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A multi-regional generalized RAS updating technique

Umed Temursho ^a, Jan Oosterhaven ^b and M. Alejandro Cardenete ^c

ABSTRACT

We present an extension of the generalized RAS (GRAS) technique to a multi-regional (MR) or multi-national setting. The framework is applicable to updating/regionalizing/balancing any partitioned matrix that needs to conform to new row sums, column sums and additional non-overlapping aggregation constraints. The technique, which we refer to as MR-GRAS, also handles non-exhaustive constraints, in which case the missing values are endogenously generated in the updating process. We derive the closed-form solution of MR-GRAS, propose a simple iterative algorithm for its computation, and discuss the main analytical properties of the method as well as the normalization and interpretation of MR-GRAS multipliers. From a wide range of possible MR-GRAS applications, several updating frameworks of national and interregional supply and use tables are examined.

KEYWORDS

RAS, multiregional GRAS, partitioned matrix updating, non-survey regionalization methods

JEL C2, C61, C80, D57

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INTRODUCTION

There is a large literature on updating, regionalizing, projecting, balancing and/or estimating input-output tables (IOTs), supply and use tables (SUTs) and social accounting matrices (SAMs). This literature mostly considers matrix estimation techniques based on (non)linear programming approaches that aim to search for a minimum ‘distance’ between a given original matrix and a new, unknown matrix subject to certain constraints on the new matrix. Extensive empirical assessments of various matrix estimation methods are carried out by Jackson and Murray (2004), Oosterhaven (2005), Huang et al. (2008) and Temurshoev et al. (2011).

The well-known RAS method (Leontief, 1941; Stone, 1961; Bacharach, 1970) arguably is the most popular updating and regionalization method, at least among practitioners. It is a *biproportional technique* used to estimate a new matrix from an existing matrix by scaling and rescaling its entries row- and column-wise so that the given row and column totals of the new matrix are respected.¹ However, the traditional RAS can only handle non-negative matrices, which limits its applicability. This is a serious limitation in practice, in particular when dealing with medium to large-scale IOTs, SUTs and SAMs as these often include negative entries in often *sizeable*

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items, such as subsidies, net exports, reductions of inventories, trade margins, transportation margins, and depreciation. The extension of RAS, called the *generalized RAS* (GRAS) method solves this problem (Günlük-Şenesen & Bates, 1988; Junius & Oosterhaven, 2003; Temurshoev et al., 2013).

However, GRAS still suffers from a limitation in the type of constraints that can be imposed on the values of the new matrix, that is, it only allows for as well as requires constraints on the row sums and on the column sums of the new matrix. In this paper we propose a further generalization of GRAS that also allows for non-overlapping aggregation constraints on arbitrary parts of the new matrix. This situation most often occurs in updating or constructing multi-regional (MR) IOTs, SUTs and SAMS, which is why we label it as the multi-regional GRAS (MR-GRAS) method. More or less comparable MR extensions have been already made by Oosterhaven et al. (1986), Cole (1992), Gilchrist and St. Louis (1999, 2004) and Lenzen et al. (2009). However, the first four papers focus on updating non-negative matrices, while the last study due to its generality loses the inherent transparency and simplicity of the GRAS approach.²

A more general multidimensional GRAS is presented by Valderas-Jaramillo and Rueda-Cantuche (2019) as an extension of the multidimensional RAS of Holý and Šafr (2017).³ However, this paper does not make our work irrelevant, because: (1) we additionally study updating with non-exhaustive constraints, normalization and interpretation of MR-GRAS multipliers, and give an analytical analysis of the main properties of MR-GRAS solution; (2) in practice one mostly encounters updating cases that are consistent with the MR-GRAS setting (i.e., requiring only row sums, column sums and non-overlapping aggregation constraints); and (3) it is likely that most practitioners will find it rather difficult to work with such concepts as hypermatrices or hypercubes when trying to implement the multidimensional (G)RAS compared with simply applying MR-GRAS to *familiar* IOT/SUT/SAM settings.

The second section first provides the complete analytical solution of the MR-GRAS approach, proposes a simple iterative algorithm for its computation and discusses the possibilities of including non-exhaustive constraints. Such transparency makes mastering advanced knowledge of complex numerical optimization techniques irrelevant and having access to high-performance solvers unnecessary as the proposed iterative approach can easily be applied with widely available software, such as Excel or R. Further, it allows for easier control of the convergence process compared with using built-in functions of the optimization solvers. In cases of non-convergence, one may derive an approximate solution simply by increasing the threshold level of only one stopping criterion of the MR-GRAS algorithm. Then by studying the approximate (non-optimal) table, including the obtained multipliers, one is generally able to find the exact source(s) of such non-convergence problems. In contrast, in case of general-purpose solvers one has to change many stopping rule criteria, which is often not straightforward, especially when the researcher has little knowledge of the complex algorithms underlying such optimization routines.

Importantly, as noted above, the MR-GRAS multipliers have economic interpretations that could very well be the focus of the research. These multipliers make up the MR-GRAS analytical solution and are directly accessible as the output of its iterative algorithm. In contrast, such information cannot readily be retrieved from the applications of numerical optimization techniques. However, given that (MR-)RAS multipliers are unique only up to a scalar, a normalization procedure is required for any interpretation of these multipliers. These issues are discussed in the second part of the second section. Its third subsection examines the main analytical properties of the MR-GRAS method. These shed more light on why bi- and tri-proportional scaling methods may be preferred to alternative IOT/SUT/SAM updating approaches.

In the third section, given the importance of regional, national, interregional, international and global SUTs, from which corresponding IOTs are derived, we separately discuss how the MR-GRAS technique can be applied in updating/balancing of national and international

SUTs. Here we will also discover that the SUT-RAS method⁴ is a particular case of the GRAS method, whereas MR-GRAS further extends the horizons of improved estimation of regional and national SUTs by providing the possibility of incorporating known non-overlapping aggregation values on the different parts of a SUT.

Finally, the supplemental online material explains in more detail the treatment of non-exhaustive constraints and provides a *detailed guide* on practical implementation of MR-GRAS. Through a worked example, it demonstrates the procedure of tri-proportional scaling of different updating frameworks with exhaustive constraints, non-exhaustive disaggregate constraints and/or non-exhaustive aggregation constraints.

The fourth section concludes.

MULTI-REGIONAL GENERALIZED RAS (MR-GRAS)

MR-GRAS with (non)exhaustive constraints

Let x_{ij}^0 and x_{ij} be the ij -th element of, respectively, the initial and the target IOT, which are denoted by rectangular $m \times n$ matrices $\mathbf{X}^0$ and $\mathbf{X}$.⁵ Denote the known row sums of $\mathbf{X}$ by $u_i = \sum_j x_{ij}$ and the known column sums of $\mathbf{X}$ by $v_j = \sum_i x_{ij}$. To obtain a feasible solution, it has to be assumed that these prespecified restrictions are mutually consistent, that is, $\sum_i u_i = \sum_j v_j$. This consistency restriction, however, is not needed with non-exhaustive row and/or column totals, the details of which are discussed below. This is the setting of the traditional GRAS method. The MR-GRAS method, additionally, handles arbitrary non-overlapping *aggregation constraints* of the form $\sum_{i \in I, j \in J} x_{ij} = w_{IJ}$, where the uppercase indices I and J indicate the aggregate (e.g., nation-level) counterparts of the relevant disaggregated (e.g., region-level) row and column indices i 's and j 's, respectively, and w_{IJ} 's are the corresponding known aggregate (e.g., national) values.

Following Junius and Oosterhaven (2003), first define the ratio of the unknown (or 'new') to the known (or 'old') entries of the corresponding IOT by $z_{ij} \equiv x_{ij}/x_{ij}^0$ whenever $x_{ij}^0 \neq 0$. For $x_{ij}^0 = 0$, this ratio should be set to unity (Lenzen et al., 2007). The MR-GRAS problem is then formalized as minimizing the weighted logarithm of the relative distance between the entries of the new and the old IOT:⁶

$$\min_{z_{ij}} f(Z) = \sum_i \sum_j |x_{ij}^0| z_{ij} \ln\left(\frac{z_{ij}}{e}\right) \quad (1a)$$

such that

$$\sum_j x_{ij}^0 z_{ij} = u_i \quad \text{for all } i = 1, \dots, m, \quad (1b)$$

$$\sum_i x_{ij}^0 z_{ij} = v_j \quad \text{for all } j = 1, \dots, n, \quad (1c)$$

$$\sum_{i \in I, j \in J} x_{ij}^0 z_{ij} = w_{IJ} \quad \text{for all } I = 1, \dots, M < m \text{ and } J = 1, \dots, N < n, \quad (1d)$$

where e is the base of natural logarithm. The last $M \times N$ constraints (1d) are additional for MR-GRAS compared with the regular GRAS. In case of updating an MR-IOT, these additional constraints ensure that the sum of the corresponding intra- and interregional cells of the MR-IOT add to the values of the known new national cells w_{IJ} 's.⁷

The following important points regarding MR-GRAS constraints need to be stressed. First, we restrict our solution of (1) to the case wherein any disaggregated item $\{i, j\}$ is considered to be a part of *only one* aggregated set $\{I, J\}$. This means that the above-presented MR-GRAS problem only allows for *non-overlapping* aggregation constraints. This choice is favoured because this type of aggregation represents the prevalent situation researchers and practitioners come across most often. Inclusion of overlapping aggregation constraints generally makes the final

solution more complicated, leading to the loss of simplicity and transparency that we wish to maintain in the MR version of GRAS, as in RAS. In case of overlapping and possibly conflicting constraints we suggest using the KRAS approach of Lenzen et al. (2009) or some general-purpose constrained optimization solver.

Following the approach of Gilchrist and St. Louis (1999, 2004), in practical implementation of the MR-GRAS method we use aggregator matrices $\mathbf{G}$ and $\mathbf{Q}$ consisting of zeros and ones in order to write compactly and easily (e.g., without any further vectorization) the aggregation constraints as $\mathbf{G}(\mathbf{X}^0 \otimes \mathbf{Z})\mathbf{Q} = \mathbf{G}\mathbf{X}\mathbf{Q} = \mathbf{W}$, where $\mathbf{W}$ is the $M \times N$ aggregation constraints matrix with typical elements w_{ij} and the symbol $\otimes$ is Hadamard product of element-wise matrix multiplication. As follows from (1d), the aggregator matrices $\mathbf{G}$ and $\mathbf{Q}$ must be of dimensions $M \times m$ and $n \times N$, respectively, where M is not necessarily equal to N . Imposing non-overlapping aggregation constraints requires that the column sums of $\mathbf{G}$ and the row sums of $\mathbf{Q}$ are all unity, that is, that $\mathbf{1}'\mathbf{G} = \mathbf{1}'$ and $\mathbf{Q}\mathbf{1} = \mathbf{1}$, where $\mathbf{1}$ is a vector of ones with appropriate dimension.

Second, if the constraints (1b)–(1d) are exhaustive, that is, when all the entries of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{W}$ are known, all constraints have to be *mutually consistent* and consistent with the initial matrix $\mathbf{X}^0$. Checking these consistency requirements is not always trivial, but the following checks will be helpful for practitioners. At least the following two requirements on the mutual consistency of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{W}$ need to hold:

- Identical aggregate row sums: $\mathbf{G}\mathbf{u} = \mathbf{W}\mathbf{1}$.
- Identical aggregate column sums: $\mathbf{v}'\mathbf{Q} = \mathbf{1}'\mathbf{W}$.

Because of the properties of $\mathbf{G}$ and $\mathbf{Q}$ discussed above, these two requirements also imply identical overall sums of the constraints, that is, $\mathbf{1}'\mathbf{W}\mathbf{1} = \mathbf{1}'\mathbf{u} = \mathbf{1}'\mathbf{v}$. Besides, the practitioner has to check whether the benchmark table $\mathbf{X}^0$ is consistent with the constraints $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{W}$. For example, if row i of $\mathbf{X}^0$ consists of only negative elements while the corresponding u_i is positive, then the (MR-)GRAS problem is infeasible and does not have solution (because of the MR-GRAS sign-preserving property, discussed below). A similar problem would arise if, for example, column j of $\mathbf{X}^0$ consists of only zeros, but the corresponding v_j is non-zero (because of the MR-GRAS zero-preserving property), or $w_{ij} = 0$ while the corresponding entries in $\mathbf{X}^0$ have only negative signs, making it impossible to be aggregated to a zero value.

Third, the aggregation constraints are allowed to be *non-exhaustive*. That is, it is *not* necessary that all w_{ij} 's are known, *some* aggregation values might be missing, which could very well happen in practice. In such cases, the corresponding aggregate values are implicitly and endogenously determined within the MR-GRAS procedure, which depends on the structure of the existing aggregation constraints and the relevant row sums and column sums constraints. Of course, in the extreme case when *all* w_{ij} 's are missing, MR-GRAS boils down to the standard GRAS.

Fourth, similar to the previous point, the framework also allows for non-exhaustive row sum constraints and/or column sum constraints, in which case the assumption $\sum_i u_i = \sum_j v_j$ is not required anymore. In such cases, the MR-GRAS benchmark table and all its constraints need to be modified, as detailed in Appendix E.3 in the supplemental online material. The MR-GRAS framework is also general enough to allow for different configurations of *both* non-exhaustive aggregation constraints and non-exhaustive column- and/or row-sums constraints.

Two simple hypothetical examples of data structures that can be handled by MR-GRAS are illustrated in Figure 1. The first example shows interregional and international trade flows for a particular commodity, and the second example shows the intermediate transactions of a hypothetical MRIO table, both also illustrating the corresponding non-overlapping aggregation totals.

To find the solution of (1), following Junius and Oosterhaven (2003), first the original matrix needs to be decomposed as $\mathbf{X}^0 = \mathbf{P}^0 - \mathbf{N}^0$, where $\mathbf{P}^0$ contains the positive elements of $\mathbf{X}^0$ and $\mathbf{N}^0$ consists of the absolute values of the negative entries in $\mathbf{X}^0$. The optimal solution of the

Figure 1. Examples of MR-GRAS consistent data structures.

Note: Row and column indices of the disaggregated data are given by i and j , respectively, whereas their corresponding aggregate sets are I and J . In each case, dark cells visualize the components of one arbitrarily chosen aggregation constraint, also highlighted in the corresponding aggregation matrix. (a) An example of trade flows of a specific good among nations (indicated by I and J) that are further disaggregated by regions (indicated by i and j); here, it is assumed that country 4 does not produce (hence cannot export) the good in question. (b) A hypothetical MRIO table with three countries a , b and c , and three sectors 1, 2 and 3; for simplicity, only the intermediate transactions are shown.

underlying Lagrangian function can then be easily derived and compactly written, making use of the decomposition $x_{ij}^0 = p_{ij}^0 - n_{ij}^0$, as follows (see Appendix A in the supplemental material):

$$x_{ij} = t_{IJ} \cdot r_i p_{ij}^0 s_j - \frac{1}{t_{IJ}} \frac{1}{r_i} n_{ij}^0 \frac{1}{s_j}, \quad (2)$$

where $r_i > 0$, $s_j > 0$ and $t_{IJ} > 0$. Hence, similar to the standard GRAS solution (Günlük-Şenese & Bates, 1988; Junius & Oosterhaven, 2003), the row and column multipliers are given by r_i and s_j , respectively. However, in contrast to the GRAS method, the MR-GRAS approach is *no longer* a bi-proportional technique as the added *aggregation multipliers* t_{IJ} 's make its solution *tri-proportional*. Not surprisingly, with $t_{IJ} = 1$ for all I and all J , that is, without aggregation constraints, the MR-GRAS solution boils down to that of the standard GRAS approach.⁸ Our choice of non-overlapping aggregation constraints implies that for any $\{i, j\}$ in (2) there is only one t_{IJ} which includes the ij -th entry in question. If for some pair $\{I, J\}$ the aggregation constraint is not specified (or is unknown), the corresponding aggregation multiplier can be considered as unity from the outset, $t_{IJ} = 1$.

The closed-form solutions of the three sets of multipliers are given by the following expressions (see Appendix A in the supplemental material):

$$r_j = \begin{cases} \frac{u_i + \sqrt{u_i^2 + 4p_i(s, t)n_i(s, t)}}{2p_i(s, t)} & \text{for } p_i(s, t) > 0, \\ -\frac{n_i(s, t)}{u_i} & \text{for } p_i(s, t) = 0, \end{cases} \quad (3)$$

$$s_j = \begin{cases} \frac{v_j + \sqrt{v_j^2 + 4p_j(r, t)n_j(r, t)}}{2p_j(r, t)} & \text{for } p_j(r, t) > 0, \\ -\frac{n_j(r, t)}{v_j} & \text{for } p_j(r, t) = 0, \end{cases} \quad (4)$$

$$t_{IJ} = \begin{cases} \frac{w_{IJ} + \sqrt{w_{IJ}^2 + 4p_{IJ}(r, s)n_{IJ}(r, s)}}{2p_{IJ}(r, s)} & \text{for } p_{IJ}(r, s) > 0, \\ -\frac{n_{IJ}(r, s)}{w_{IJ}} & \text{for } p_{IJ}(r, s) = 0, \end{cases} \quad (5)$$

where

$$p_i(s, t) = \sum_j p_{ij}^0 s_j t_{IJ} \quad \text{and} \quad n_i(s, t) = \sum_j \frac{n_{ij}^0}{s_j t_{IJ}}, \quad (6a)$$

$$p_j(r, t) = \sum_i r_i p_{ij}^0 t_{IJ} \quad \text{and} \quad n_j(r, t) = \sum_i \frac{n_{ij}^0}{r_i t_{IJ}}, \quad (6b)$$

$$p_{IJ}(r, s) = \sum_{i \in I, j \in J} r_i p_{ij}^0 s_j \quad \text{and} \quad n_{IJ}(r, s) = \sum_{i \in I, j \in J} \frac{n_{ij}^0}{r_i s_j}. \quad (6c)$$

Note that the appearance of r , s and/or t within the parentheses in (3)–(6) explicitly indicates the dependence of the corresponding terms on the row, column and/or aggregation multipliers, respectively. For simplicity, however, the dependence of these terms on the observed, exogenous elements, that is, p_{ij}^0 or n_{ij}^0 , is suppressed.

Each multiplier in (3)–(5) is defined by two expressions: the first is valid when the corresponding row(s) and/or column(s) of the original matrix include(s) at least one positive element, while the second solution is used instead when the relevant elements of the old matrix consist(s) of only non-positive entries with at least one negative element (for details on the relevance of this issue, which becomes even more relevant in a multiregional setting, see Temurshoev et al., 2013). Note from (3) that, for example, when $p_i(s, t) = 0$ but $n_i(s, t) > 0$ (i.e., row i in $\mathbf{X}^0$ includes only negative non-zero entries), the row multiplier r_i is also positive (as it should be), because then it also must be true from the requirement of consistent benchmark matrix and constraints that $u_i < 0$. Similarly, if for a particular aggregate set $\{I, J\}$ all the corresponding elements in the original matrix are non-positive with at least one negative entry, then the consistent aggregation constraint must have $w_{IJ} < 0$ for the set $\{I, J\}$ under consideration. The last will also ensure that the corresponding aggregation multiplier t_{IJ} is positive.

There is, however, one special case – uninteresting and unlikely to be encountered in an IOT/SUT/SAM updating – when the MR-GRAS iterative procedure results in *zero* multipliers. We refer to this case as the *weak sign preservation* property of the MR-GRAS method. This possibility is discussed in the third subsection.

In order to compute the required multipliers and the new adjusted matrix $\mathbf{X}$, we propose the following simple iterative algorithm (see Appendix F in the supplemental online material for the Matlab code).

- Iteration $iter = 0$: set $r_i(iter = 0) = 1$ for all i and $t_{IJ}(iter = 0) = 1$ for all I and all J . This initialization essentially means that one starts the adjustment procedure from the original matrix.
- Iteration $iter = 1, 2, \dots, k$: perform the following sequence of computations:
 - (a) Calculate $s_j(iter)$ using (4) as a function of $r_i(iter - 1)$ and $t_{IJ}(iter - 1)$, both derived from the previous iteration.

- (b) Calculate $r_i(\text{iter})$ using (3) as a function of $s_j(\text{iter})$ of the current iteration and $t_{IJ}(\text{iter} - 1)$ from the previous iteration.
- (c) Calculate $t_{IJ}(\text{iter})$ using (5) as a function of $r_i(\text{iter})$ and $s_j(\text{iter})$, both of current iteration.
- Iteration $\text{iter} = k$: stop when the multipliers converge for a sufficiently small tolerance level $\epsilon > 0$, that is, when $s_j(k) - s_j(k - 1) < \epsilon$ for all j , $r_i(k) - r_i(k - 1) < \epsilon$ for all i , and $t_{IJ}(k) - t_{IJ}(k - 1) < \epsilon$ for all I and J .
- Finally, using the last iteration values of multipliers $r_i(k)$, $s_j(k)$ and $t_{IJ}(k)$ derive the adjusted entries of the new matrix, x_{ij} 's, using the MR-GRAS solution (2).

Normalization and interpretation of MR-GRAS multipliers

How to interpret the row, column and aggregation multipliers of the MR-GRAS solution? In the IO literature, the uniform changes along any row and down any column of an intermediate input coefficients matrix are interpreted to reflect the *substitution effects* and *fabrication effects*, respectively (Stone, 1961). That is, it is often claimed that the substitution factor r_i measures whether input i has been replaced by other inputs (if $r_i < 1$) or has replaced other inputs (if $r_i > 1$) in the new versus the old IO matrix. The fabrication factor s_j indicates whether sector j absorbs more (if $s_j > 1$) or less (if $s_j < 1$) intermediate inputs compared with primary inputs (Miller & Blair, 2009, ch. 7.4.4). Oosterhaven et al. (1986, p. 62) interpret the aggregation multiplier t_{IJ} as *national cell-specific technology effect* indicating the rise or fall in importance of product i for sector j within the national economy at hand. More generally, in a world MR-IOT setting, the aggregation multiplier t_{IJ} could (better) be termed as *interindustry technology effect* which captures a pure technology effect, as opposed to the substitution effect that may, and in most cases, in fact, will contain a spatial substitution element.⁹

Depending on the research focus and/or nature of the underlying data, the interpretation of multipliers could vary. For example, Zhao and Squibb (2019) applied the MR-GRAS approach to estimate interregional trade flows among different regions in the United States, and interpreted the MR-GRAS aggregation multiplier as *regional effect* which ‘indicates the rise or fall in importance of input i in region j ’ (p. 6), or equivalently, ‘captures trade flows between regions, and also control the aggregated trade flows to the total regional supply and use’ (p. 1). As a side note, the MR-GRAS has been also used by the Joint Research Centre of the European Commission is projecting MR-IOTs for the baseline scenario that represents a projection of the world economy under the assumption of current climate and energy policies and also realization of nationally determined contributions in line with Paris agreement (for details, see Rey Los Santos et al., 2018).

However, any interpretation of the standard (G)RAS multipliers requires a normalization since r_i 's and s_j 's are unique only up to a scalar (e.g., Toh, 1998; De Mesnard, 2002, 2004; Lahr & De Mesnard, 2004). One encounters the same non-uniqueness problem with MR-GRAS multipliers. From (2) it is easy to verify that if r_i , s_j and t_{IJ} are the MR-GRAS multipliers, so are, for example, $\delta \times r_i$, s_j/δ and t_{IJ} , or alternatively $\delta^{1/2} \times r_i$, s_j/δ and $\delta^{1/2} \times t_{IJ}$ for any $\delta > 0$. In all these cases the updated matrix $\mathbf{X}$ is exactly the same, but the underlying multipliers are unique only up to a scalar. Therefore, as follows from (2), what is uniquely identified is only the product $t_{IJ}r_i s_j$ for all $i \in I$ and all $j \in J$.

To overcome the problem of the non-uniqueness of multipliers, the use of a scaling equation (i.e., *normalization*) is required. In general, it is preferred that the choice of normalization is justifiable on theoretical and/or empirical grounds. For example, within the IO framework, Van der Linden and Dietzenbacher (2000) propose a normalization based on the economic reasoning that for the whole system under consideration the aggregate substitution effect should be zero. The latter simply means that the system-wide use of intermediate inputs with substitution effects is equal to the system-wide intermediate use without substitution.

In case of MR-GRAS, this normalization proceeds as follows. If we denote the normalized multipliers by an asterisk superscript, then the first normalization task is to convert r_i into r_i^* for all i such that $\sum_i (u_i/r_i^*) = \sum_i u_i$. Let us denote the weighted harmonic mean of *non*-normalized row multipliers r_i 's by

$$\bar{r}_H \equiv \frac{\sum_i u_i}{\sum_i (u_i/r_i)},$$

which will generally be different from unity. After the MR-GRAS algorithm, the normalized multipliers are then obtained as follows:

$$r_i^* = \frac{r_i}{\bar{r}_H}, \quad s_j^* = s_j \times \bar{r}_H \quad \text{and} \quad t_{IJ}^* = t_{IJ}. \quad (7)$$

It is easy to show that the substitution effects r_i^* 's in (7) indeed satisfy $\sum_i (u_i/r_i^*) = \sum_i u_i$, that is, $\bar{r}_H^* = 1$. Hence, by scaling the substitution effects, the normalized column multipliers, $s_j^* = s_j \times \bar{r}_H$, would cover the fabrication effects of substitution of intermediate inputs for primary factor inputs.¹⁰

In other settings, for example, when updating the flows of migrants, social networks, flows of financial assets, etc., the interpretation of the MR-GRAS multipliers would be different which implies that a different normalization (most likely) could be more appropriate. That is, it is the multipliers interpretation which is critical in choosing the most appropriate scaling for the problem at hand.

The main properties of the MR-GRAS method

The (G)RAS method has several attractive properties that make it popular and (arguably) the most widely used updating method among practitioners. One of these properties is the existence of a closed-form solution, which makes GRASing a rather simple procedure to implement, requiring no advanced optimization and programming knowledge. As follows from the first subsection above, this property also holds for the MR-GRAS approach. Being a rescaling method, the (MR-G)RAS is also *transparent* as there is a simple relation (2) between the new and old matrices.

Two important economic structure-maintaining properties of the (MR-G)RAS technique are its *sign-preserving* and *zero-preserving* properties. This follows from the MR-GRAS analytical solution (2) and the implied strictly positive scaling multipliers. We refer to this property as *strong sign preservation*. However, compared with the analytical solution, the (MR-G)RAS iterative procedure adds a possibility of what we call as *weak sign preservation*. This occurs, for example, when one has a row with at least one strictly positive number and a constraint for that row equal to zero. The (MR-G)RAS solution then sets that whole row equal to zero. This can be verified using (3): without any negative element along the non-zero row i , that is, $p_i(s, t) > 0$ but $n_i(s, t) = 0$, the zero row sum requirement $u_i = 0$ implies zero multiplier, $r_i = 0$. Hence, all strictly positive numbers along row i of the original table change into a semi-positive number (actually zero). Thus, 'weak sign preservation'. Using (4), similar possibility can be seen for a non-zero and non-negative column j that is constrained to sum to zero, $v_j = 0$. The same holds for the additional aggregation constraints of the MR-GRAS procedure: if in the original table all the entries $\{i, j\}$ corresponding to one specific aggregate set $\{I, J\}$ are non-negative with at least one positive element and are constrained to sum to zero, $w_{IJ} = 0$, then all the original non-zero elements turn into a semi-positive number, zero, as follows from (5).

However, it must be recognized that such cases of turning positive elements into zeros due to zero row, column and/or aggregate sums is uninteresting for updating purposes. Why would one ever want to nullify a set of positive entries in an IOT/SUT/SAM update? Economies grow and become more complex over time, thus positive flows turning into zero is not very likely. But even if such cases arise (e.g., old industries cease to exist), simply nullifying (manually) the corresponding original entries also does the job.

A case that is relevant for updating purposes is constraining elements of both signs to sum to zero. As follows from (3) to (5), in the presence of both positive and negative elements with corresponding zero summation constraints, the multipliers will always be strictly positive. Thus, the underlying positive and negative elements will keep their signs in its strict sense and will be updated (and thus equally contribute) to sum to zero.

To sum, positive and negative elements in the old matrix keep their original signs after the updating procedure, while zero old entries remain zeros in the new matrix. The usefulness of these properties could be assessed from two perspectives. On the one hand, if one is interested in keeping the structure of the old matrix in terms of its elements' signs and zero values in the projected matrix, then MR-GRAS is an ideal updating technique to use. On the other hand, if the problem at hand finds it *critical* to allow for changing signs and switching between zero to non-zero values, then applying MR-GRAS is not recommended. However, if one has information on which entries would change their signs or become (non)zero, this could readily be introduced within the old table and the 'desirable' outcome would be obtained from the MR-GRAS procedure.

The MR-GRAS zero-preserving property also allows one to incorporate extra information on the updated matrix in exactly the same way as is implemented in the so-called modified RAS approach.¹¹ This property of introducing additional exogenous data is critically important since it is generally recommended to use all available information in the updating procedure. As De Mesnard and Miller (2006) put it, '[a]s a general rule, introduction of accurate exogenous information into RAS improves the resulting estimates, and counterexamples should probably not be taken too seriously' (p. 517).

McDougall (1999) makes a detailed comparison of RAS and other entropy-theoretic methods, including the minimum sum of cross-entropies (MSCE) technique (Golan et al., 1994; Golan & Vogel, 2000), and argues that, in general, RAS remains the preferable matrix balancing technique. In particular, he states that there is 'one important and desirable property that the RAS method has and MSCE does not: ... the RAS preserves the ordering of input intensities across industries, ... [while] in general, the MSCE estimates do not preserve the intensity ordering' (McDougall, 1999, p. 10, Proposition 5). By preserving input intensity ordering it is meant that if for any pair of industries b and k in the base table one has the relation:

$$\frac{x_{i1b}^0}{x_{i2b}^0} \geq \frac{x_{i1k}^0}{x_{i2k}^0}, \quad \text{then in the target table it is also valid that} \quad \frac{x_{i1b}}{x_{i2b}} \geq \frac{x_{i1k}}{x_{i2k}}.$$

Note that not only are the input intensities maintained but also the output intensities, which may be important if applications of the supply-driven IO model are foreseen (see Oosterhaven, 2019, ch. 6, for the latest warning to do so). Applied to RASed value added components, it means that the ordering of the capital-to-labour ratios across industries in the original data is preserved after RAS balancing. This is indeed an important property, since 'there is nothing in the new data to support any reversal in relative input intensities; there cannot be, since the new data contain no industry-specific information about cost structures' (McDougall, 1999, p. 11). However, the same *inputs-intensity-ordering preserving* property *only partially* holds for the MR-GRAS approach: it only holds for MR-GRAS if the industries b and k under

consideration are subject to the same aggregation constraint set J of purchasers of inputs (see Appendix B in the supplemental material online).

There are two other properties of the traditional RAS that are not kept in its generalized versions when the benchmark matrix includes at least one negative element. First, Dietzenbacher and Miller (2009) formally showed that for the RAS outcome it does *not* make a difference whether one is updating a transactions matrix or the corresponding input or output coefficients matrices. In fact, the proof is also valid for MR-RAS because their main conclusion is not affected by adding other restrictions to the basic RAS problem, such as when ‘the sum of a set of elements may be known *a priori*, or inequalities for sets of elements may be imposed’ (p. 559). This ‘uniqueness’ property of ‘generating the same answer whether updating the transactions or the coefficients is a very attractive property that holds exclusively for RAS, at least within the set of commonly applied updating procedures’ (p. 564). This property of RAS, however, does not hold for the GRAS technique when negative elements are present in $\mathbf{X}^0$. Therefore, it will not hold for MR-GRAS either. The proof is given in Appendix C in the supplemental online material.

Finally, on the base of the principle of insufficient reason (or Laplace criterion), it could be argued that another desirable property of a matrix updating technique might be a *homotheticity property*: if all the exogenous constraints are multiplied by the same scalar k , then the new/updated matrix equals the same multiple of the old matrix, that is, $\mathbf{X} = k\mathbf{X}^0$. Motorin (2017) showed that while RAS passes this homothetic test, GRAS does not. Hence, this statement also holds true for MR-GRAS when there is at least one negative element in the benchmark table.

UPDATING NATIONAL AND MULTI-REGIONAL SUTs

It is clear that the MR-GRAS method can be used for constructing/balancing/updating *any* partitioned matrix to conform to new exhaustive or non-exhaustive constraints (where the aggregation constraints are non-overlapping). Given the increased importance of SUTs, this section discusses MR-GRAS applied to updating SUTs.

With the appropriate formulation of the old matrix $\mathbf{X}^0$ and the constraints $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{W}$ for the new matrix $\mathbf{X}$, MR-GRAS can be used to update SUTs, both within national and inter-country/regional settings. This applies to any SUTs framework, be it in basic prices or in purchasers prices, or where total use is separated into domestic and imported use, or any of the four types of regionalized national SUTs (Oosterhaven, 2019, ch. 3). In fact, it turns out that one may already use the GRAS approach to estimate SUTs when total outputs by product are not available. This was one of the main motivations for introducing the SUT-RAS approach by Temurshoev and Timmer (2011).

Consider as an example the national SUTs framework in basic prices. For such a setting we can formulate the benchmark matrix as follows (for simplicity, zero superscripts are suppressed on the right-hand side):

$$\mathbf{X}^0 = \begin{bmatrix} -\mathbf{S}^d & \mathbf{0} & \mathbf{U}_b^d & \mathbf{Y}_b^d \\ \mathbf{O} & -\mathbf{m} & \mathbf{U}_b^m & \mathbf{Y}_b^m \\ \mathbf{0}' & 0 & \mathbf{t}'_u & \mathbf{t}'_y \end{bmatrix}, \quad (8)$$

where $\mathbf{O}$ and $\mathbf{0}$ are, respectively, the null matrix and null vector of appropriate dimensions; $\mathbf{S}^d$ denotes the (domestic) supply matrix of dimension product by industry ($p \times s$), which is a transpose of the make matrix often denoted by $\mathbf{V}$; $\mathbf{m}$ is the p -dimensional column vector of total imports priced at CIF; $\mathbf{U}_b^d$ and $\mathbf{U}_b^m$ are $p \times s$ matrices of, respectively, domestic and imported intermediate uses at basic prices; $\mathbf{Y}_b^d$ and $\mathbf{Y}_b^m$ are, respectively, $p \times f$ matrices of domestic and

imported final uses at basic prices (where f is the number of final use categories); and $\mathbf{t}_u$ and $\mathbf{t}_y$ are, respectively, s - and f -dimensional vectors of total taxes less subsidies on products for intermediate and final uses, and transposition is indicated by a prime ($'$).

Note that for any balanced SUTs framework, the row sums of the above benchmark matrix, except for its last row, are all zeros because of product-level supply-use accounting balances of domestic production and imports, that is, $\mathbf{S}^d \mathbf{t} = \mathbf{U}_b^d \mathbf{t} + \mathbf{Y}_b^d \mathbf{t}$ and $\mathbf{m} = \mathbf{U}_b^m \mathbf{t} + \mathbf{Y}_b^m \mathbf{t}$. Obviously, the negative values in (8) are of the same relative size as the positive values, which makes the use of (MR-)GRAS indispensable.

Now assume, similar to Temurshoev and Timmer (2011), that for a new matrix (to be projected on base of $\mathbf{X}^0$) only the following information is available: total output by industry at basic prices $\mathbf{x}' = \mathbf{t}' \mathbf{S}^d$, the vector of gross value added (GVA) by industry at basic prices $\mathbf{w}$, the total (domestic and imported) final uses by final demand category at purchasers' prices $\mathbf{y}_f$, and the economy-wide totals of imports (m_{tot}) and net taxes on products (t_{tot}). To be able to use the GRAS method for the updating purpose, the row and column sums of the new matrix $\mathbf{X}$ are defined as follows (the subscript $2p$ of the null vector indicates its dimension):

$$\mathbf{u}' = [\mathbf{0}'_{2p} \quad t_{tot}] \quad \text{and} \quad \mathbf{v}' = \begin{bmatrix} -\mathbf{x}' & -m_{tot} & \mathbf{x}' - \mathbf{w}' & \mathbf{y}'_f \end{bmatrix}. \quad (9)$$

GRASing $\mathbf{X}^0$ in (8) to satisfy (9) yields exactly the same SUTs as those produced by the SUT-RAS approach: the outcome of these two approaches must be the same as the two problems are equivalent, that is, they have the same objective function and the same constraints (for the proof, see Appendix D in the supplemental online material).

All other frameworks of national SUTs can be updated/estimated using the GRAS approach by appropriate reformulation of $\mathbf{X}^0$, $\mathbf{u}$ and $\mathbf{v}$, including the cases when some components of SUTs are not known. If there is additional information on (non-overlapping) aggregate values of certain parts of national SUTs, then it is preferable to use MR-GRAS instead of GRAS or SUT-RAS, as incorporating additional information on the new SUTs (generally) improves the final estimates.

Moreover, the MR-GRAS is even more useful for updating inter-national/regional or global SUTs. Consider a global SUTs framework in basic prices, which distinguishes between the origin and destination countries of the intermediate and final uses (i.e., the 'purchases only' member of the family of interregional SUTs in Oosterhaven, 1984). Within the MR-GRAS setting, such a SUT with c countries (regions) can be compactly written as:

$$\mathbf{X}^0 = \begin{bmatrix} -\mathbf{S}^1 & \mathbf{O} & \dots & \mathbf{O} & \mathbf{U}^{11} & \mathbf{U}^{12} & \dots & \mathbf{U}^{1c} & \mathbf{Y}^{11} & \mathbf{Y}^{12} & \dots & \mathbf{Y}^{1c} \\ \mathbf{O} & -\mathbf{S}^2 & \dots & \mathbf{O} & \mathbf{U}^{21} & \mathbf{U}^{22} & \dots & \mathbf{U}^{2c} & \mathbf{Y}^{21} & \mathbf{Y}^{22} & \dots & \mathbf{Y}^{2c} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \dots & -\mathbf{S}^c & \mathbf{U}^{c1} & \mathbf{U}^{c2} & \dots & \mathbf{U}^{cc} & \mathbf{Y}^{c1} & \mathbf{Y}^{c2} & \dots & \mathbf{Y}^{cc} \\ \mathbf{0}' & \mathbf{0}' & \dots & \mathbf{0}' & (\mathbf{t}_u^1)' & (\mathbf{t}_u^2)' & \dots & (\mathbf{t}_u^c)' & (\mathbf{t}_u^1)' & (\mathbf{t}_u^2)' & \dots & (\mathbf{t}_u^c)' \end{bmatrix}, \quad (10)$$

where $\mathbf{S}^r$ is the product-by-industry domestic supply matrix of country r , the ij -th element of $\mathbf{U}^{rs}$ indicates the amount of intermediate product i from country r used by industry j in country s , while the if -th element of $\mathbf{Y}^{rs}$ indicates the amount of final product i from country r used by final user f in country s , and the j -th element of $\mathbf{t}_u^s$ (respectively the f -th entry of $\mathbf{t}_y^s$) denotes total taxes less subsidies on products paid by intermediate user j (respectively the final user f) in country s . Note that compared with (8) imports from the rest of the world $\mathbf{m}$ are absent here.

The MR-GRAS row and column sums constraints corresponding to (10) then take the form:

$$\mathbf{u}' = [\mathbf{0}'_{2c} \quad t_{\text{world}}] \quad \text{and} \\ \mathbf{v}' = \begin{bmatrix} -(\mathbf{x}^1)' & -(\mathbf{x}^2)' & \cdots & -(\mathbf{x}^c)' & (\mathbf{u}^1)' & (\mathbf{u}^2)' & \cdots & (\mathbf{u}^c)' & (\mathbf{y}_f^1)' & (\mathbf{y}_f^2)' & \cdots & (\mathbf{y}_f^c)' \end{bmatrix},$$

where t_{world} is the value of net taxes on products at the world level; and $\mathbf{u}^c$ and $\mathbf{y}_f^c$ denote, respectively, the vectors of total intermediate inputs and total final uses at purchasers' prices in country c . In such a setting, imposing non-overlapping aggregation constraints is particularly relevant, especially to ensure that the different (parts of) components of SUTs of each individual country sum to the corresponding aggregate value. This information then needs to be incorporated in the aggregator matrices $\mathbf{G}$ and $\mathbf{Q}$ with the corresponding aggregation values included in $\mathbf{W}$, also when there is only partial information available.

Assume one has information on $\mathbf{u}'\mathbf{U}^{rs}\mathbf{u} = u^{rs}$, $\mathbf{u}'\mathbf{Y}^{rs}\mathbf{u} = y^{rs}$, $\mathbf{u}'\mathbf{t}_u^s = t_u^s$ and $\mathbf{u}'\mathbf{t}_y^s = t_y^s$ for all r and all s . The corresponding aggregator matrices then take the following form:

$$\mathbf{G}_{(c+1) \times (cp+1)} = \begin{bmatrix} \mathbf{I}_c \otimes \mathbf{u}'_p & \mathbf{0}_c \\ \mathbf{0}'_{cp} & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{Q}_{c(2s+f) \times (c+f)} = \begin{bmatrix} \mathbf{O}_{cs \times c} & \mathbf{O}_{cs \times f} \\ \mathbf{I}_c \otimes \mathbf{u}_s & \mathbf{O}_{cs \times f} \\ \mathbf{O}_{cf \times c} & \mathbf{I}_c \otimes \mathbf{u}_f \end{bmatrix},$$

where subscripts denote the dimension of the corresponding vector/matrix and $\otimes$ is the Kronecker product. Appendix E in the supplemental online material presents a detailed guide on practical implementation of MR-GRAS, which, among other issues, explains how to deal with cases of missing w_{IJ} 's.

In practice, it is often the case that for each country only the overall total taxes less subsidies on products are known without the corresponding totals on intermediate and final uses, that is, only t^s are given for each s but not its two individual components t_u^s and t_y^s (where $t^s = t_u^s + t_y^s$). In such a setting, the net taxes-related part of the global SUTs in (10) can be further expanded as follows:

$$\mathbf{X}^0 = \begin{bmatrix} -\mathbf{S}^1 & \mathbf{O} & \cdots & \mathbf{O} & \mathbf{U}^{11} & \mathbf{U}^{12} & \cdots & \mathbf{U}^{1c} & \mathbf{Y}^{11} & \mathbf{Y}^{12} & \cdots & \mathbf{Y}^{1c} \\ \mathbf{O} & -\mathbf{S}^2 & \cdots & \mathbf{O} & \mathbf{U}^{21} & \mathbf{U}^{22} & \cdots & \mathbf{U}^{2c} & \mathbf{Y}^{21} & \mathbf{Y}^{22} & \cdots & \mathbf{Y}^{2c} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & -\mathbf{S}^c & \mathbf{U}^{c1} & \mathbf{U}^{c2} & \cdots & \mathbf{U}^{cc} & \mathbf{Y}^{c1} & \mathbf{Y}^{c2} & \cdots & \mathbf{Y}^{cc} \\ \mathbf{0}' & \mathbf{0}' & \cdots & \mathbf{0}' & (\mathbf{t}_u^1)' & \mathbf{0}' & \cdots & \mathbf{0}' & (\mathbf{t}_u^1)' & \mathbf{0}' & \cdots & \mathbf{0}' \\ \mathbf{0}' & \mathbf{0}' & \cdots & \mathbf{0}' & \mathbf{0}' & (\mathbf{t}_u^2)' & \cdots & \mathbf{0}' & \mathbf{0}' & (\mathbf{t}_y^2)' & \cdots & \mathbf{0}' \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}' & \mathbf{0}' & \cdots & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \cdots & (\mathbf{t}_u^c)' & \mathbf{0}' & \mathbf{0}' & \cdots & (\mathbf{t}_y^c)' \end{bmatrix}, \quad (11)$$

which ensures that the known country-level totals of net taxes on products are explicitly incorporated in the corresponding row sums vector $\mathbf{u}$.

All in all, different formulations of MR-GRAS settings for national, interregional or global SUTs are possible, which depend on what components of the new SUTs are known or not known, and/or on the aims of the researcher.

CONCLUSIONS

In this paper we have presented the MR-GRAS method, which is an extension of the GRAS technique to a multi-regional setting. The framework is applicable to updating/balancing/constructing regional, national, interregional and global IO tables, SUTs, SAMs or generally any

partitioned matrix that needs to conform the new row sums, column sums and the additional non-overlapping aggregation constraints. The focus on non-overlapping aggregation constraints is motivated by (1) the aim of maintaining the simplicity and transparency properties of the GRAS technique in its ‘multi-regional’ setting and (2) the fact that in practice such settings are the cases practitioners come across most often.¹²

We derived the complete analytical solution of the method, proposed a simple iterative algorithm for its computation, and explained the possibilities of including *non-exhaustive* row sums and/or column sums and/or the (additional) aggregation constraints. Having such flexibility is critical as in real-life applications often not all the values of the three types of constraints are available, in which case the missing values are endogenously derived within the MR-GRAS updating procedure. In addition, we have elaborated on the main properties of the method, (some of) which also explain the popularity and attractiveness of the RAS-type balancing methods in practice. Further, we have discussed the normalization and interpretation of MR-GRAS multipliers. Next, from a wide range of possible MR-GRAS applications, we have examined a few updating settings, including national and global SUTs. Finally, the supplemental online material provides a detailed guide on practical implementation of MR-GRAS, which through a worked example demonstrates this procedure of tri-proportional scaling of different updating frameworks with exhaustive and non-exhaustive constraints, using MATLAB programming language. Our MATLAB code of the method is also made available in the Appendix in the supplemental material.

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DISCLOSURE STATEMENT

No potential conflict of interest was reported by the authors.

NOTES

¹ See Lahr and De Mesnard (2004) for details of the RAS method (including its history), which also gives an extensive set of references on the topic.

² That is, the most flexible framework, the so-called KRAS (K for *Konfliktfreies*) of Lenzen et al. (2009), generalizes the GRAS method to: (1) incorporate constraints on *arbitrary subsets* of matrix elements, including cases of constraints’ coefficients being different from 1 or –1; (2) include *reliability* of the initial estimate and the external constraints; and (3) find a *compromise solution* between conflicting constraints. It is, however, not surprising that such flexibility comes at the cost of losing update control and requires rather substantial programming and computational skills.

³ We were not aware of Holý and Šafr (2017), while MR-GRAS was developed and used in empirical studies well before Valderas-Jaramillo and Rueda-Cantuche (2019) (see e.g., Rey Los Santos et al., 2018; Zhao & Squibb, 2019).

⁴ The SUT-RAS approach (Temurshoev & Timmer, 2011) applies the GRAS updating idea to the *joint* estimation of national SUTs with different settings. This method is used in the World Input-Output Database project (www.wiod.org) to construct time series of national SUTs, which are one of the building blocks of the WIOD’s world IOTs.

⁵ Matrices are given in bold capitals, vectors in bold lower case and scalars in italicized lower case letters. Vectors are columns by definition; row vectors are obtained by transposition, indicated by a prime.

⁶ The (MR-)GRAS objective function is sometimes written *without* the base e of the natural logarithm. This omission would *not* cause any problem and the two formulations would be equivalent *as long as* the incorporated constraints fix the overall sum of the adjusted matrix elements. This, however, might *not* always be the case with non-exhaustive constraints, in which case (1a) should be used (see Appendix E.4 in the supplemental online material).

⁷ For simplicity of exposition, in (1a)–(1d) we have *not* used additional superscripts to explicitly distinguish between the disaggregated (e.g., intra- and interregional) variables and the aggregated (e.g., national) data as is done by Oosterhaven et al. (1986).

⁸ Hence, within the GRAS framework, ‘the procedure RAS is appropriate for positive elements, but needs to be replaced by $(1/R)A(1/S)$ for negative elements’ (Günlük-Şenesen & Bates, 1988, p. 476). The *complete* analytical expressions of the GRAS multipliers r_i and s_j are presented in Temurshoev et al. (2013).

⁹ The substitution effect is an across-a-row uniform substitution effect, for example, all industries everywhere are using more plastic parts from China. Simultaneously, all industries everywhere might be using less plastic parts from Japan (pure trade, or spatial substitution effect), or less metal parts from China (pure technology effect) or less metal parts from Japan (combined trade and technology effects). The interindustry technology effect means that plastic parts from *everywhere* are less used by, say, the food industry everywhere.

¹⁰ In general, there are two options for normalization: (1) a chosen scaling equation is used within each individual iteration of the (MR-G)RAS procedure; or (2) normalization is applied to multipliers after the (MR-G)RAS iterative procedure. De Mesnard (2004) calls these approaches, respectively, as *ex ante* and *ex post* normalization. As shown by De Mesnard (2004), for *ex ante* normalization it is impossible to analytically derive the normalized solution and convergence must be proved at each step of normalization. For these reasons, here the *ex post* normalization approach is adopted.

¹¹ The modified MR-GRAS approach would work as follows: nullify all cells of the benchmark table, which are known in $\mathbf{X}$; adjust the corresponding row, column and aggregation totals for this existing information; run the MR-GRAS procedure; and finally add back the known information into the updated matrix (e.g., Miller & Blair, 2009, ch. 7.4.5).

¹² For example, within a MRIO setting, the aggregation constraints will guarantee that the unknown inter- and intra-regional sectoral transactions are consistent with (i.e., add up to) the relevant exogenously specified aggregate (e.g., national accounts and aggregate trade) data. Moreover, updating regional, national, inter-country/regional or global SUTs very often can be easily formulated in a setting that fits in the MR-GRAS framework.

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