_s = 0.01$.

by computing the integral in Eq.(4.45) [183]

$$c_n \simeq \sqrt{\frac{\pi}{2}} \frac{\tau}{2a} \exp \left[i\omega_n t_0 - \frac{\tau^2}{2} (\omega_n - \omega_0)^2 \right]. \quad (4.47)$$

Hence, we can now use Eq.(4.44) with the coefficients given in Eq.(4.47) to plot the time dependent profile of the signal, see Fig. 2. Note that, because only the long-lived QNMs (QNFs with small negative imaginary part) led to the echoes in the late time [191], thus it is also sufficient to analyze these results using Eq.(4.43). In the regime $\lambda \ll 1/\ell_s \equiv M_s$, we have already seen that the QNMs are very similar to the local case, therefore the same applies for the echoes. It means that when the height of the potential barriers is below the energy scale $1/\ell_s \equiv M_s$, nonlocality does not play a relevant role, i.e. the echoes are almost the same as the local case. However, when the height of the potential barrier is either comparable or larger than the nonlocal energy scale, i.e. $\lambda \gtrsim M_s$, the damping time of the nonlocal case is much longer at low frequency. So a very distinct difference arises between local and nonlocal cases. As shown in Fig. 2,

the amplitudes of the late time echoes are amplified with respect to the local ones, meaning that nonlocality acts as an *amplifier*. Actually, we can adjust the degree of amplification by adjusting the height of the potential λ . Such a behavior is consistent with the fact that the nonlocal interaction makes the life-times of the waves longer [74], which means they are less damped in time, therefore the echo signal is more amplified. The presence of a smaller damping scale also implies that in the process of transmission less energy is dissipated when the interaction between the wave and the potential barrier is nonlocal, therefore the echoes are louder.

4.3 Discussion

Let us conclude this Chapter by emphasizing that nonlocal effects can be amplified in presence of double delta potential barriers, which can be experimentally probed. In our opinion, gravitational waves in the merger of two UCOs, or worm-holes, might yield the secret of nonlocality, which goes in conjunction with the avoidance of an event horizon to explain that black hole information loss paradox. It is worthwhile mentioning that this kind of nonlocal effects could be captured in models of condensate matter system, in which the reflection and transmission properties of quantum mechanical barriers can be potentially tested in a table-top experiment, or nonlocal effects on quantum mechanical oscillators have been studied previously [192].

Our analysis has been made in the simple case of flat background but future studies are needed to take into account the presence of a curvature and of a more realistic effective potential, but we believe that the essence will be very similar to what we have presented here. The key smoking-gun signature will be the presence of a smaller damping scale and the amplification of echoes, which is a distinctive feature of nonlocality and is surely absent in a local theory.

Chapter 5

Bouncing universe: theory and problems

In this Chapter, we give a brief overview of bouncing universe model and the cosmology perturbations in the context of GR, which can be seen as a foundation of the later Chapter 6. In Section 5.1, we introduce the bouncing universe model in GR as one way of resolving the cosmology singularities and discuss the unsolved problems of this bouncing model. In Section 5.2, we study the cosmology perturbation in GR, which is the first step for studying the formation of the structure. Following a similar procedure, we will investigate the cosmology perturbations in the context of nonlocal gravity, which will be illustrated in Chapter 6.

5.1 Bouncing universe in general relativity

The universe is an ancient and mysterious topic. Countless sages have explored this and reached their conclusions. Nevertheless, cosmology can only be regarded as a subject after the birth of GR. The basic assumption of the universe is called the cosmological principle, which is stated as follows: the universe is isotropic and homogeneous on large scales. The big bang theory of cosmology is extremely successful except very early period. According to the singularity theorems proposed

by Hawking-Penrose, if the matter which couples minimally to gravity obeys the null energy condition (NEC), an initial cosmological singularity from which our universe is born inevitably arises in a homogeneous and isotropic model in the context of GR. In cosmology, the inevitable existence of the Big Bang singularity in the presence of standard matter fluids is a fundamental problem in General Relativity (GR) [14]. Even in vacuum, GR has Kasner type solutions which lead to the (in)famous Belinsky-Khalatnikov-Lifshitz (BKL) singularity [115, 117, 118]. The problem of cosmological singularity can be addressed in quantum cosmology through the no boundary proposal [193, 194]. Another interesting way of resolving the cosmological singularity is through a non-singular bouncing scenario [86, 87, 195, 196] which has been widely investigated in several ultraviolet (UV) completions of GR such as string theory [119, 197, 198], string gas cosmology [199] and loop quantum gravity [200]. In this section, we introduce the basic theory of bouncing universe and existing problems in GR.

5.1.1 Friedmann equations

Based on the cosmology principle, the metric of the universe on large scales is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right), \quad (5.1)$$

where t is called the cosmological time, $a(t)$ is the scale factor, and $K = -1, 0, 1$, correspond to hyperbolic, flat, spherical three dimensional space respectively.

On the other hand, because the universe is homogeneous and isotropic, the matters in it can be viewed as perfect fluid with energy-momentum tensor given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (5.2)$$

where the energy density ρ and pressure p are only functions of t , and u^a is the four-velocity of an observer comoving with the expansion of cosmology, i.e.

$u^a = (\frac{\partial}{\partial t})^a$, or in components

$$u^0 = 1, \quad u^i = 0. \quad (5.3)$$

According to the Einstein equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_p^2} T_{\mu\nu}, \quad (5.4)$$

we can obtain the Friedmann equations which determine the evolution of the scale factor $a(t)$

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} = \frac{\rho}{3M_p^2} + \frac{\Lambda}{3}, \quad (5.5)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p) + \frac{\Lambda}{3}, \quad (5.6)$$

where $\dot{}$ denotes the derivative with respect to the cosmological time t . Defining the Hubble parameter

$$H \equiv \frac{\dot{a}}{a}, \quad (5.7)$$

which describe the expansion (contraction) rate of the universe, the Friedmann equations can be cast into

$$H^2 = \frac{\rho}{3M_p^2} - \frac{K}{a^2} + \frac{\Lambda}{3}, \quad (5.8)$$

$$\dot{H} = -\frac{1}{2M_p^2}(\rho + p) + \frac{K}{a^2}. \quad (5.9)$$

The conservation of energy-momentum tensor gives

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \quad (5.10)$$

which is consistent with the Friedmann equations. Now we have two independent equations for three variables $a(t)$, $\rho(t)$ and $p(t)$, so we need supplement a equation

of state which describes the relation between ρ and p

$$\omega = \frac{p}{\rho}. \quad (5.11)$$

If ω is a constant, the conservation equation gives

$$\rho \propto \frac{1}{a^{3(1+\omega)}}. \quad (5.12)$$

For non-relativistic cold matter (dust), $\omega = 0, \rho \propto a^{-3}$; For relativistic hot matter (radiation), $\omega = \frac{1}{3}, \rho \propto a^{-4}$; For cosmological constant, $\omega = -1, \rho = \text{const}$. From the right hand side of (5.8) it can be seen that the curvature term K/a^2 acts like an energy component with an equation of state $\omega = -\frac{1}{3}$, which can be called k matter.

For a non-singular bouncing universe, a decreases in the contracting phase, reaches a finite value at the bouncing point, then increases in the expanding phase. Assuming the bounce takes place at $t = 0$, we have

$$a(0) > 0, \quad \dot{a}(0) = 0, \quad \ddot{a}(0) > 0, \quad (5.13)$$

i.e.

$$H(0) = \frac{\dot{a}(0)}{a(0)} = 0, \quad \dot{H}(0) = \frac{\ddot{a}(0)}{a(0)} - \frac{\dot{a}(0)^2}{a(0)^2} > 0. \quad (5.14)$$

Thus for open universe (flat and hyperbolic $K \leq 0$), Friedmann equation (5.9) implies

$$\rho + p < 0, \quad (5.15)$$

which violates the null energy condition (NEC). If the energy density $\rho > 0$, this means the equation of state $\omega < -1$. In order to obtain non-singular bouncing cosmology in the context of GR, we need to introduce modified matter which violates the NEC, that typically would result in a gradient or ghost instability [90, 91].

The simplest way to obtain a non-singular bouncing model with modified matter is to introduce a scalar field with negative kinetic term, i.e. a “ghost” field, at the cost of bringing ghost instability. An improved non-singular bounce

can be obtained by the “ghost condensate” mechanism [201]. Ghost condensation is the kinetic sector analog of the Higgs mechanism in the potential sector, in which a scalar field is assumed to have a noncanonical Lagrangian that allows it to have $\omega < -1$. However in this model a new problem is gradient instability [90]. We can also obtain non-singular bouncing cosmology from modified gravity [202–204], in which generally there would be ghost instability in the gravitational sector.

5.1.2 Anisotropy and Belinskii-Khalatnikov-Lifshitz singularity

In the early time of big bang, the universe may not be isotropic. In the absence of spatial curvature, anisotropy can be introduced by generalizing the FLRW metric to Bianchi-I spacetime metric [114]

$$ds^2 = -dt^2 + a(t)^2 \left(e^{2\beta_1(t)} dx_1^2 + e^{2\beta_2(t)} dx_2^2 + e^{2\beta_3(t)} dx_3^2 \right), \quad (5.16)$$

where the exponents are chosen to satisfy $\beta_1 + \beta_2 + \beta_3 = 0$ so that a is the average scale factor. Here β_i representing the differences between the FLRW metric implies the space is expanding or contracting at different rates in different directions. Substituting this metric ansatz into the Einstein equation, we have

$$\left(\frac{\dot{a}}{a} \right)^2 - \frac{1}{6}(\dot{\beta}_1^2 + \dot{\beta}_2^2 + \dot{\beta}_3^2) = \frac{\rho}{3M_p^2}, \quad (5.17)$$

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a} \right)^2 + \frac{1}{2}(\dot{\beta}_1^2 + \dot{\beta}_2^2 + \dot{\beta}_3^2) = -\frac{1}{2M_p^2}(\rho + p), \quad (5.18)$$

and another equation for β_i

$$\ddot{\beta}_i + 3\frac{\dot{a}}{a}\dot{\beta}_i = 0, \quad (5.19)$$

which implies $\dot{\beta}_i = b_i/a^3$, where b_i are constants. Define

$$\sigma_0^2 = \frac{1}{2}(b_1^2 + b_2^2 + b_3^2). \quad (5.20)$$

The equations (5.17) and (5.18) become

$$H^2 = \frac{1}{3M_p^2} \left(\rho + \frac{\sigma_0^2}{a^6} \right), \quad (5.21)$$

$$\dot{H} = -\frac{1}{2M_p^2}(\rho + p) - \frac{\sigma_0^2}{a^6}. \quad (5.22)$$

The term σ_0^2/a^6 shows that the anisotropy acts as a stiff matter with the equation of state $\omega = 1$.

Neglecting ρ in Friedmann equation (5.21), the scale factor is solved to be $a \sim (-t)^{1/3}$, where t is negative and increasing. Then equation (5.19) gives $a^2 e^{2\beta_i} = (-t)^{2p_i}$, which recovers the Kasner metric with constant exponents $p_i = \frac{1}{3} + \frac{b_i}{\sqrt{3}\sigma_0}$. Since these exponents satisfy $p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1$, so one of them must have the opposite sign with the other two, which means two spatial dimensions contract while the third expands as $t \rightarrow 0$. This kind of phenomenon is known as Belinskii-Khalatnikov-Lifshitz (BKL) instability [115, 117, 118]. The chaotic switching of directions occurs if there is spatial curvature, in which case a Kasner solution works as an approximation during each epoch between consecutive switches.

Therefore, if the equation of state of all the energy component is $\omega < 1$, then the anisotropy will be amplified in the contracting phase which leads to a chaotic mixmaster, and the bounce will be spoiled. Hence there has to be an energy component with an equation of state $\omega > 1$, termed super-stiff, which will dominate over the anisotropy in the contracting phase.

To summarise, the most general Friedmann equations describing the evolution of bouncing universe can be written as

$$H^2 = \frac{1}{3M_p^2} \left(\Lambda - \frac{\rho_{k0}}{a^2} + \frac{\rho_{m0}}{a^3} + \frac{\rho_{r0}}{a^4} + \frac{\rho_{s0}}{a^6} + \frac{\rho_0}{a^{3(1+\omega)}} \right), \quad (5.23)$$

including energy components:

- large negative pressure matter: $\omega < -1$;
- cosmological constant: $\omega = -1$;

- spatial curvature or k matter: $\omega = -\frac{1}{3}$;
- non-relativistic cold matter (dust): $\omega = 0$;
- relativistic hot matter (radiation): $\omega = \frac{1}{3}$;
- anisotropy or stiff matter: $\omega = 1$;
- super-stiff matter: $\omega > 1$.

The necessity of large negative pressure matter and super-stiff matter make the existence of bouncing universe hardly possible, at least instability in the framework of GR. An interesting candidate of bouncing universe motivated by super-string theory is the so called Ekpyrotic scenario [119]. The low energy effective field theory of the Ekpyrotic scenario is given by Einstein gravity minimally coupled to a scalar field. The key feature of the Ekpyrotic scenario is that the equation of state ω of the scalar field is much larger than 1 during the contracting phase. In the next section, we further analyze the stability of bounce and evolution of small fluctuations based on cosmological perturbation.

5.2 The cosmological perturbation

The assumption of a homogeneous and isotropic universe ignored many exciting things, such as galaxies and their clusters. So in order to investigate the formation of the structures and the stability of bouncing universe, we need to consider perturbations around the FLRW metric. In this section, we will lay a foundation by deriving the general relativistic equations that govern small fluctuations and drawing general conclusions about their implications.

5.2.1 Perturbation of the energy momentum tensor

Considering a more general energy-momentum tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} + \pi_{\mu\nu}, \quad (5.24)$$

where the anisotropic pressure $\pi^\mu{}_\nu = (0, \pi^1{}_1, \pi^2{}_2, \pi^3{}_3)$ is traceless $\pi^\mu{}_\mu = 0$ and transverse to the velocity vector $\pi_{\mu\nu}u^\mu = 0$. The anisotropic pressure is sourced by the quadrupole moments of photons and neutrinos distributions, see [205]. For this form of energy momentum tensor (5.24), the heat transfer contribution is not considered.

The first-order perturbation to the energy momentum tensor (5.24) is given by

$$\delta T_{\mu\nu} = (\delta p + \delta \rho)\bar{u}_\mu\bar{u}_\nu + (\bar{p} + \bar{\rho})\delta u_\mu\bar{u}_\nu + (\bar{p} + \bar{\rho})\bar{u}_\mu\delta u_\nu + \bar{p}\delta g_{\mu\nu} + \delta p\bar{g}_{\mu\nu} + \pi_{\mu\nu}. \quad (5.25)$$

Note that the anisotropic pressure $\pi_{\mu\nu}$ is purely perturbed quantity and $\delta g_{\mu\nu} = a^2 h_{\mu\nu}$. Choosing the comoving coordinates, so one has $\bar{u}^i = 0$. Using the normalisation of the four-velocity, i.e. $\bar{g}_{\mu\nu}\bar{u}^\mu\bar{u}^\nu = -1$, we can obtain

$$\bar{u}^0 = 1/a, \quad \bar{u}_0 = a. \quad (5.26)$$

If we expand the normalisation relation of the four-velocity up to the first order, then

$$\delta g_{\mu\nu}\bar{u}^\mu\bar{u}^\nu + 2\bar{g}_{\mu\nu}\delta u^\mu\bar{u}^\nu = 0. \quad (5.27)$$

Thus we can get the relation between metric perturbation h_{00} and δu^0 , i.e.

$$\delta u^0 = \frac{h_{00}}{2a^3}. \quad (5.28)$$

Expanding the four-velocity up to the first order, we get

$$\delta u_\mu = \bar{g}_{\mu\nu}\delta u^\nu + h_{\mu\nu}\bar{u}^\nu, \quad \delta u^\mu = \bar{g}^{\mu\nu}\delta u_\nu - \bar{g}^{\mu\rho}h_{\rho\nu}\bar{u}^\nu. \quad (5.29)$$

So from eq.(5.29), we can get

$$\delta u_0 = \frac{h_{00}}{2a}, \quad a\delta u^i = \frac{\delta u_i}{a} - \frac{1}{a^2}h_{i0}. \quad (5.30)$$

Basing on the eq.(5.26), eq.(5.28),eq.(5.30), we can finally obtain

$$\delta T_{00} = a^2 \delta \rho - \bar{\rho} h_{00}, \quad (5.31)$$

$$\delta T_{0i} = -a(\bar{\rho} + \bar{p})\delta u_i + \bar{p} h_{0i}, \quad (5.32)$$

$$\delta T_{ij} = \bar{p} h_{ij} + a^2 \delta_{ij} \delta p + \pi_{ij}. \quad (5.33)$$

The energy-momentum tensor with mixed components can be caculated from

$$\delta T^\mu{}_\nu = \bar{g}^{\mu\rho} \delta T_{\rho\nu} + \delta g^{\mu\rho} \bar{T}_{\rho\nu}. \quad (5.34)$$

Thus the mixed components of the energy-momentum tensor are

$$\delta T^0{}_0 = -\delta \rho, \quad (5.35)$$

$$\delta T^0{}_i = a^{-1}(\bar{\rho} + \bar{p})\delta u_i, \quad (5.36)$$

$$\delta T^i{}_j = \delta^i{}_j \delta p + \pi^i{}_j, \quad (5.37)$$

where $\pi^i{}_j = a^{-2} \delta^{il} \pi_{lj}$. The scalar-vector-tensor decomposition of the perturbed energy-momentum tensor takes the form

$$\delta T^0{}_0 = -\delta \rho, \quad (5.38)$$

$$\delta T^0{}_i = a^{-1}(\bar{\rho} + \bar{p})(D_i \delta u^S + \delta u_i^V), \quad (5.39)$$

$$\delta T^i{}_j = \delta^i{}_j \delta p + a^{-2}(D^i D_j \pi^S + D^i \pi_j^V + D_j \pi^{iV} + \pi_j^{iT}). \quad (5.40)$$

5.2.2 Scalar-Vector-Tensor decomposition of the metric perturbation

In the following we take $K = 0$ as example for simplicity. When we study the cosmological perturbation, it is convenient to transform into the conformal time coordinates, defined by $d\tau = dt/a$. The spatially-flat FLRW metric becomes

$$ds^2 = a(\tau)^2 [-d\tau^2 + \delta_{ij} dx^i dx^j]. \quad (5.41)$$

In our scenario of cosmological perturbations, we perturb the background FLRW metric up to first order as

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu}). \quad (5.42)$$

The perturbed metric can be decomposed as

$$ds^2 = a^2 \left[-(1 + 2\phi)d\tau^2 + 2B_i dx^i d\tau + (\delta_{ij} + h_{ij})dx^i dx^j \right], \quad (5.43)$$

where ϕ is a scalar, B_i is a 3-vector and E_{ij} is a symmetric 3-tensor. As we already discussed in (2.25), B_i contains a scalar mode B in addition to a pure transverse vector mode $\hat{B}_i$; we can always perform the decomposition

$$B_i = \hat{B}_i + D_i B, \quad (5.44)$$

where $D^i \hat{B}_i = 0$. Similarly, as (2.27), E_{ij} contains, in addition to a pure (transverse and traceless) tensor mode $\hat{h}_{ij}$, a pure (divergenceless) vector mode $\hat{E}_i$ and two scalars E and ψ :

$$E_{ij} = \hat{h}_{ij} + D_i \hat{E}_j + D_j \hat{E}_i + \left(D_i D_j - \frac{1}{3} \delta_{ij} \Delta \right) E - \psi \delta_{ij}, \quad (5.45)$$

where $D^i \hat{E}_i = 0$, $D^i \hat{h}_{ij} = 0$, $\hat{h}^i_i = 0$, and $\Delta = \delta^{ij} D_i D_j$ is the Laplacian on three dimensional Euclidean space $\mathbb{R}^3$. Effectively, we have decomposed the ten degrees of freedom in $h_{\mu\nu}$ into four scalars ϕ, ψ, B, E with 1 degree of freedom each, two divergenceless 3-vectors $\hat{B}_i, \hat{E}_i$ with 2 degrees of freedom each, and one transverse traceless 3-tensor $\hat{h}_{ij}$ with 2 degrees of freedom.

The perturbations (5.43) are not independent of general coordinate transformations. There are four gauge invariant variables, i.e. 2 gauge invariant scalar perturbations, 1 gauge invariant vector perturbation, 1 gauge invariant tensor perturbation. Thanks to gauge freedom we can set any 4 components of the metric to zero. We will choose the longitudinal gauge (Newtonian gauge), where we have [94, 206]:

$$B = 0, \quad E = 0, \quad \hat{E}_i = 0, \quad (5.46)$$

which completely fixes the coordinate system, leaving 6 ($2S + 2V + 2T$) dofs. In this gauge then, the perturbed metric is given by

$$ds^2 = a^2 \left[-(1 + 2\phi)d\tau^2 + 2\hat{B}_i dx^i d\tau + \left((1 - 2\psi)\delta_{ij} + 2\hat{h}_{ij} \right) dx^i dx^j \right]. \quad (5.47)$$

Finally, in component form, $h_{\mu\nu}$ can be written as

$$h_{00} = a^2(-2\phi), \quad h_{0i} = a^2(\hat{B}_i), \quad h_{ij} = a^2(-2\psi\delta_{ij} + 2\hat{h}_{ij}). \quad (5.48)$$

An interesting fact is that the metric perturbations ψ and ϕ , which are known as Bardeen's potential, become gauge invariant after applying the longitudinal gauge [207].

5.2.3 Perturbation equations

The evolution equations for the perturbations are given by the linearized Einstein equation

$$\delta G^\mu{}_\nu = \frac{1}{M_p^2} \delta T^\mu{}_\nu. \quad (5.49)$$

Field Equations for Scalar Perturbations

The perturbed metric of the scalar modes in the longitudinal gauge is

$$ds^2 = a^2 \left[-(1 + 2\phi)d\tau^2 + (1 - 2\psi)\delta_{ij} dx^i dx^j \right]. \quad (5.50)$$

Here we list the field equations for the scalar perturbation:

$$a^2 \delta G^0{}_0 = 6\mathcal{H}\psi' + 6\mathcal{H}^2\phi - 2\Delta\psi = -a^2 \frac{1}{M_p^2} \delta\rho, \quad (5.51)$$

$$a^2 \delta G^0{}_i = 2D_i(-\psi' - \mathcal{H}\phi) = a \frac{1}{M_p^2} (\bar{\rho} + \bar{p}) D_i \delta u, \quad (5.52)$$

$$\begin{aligned} a^2 \delta G^i{}_j &= [2\psi'' + 4\mathcal{H}\psi' + 2\mathcal{H}\phi' + 4\mathcal{H}'\phi + 2\mathcal{H}^2\phi + \Delta(-\psi + \phi)] \delta^i{}_j, \\ &\quad - D^i D_j(-\psi + \phi) = a^2 \frac{1}{M_p^2} (\delta^i{}_j \delta p + D^i D_j \pi^S), \end{aligned} \quad (5.53)$$

where $\mathcal{H} = \frac{a'}{a}$, and ' denotes derivative with respect to the conformal time τ . The δG^i_j can be separated into trace and traceless part. So the field equations of the scalar perturbation can be rewrite as following

$$6\mathcal{H}\psi' + 6\mathcal{H}^2\phi - 2\Delta\psi = -a^2 \frac{1}{M_p^2} \delta\rho, \quad (5.54)$$

$$2D_i(-\psi' - \mathcal{H}\phi) = a \frac{1}{M_p^2} (\bar{\rho} + \bar{p}) D_i \delta u, \quad (5.55)$$

$$6\psi'' + 12\mathcal{H}\psi' + 6\mathcal{H}\phi' + 12\mathcal{H}'\phi + 6\mathcal{H}^2\phi + 3\Delta(-\psi + \phi) = a^2 \frac{3}{M_p^2} \delta p, \quad (5.56)$$

$$-(D^i D_j - \frac{1}{3} \delta^i_j \Delta)(-\psi + \phi) = \frac{a^2}{M_p^2} D^i D_j \pi^S. \quad (5.57)$$

From the Eq.(5.57), we shall see $\phi = \psi$ when the anisotropic pressure π^μ_ν is absent. The anisotropic pressure is sourced by the quadrupole moments of photons and neutrinos distributions[208], so $\phi \neq \psi$ in the early universe. But $\phi = \psi$ holds, for example, when cold dark matter dominated the universe. Note that $\phi \neq \psi$ may happen, when we are not working in the frame of GR, even though cold dark matter or dark energy dominates.

Reformulating the field equations of the scalar perturbation using the condition $\phi = \psi$, then

$$6\mathcal{H}\phi' + 6\mathcal{H}^2\phi - 2\Delta\phi = -a^2 \frac{1}{M_p^2} \delta\rho, \quad (5.58)$$

$$2D_i(-\phi' - \mathcal{H}\phi) = a \frac{1}{M_p^2} (\bar{\rho} + \bar{p}) D_i \delta u, \quad (5.59)$$

$$2\phi'' + 6\mathcal{H}\phi' + 4\mathcal{H}'\phi + 2\mathcal{H}^2\phi = a^2 \frac{1}{M_p^2} \delta p. \quad (5.60)$$

If the energy perturbation is expressed in the comoving gauge, i.e. $\delta T^0_i = 0$, we can then derive $\Delta\phi = a^2 \frac{1}{2M_p^2} \delta\rho$, using Eq.(5.58, 5.59). This is the Poisson equation which means that the Bardeen potential is the ordinary Newton potential in the appropriate limit.

Considering the pressure is a thermodynamical function of energy density and

entropy S , then

$$\delta p = c_s^2 \delta \rho + T \delta S. \quad (5.61)$$

Using Eq.(5.61), and substituting Eq.(5.60) into Eq.(5.58), we get

$$\phi_k'' + 3\mathcal{H}(1 + c_s^2)\phi_k' - c_s^2 k^2 \phi_k + [2\mathcal{H}' + (1 + 3c_s^2)\mathcal{H}^2]\phi_k = \frac{a^2}{2M_p^2} T \delta S, \quad (5.62)$$

where we get this result in Fourier space. If we consider adiabatic perturbations, i.e. $\delta S = 0$, then Eq.(5.62) just describe a simple parametric oscillator.

Field Equations for vector perturbations

The perturbed metric of the vector modes in the longitudinal gauge is

$$ds^2 = a^2 \left[-(1 + 2\phi)d\tau^2 + 2\hat{B}_i dx^i d\tau \right]. \quad (5.63)$$

So the field equations for the vector perturbation are

$$a^2 G^0_0 = \Delta \hat{B}_i = -\frac{2a}{3M_p^2} (\bar{\rho} + \bar{p}) \delta u_i^V, \quad (5.64)$$

$$a^2 G^0_j = \hat{B}_i{}' + 2\mathcal{H}\hat{B}_i = \frac{a^2}{3M_p^2} \bar{p} (D^i \pi_j^V + D^j \pi_i^V). \quad (5.65)$$

If there is no anisotropic pressure $\pi_j^V = 0$, Eq.(5.65) will tell us $\hat{B}_i \propto a^{-2}$. So it is well known that vector perturbations decay as a^{-2} in the context of an expanding universe, we can then set them to zero. But this is not the case when we consider contracting universe in the bouncing scenario [92].

Field Equations for tensor perturbations

The perturbed metric of the tensor modes in the longitudinal gauge is

$$ds^2 = a^2 \left[-(1 + 2\phi)d\tau^2 + (1 + 2\hat{h}_{ij})dx^i dx^j \right]. \quad (5.66)$$

The field equation of the tensor part is

$$h''_{ij} + 2\mathcal{H}h'_{ij} + \Delta h_{ij} = \frac{a^2}{M_p^2} \pi_j^{iT}. \quad (5.67)$$

In agreement of the observational facts, we can set π_j^{iT} to zero. Considering non-expanding case for example, i.e. $\mathcal{H} = 0$, so

$$\square \hat{h}_{ij} = 0. \quad (5.68)$$

This is just the linearized gravitational wave equation in Minkowski background.

5.3 Discussion

In the context of GR or generic higher derivative gravity, it is difficult to obtain a stable non-singular bouncing universe since the ghost problems in matter or gravitational sector. Even if the theory is absent of ghost, we still need study the cosmological perturbation. A stable bounce requires that all perturbation modes are bounded across the bounce. On the other hand, we need introduce super-stiff matter to suppress the growth of anisotropy during the contracting phase, which is known as Ekpyrotic mechanism [119] as we already discussed. We will show that nonlocal gravity provides new insight in these problems in the next Chapter.

Chapter 6

Bouncing universe in nonlocal gravity

This Chapter is mainly on studying the exact bouncing solutions in the nonlocal quadratic scalar curvature gravity model. In Section 6.1, we found exact bouncing solutions without adding matter around homogeneous and isotropic bouncing cosmological backgrounds. Moreover we study the scalar-vector-tensor perturbations to verify the stability of these solutions. In Section 6.2, we show that it is possible to realize a cosmological bouncing solution in an anisotropic but homogeneous Bianchi-I background [114] with proper matter sector. This Chapter is mostly based on my publications **P2** and **P5**.

6.1 Stable and isotropic bouncing universe

As we already illustrated in the last Chapter, bouncing solutions are typically found by modifying the matter sector by adding fields which violate the null energy condition (NEC), and/or by modifying GR, for instance, by adding higher derivative terms to the Einstein-Hilbert (EH) action [87–89, 92, 202–204]. NEC violation and higher derivative actions generally imply sicknesses in the form of ghost (fields with wrong-sign kinetic terms) and gradient instabilities.

In this Section, we introduce a class of nonlocal $R+R\mathcal{F}R$ theory with a cosmological constant which is ghost-free around homogeneous and isotropic bouncing cosmological backgrounds. $\mathcal{F}$ is the nonlocal term (discussed below) essential for avoiding ghosts. To the best of our knowledge, we show for the first time that a class of higher derivative gravity can give stable, non-singular bouncing scenarios, in the absence of matter, and without any vector or tensor modes at the linearized level around a time dependent background. Only one scalar mode survives, which can be made ghost-free, and exhibits bounded, oscillatory evolution across the bounce.

6.1.1 Nonlocal $R\mathcal{F}R$ gravity without matter

Nonlocal generalization of $R + R^2$ by adding an infinite series of covariant derivatives reads [41]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + R\mathcal{F}(\square)R - \Lambda \right] \quad (6.1)$$

where Λ is the cosmological constant, the form factor $\mathcal{F}(\square)$ is an arbitrary analytic function of d'Alembertian, $\square = g_{\mu\nu} \nabla^\mu \nabla^\nu$, containing all orders of $\square$. It has the power series expansion

$$\mathcal{F}(\square) = \sum_{n=0}^{\infty} f_n \square_s^n, \quad (6.2)$$

where f_n are dimensionless coefficients, dimensionless $\square_s = \square/M_s^2$, and $M_s (< M_p \sim 10^{18} \text{ GeV})$ is a new high energy scale of nonlocality at which higher derivative terms become important. The equations of motion (EOM) of Eq.(6.1) are [52]

$$\begin{aligned} E^\mu{}_\nu &\equiv - [M_p^2 + 4\mathcal{F}(\square)R] G^\mu{}_\nu - R\mathcal{F}(\square)R\delta^\mu{}_\nu \\ &\quad + 4(\nabla^\mu \partial_\nu - \delta^\mu{}_\nu \square) \mathcal{F}(\square)R + 2\mathcal{K}^\mu{}_\nu \\ &\quad - \delta^\mu{}_\nu (\mathcal{K}^\sigma{}_\sigma + \tilde{\mathcal{K}}) - \Lambda \delta^\mu{}_\nu = 0, \end{aligned} \quad (6.3)$$

where

$$\mathcal{K}^\mu{}_\nu = \frac{1}{M_s^2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (\partial^\mu \square_s^l R) (\partial_\nu \square_s^{n-l-1} R), \quad \tilde{\mathcal{K}} = \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (\square_s^l R) (\square_s^{n-l} R). \quad (6.4)$$

The trace of the EOM is

$$(M_p^2 - 12\square\mathcal{F}(\square)) R - 2\mathcal{K}^\mu{}_\mu - 4\tilde{\mathcal{K}} - 4\Lambda = 0. \quad (6.5)$$

The structure of the form factor $\mathcal{F}(\square)$ is determined by finding the second order action around a fixed background and demanding that all perturbations be ghost-free. Note that most of the upcoming computations are carried out covariantly. Solving the EOM Eq.(6.31) in full generality is non-trivial but the following ansatz for the background Ricci scalar helps in making headway [41, 61, 97]

$$\bar{\square}\bar{R} = r_1\bar{R} + r_2 \quad (6.6)$$

where r_1 and r_2 are dimensionful constant parameters. The ansatz Eq.(6.6) is essentially the trace EOM of local $R + R^2$ theory with a cosmological constant given by [41]

$$\mathcal{L} = \frac{M_p^2}{2} R + f_0 R^2 - \Lambda. \quad (6.7)$$

The ansatz is solved by bouncing backgrounds with cosine hyperbolic and exponential type scale factors, as we will see soon. Straightforwardly, we then obtain

$$\bar{\square}^n \bar{R} = r_1^n \left(\bar{R} + \frac{r_2}{r_1} \right) \implies \mathcal{F}(\bar{\square})\bar{R} = \mathcal{F}_1 \bar{R} + \mathcal{F}_2 \quad (6.8)$$

where

$$\mathcal{F}_1 = \mathcal{F}(r_1) \quad \text{and} \quad \mathcal{F}_2 = \frac{r_2}{r_1} (\mathcal{F}_1 - f_0) \quad (6.9)$$

are constants. Substituting the ansatz Eq.(6.6) in the EOM Eq.(6.3) and trace EOM Eq.(6.5) gives us the following unique conditions on the form factor $\mathcal{F}(\bar{\square})$

$$\mathcal{F}_1 = \mathcal{F}'(r_1) = 0, \quad \mathcal{F}_2 = -\frac{M_p^2}{4}, \quad \Lambda = -\frac{M_p^2}{16f_0} \quad (6.10)$$

where $\mathcal{F}'(r_1)$ denotes the derivative of $\mathcal{F}(\bar{\square})$ with respect to $\bar{\square}_s$, evaluated at $\bar{\square} = r_1$. From Eqs. (6.9) and (6.10), we can deduce that

$$f_0 < 0 \quad \text{for} \quad \Lambda > 0, \quad f_0 > 0 \quad \text{for} \quad \Lambda < 0. \quad (6.11)$$

Note that in Ref.[61, 97], bouncing solutions and linear perturbations were studied in the presence of fluid radiation, which corresponds to the case when $\mathcal{F}_1 < 0$. In this Chapter, we emphasize that a bounce can be achieved in the theory satisfying the conditions in Eq.(6.10), without requiring any radiation/matter.

6.1.2 Perturbations

We now study perturbations and their stability around backgrounds satisfying the ansatz Eq.(6.6) in two, equivalent ways - both at the linearized level of EOM, and at the quadratic level of the action. Linearizing the full EOM Eq.(6.31) and trace EOM Eq.(6.32) around the background given by Eq.(6.6), and substituting the conditions in Eq.(6.10), we obtain, respectively

$$\begin{aligned} \delta E^\mu_\nu &= \left[\left(4\bar{G}^\mu_\nu + 2\delta^\mu_\nu \bar{R} - 4(\bar{\nabla}^\mu \bar{\nabla}_\nu - \delta^\mu_\nu \bar{\square}) \right) (\bar{\square} - r_1) \right. \\ &\quad \left. - 2(\partial_\nu \bar{R} \partial^\mu + \partial^\mu \bar{R} \partial_\nu) + 2\delta^\mu_\nu (\partial_\sigma \bar{R} \partial^\sigma + \bar{\square} \bar{R}) \right] \mathcal{Z}(\bar{\square}) \zeta = 0 \\ \delta E &= \left[\partial^\mu \bar{R} \partial_\mu + 2(r_1 \bar{R} + r_2) + 3(\bar{\square} - r_1)^2 + (\bar{R} + 3r_1)(\bar{\square} - r_1) \right] \mathcal{Z}(\bar{\square}) \zeta = 0. \end{aligned} \quad (6.12)$$

where

$$\zeta = \delta(\bar{\square}) \bar{R} + (\bar{\square} - r_1) \delta \bar{R} \quad (6.13)$$

and

$$\mathcal{Z}(\bar{\square}) = \frac{\mathcal{F}(\bar{\square})}{(\bar{\square} - r_1)^2}. \quad (6.14)$$

Note that the perturbed EOM Eq.(6.12) has only one perturbation variable ζ , and we will shortly see that this leads to interesting physics. One may wonder about the absence in Eq.(6.12) of the perturbation of Einstein tensor from GR.

This can be easily understood since the following coefficient of Einstein tensor in Eq.(6.31) vanishes upon using the conditions in Eq.(6.10)

$$[M_p^2 + 4\mathcal{F}(\bar{\square})\bar{R}] = 0. \quad (6.15)$$

From the $i \neq j$ perturbed EOM in Eq.(6.12), we ultimately obtain the simple EOM for ζ

$$\mathcal{Z}(\bar{\square})\zeta = 0. \quad (6.16)$$

To deduce the kinetic structure of ζ and, therefore, the (background dependent) ghost-free form factor $\mathcal{F}(\bar{\square})$, we compute the second variation of the action Eq. (6.1) around Eq. (6.6), imposing further the conditions $\mathcal{F}_1 = 0$, $\mathcal{F}_2 = -\frac{M_p^2}{4}$

$$\begin{aligned} \delta^2 S = \int d^4x \sqrt{-\bar{g}} \left\{ \frac{M_p^2}{4} (\delta_{GR} - \delta^{(2)}R) - \Lambda \left(\frac{h^2}{8} - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} \right) + \frac{h}{2} \bar{R} \mathcal{F}(\bar{\square}) \delta R \right. \\ \left. + \frac{h}{2} \bar{R} \delta \mathcal{F}(\bar{\square}) \bar{R} + \delta R \delta \mathcal{F}(\bar{\square}) \bar{R} + \bar{R} \delta \mathcal{F}(\bar{\square}) \delta R + \bar{R} \delta^{(2)} \mathcal{F}(\bar{\square}) \bar{R} \right\} \end{aligned} \quad (6.17)$$

where $\sqrt{-\bar{g}} \delta_{GR} \equiv \delta^{(2)}(\sqrt{-\bar{g}}R)$ and $\delta^{(2)}R$ is the second variation of R . After considerable manipulation of terms in Eq.(6.17), the quadratic action remarkably simplifies to

$$\delta^2 S = \int d^4x \sqrt{-\bar{g}} \zeta \mathcal{Z}(\bar{\square}) \zeta. \quad (6.18)$$

As expected, at the quadratic level of the action, the presence of the sole variable, ζ , is consistent with our earlier result obtained from the linearized EOM in Eq.(6.16)¹ - by varying $\delta^2 S$ with respect to ζ , we exactly recover the linearized EOM for ζ .

Simplifying the quadratic action Eq.(6.17) and the linearized EOM Eq.(6.12) involves using the formulas

$$\delta \mathcal{F}(\square) = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \square_s^l \delta(\square_s) \square_s^{n-l-1} \quad (6.19)$$

¹Our result Eq.(6.18) turns out to be the same as the nonlocal part of the quadratic action around an inflationary background satisfying $\bar{\square}\bar{R} = r_1\bar{R}$ [209]

and

$$\begin{aligned} \delta^{(2)}\mathcal{F}(\square) &= \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \square_s^l \delta^{(2)}(\square_s) \square_s^{n-l-1} \\ &+ 2 \sum_{n=2}^{\infty} \sum_{l=0}^{n-2} \sum_{k=0}^{n-l-2} f_n \square_s^l \delta(\square_s) \square_s^k \delta(\square_s) \square_s^{n-k-l-2}. \end{aligned} \quad (6.20)$$

Using these, the background ansatz Eq.(6.6) and conditions in Eq.(6.10), we can simplify the last five terms in the quadratic action by using the following relations

$$\begin{aligned} \delta(\mathcal{F}(\square)R) &= (\bar{\square} - r_1) \mathcal{Z}(\bar{\square}) \zeta \\ \int d^4x \sqrt{-\bar{g}} \bar{R} \delta \mathcal{F}(\square) \delta R &= \int d^4x \sqrt{-\bar{g}} \left[\left(\bar{R} + \frac{r_2}{r_1} \right) \delta(\square) \mathcal{Z}(\bar{\square}) (\bar{\square} - r_1) \delta R \right. \\ &\quad \left. + \frac{h}{2} \frac{r_2}{r_1} (\mathcal{F}(\bar{\square}) - f_0) \delta R \right]. \\ \int d^4x \sqrt{-\bar{g}} \bar{R} \delta^{(2)}\mathcal{F}(\square) \bar{R} &= \int d^4x \sqrt{-\bar{g}} \left[\left(\bar{R} + \frac{r_2}{r_1} \right) \delta(\square) \mathcal{Z}(\bar{\square}) \delta(\square) \bar{R} \right. \\ &\quad \left. + \frac{h}{2} \frac{r_2}{r_1} (\bar{\square} - r_1) \mathcal{Z}(\bar{\square}) \delta(\square) \bar{R} + \frac{M_p^2}{4r_1} \frac{h}{2} \left(\delta(\square) \bar{R} + \delta^{(2)}(\square) \bar{R} \right) \right]. \end{aligned}$$

Substituting Eq.(6.21) in Eq.(6.17), using the following identity Eq.(6.21) for some scalars X and Y , performing numerous integration by parts and omitting total derivatives, we finally obtain the result in Eq.(6.18)

$$\int d^4x \sqrt{-\bar{g}} Y \delta(\square) X = \int d^4x \sqrt{-\bar{g}} \left[X \delta(\square) Y - \frac{h}{2} (Y \bar{\square} X - X \bar{\square} Y) \right]. \quad (6.21)$$

To clearly expose the physical degrees of freedom in our theory at the level of the quadratic action, we study SVT modes in the context of (1+3) cosmological perturbation theory [206]. We fix the background $\bar{g}_{\mu\nu}$ to be spatially flat FLRW metric given by the line element

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad (6.22)$$

where $a(t)$ is the scale factor, satisfying the ansatz Eq.(6.6). We write the full metric as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is the perturbation. In longitudinal gauge, $h_{\mu\nu}$ has components

$$h_{00} = a^2(-2\phi), \quad h_{0i} = a^2(\hat{B}_i), \quad h_{ij} = a^2(-2\psi\delta_{ij} + 2\hat{h}_{ij}). \quad (6.23)$$

Note that

$$\partial^i \hat{B}_i = \partial^i \hat{h}_{ij} = \hat{h}_i{}^i = 0, \quad (6.24)$$

so $h_{\mu\nu}$ has 6 degrees of freedom (2S+2V+2T) after gauge fixing² [98]. Given this decomposition, it is straightforward to see that ζ (defined in Eq.(6.13)) is a linear combination of ϕ and ψ (see A.13), and therefore is effectively a scalar under $SO(3)$. The quadratic action Eq.(6.18) then solely has one scalar propagating degree of freedom ζ . It is easy to understand why the vector $\hat{B}_i$ and tensor mode $\hat{h}_{ij}$ are absent. The background conditions we imposed, see Eq.(6.10), lead to the removal of vector and tensor modes that could potentially come from the term proportional to M_p^2 in Eq.(6.17). All other terms in $\delta^2 S$ generate only scalars in terms of ϕ and ψ . This confirms the result we had obtained earlier from the linearized EOM.

For the theory to have no extra degrees of freedom or ghosts at the quadratic level of the action in Eq.(6.18), the kinetic operator $\mathcal{Z}(\bar{\square})$ can have at most a single zero according to the Weierstrass product theorem. For the case of $\Lambda > 0$ (see Eq.(6.11)), we therefore make the following minimal choice for $\mathcal{F}(\bar{\square})$ (see Eq.(6.14)) satisfying all the necessary conditions in Eq.(6.10)

$$\mathcal{F}(\bar{\square}) = \frac{1}{M_s^6} (\bar{\square} - m^2) (\bar{\square} - r_1)^2 e^{\gamma(\bar{\square})} \quad (6.25)$$

where $m^2 \geq 0$ is some arbitrary mass scale and γ is an arbitrary entire function of $\bar{\square}/M_s^2$. This choice ensures that the kinetic term for ζ has only one zero at $\bar{\square} = m^2$. Note that the kinetic term for ζ has the correct sign to avoid ghosts. Similarly, for the case of $\Lambda < 0$, the minimal choice for $\mathcal{F}(\bar{\square})$ would be one

²The longitudinal gauge removes 2 scalar and 2 vector degrees of freedom from the original total of 10 in $h_{\mu\nu}$

without any zeros for $\mathcal{Z}(\bar{\square})$

$$\mathcal{F}(\bar{\square}) = \frac{1}{M_s^4} (\bar{\square} - r_1)^2 e^{\gamma(\bar{\square})}. \quad (6.26)$$

In this case, ζ contains no zeros in the kinetic operator and acts like a p -adic scalar [35].

6.1.3 Stable bouncing universe

We now specialize to known bouncing solutions [41, 61, 210]. They all satisfy the ansatz Eq.(6.6), are vacuum solutions of the nonlocal theory Eq.(6.1), and possess only a single ghost-free scalar mode (and no vector or tensor modes) around the bouncing background at the linearized level when the nonlocal form factor $\mathcal{F}(\bar{\square})$ is chosen appropriately as just discussed

1. Cosine hyperbolic bounce with $a(t) = a_0 \cosh(\sqrt{r_1/2}t)$. In this case, $\Lambda > 0$.
2. Exponential bounce with $a(t) = a_0 e^{\frac{\lambda}{2}t^2}$. In this case, $\Lambda > 0$ for $\lambda > 0$ and $\Lambda < 0$ for $\lambda < 0$. For $\lambda > 0$ we have contraction followed by expansion like in a usual bounce. For $\lambda < 0$, the universe reaches a maximum scale factor at $t = 0$ and then starts to decrease in size. This is also a possible way the Big Bang singularity could be resolved.
3. Cyclic universe: This scenario was numerically studied in [61]. It was found that in the case of $\Lambda < 0$, there exists a cyclic scenario where the universe undergoes multiple bounces.

Let us restrict to the case of $\Lambda > 0$ and a cosine hyperbolic bounce, in which case $\mathcal{F}(\bar{\square})$ is given by Eq.(6.25), and the corresponding quadratic action for ζ reads

$$\delta^2 S = \frac{1}{M_s^6} \int d^4x \sqrt{-\bar{g}} \zeta e^{\gamma(\bar{\square})} (\bar{\square} - m^2) \zeta. \quad (6.27)$$

For a stable bounce, we would require the perturbation ζ to be well behaved in time. All solutions of the nonlocal EOM for ζ in Eq.(6.16), for $\mathcal{F}(\bar{\square})$ given in

Eq.(6.25), are captured by solutions of the local equation

$$(\bar{\square} - m^2)\zeta = 0 \quad (6.28)$$

because the nonlocal factor $e^{\gamma(\bar{\square})}$ does not introduce any new zeros in the kinetic operator [35, 41]. Setting $a_0 = 1, r_1 = m^2 = M_s^2$ for the cosine hyperbolic bounce, the generic solution of Eq.(6.28) in Fourier space looks like

$$\zeta_k(z) = c_1 H_k^{(+)}(z) + c_2 H_k^{(-)}(z), \quad (6.29)$$

where

$$H_k^{\pm} = \frac{\sqrt{e^{\sqrt{2}z}}}{1 + e^{\sqrt{2}z}} \left[\frac{\left(1 \pm i\sqrt{e^{\sqrt{2}z}}\right)^2}{1 + e^{\sqrt{2}z}} \right]^{\sqrt{\frac{2k^2}{M_s^2} + 1}}, \quad (6.30)$$

where $z = M_s t$ and $c_{1,2}$ are constants. We can easily deduce that ζ_k is real for $c_1 = c_2$. Here, we do not quantize ζ and merely wish to observe how it evolves classically through the bouncing phase. We have shown the solution ζ_k in Figure. 6.1, where we can see that it is oscillatory and bounded for the chosen values of $c_{1,2}$ and k . This behaviour persists to hold for any value of k with increasing number of bounded oscillations as $k \rightarrow \infty$. We do not present here the solutions for ζ around other bouncing backgrounds mentioned earlier since the nature of perturbations remains very similar to that of cosine hyperbolic bounce.

For the case of cosine hyperbolic bounce, we will have a de Sitter (inflationary) evolution as a late time attractor [41, 62, 97, 211]. We can observe from Fig. 6.1 that $\zeta \rightarrow 0$ in the late time leading to $(\bar{\square} - r_1)\delta R \rightarrow 0$. This observation confirms that we exactly recover the late time behavior of scalar fluctuations found in Ref.[97].

There is also the intriguing possibility of a phase of super inflation after cosine hyperbolic bounce, and before cosmic inflation. This makes this type of bounce an interesting framework for explaining low multipoles in the observed data for cosmic microwave background radiation [212]. It is important to note that the

Figure 6.1: Evolution in cosmic time $z = M_s t$ of a Fourier component of the scalar mode ζ_k for cosine hyperbolic bounce. We have set $a_0 = 1, r_1 = m^2 = M_s^2, c_1 = c_2 = 0.1M_s$, and $k = 20M_s$ (blue curve) and $k = 0$ (red curve).

late time de Sitter stage is significantly different in our case compared to [97], because of the absence of any tensor modes due to our background conditions in Eq.(6.10). Note that one can take the GR limit at the level of the action in Eq.(6.1), but not at the level of our background solution satisfying EOM in Eq.(6.3) with the uniquely tuned conditions in Eq.(6.10).

It is possible to have special points in the theory space of a gravity model where the physical spectrum differs from the usual expectations - for example, pure R^2 theory propagates a massless scalar and tensor modes around (A)dS background which can be seen by conformally transforming pure R^2 to Einstein frame where the action turns out to be GR with a massless scalar and cosmological constant[213]. However, around flat space, the spectrum contains only a massless scalar and no tensor modes (here, the conformal transformation into Einstein frame is singular). Another example would be Critical Gravity described by $\mathcal{L} = \frac{M_p^2}{2}(R - 2\Lambda + \alpha R^{\mu\nu} R_{\mu\nu} + \beta R^2)$ - here, the tunings $\alpha = -3\beta$ and $\beta = -1/2\Lambda$ remove the massive scalar mode and make the massive spin-2 mode massless [214]. In the context of nonlocal theories, the theory described by action in Eq. (6.1) permits special classes of (A)dS and non-maximally symmetric covariantly constant curvature backgrounds around which the physical spectrum consists of

only a scalar mode, and no tensor or vector modes [215]. In this Section, we find a special class of dynamical backgrounds where we can have such a physical spectrum with only a scalar mode as a result of Eq.(6.10).

6.2 Anisotropic bouncing universe

As we already mentioned in the last Chapter, one of the problems of bouncing universe in GR is that anisotropy dominates in contraction phase, which results in BKL instability [115, 116]. So we need introduce super-stiff matter to suppress anisotropy, which is the so called ekpyrotic mechanism [119–123].

In this Section, we restrict ourselves to the study of Bianchi-I type cosmology where there are three scale factors in the three spatial directions, respectively. We find a new non-singular bouncing solution in a Bianchi-I background in nonlocal, infinite derivative gravity, whose action includes the GR term (with a cosmological constant) and a nonlocal quadratic in Ricci scalar term (that is associated with a form factor which is an analytic non-polynomial function of the d'Alembertian $\square$ operator), and a matter sector (which is later determined from the equations of motion). We show that nonlocal gravity can ameliorate the challenges faced in a local theory of gravity when it comes to simultaneously satisfying positivity of energy density and absence of instabilities (ghost/tachyonic). Furthermore, we show that the problem of anisotropy dominating in contraction phase in GR will also be resolved in the frame of nonlocal gravity.

6.2.1 Nonlocal $R\mathcal{F}R$ gravity with matters

In order to find anisotropic bouncing solution in nonlocal gravity, now we add matter degrees of freedom into the theory. The equations of motion (EOM) become [41, 52, 61, 97]

$$\begin{aligned}
 & - \left[M_p^2 + 4\mathcal{F}(\square)R \right] G^\mu{}_\nu - R\mathcal{F}(\square)R\delta^\mu{}_\nu + 4(\nabla^\mu\partial_\nu - \delta^\mu{}_\nu\square)\mathcal{F}(\square)R \\
 & + 2\mathcal{K}^\mu{}_\nu - \delta^\mu{}_\nu(\mathcal{K}^\sigma{}_\sigma + \tilde{\mathcal{K}}) - \Lambda\delta^\mu{}_\nu + T^\mu{}_\nu = 0
 \end{aligned} \tag{6.31}$$

where T^μ_ν is the energy-momentum tensor. The trace EOM is

$$(M_P^2 - 12\Box\mathcal{F}(\Box))R - 2\mathcal{K}^\mu_\mu - 4\tilde{\mathcal{K}} - 4\Lambda = -T. \quad (6.32)$$

Substituting the ansatz (6.6) in the EOM (6.31), we obtain a simple expression for the energy-momentum tensor [61, 97]

$$\begin{aligned} T^\mu_\nu = & \left[M_P^2 + 4(\mathcal{F}_1 R + \mathcal{F}_2) \right] G^\mu_\nu + \Lambda \delta^\mu_\nu \\ & - 2\mathcal{F}'_1 \left[\partial^\mu R \partial_\nu R - \frac{\delta^\mu_\nu}{2} \left(g^{\sigma\rho} \partial_\sigma R \partial_\rho R + r_1 \left(R + \frac{r_2}{r_1} \right)^2 \right) \right] \\ & - 4\mathcal{F}_1 [\nabla^\mu \partial_\nu R - \delta^\mu_\nu (r_1 R + r_2)] + \delta^\mu_\nu \left[\mathcal{F}_1 R^2 - \frac{r_2}{r_1} \mathcal{F}_2 \right] \end{aligned} \quad (6.33)$$

where $\mathcal{F}'_1 \equiv \mathcal{F}'(r_1)$ denotes the first derivative of $\mathcal{F}(\Box)$ with respect to $\Box$, evaluated at $\Box = r_1$. In particular, for traceless matter with $T^\mu_\mu = 0$, substituting the ansatz (6.6) in the trace EOM (6.32) gives us the following unique conditions on $\mathcal{F}(\Box)$ [61, 97]

$$\mathcal{F}'_1 = 0, \quad \mathcal{F}_2 = -\frac{M_P^2}{4} + 3r_1 \mathcal{F}_1, \quad \Lambda = -\frac{M_P^2}{4} \frac{r_2}{r_1} \quad (6.34)$$

which substituting in (6.33) provides an even simpler expression for the energy-momentum tensor

$$T^\mu_\nu = 4\mathcal{F}_1 \left[(R + 3r_1) G^\mu_\nu - \nabla^\mu \partial_\nu R + \frac{1}{4} \delta^\mu_\nu (R^2 + 4r_1 R + r_2) \right]. \quad (6.35)$$

Everything discussed until now is covariant, that is, we did not choose any specific metric. In our work, we will consider cosmological spacetimes, particularly bouncing scenarios. As a specific example, for spatially flat, homogeneous and isotropic FLRW, the scale factor [41]

$$a(t) = a_0 \cosh \left(\sqrt{\frac{r_1}{2}} t \right), \quad (6.36)$$

where a_0 is the scale factor at the bounce point $t = 0$, describes a bouncing universe which solves the ansatz (6.6) with $r_2 = -6r_1^2$ [41, 61, 63, 97]. It is therefore also a solution of the EoM (6.31) of quadratic curvature nonlocal gravity (6.1). In the case of traceless matter, this cosh bounce is an exact solution of the EoM (6.35) with an energy density arising from radiation and given by $\rho_r = -27\mathcal{F}_1 r_1^2$, which implies $\mathcal{F}_1 < 0$ if we want positive energy density $\rho_r > 0$ for stability [97].

In a local $R + f_0 R^2 - \Lambda$ theory, $\mathcal{F}_1 = f_0$ and $f_0 < 0$ leads to a tachyonic instability [216]. In nonlocal gravity, on the other hand, it is possible to fix $\mathcal{F}_1 < 0$ without leading to any instabilities. Note that the cosh solution approaches de Sitter (dS) asymptotically at early ($t \rightarrow -\infty$) and late ($t \rightarrow +\infty$) times, with a constant Ricci scalar $R = 6r_1^3$.

6.2.2 Anisotropic bouncing solutions

As just mentioned, cosmological bouncing solutions which satisfy the ansatz (6.6) are already known when the background is flat, homogeneous and isotropic (FLRW). But a bounce need not happen in an isotropic manner. Therefore, we choose a bit more complicated spacetime such as the homogeneous but anisotropic Bianchi-I background. The Bianchi-I spacetime metric can in general be expressed as [114]

$$ds^2 = -dt^2 + a^2 [e^{2\beta_1} dx_1^2 + e^{2\beta_2} dx_2^2 + e^{2\beta_3} dx_3^2] \quad (6.37)$$

where $a = a(t)$ is the average scale factor (geometric mean of scale factors in the three spatial directions), and where deviation from FLRW is parameterized by $\beta_i = \beta_i(t)$ (in each spatial direction $i = 1, 2, 3$) which satisfy

$$\beta_1 + \beta_2 + \beta_3 = 0. \quad (6.38)$$

³Another interesting bouncing solution (in spatially flat FLRW) of the ansatz (6.6), and hence of quadratic curvature nonlocal gravity (6.1), is $a(t) = a_0 e^{\frac{\lambda}{2}t^2}$ with $r_1 = -6\lambda$, $r_2 = 12\lambda^2$ and $\Lambda = \lambda M_P^2/2$ [63]. It is worth noting here that bouncing cosmology has also been studied in the context of Deser-Woodard infrared nonlocal modification of gravity [217].

A quantity called shear σ^2 defined as

$$\sigma^2 = \dot{\beta}_1^2 + \dot{\beta}_2^2 + \dot{\beta}_3^2, \quad (6.39)$$

essentially parameterizes the extent of anisotropic deviation of the background away from FLRW. The Ricci scalar for the anisotropic metric (6.37) is

$$R = 12H^2 + 6\dot{H} + \sigma^2 = \bar{R} + \sigma^2, \quad (6.40)$$

where $\bar{R}$ is the Ricci scalar for an FLRW metric with the scale factor a , and $H = \dot{a}/a$ is the average of the three Hubble factors in the three spatial directions. Particularly, from the metric (6.37), we can define the three scale and Hubble factors in the three spatial directions ($i = 1, 2, 3$) as

$$a_i = ae^{\beta_i}, \quad H_i = H + \dot{\beta}_i, \quad H = \frac{1}{3}(H_1 + H_2 + H_3). \quad (6.41)$$

The $\square$ operator for the Bianchi-I metric (6.37) acting on a time-dependent scalar is

$$\square = -\frac{\partial^2}{\partial t^2} - 3H\frac{\partial}{\partial t}, \quad (6.42)$$

which is exactly the same as in FLRW. In this paper, we generalize the known cosh bounce in FLRW to Bianchi-I backgrounds (6.37). We take the average scale factor to be cosh, see (6.36), where a_0 is now the average scale factor at the bounce point $t = 0$. Henceforth, we choose a convenient normalization of $a_0 = 1$ in (6.36) so that the minimum value of the average scale factor is 1. Just as in FLRW, we assume the following ansatz to hold for the Bianchi-I metric (6.37)

$$\square R = s_1 R + s_2, \quad (6.43)$$

where $s_{1,2}$ are some dimensionful constants. Substituting (6.40) in (6.43), we obtain $s_1 = r_1, s_2 = r_2$ (where $r_{1,2}$ are the dimensionful constants in FLRW cosh bounce with $\square \bar{R} = r_1 \bar{R} + r_2$) and the following second order differential equation

$$\square \sigma^2 = s_1 \sigma^2, \quad (6.44)$$

which can be solved to obtain the shear as

$$\sigma^2 = \sigma_0^2 \operatorname{sech}^2 \left(\sqrt{\frac{r_1}{2}} t \right) = \frac{\sigma_0^2}{a^2} \quad (6.45)$$

subject to boundary conditions $\sigma^2|_{t=0} = \sigma_0^2$ and $\frac{d}{dt}(\sigma^2)|_{t=0} = 0$. The first condition is normalization, while the second is required to obtain positive σ^2 .

In GR, shear goes like $\sigma^2 \sim a^{-6}$, and the effective energy density coming from the anisotropic background $\rho_\sigma = \sigma^2/2$ therefore dominates over matter, radiation and spatial curvature when the scale factor a becomes very small [86]. This leads the geometry during the contraction phase into a Kasner solution, and also causes BKL instability [115].

On the other hand, in a cosh bounce (6.36) satisfying the ansatz (6.43) in a Bianchi-I universe (6.37), $\sigma^2 \sim a^{-2}$ increases much slower in comparison during contraction, reaches a maximum at the bounce point $t = 0$, and then decreases during the expansion phase (and, moreover, exponentially as $\sim e^{-2t}$ at late times). From (6.39) and (6.45), we obtain (for $i = 1, 2, 3$)

$$\dot{\beta}_i = \sigma_0 b_i \operatorname{sech} \left(\sqrt{\frac{r_1}{2}} t \right) \quad \text{or} \quad \beta_i = \sqrt{\frac{2}{r_1}} \sigma_0 b_i \operatorname{gd} \left(\sqrt{\frac{r_1}{2}} t \right) \quad (6.46)$$

where b_i are dimensionless coefficients and $\operatorname{gd}(t)$ is the Gudermannian function which is defined as $\operatorname{gd}(t) = \int_0^t dx \operatorname{sech}(x)$. Any constants of integration can be absorbed into coordinate redefinitions. From (6.38) and (6.39), we get the following constraints

$$b_1 + b_2 + b_3 = 0, \quad b_1^2 + b_2^2 + b_3^2 = 1 \quad (6.47)$$

which admit an infinite number of solutions for the triplet (b_1, b_2, b_3) .

Background anisotropy is quantified by the shear $\sigma^2(t)$ which from (6.45) becomes maximum at the bounce point $t = 0$, and goes to zero exponentially as $t \rightarrow \pm\infty$ (see Fig.(6.2)). The universe therefore approaches isotropic and homogeneous FLRW asymptotically as $t \rightarrow \pm\infty$, along with entering a dS phase. We can understand better the isotropic and anisotropic limits of spacetime by

Figure 6.2: Time evolution (in corresponding units of M_P) of anisotropy factors $\beta_i(t)$ (top plot) and $\dot{\beta}_i(t)/M_P$ (middle plot) given in (6.46), shear $\sigma^2(t)/M_P^2$ given in (6.45) (middle plot), and Hubble factors in the three spatial directions H_i/M_P given in (6.41) (bottom plot). We have chosen $r_1 = M_s^2 = 0.1M_P^2$ and $\sigma_0 = 1M_P$ such that the universe is dominated by anisotropy as $t \rightarrow 0$. We chose $(b_1, b_2, b_3) = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}\right)$ in accordance with (6.47). In the middle plot, we see that the magnitude of background anisotropy (in each spatial direction $i = 1, 2, 3$) reaches a maximum at the bounce point $t = 0$, and then asymptotes to zero before and after, as the universe asymptotes to FLRW (and eventually to dS, as can be seen from the bottom plot where H_i approach constant values for $t \gg t_{\text{iso}}$ (see (6.48))). The cosmological constant $\Lambda = \frac{3}{2}M_P^2 M_s^2$ (see (6.54)). In models based on GR, the shear σ^2 decays like a^{-6} leading to strong growth of anisotropies during the contraction phase [86]. On the other hand, in nonlocal quadratic scalar curvature theory (6.1), since σ^2 evolves like a^{-2} , we can have a smoother transition from contraction to expansion.

looking at the structure of the Ricci scalar given in (6.40), where we can compare the FLRW $\bar{R}$ part of the full Ricci scalar R that depends on the average Hubble parameter H with the part corresponding to the anisotropic shear σ^2 . We can say that the spacetime anisotropy-dominated phase of evolution is when $\sigma^2(t) \gg \bar{R}$ (i.e., when the maximum contribution to the curvature scalar R comes from the shear) while it is isotropy-dominated when $\sigma^2(t) \ll \bar{R}$ (i.e., when the average evolution dominates over anisotropies). In Fig. (6.2), we depict the evolution of the universe which is extremely anisotropic towards $t \rightarrow 0$, reaching towards isotropy in the limit $t \rightarrow \pm\infty$. We can determine the time scale t_{iso} at which the universe starts turning towards isotropic evolution as follows

$$\bar{R} \sim \sigma^2 \implies t_{\text{iso}} \sim \pm \sqrt{\frac{2}{r_1}} \cosh^{-1} \left(\sqrt{\frac{3r_1 + \sigma_0^2}{6r_1}} \right). \quad (6.48)$$

Note that for the quadratic curvature theory in (6.1) with the nonlocality scale M_s , we can choose⁴ $r_1 = M_s^2$ as in Fig.(6.2). For the values chosen in Fig.(6.2), $r_1 = 0.1M_P^2$ and $\sigma_0 = 1M_P$, we obtain $t_{\text{iso}} \approx \pm 4.19M_P^{-1}$ at which point the shear drops to $\sigma^2(t = t_{\text{iso}}) \approx 0.46M_P^2$, down from its maximum value of $1M_P^2$ at $t = 0$. As time goes on, the shear drops to $\sigma^2(t = 2t_{\text{iso}}) \approx 0.09M_P^2$ and $\sigma^2(t = 4t_{\text{iso}}) \approx 0.002M_P^2$.

6.2.3 Matter sources for anisotropic bounces

Having completely fixed the anisotropic bouncing background in the previous section, we now determine the corresponding possible matter sectors which can drive such a non-singular evolution of the universe (6.36) with controlled growth of anisotropies (6.46). We show how anisotropic shear σ^2 effectively contributes to fluid energy densities and pressures. We also determine the conditions where all component energy densities are non-negative, ensuring a physically healthy

⁴This choice is just for simplicity. An exact expression for r_1 in terms of M_s^2 cannot be found just from the EOM of the nonlocal theory in (6.33) because from there, due to the presence of the form factor $\mathcal{F}(\Box)$ defined in (6.2), one obtains (in general) an infinite order algebraic equation for r_1 . Moreover, for an arbitrary $\mathcal{F}(\Box)$, there are an infinite number of undetermined coefficients f_n in its power series expansion, preventing us from uniquely fixing r_1 in terms of M_s^2 .

bounce mechanism. Moreover, we also show that under certain conditions, it is possible to have a bounce in vacuum (but with Λ).

Without loss of generality, the energy-momentum tensor T^μ_ν for quadratic curvature nonlocal gravity (6.1) around a Bianchi-I cosmological background (6.37) has the diagonal form

$$T^\mu_\nu = (-\rho, p_1, p_2, p_3) \quad (6.49)$$

where p_i (for $i = 1, 2, 3$) are pressures in x, y, z directions. We will soon see that anisotropic pressures necessarily arise in some (but not all) cases of possible matter sectors. For a perfect fluid, $p_1 = p_2 = p_3$ and the energy density of each component evolves as

$$\rho_I = \rho_{I0} a^{-3(1+w_I)} \quad (6.50)$$

where I labels the different species, ρ_{I0} denotes the energy density at the bounce point of component I , and w_I is its equation of state. The zero component of energy-momentum conservation $\nabla_\mu T^\mu_\nu = 0$ in a Bianchi-I background (6.37) takes the form

$$\dot{\rho} + 3H\rho + H(p_1 + p_2 + p_3) + \dot{\beta}_1 p_1 + \dot{\beta}_2 p_2 + \dot{\beta}_3 p_3 = 0. \quad (6.51)$$

We now find the energy-momentum tensor responsible for the bounce. We use the EoM in (6.33) and substitute in it the cosh bouncing solution in a Bianchi-I background (6.37), (6.36), (6.45) and (6.46) to obtain an energy density with three components

$$\rho = \frac{\rho_{k0}}{a^2} + \frac{\rho_{r0}}{a^4} + \frac{\rho_{s0}}{a^6} \quad (6.52)$$

where

$$\begin{aligned} \rho_{k0} &= -\frac{1}{2}\sigma_0^2 (60\mathcal{F}_1 r_1 + 4\mathcal{F}_2 + M_P^2) - \frac{3r_1}{2} (-12\mathcal{F}_1 r_1 + 4\mathcal{F}_2 + M_P^2) \\ \rho_{r0} &= -3(\sigma_0^2 - 3r_1)^2 (\mathcal{F}_1 + \mathcal{F}'_1 r_1) \\ \rho_{s0} &= 2\mathcal{F}'_1 r_1 (\sigma_0^2 - 3r_1)^2 \end{aligned} \quad (6.53)$$

where the first component is ‘ k -matter’ whose energy density scales as a^{-2} (just like spatial curvature in GR), second is radiation labeled by ‘ r ’, and the last

is stiff matter labeled by ‘ s ’ whose energy density scales as a^{-6} . We see that anisotropic shear, parameterized by σ_0 , effectively contributes to all the three matter sectors. Note that the cosmological constant Λ is fixed to be

$$\Lambda = \frac{3}{2}M_P^2 r_1 \quad (6.54)$$

in order to absorb the constant term in ρ in (6.52). Pressure is anisotropic in general (due to k -matter) and given by (in each spatial direction $i = 1, 2, 3$)

$$p_i = \left(\frac{p_{k0}}{a^2} + \pi^i_i \right) + \frac{p_{r0}}{a^4} + \frac{p_{s0}}{a^6} \quad (6.55)$$

where

$$\begin{aligned} p_{r0} &= w_r \rho_{r0} \quad \text{with} \quad w_r = 1/3, & p_{s0} &= w_s \rho_{s0} \quad \text{with} \quad w_s = 1, \\ p_{k0} &= w_k \rho_{k0} \quad \text{with} \\ w_k &= \frac{-\frac{1}{2}\sigma_0^2 (12\mathcal{F}_1 r_1 + 4\mathcal{F}_2 + M_P^2) + \frac{r_1}{2} (-12\mathcal{F}_1 r_1 + 4\mathcal{F}_2 + M_P^2)}{-\frac{1}{2}\sigma_0^2 (60\mathcal{F}_1 r_1 + 4\mathcal{F}_2 + M_P^2) - \frac{3r_1}{2} (-12\mathcal{F}_1 r_1 + 4\mathcal{F}_2 + M_P^2)}, \end{aligned} \quad (6.56)$$

and the k -matter anisotropic pressure π^i_i (in each spatial direction $i = 1, 2, 3$) is given by

$$\pi^i_i = \pm \sqrt{2r_1} b_i \sigma_0 (4\mathcal{F}_2 + M_P^2 + 24\mathcal{F}_1 r_1) \sqrt{\frac{1}{a^2} - \frac{1}{a^4}} \quad (+ \text{ for } t > 0, - \text{ for } t < 0) \quad (6.57)$$

where there is no summation over the index i in π^i_i . The appearance of π^i_i is a consequence of purely $\dot{\beta}_i$ and $\ddot{\beta}_i$ -dependent terms in the EoM⁵. The σ^2 -dependent contributions to the energy density and pressure, on the other hand, are isotropic in all spatial directions. This shows that, in general, the matter sector consists of two perfect fluids (radiation and stiff matter) and one fluid

⁵In GR, one obtains a second order differential equation for β_i by using the Friedmann equations and the constraint on β_i (6.38) in the presence of a perfect fluid matter sector. Essentially, the purely $\dot{\beta}_i$ and $\ddot{\beta}_i$ -dependent terms in $T^i_i (= G^i_i)$ put to zero gives the sought-after differential equation. In our case, we already determined β_i (6.46) by solving the Ricci scalar ansatz (6.43) for a fixed average scale factor (6.36) in a Bianchi-I background. However, this β_i does not put the purely $\dot{\beta}_i$ and $\ddot{\beta}_i$ -dependent terms in T^i_i (6.33) to zero, as is evident from non-zero π^i_i (6.57).

with anisotropic pressures (k -matter). Note from (6.56) that k -matter, in general, does not have the correct, constant equation of state $w_k = -1/3$ for it to be a perfect fluid, necessitating anisotropic pressures π^i_i in order to ensure energy-momentum conservation (6.51). In particular, we have the energy-momentum tensors of perfect fluid and k -matter as, respectively,

$$T_{\mu\nu}^I = (p_I + \rho_I)u_\mu u_\nu + p_I g_{\mu\nu}, \quad T_{\mu\nu}^k = (p_k + \rho_k)u_\mu u_\nu + p_k g_{\mu\nu} + \pi_{\mu\nu} \quad (6.58)$$

where $I = r$ or s , and u_μ is the velocity vector. The anisotropic pressure contribution $\pi^\mu_\nu = (0, \pi^1_1, \pi^2_2, \pi^3_3)$ is traceless $\pi^\mu_\mu = 0$ and transverse to the velocity vector $\pi_{\mu\nu}u^\mu = 0$. Background anisotropy, parameterized by σ_0 , does not introduce any new kinds of (effective) matter apart from those already present in the known FLRW cosh bouncing model. The FLRW limit can be taken by putting $\sigma_0 \rightarrow 0$ in (6.53), (6.56) and (6.57) [41]. We enumerate all possible matter sectors in Table (6.1), where the constraints on $\mathcal{F}_1, \mathcal{F}'_1, \mathcal{F}_2$ and σ_0 are determined by requiring that the energy density of each matter component be non-negative. In particular, requiring $\rho_{s0} \geq 0, \rho_{r0} \geq 0$ and $\rho_{k0} \geq 0$ constrains $\mathcal{F}'_1, \mathcal{F}_1$ and $\mathcal{F}_2$, respectively (see (6.53)). In some special cases, however, upon constraining the parameters of the theory and solution, it is possible to turn k -matter into a perfect fluid. From (6.57), we see that imposing a constraint

$$4\mathcal{F}_2 + M_P^2 + 24\mathcal{F}_1 r_1 = 0 \quad (6.59)$$

makes anisotropic pressures vanish $\pi^i_i = 0$, and recovers the correct, constant equation of state for k -matter $w_k = -1/3$, leaving us with three perfect fluids in the matter sector. Note that (for $\mathcal{F}_1 \neq 0$) it is impossible to satisfy both the perfect fluid constraint (6.59) and the second condition in (6.10) for traceless matter. This is another way of saying that a traceless matter sector driving an anisotropic cosh bounce must necessarily have anisotropic pressures (in the k -matter component).

It is interesting to consider the simplest case of k -matter being represented by a perfect fluid, canonical scalar field in a Bianchi-I background (6.37) with

the Lagrangian $\mathcal{L} = -(\partial\phi)^2/2 - V(\phi)$. See Table (6.1) for a list of cases where k -matter can be described by a perfect fluid. We assume that it is described by a homogeneous and time-dependent scalar field $\phi(t)$. Using

$$\begin{aligned} \rho &= \dot{\phi}^2/2 + V(\phi), & p &= \dot{\phi}^2/2 - V(\phi), \\ \rho_k &= \rho_{k0} a^{-2}, & p_k &= w_k \rho_k, \quad \text{where } w_k = -1/3, \\ H &= \sqrt{\frac{r_1}{2}} \tanh\left(\sqrt{\frac{r_1}{2}} t\right), & \text{for } a &= \cosh\left(\sqrt{\frac{r_1}{2}} t\right), \end{aligned} \quad (6.60)$$

we can deduce an oscillatory potential given by

$$V(\phi) = \frac{2\rho_{k0}}{3} \cos^2\left(\sqrt{\frac{3r_1}{4\rho_{k0}}} \phi\right). \quad (6.61)$$

The solution for $\phi(t)$ is

$$\phi(t) = \sqrt{\frac{4\rho_{k0}}{3r_1}} \operatorname{gd}\left(\sqrt{\frac{r_1}{2}} t\right) \quad (6.62)$$

where $\phi(t)$ runs from $-\frac{\pi}{2}\sqrt{\frac{4\rho_{k0}}{3r_1}}$ at $t = -\infty$ to $\frac{\pi}{2}\sqrt{\frac{4\rho_{k0}}{3r_1}}$ at $t = \infty$.

Most general anisotropic bounce

Consider as an example the most general case when all three matter components (k, r, s) are present in the model. A priori, $\mathcal{F}_1, \mathcal{F}'_1, \mathcal{F}_2, r_1$ and σ_0 are independent of each other. Relations between them are derived by invoking physical principles like positivity of energy density. Imposing this positivity for each component in (6.53), $\rho_{s0} > 0$, $\rho_{r0} > 0$ and $\rho_{k0} > 0$, we obtain the following conditions to be satisfied by $\mathcal{F}'_1, \mathcal{F}_1$ and $\mathcal{F}_2$, respectively (see also Table (6.1))

$$\mathcal{F}'_1 > 0, \quad \mathcal{F}_1 < -r_1 \mathcal{F}'_1 < 0, \quad \mathcal{F}_2 < -\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right). \quad (6.63)$$

Upon imposing the perfect fluid condition on $\mathcal{F}_2$ in (6.59), the energy density of k -matter at the bounce point becomes

$$\rho_{k0} = 18\mathcal{F}_1 r_1 (3r_1 - \sigma_0^2) \quad (6.64)$$

which in turn implies a lower bound on background anisotropy $\sigma_0^2 > 3r_1$ if we wish to have $\rho_{k0} > 0$. No such bound on σ_0 exists in the general case with two perfect radiation and stiff matter fluids and one anisotropic k -matter fluid.

Let us go back to the most general case with three fluids (6.53) and (6.63). Interestingly, for the specific value $\sigma_0^2 = 3r_1$, radiation and stiff matter vanish, leaving only anisotropic k -matter with the following energy density at the bounce point

$$\rho_{k0} = -\sigma_0^2(4\mathcal{F}_2 + M_P^2 + 8\mathcal{F}_1\sigma_0^2) \quad (6.65)$$

which implies

$$\mathcal{F}_2 < -\frac{M_P^2}{4} - 2\mathcal{F}_1\sigma_0^2 \quad (6.66)$$

if we require $\rho_{k0} > 0$. Moreover, if we now impose the perfect fluid condition (6.59) on $\mathcal{F}_2$ with this fixed value $\sigma_0^2 = 3r_1$, the energy densities of all fluids vanish and we obtain an anisotropic bounce in vacuum (but with Λ) (see Table (6.1)).

There exists another set of conditions where the bounce is in vacuum, but now with unconstrained σ_0^2 . Consider again the most general case (6.53). We fix $\mathcal{F}'_1 = \mathcal{F}_1 = 0$ to remove stiff matter and radiation from the model, and obtain

$$\rho_{k0} = -\frac{1}{2}(4\mathcal{F}_2 + M_P^2)(3r_1 + \sigma_0^2) \quad (6.67)$$

which implies that $\mathcal{F}_2 = -\frac{M_P^2}{4}$ if we want to have a bounce in vacuum. This vacuum solution is the same one we would obtain if we start from a traceless matter theory, already satisfying the tracelessness conditions in (6.10), and then put $\mathcal{F}_1 = 0$ in (6.35). These two vacuum solutions identify with each other for one specific tuning $\mathcal{F}'_1 = \mathcal{F}_1 = 0$, $\mathcal{F}_2 = -\frac{M_P^2}{4}$ and $\sigma_0^2 = 3r_1$. They are mutually exclusive otherwise.

a^{-6}	a^{-4}	a^{-2}	$\mathcal{F}_1$	$\mathcal{F}'_1$	$\mathcal{F}_2$	Perfect fluid possible?
✓	✓	✓	$< -r_1 \mathcal{F}'_1 < 0$	> 0	$< -\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right)$	if (6.59) is true; $\sigma_0^2 > 3r_1$
✗	✓	✓	< 0	0	$< -\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right)$	if (6.59) is true; $\sigma_0^2 > 3r_1$
✓	✗	✓	$-r_1 \mathcal{F}'_1 < 0$	> 0	$< -\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right)$	if (6.59) is true; $\sigma_0^2 > 3r_1$
✓	✓	✗	$< -r_1 \mathcal{F}'_1 < 0$	> 0	$-\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right)$	always
✗	✗	✓	0	0	$< -\frac{M_P^2}{4}$	never
✗	✓	✗	< 0	0	$-\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right)$	always
✓	✗	✗	$-r_1 \mathcal{F}'_1 < 0$	> 0	$-\frac{M_P^2}{4} + 3\mathcal{F}_1 r_1 \left(1 - \frac{6\sigma_0^2}{3r_1 + \sigma_0^2}\right)$	always
✗	✗	✓	-	-	$< -\frac{M_P^2}{4} - 2\mathcal{F}_1 \sigma_0^2$	never; $\sigma_0^2 = 3r_1$
✗	✗	✗	-	-	see (6.59)	vacuum; $\sigma_0^2 = 3r_1$
✗	✗	✗	0	0	$-\frac{M_P^2}{4}$	vacuum

Table 6.1: Possible matter sectors for a bounce in a Bianchi-I background for the quadratic curvature theory (6.1) following the EoM in (6.33) and the background Ricci scalar ansatz (6.43). At the top of the first three columns, a^{-6} represents stiff matter, a^{-4} represents radiation and a^{-2} represents k -matter. Conditions on $\mathcal{F}_1, \mathcal{F}'_1$ and $\mathcal{F}_2$ arise from imposing non-negativity of energy density of each matter component, $\rho_{r0} \geq 0, \rho_{s0} \geq 0$ and $\rho_{k0} \geq 0$, respectively (see (6.53)). Note that the inequality for $\mathcal{F}_2$ can be rewritten as an inequality for f_0 instead, using the definition of $\mathcal{F}_2$ in (6.9). A hyphen ‘-’ indicates that the value is unconstrained. The value of background anisotropy σ_0 is unconstrained, unless mentioned otherwise. The last column enumerates the cases when k -matter (and therefore the entire matter sector) can be a perfect fluid.

It is instructive to mention the cosh bouncing model (6.36) in a Bianchi-I background (6.37) when the matter sector is traceless. For this purpose, we substitute the tracelessness conditions (6.10) in the most general EoM (6.33) to obtain the simplified EoM (6.35). In FLRW, the only matter component is radiation [61]. In a Bianchi-I background, however, we see that background anisotropy σ_0 introduces k -matter, effectively. Upon imposing the tracelessness conditions (6.10) in the general expression for ρ_{k0} and p_{k0} in (6.53) and (6.56), we obtain $\rho_{k0} = -36\mathcal{F}_1 r_1 \sigma_0^2$ and $w_k = 1/3$. Note that the condition $\mathcal{F}'_1 = 0$ in (6.10) removes stiff fluid from the model. The equation of state corresponds to radiation, even though the fluid has an energy density which decays like a^{-2} . This necessitates anisotropic pressures π^i_i in order to conserve energy-momentum

(6.51), which are given by

$$\pi^i_i = \pm 36 b_i \mathcal{F}_1 \sigma_0 \sqrt{2 r_1^3} \sqrt{\frac{1}{a^2} - \frac{1}{a^4}} \quad (+ \text{ for } t > 0, - \text{ for } t < 0). \quad (6.68)$$

Local versus nonlocal gravity

We can now use our general results in the previous sections, apply them to specific forms of $\mathcal{F}(\square)$, and compare local and nonlocal quadratic curvature gravity. What we aim to show is that nonlocal gravity admits bouncing solutions with both positive matter energy density, and no instabilities (ghost/tachyonic). We have ensured that the energy density of each matter component remains non-negative. It is worth checking whether the gravitational part of the action remains ghost-free or not.

Note that both these criteria, that the gravity and matter sectors remaining ghost-free, may not be satisfied simultaneously in a local theory of gravity. Since our cosh bouncing solution (6.36) is dynamical, we will look at its late time behavior instead for simplicity. At late times, the Ricci scalar for a cosh bouncing background goes to a constant given by $R = 6r_1$ (see end of Sec.(6.2.1)). Since $r_1 > 0$, this corresponds to dS. The correct form of $\mathcal{F}(\square)$ which avoids the introduction of propagating ghost or tachyonic degrees of freedom was already derived for maximally symmetric spacetimes in [43], and is given by

$$\mathcal{F}(\square) = \frac{1 + \frac{24r_1}{M_P^2} f_0 - e^{\alpha(\square)}}{\frac{24r_1}{M_P^2} \left(1 + \frac{\square}{2r_1}\right)} \quad (6.69)$$

where $\alpha(\square)$ is some entire function of the $\square$ operator. This choice of $\mathcal{F}(\square)$ ensures that the spin-2 quadratic action has no new zeros apart from the one present in the GR limit. Moreover, this choice introduces no new zeros in the spin-0 quadratic action, and so there is no Brans-Dicke type scalar. The above choice of ghost-free $\mathcal{F}(\square)$ can satisfy the conditions required to ensure positive energy densities of all matter components driving a cosh bouncing universe (6.36) in a Bianchi-I background (6.37). For concreteness, we assume the simplest choice of $\alpha(\square) = \square/M_s^2$.

Let us consider the case where all the three matter components (k -matter, radiation and stiff matter) are present in the model. For $\rho_{k0} > 0, \rho_{r0} > 0$ and $\rho_{s0} > 0$, we need to satisfy the conditions in (6.63) (see also first row in Table (6.1)). It is easy to verify that these conditions can be satisfied for a range of (f_0, r_1, σ_0) values. One such choice is, for instance $f_0 = -1698.43, r_1 = M_s^2 = 0.01M_P^2$ and $\sigma_0 = 0.01M_P$.

Now consider a fourth order local theory where the form factor $\mathcal{F}(\Box)$ is truncated at a finite order in derivatives, like $\mathcal{F}(\Box) = f_0$. Now, we can again satisfy the conditions for positivity of energy density in the sixth row in Table (6.1), for which $\mathcal{F}_1 = f_0 < 0, \mathcal{F}'_1 = 0$ and $\mathcal{F}_2 = 0$. Then, for a chosen value of σ_0^2 , these can be satisfied for $0 < r_1 < \frac{5\sigma_0^2}{3}$ and $\mathcal{F}_1 = \frac{3r_1 + \sigma_0^2}{36r_1^2 - 60r_1\sigma_0^2}$. However, $f_0 < 0$ signals the presence of a tachyonic instability [216]; we cannot simultaneously satisfy positivity of energy density and absence of tachyonic instability in this example local theory.

6.3 Discussion

In this Chapter, we have found a simple model of non-singular, homogeneous and isotropic bouncing cosmology (with a cosmological constant) in vacuum with spatially flat geometry. Moreover, for the background satisfying certain conditions, see Eq.(6.10), there exists only a scalar perturbation at the linearized level, and no propagating vector or tensor modes. This is contrary to the case in GR and infinite derivative gravity in general, which propagate tensor modes. Finding the curious ghost-free form factor $\mathcal{F}(\Box)$ around a dynamical background that leads to only a scalar propagating mode is non-trivial and it is the main highlight of this paper. To our knowledge, this is the first such study in infinite derivative theories of gravity. Moreover, in a cosine hyperbolic bouncing scenario, we have explicitly shown the lone scalar propagating degree of freedom is stable and its evolution is oscillatory and bounded. Of course, these results are valid at the linearized level in perturbation theory and it would be interesting to see what happens at higher orders, which we leave for future discussions.

We also have studied for the first time anisotropic but homogeneous Bianchi-I

background solutions in nonlocal quadratic in Ricci scalar gravity in the presence of a cosmological constant, with a matter sector that turns out to include both perfect fluids and a fluid with anisotropic pressures. We show that this theory admits a bouncing scenario in the anisotropic background without violating positivity of energy density of each matter source. Unlike local higher derivative gravity, an anisotropic bounce in nonlocal gravity leads to late time FLRW (and eventually dS) without ghost/tachyonic modes in the late time dS phase. Space-time anisotropies grow slower than in GR during the contraction phase, peak to a finite value at the bounce point, and then decrease during the expansion phase as the universe asymptotes to isotropic and homogeneous FLRW, and eventually dS. We found that driving a bounce in a Bianchi-I background necessarily requires anisotropic pressures in the k -matter sector, unless the form factor $\mathcal{F}(\Box)$ in the action obeys a certain constraint.

Chapter 7

Conclusions and Outlook

General Relativity is the fundamental theory of gravity and plays significant role in astrophysics and cosmology. The gravitational wave and black hole photo verify that GR is correct even in strong gravitational regime. However some key problems are unsolved in the context of GR, such as BH singularity and cosmological singularity and non-renormalization. It is believed that these problems would be resolved in ultraviolet complete quantum gravity, though it has not been fully established.

String theory is one of the most prospective candidates of UV complete quantum gravity. In the low energy effective theory of string theory, the higher curvature interactions naturally arise. Thus GR should be modified by higher derivative gravity in strong gravitational regime. It has been proved that curvature-quadratic gravity is renormalizable at the tree level. Unfortunately general high derivative gravity typically owns additional ghost massive spin-2 graviton which leads to instability. Thus the compatibility of renormalization and unitarity is extremely non-trivial for quantum gravity.

An elegant proposal to resolve the unitarity of generic higher derivative gravity is generalizing the local theory to nonlocal case by adding infinite derivative operator which does not introduce extra poles in the propagator. In this thesis, we study some applications of nonlocal gravity in astrophysics and cosmology,

specifically including ultracompact objects and bouncing universe.

We study the observational signatures of UCOs in both optical and acoustic aspects. When UCO or BH is illuminated by a thin accretion disk, we will observe interesting "photon rings" structure in its optical appearance. The "photon rings" are precisely associated with the photon sphere of UCO or BH. For BH, the effective potential of radial null geodesic has only one peak located at the photon sphere. For UCO we have additional peak near the effective surface of UCO (or photon sphere in another spacetime for wormhole). So photons with an impact factor less to the critical value may be bounced back. This reflectivity of photon in UCO results in extra "photon rings" in the observational appearance. We show this feature explicitly in asymmetric thin-shell wormhole.

The ringdown is the final stage of a compact binary coalescence when the remnant relaxes to an equilibrium solution. The ringdown signal is determined by the complex eigenfrequencies, termed as quasi-normal modes, of the perturbation equation which can be arranged into a Schrodinger-like wave equation in GR. For BH, the effective potential of gravitational perturbation has only one peak located approximately at the photon sphere. The boundary conditions are purely outgoing in infinity and purely ingoing in horizon. For UCO, the effective potential of gravitational perturbation features a cavity between the photon sphere and the effective surface of the UCO (or photon sphere in another spacetime for wormhole). This potential cavity can support long-lived semi-trapped modes which are responsible not only for a completely different QNM spectrum with respect to the BH case, but also for the gravitational wave echoes.

We study the nonlocal effect in GW echoes by generalizing the gravitational perturbation equation to a nonlocal version of Schrodinger-like equation and simplifying the effective potential into double delta potential barriers. We found that nonlocality amplifies echoes and analyzed its influence on the damping time of echoes. Though the model is very idealized, our analysis will provide smoking-gun signatures of nonlocal interactions in GW echoes.

From the cosmology point of view, we study a class of higher derivative, nonlocal gravity which admits homogeneous and isotropic non-singular, bouncing universes in the absence of matter. At the linearized level, the theory propagates

only a scalar degree of freedom, and no vector or tensor modes. The scalar can be made free from perturbative ghost instabilities, and has oscillatory and bounded evolution across the bounce. We also generalize the bouncing solution to an anisotropic but homogeneous Bianchi-I background by adding k -matter, radiation and stiff matter. Generically, k -matter exerts anisotropic pressures. We show that the anisotropic shear grows slower than in general relativity during the contraction phase, peaks to a finite value at the bounce point, and then decreases as the universe asymptotes towards isotropy and homogeneity, and ultimately to de Sitter. Thus we don't need to introduce super-stiff matter to suppress the growth of anisotropy which leads to BKL instability in contraction phase. We have a smoother transition from contraction to expansion compared to bouncing solutions in GR. We will also test the bouncing solution in local and nonlocal gravity and show that in the latter case it is possible to simultaneously satisfy positivity of energy density and, at least in the late time de Sitter phase, avoid the introduction of propagating ghost/tachyonic modes.

Open issues:

Here I list some open questions for future researches:

- In our analysis of nonlocal effect in echoes, we only generalize the Schrodinger-like perturbation equation to a nonlocal version and simplify the effective potential by a double delta potential barrier. Further study is implementing complete gravitational perturbation around certain curved spacetime for nonlocal gravity, which requires seeking exact spherically symmetric solutions in the non-linear regime;
- We illustrate the photon rings of asymmetric thin-shell wormhole which is comparatively artificial. It is interesting to discuss more realistic UCOs and more realistic emission models, in which the reflective mechanism of photons is more complicated. For example, the analysis should include the inclination angle between the observer's line of view and the plane of the disk. In such a case, the rotation of the disk around the wormhole also

plays an interesting role. It would also be quite interesting if we can study the observational signature of the accretion disk around the nonlocal star such that we can see the influence of the nonlocal interaction on the photon rings. Moreover, the detection of "photon rings" is still lacking, which will become possible with longer interferometric baselines (likely requiring a space mission) that reach the resolution;

- In obtaining the anisotropic bouncing universe, we don't discuss the stability. To investigate the stability, we should analyze the perturbative spectrum around the anisotropic background. The perturbative spectrum is expected to be made free from ghosts/tachyons and has oscillatory and bounded evolution across the bounce. Also, it would be interesting to see what the corresponding matter sector looks like for an exponential bounce in Bianchi-I spacetimes where $a(t) \sim e^{t^2}$, another known solution of the ansatz (6.6) used to simplify the EoM (6.31). It is also important to check if there exists a non-singular cosmological evolution in nonlocal gravity in other Bianchi spacetimes. Furthermore, we have only studied the effect of having a nonlocal quadratic Ricci scalar term in action, but it will be curious to see how bouncing scenarios evolve once we include nonlocal quadratic in Ricci and Weyl tensor terms too. We leave all these interesting questions for future investigations.

Appendix A

Riemann tensor and variations

A metric $g_{\mu\nu}$ is a covariantly constant, which naturally determines a derivative operator ∇_a , i.e. $\nabla_\mu g_{\nu\rho} = 0$. The Christoffel symbol (affine connection) is given by

$$\Gamma^\mu{}_{\nu\rho} = \frac{1}{2}g^{\mu\sigma}(\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\sigma\nu} - \partial_\sigma g_{\nu\rho}). \quad (\text{A.1})$$

The Riemann tensor is

$$R^\mu{}_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu{}_{\sigma\nu} - \partial_\sigma \Gamma^\mu{}_{\rho\nu} + \Gamma^\mu{}_{\rho\lambda} \Gamma^\lambda{}_{\sigma\nu} - \Gamma^\mu{}_{\sigma\lambda} \Gamma^\lambda{}_{\rho\nu}. \quad (\text{A.2})$$

The Ricci tensor is

$$R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}. \quad (\text{A.3})$$

And the Ricci scalar is

$$R = g^{\mu\nu} R_{\mu\nu}. \quad (\text{A.4})$$

The Einstein tensor is defined by

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \quad (\text{A.5})$$

Considering a linear variation $\delta g_{\mu\nu}$ around arbitrary background, the variation of metric determinant reads

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}. \quad (\text{A.6})$$

The variation of affine connection reads

$$\delta\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2}g^{\lambda\rho}(\nabla_{\nu}\delta g_{\mu\rho} + \nabla_{\mu}\delta g_{\nu\rho} - \nabla_{\rho}\delta g_{\mu\nu}). \quad (\text{A.7})$$

The variation of Riemann tensor reads

$$\delta R^{\rho}_{\sigma\mu\nu} = \nabla_{\mu}\delta\Gamma_{\nu\sigma}^{\rho} - \nabla_{\nu}\delta\Gamma_{\mu\sigma}^{\rho}. \quad (\text{A.8})$$

If we denote the variation by $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, i.e.

$$h_{\mu\nu} \equiv \delta g_{\mu\nu}, \quad h^{\mu\nu} = -\delta g^{\mu\nu}, \quad h = \bar{g}^{\mu\nu}h_{\mu\nu}, \quad (\text{A.9})$$

the perturbations of the curvature quantities up to second-order read

$$\begin{aligned} \delta\sqrt{-g} &= \sqrt{-\bar{g}}\frac{h}{2}, \quad \delta^{(2)}\sqrt{-g} = \sqrt{-\bar{g}}\left(\frac{h^2}{8} - \frac{1}{4}h_{\mu\nu}h^{\mu\nu}\right), \\ \delta R &= -h^{\alpha\beta}\bar{R}_{\alpha\beta} + \bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}h^{\alpha\beta} - \bar{\square}h, \\ \delta(\square)\bar{R} &= \frac{1}{2}\bar{\nabla}_{\alpha}h\bar{\nabla}^{\alpha}\bar{R} - \bar{\nabla}^{\alpha}\bar{R}\bar{\nabla}_{\beta}h^{\beta}_{\alpha} - h_{\alpha\beta}\bar{\nabla}^{\beta}\bar{\nabla}^{\alpha}\bar{R}. \end{aligned} \quad (\text{A.10})$$

Hence the second-order variation of the Einstein-Hilbert action around arbitrary background reads

$$\begin{aligned} \delta^{(2)}S_{EH} &= \frac{M_p^2}{2} \int d^4x \sqrt{-\bar{g}} \left[\frac{1}{4}h_{\mu\nu}\bar{\square}h^{\mu\nu} - \frac{1}{4}h\bar{\square}h + \frac{1}{2}h\bar{\nabla}_{\mu}\bar{\nabla}_{\rho}h^{\mu\rho} + \frac{1}{2}\bar{\nabla}_{\mu}h^{\mu\rho}\bar{\nabla}_{\nu}h^{\nu}_{\rho} \right. \\ &\quad \left. + (hh^{\mu\nu} - 2h^{\mu}_{\sigma}h^{\sigma\nu})\left(\frac{1}{8}\bar{g}_{\mu\nu}\bar{R} - \frac{1}{2}\bar{R}_{\mu\nu}\right) \right. \\ &\quad \left. - \left(\frac{1}{2}\bar{R}_{\sigma\nu}h^{\sigma}_{\rho}h^{\nu\rho} + \frac{1}{2}\bar{R}^{\sigma}_{\rho\nu\mu}h^{\mu}_{\sigma}h^{\nu\rho}\right) \right]. \end{aligned} \quad (\text{A.11})$$

If we consider the perturbations around the spatially-flat FLRW background in the SVT form, i.e.

$$ds^2 = a(\tau)^2 \left[-(1 + 2\phi)d\tau^2 + 2\hat{B}_i dx^i d\tau + \left((1 - 2\psi)\delta_{ij} + 2\hat{h}_{ij} \right) dx^i dx^j \right], \quad (\text{A.12})$$

then the useful perturbations in the SVT form read

$$\begin{aligned} h &= \bar{g}^{\mu\nu} h_{\mu\nu} = \bar{g}^{00} h_{00} + \bar{g}^{ij} h_{ij} = 2\phi - 6\psi, \\ h^{\alpha\beta} \bar{R}_{\alpha\beta} &= \frac{1}{a^2} [6\mathcal{H}'\phi - (6\mathcal{H}' + 12\mathcal{H}^2)\psi], \\ \bar{\square} h &= -\frac{1}{a^2} (\partial_0^2 - \Delta + 2\mathcal{H}\partial_0)(2\phi - 6\psi), \\ \delta R &= -\frac{2}{a^2} (6\mathcal{H}^2\phi + 6\mathcal{H}'\phi + 3\mathcal{H}\phi' + 9\mathcal{H}\psi' + 3\psi'' + \Delta\phi - 2\Delta\psi), \\ \delta^{(2)}\sqrt{-g} &= \frac{a^4}{2} (\hat{B}_i \hat{B}^i - 2\hat{h}_{ij} \hat{h}^{ij} - \phi^2 + 3\psi^2 - 6\phi\psi), \\ \delta(\square)\bar{R} &= \frac{1}{a^2} [\bar{R}'(\phi' + 4\mathcal{H}\phi + 3\psi') + 2\bar{R}''\phi], \end{aligned} \quad (\text{A.13})$$

where $'$ denotes derivative with respect to the conformal time τ and $\Delta = \delta^{ij} D_i D_j$ is the Laplacian on three dimensional Euclidean space $\mathbb{R}^3$.

Appendix B

Quasi-normal mode

For one dimensional scattering problem in local theory, we need solve the Schrodinger-like wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + (\omega^2 - V)\psi = 0 \quad (\text{B.1})$$

for certain potential $V(x)$. In general we assume the potential $V(x)$ is positive and satisfies

$$V(x) \rightarrow 0 \quad \text{as} \quad x \rightarrow \pm\infty. \quad (\text{B.2})$$

The general solution of $\psi(x)$ in this case has the form

$$\psi(x) = \begin{cases} Ae^{i\omega x} + Be^{-i\omega x}, & x \rightarrow -\infty; \\ Ce^{i\omega x}, & x \rightarrow +\infty. \end{cases} \quad (\text{B.3})$$

The reflection coefficient r and transmission coefficient t is defined respectively as

$$\mathcal{R} = \frac{B}{A}, \quad \mathcal{T} = \frac{C}{A}. \quad (\text{B.4})$$

If we change $V \rightarrow -V$ and $\omega \rightarrow i\omega$, the solution becomes

$$\psi(x) = \begin{cases} A'e^{-\omega x} + B'e^{\omega x}, & x \rightarrow -\infty; \\ C'e^{-\omega x}, & x \rightarrow +\infty. \end{cases} \quad (\text{B.5})$$

In order to have bound states we demand the wave function to converge to zero as $x \rightarrow \pm\infty$. This requires $A' = 0$ which in turn are the poles of $\mathcal{R}'$ and $\mathcal{T}'$ at negative energies $E = -\omega^2$.

If we directly find the poles of $\mathcal{R}$ and $\mathcal{T}$ that amounts to finding the zeros of A , we obtain the so-called quasi-normal modes which satisfy the purely outgoing wave boundary conditions at infinity [186]

$$\psi \rightarrow e^{-i\omega x} \quad \text{as } x \rightarrow -\infty \quad \text{and} \quad \psi \rightarrow e^{i\omega x} \quad \text{as } x \rightarrow +\infty. \quad (\text{B.6})$$

Under these boundary conditions, the frequency is complex

$$\omega = \omega_{\text{R}} + i\omega_{\text{I}}. \quad (\text{B.7})$$

The time evolution of wave function is determined by $e^{-i\omega t}$, so a stable mode has $\omega_{\text{I}} < 0$, whereas an unstable mode has $\omega_{\text{I}} > 0$, with damping (instability) time scale $\tau_{\text{damping(inst)}} \equiv 1/|\omega_{\text{I}}|$ for the former (latter) case. The quasi-normal modes are especially meaningful for a black hole spacetime, because there shall only be an ingoing wave at the horizon, while there will only be the outgoing wave at spatial infinity [80].

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