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Numerical analysis of forward and inverse fluid mechanics problems

Cristian Esteban Cárcamo Sánchez

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