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Abstract

Stochastic actor-based models for network dynamics have the primary aim of statistical inference about processes of network change, but may be regarded as a kind of agent-based models. Similar to many other agent-based models, they are based on local rules for actor behavior. Different from many other agent-based models, by including elements of generalized linear statistical models they aim to be realistic detailed representations of network dynamics in empirical data sets. Statistical parallels to micro–macro considerations can be found in the estimation of parameters determining local actor behavior from empirical data, and the assessment of goodness of fit from the correspondence with network-level descriptives. This article studies several network-level consequences of dynamic actor-based models applied to represent cross-sectional network data. Two examples illustrate how network-level characteristics can be obtained as emergent features implied by micro-specifications of actor-based models.

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statistical inference, agent-based simulation, social networks, micro-macro link, emergence

The contribution of inferential statistical techniques to agent-based modeling has been moderate so far. Statistical analysis, when viewed as the science of how to generalize features of random samples to populations, tends to rule out, overlook, or de-emphasize systematic differences in dependence structure that exist between samples and populations. Indeed, phenomena of *emergence* that occur in larger social systems, and that may reflect the functioning of the system as an organic whole, remain outside the scope of classical survey sampling (Barton 1968). Agent-based simulation models, on the other hand, have been developed for exactly this purpose. They can be employed when one wants to study those phenomena on the system level that emerge, typically as unintended consequences, from the dynamic interplay of the system's lower level constituents. One could pointedly speak of a division of labor between the two disciplines: Statistical inference from random samples is appropriate to the degree that system-level properties can be inferred by mere aggregation of independent sampling units, while agent-based modeling is appropriate to the degree that this is not the case.

Not all techniques of statistical inference, however, are based on random sampling, and in consequence, some statistical models and techniques may actually lend themselves quite well for studying micro-macro questions. In this article, we show how existing models for the empirical analysis of network dynamics can be used to study emergent, system-level (macro) properties in social networks, achieving an empirical orientation of micro-to-macro modeling. We use models for statistical inference about network dynamics (Snijders 2001; Snijders, van de Bunt, and Steglich 2010; Steglich, Snijders, and Pearson 2010) that are defined in terms of choices made by the actors in the network concerning their outgoing ties, and can be regarded as agent-based simulation models. Indeed, the complex interdependence of social actors in a network context could hardly be represented otherwise than by simulation models. Our approach further uses ideas from agent-based modeling by focusing on emergent properties as criteria for model fit; and it uses statistical ideas in its way of estimating free parameters in the model and attempting to obtain a model fitting well to the empirical data set. In this way,

we hope to contribute to the stream in the literature on agent-based modeling that stresses the need for calibration of agent-based models to selected features of real-world data (Boero and Squazzoni 2005; Moss 2008), in particular to the combination described by Manzo (2007:56; also see Hedström and Bearman 2009) as “describe by means of variables → explain by means of mechanisms → formalize by means of simulations.” More generally, we champion a combination of theoretical and statistical approaches in the study of real-life complex systems and hope that readers will see some merit in our contribution.

This article is about modeling cross-sectional observations of social networks (where “cross-sectional” means that an observation was made at a single moment in time) with a focus on the representation of network-level properties by an individual-based model; such properties may be called *emergent* because they follow from the complex interdependence between the individual actors situated in a common social network. Like many other stochastic models for dependencies in a cross-sectionally observed network, we represent the network by a probability distribution obtained as a stationary distribution in a dynamic interaction process. The currently most used statistical model of this kind is the exponential random graph model (ERGM; Snijders et al. 2006; Wasserman and Pattison 1996). This is, however, a tie-based model and does not represent the agency of the individual actors represented by the nodes in the network. Therefore, we consider the stochastic actor-based model (Snijders 1996, 2001; Snijders, van de Bunt, et al. 2010) that represents changes in the network as following from choices made by the individual actors, depending on their embeddedness in the network as well as the attributes of themselves and of all other actors. This model was proposed as a model for analyzing longitudinal network data, but here we use it to model cross-sectionally observed networks (Steglich 2006; Quintane et al. 2011).

In two very different example data sets, an iterative model selection procedure is employed that explicitly aims at the detailed specification of this model so as to faithfully reproduce the data sets’ macro features as emergent properties following from the behavior of the individuals in the context of the social network. The first of these concerns the friendship network of secondary schoolchildren from the *Teenage Friends & Lifestyle Study* (West and Michell 1996). The most striking global network features here are related to community structures among the pupils. The second example is an analysis of *Lazega’s Lawyers* data, the advice seeking network between the lawyers employed in a New England-based law firm (Lazega 2001; Lazega and van Duijn 1997). As will be seen, the global

network structure here primarily reflects the hierarchy between the lawyers. Interestingly, for both data sets, the local mechanism of *transitive closure* is crucial for obtaining a micro model that reproduces the macro features. It engenders both community structure (in the first data set: friends of friends being friends) and hierarchy (in the second data set: advisors of advisors being advisors).

Because nuances in the operationalization of transitive closure turn out to be quite important, we propose and illustrate a general method for studying the sensitivity of the network structures produced by the model to crucial parameters; in this case, two parameters that determine, in different ways, the importance to the actors of transitive closure. The method aims to study the sensitivity to one parameter in a model that contains other parameters, where not the other parameters are being kept constant, but what is kept constant are rather the features in the generated network structures corresponding to these other parameters (Steglich 2007). This sideline is elaborated in an own section because it responds to typical challenges and fundamental dangers that researchers face when studying complex phenomena of emergence.

In the following sections, we first position our research in the field and then introduce the research domain of network dynamics, with an emphasis on emergence of macro-level characteristics. We next give a brief sketch of stochastic actor-based modeling as a statistical technique that instantiates agency of social actors on the micro level, yet may also explain the macro-level outcome of a complete network. We proceed with the empirical analysis of the two networks introduced above, guided by concerns of the macro-level adequacy of the micro-level agency model, and the study of sensitivity to parameters. The article finishes with a discussion of our results.

Background

Spurred by the increased availability of computational capacities also to social scientists, the conceptual approach of *agent-based modeling* is increasingly being used to arrive at a deeper understanding of the functioning of social and economic systems. This approach can be used, for example, to study how system-level (macro) characteristics can be explained as emergent consequences of interdependent actor-level (micro) processes, exploring the consequences of these micro-level processes computationally by running *multi-agent simulations* in which the agents are used to represent the individual social actors. This allows extending work in the traditions of, for example,

rational choice sociology and *game theory*, and relaxing assumptions usually made in these approaches. These traditions employ an analytical perspective and derive macro-level consequences from micro-level assumptions, often by way of equilibrium concepts. The added value of the computational approach is the flexibility and transparency with which explicit assumptions about individual behavior on one hand and systemic interdependencies of individuals (bridge assumptions) on the other hand can be related to the system-level outcomes, and the complexity of models that can be handled (Helbing and Baretto 2011; Macy and Willer 2002; Raub, Buskens, and van Assen 2011).

The strength of agent-based simulation is the wealth of possibilities offered for theoretical explorations that can chart the precise correspondence between combinations of assumptions on one hand, and system-level outcomes on the other (Boero and Squazzoni 2005). The risk is that empirical data are used too loosely as a basis for model calibration and parameter values are selected based on convenience or mathematical interest, leading to limited empirical relevance of model assumptions as well as results. These topics are the traditional domain of statistical modeling and inference. Like for agent-based simulation, also in statistical inference parsimony of modeling (Occam's razor) is considered important, but the constraints are different. For agent-based modeling, the main constraint is that the model contains as few auxiliary elements as possible next to those that are essential to express the studied theory; for statistical modeling, the model should not contain more elements than necessary to express the studied research questions and to achieve an adequate fit between model and empirical data. The latter requirement generally leads to statistical models being more complicated than agent-based models—associated with the inclusion of control variables, perhaps the representation of several competing or complementary theories, and so on. The strength of statistical modeling resides in the possibilities offered to bring models closer to empirical data (by goodness-of-fit testing in various guises), leading to the important place of statistical modeling in the theoretical–empirical cycle. Its weakness is that often theories are represented in watered-down versions, and although predictions (hypotheses) derived from theories are tested, the core elements of social science theories—for example, agency—are not directly represented. Thus, the usual forms of statistical modeling favor the language of variables rather than the language of agency (Macy and Willer 2002).

In this article, we plead for attempting to combine the strong points of both traditions, and to develop and use statistical models that explicitly incorporate agency—or, expressed differently, to use agent-based models in a

statistical paradigm, which includes the estimation and testing of parameters as well as the assessment and cumulative improvement of goodness of fit. More specifically, we elaborate the combination of agent-based and statistical modeling for the case of actor-based models for network evolution (Snijders 1996, 2001). A case in point for statistical models of agency are discrete choice models in econometrics (e.g., McFadden 1974; Train 2003). However, econometrics has a strong atomistic prejudice, ruling out by way of assumption the kind of interdependencies that we are interested in studying—and accordingly, estimation of these comparatively simple models does not necessarily require computer simulation. We think that agent-based simulation models can demonstrate their strength especially in the representation of the detailed interactions between social actors that are the topic of social network analysis.

Social networks, due to the characteristic interdependence of network ties, can be analyzed adequately perhaps only by computational models. Actor-based models for network dynamics were originally presented in a plea for the combination of theoretical and statistical models (Snijders 1996). However, one of the characteristics of the agent-based modeling approach, the study of emergent properties at a system level, has been pursued for these models only marginally (Steglich 2007; Steglich et al. 2010). In the present article, we want to give it full attention. As was argued already by Robins, Woolcock, and Pattison (2005), stochastic network models implemented by computational algorithms—in their case *ERGMs*—can be a good choice to generate networks with desired macro features. Related to this article is the work by Hunter, Goodreau, and Handcock (2008) who proposed methods to assess and improve the fit of statistical network models by considering relevant macro features. The present investigation follows the lead of these two articles, now considering models for network evolution that are based on the explicit representation of agency, in contrast to the *ERGMs* that are tie based. Our focus is on the system-level properties of the networks generated by actor-level models and on the identification of the ingredients of the model specification that are operative to bring these system-level properties closer to empirical reality.

Modeling Dynamic Social Networks

Social networks are representations of the interdependencies between the social actors constituting a social system (Wasserman and Faust 1994). Examples can be found everywhere in society: in the primary social order (friendship in school classes, informal relations at the work place), in markets

(buyer–seller interaction, supply chains, structural competition), but also in the field of organizations (contracts between firms, alliance formation) and in government (governance, institutional design, and cooperation). Networks can be expressed, in their most simple form, by binary relational variables defined on the set of all pairs of actors, indicating for any two actors in the group under study whether they stand in a relation. Most social networks are dynamic by nature; new ties can be established and old ones can be terminated. These changes can often be considered to be the result of agency, that is, actors in the network deliberately changing the way in which they relate to other actors. In this view, network dynamics is represented as the result of micro mechanisms, which will be specified below as components of the behavioral rules followed by social actors when they forge or terminate their social ties. The outcome of these “bridging and bonding” decisions can be interesting to study on the macro level of global network structure. Macro features like the *small world* property (Watts 1999) or the *scale-free* property (Barabási and Albert 1999; de Solla Price 1976) have been explained, respectively, with reference to micro mechanisms such as *random rewiring* of locally clustered networks and *preferential attachment* shown by newcomers in growing networks.

Stochastic actor-based models for network dynamics (Snijders 1996, 2001; Snijders, Koskinen, and Schweinberger 2010; Snijders, van de Bunt, et al. 2010) have the purpose to represent network dynamics on the basis of observed longitudinal data, and evaluate these according to the paradigm of statistical inference. These models represent network dynamics as being driven by many different tendencies, among which can be the micro mechanisms alluded to above. Such tendencies may well operate simultaneously. Some examples are reciprocity (“you scratch my back and I’ll scratch yours”), transitivity (“friends of my friends are my friends”), homophily (“birds of a feather flock together”), and assortative matching (choice of network ties based on similarity of network position). By including several of such tendencies simultaneously, the models aim to give a good representation of the stochastic dependence between the different network ties. This permits testing hypotheses about these tendencies, and estimating parameters expressing their strengths, while controlling for other tendencies (which in statistical terminology might be called *confounders*). The actor-based nature of the model implies that changes in the network are modeled as choices by the actors. This leads to a model combining agency and structure, which is well suited for expressing theories based on purposeful behavior by social actors conditioned by their network context, but also for exploring the macro-level consequences of these theories.

Statistical methods for parameter estimation in these models are based on simulation, as will be elaborated below. They have been implemented in the *R* package¹ RSiena (Ripley, Snijders, and Preciado 2012). The parameters in the statistical model represent the strengths of the various micro mechanisms included in the model, and govern the behavior of actors in their local network context. The estimation procedures imply that those characteristics of the high-dimensional network space that are directly used to estimate these parameters must be represented well by the model, and it is not surprising that they will have a good fit between data and networks simulated from the model. These characteristics describe features of the local networks of the actors, aggregated over all actors in the data set, and correspond to parameters in the model that are empirically estimated. The case is different for the features of the network that are not a direct component of the local decision making of social actors, but emerge over time from their interplay in the network context. This highlights an important distinction that needs to be made concerning the emergence of macro-level characteristics of the social network with respect to a given model.

A Taxonomy of Network Macro Features

“Emergence” can be characterized shorthand by the adage that “the total is more than the sum of its parts.” But what actually is the sum of the parts? Assuming a given actor model on the micro level, we here distinguish computationally between three classes of macro properties. Two of them are *aggregated micro features*, aggregates of functions of the local network neighborhood of the individual actors. First, there are those aggregate micro features that are a trivial consequence of the micro-level model and its estimated parameters (in our case, the parameters are the α_k and β_k in equations (1) and (3) below). They express dynamic tendencies in the network that can be fully understood from the viewpoint of the actor as represented in the micro model. In this sense, they are not strictly emergent but an explicit part of the model design. Second, there are those aggregated micro features that do not correspond to estimated parameters in the current micro model, but could be fully represented by an enriched one. They are emergent conditional on the current micro model. The third class are *proper macro features* that cannot be readily defined by reference to local network neighborhoods alone and that therefore will never be trivial consequences of any micro-level model. In this sense, they are “unconditionally emergent,” irrespective of what the micro model is.

To illustrate these concepts, first consider the examples of the *reciprocity index* of a network and the *characteristic path length* (or median geodesic distance) in the network. The former is an aggregated micro feature that could be part of the model design, depending on whether or not a reciprocation tendency is added to the micro-level actor model. It is defined as the proportion of ties in the network that are reciprocated, that is, the ratio of the sum of the number of reciprocated ties that every actor is involved in to the sum of the number of ties that every actor is involved in. Local calculations involving only the immediate network neighborhood of each individual actor suffice, and the models presented below contain a specific parameter determining the extent of tie reciprocation. This is not true for the characteristic path length, which would be a proper macro feature. Here, first a matrix of pairwise geodesic distances needs to be calculated, which essentially requires the whole network data set; no local calculations can be substituted (although sometimes semilocal approximations may be possible). In consequence, a desired reciprocity level will be much easier to achieve in simulations by formulating local rules for agent behavior than a desired characteristic path length.

As an example for conditional emergence of an aggregated micro feature, consider a micro model that incorporates a strong tendency toward homophily on an individual variable (say, the race or the gender of the actor). Such a model will imply transitivity in simulated networks (Goodreau, Kitts, and Morris 2009; Steglich 2007), even without explicitly including tendencies toward transitivity in the actor model. Transitivity in such a situation is “conditionally emergent,” but still of the aggregated micro type, as it can be calculated from the triad census which is a local aspect of network structure (Holland and Leinhardt 1976). By adding an explicit transitivity tendency to the micro model, transitivity would lose its status as “conditionally emergent” but become a part of the model design instead.

For fitting a stochastic actor-based model to a data set, the difference between “aggregated micro” and “proper macro” features is essential. The latter may be criteria on which we want to evaluate goodness of fit of our model to the data set; for example, we may want to have a model that produces (in expected value over simulations) the observed characteristic path length. This macro-level, global fit criterion cannot readily be tied to any local network characteristic in the actors’ personal network neighborhoods. At the micro level of modeling actor behavior, this implies that there is no model parameter, the inclusion of which would guarantee perfect fit on this macro dimension. In this situation, aggregated micro features can play an “intervening” role in model construction. Because they are tied to

micro-level model parameters, they can be represented well by definition. But they also might be associated (algorithmically or correlationally) to a proper macro feature. Whenever this is the case, goodness of fit on this macro feature will likely depend on the inclusion (and size) of the micro-level model parameter/parameters tied to its associated aggregated micro feature/micro features. To obtain a model that satisfactorily represents the proper macro feature, we therefore propose an indirect approach, namely, the identification of appropriate local mechanisms that likely affect these proper macro properties. A case in point is the role that micro model parameters expressing transitivity play for representing proper macro features expressing community structure and hierarchy, which will be elaborated in detail in the empirical sections of this article.

Sensitivity of macro-level features to micro-level parameters will be studied in the context of fitting stochastic actor-based network evolution models to two empirical data sets. In a stepwise model construction procedure, the models will be partially fitted (or “empirically calibrated”) to a number of “aggregate micro” features of the data. At each step, the partially fitted model defines a probability distribution of networks. By generating a sample from this distribution, the quality of reproduction of the empirical data can be evaluated also on those macro features that were not included in the partial fitting of the model, paying special attention to “proper macro” features. Based on this evaluation, model enlargements are identified that have the potential to increase fit on poorly modeled network dimensions, if those exist. In the following section, the model family will be introduced as well as the procedure of partial fitting.

Actor-based Model for Network Evolution

In its basic form, the stochastic actor-based network evolution model is defined as a stochastic process $X(t)$ on the state space of all binary directed networks on a set of n actors, over a time interval $[t_{\text{begin}}; t_{\text{end}}]$. Time t is a continuous parameter. The modeling relies on two basic assumptions. First, it is assumed that the actors in the (directed) network have control over their outgoing ties, that is, they can decide which other actors to link to; their freedom is not absolute, and the way in which they can change their outgoing ties is detailed below. They do not have direct control over their incoming ties, so they have no say on who may link to them. Second, it is assumed that change happens in smallest possible steps, so-called *micro steps*, explained below. This means that the compound change between $X(t_{\text{begin}})$ and $X(t_{\text{end}})$ is the aggregate result of (and hence decomposable into) a sequence of micro steps

that happened in the period between moments t_{begin} and t_{end} . The model is specified by two main components: the *rate function* λ modeling *how frequently* an actor $i \in 1, \dots, n$ has the opportunity to apply a change to the network at any time point $t \in (t_{\text{begin}}; t_{\text{end}})$, and the *objective function* f , modeling *what* that change looks like.

For a detailed model description, we refer the reader to the publications by Snijders (2001, 2005) and colleagues (Snijders, van de Bunt & Steglich, 2010). We here focus on the main model components: the simulation algorithm generating the distribution of $X(t_{\text{end}})$ and the estimation algorithm that can be used to estimate parameters and establish partial fit of this distribution to an observed network x_{end} . Finally, the idea of assessing goodness of fit on dimensions other than those included in the partial fitting is addressed (cf. Hunter et al. 2008; Lospinoso 2012). In the empirical section that follows, this goodness-of-fit approach will be employed in the assessment of how well the actor-based model succeeds in reproducing some important macro properties of a network, and what are the elements in a model specification that are responsible for the quality of this reproduction.

Model Components: Rate Function and Objective Function

The smallest change possible in a binary directed network is the change of a *tie variable* X_{ij} from the state “tie is present” ($X_{ij} = 1$) to “tie is absent” ($X_{ij} = 0$), or vice versa. This is called a *micro step*, and the stochastic actor-based model assumes that all observed change in a network results from a sequence of such micro steps. This rules out simultaneous change of multiple tie variables, and it uniquely identifies the *sender* of the tie variable as the actor in control of this particular micro step; if tie variable X_{ij} is changing, the actor responsible would be i . If the previous state of the network is x , the resulting network after this micro step, that is, after toggling the tie variable X_{ij} , is denoted as $x^{\pm ij}$.

The micro step is modeled as a decision taken by the actor responsible for it, that is, the sender of the tie variable in question. This is elaborated in two steps. First, a model component identifies, at any given time point, the actor that has the opportunity to make the next decision and the associated waiting time, and second another model component identifies the result of this actor’s decision making. These components are determined by the *rate function* and the *objective function*, respectively.

The rate function λ models the rate (or speed) at which actors get opportunities to change their outgoing ties. After the previous micro step has been made, for each actor i waiting times τ_i are drawn from the exponential

distribution with parameter λ_i , according to the probability density $p(t; \lambda_i) = \lambda_i \exp(-t\lambda_i)$. The expected value of this waiting time is $1/\lambda_i$. The actor with the shortest waiting time then is the first to get an opportunity to take a micro step, the longer waiting times are discarded. The rate function is defined by

$$\lambda_i(\rho, \alpha, x) = \rho \exp\left(\sum_k \alpha_k a_{ki}(x)\right), \quad (1)$$

where $a_i = (a_{1i}, \dots, a_{Ki})$ is a vector of actor-specific statistics expressing attributes and/or position of the actor in the network, α is a vector of weights attached to these statistics, and $\rho > 0$ is the “basic rate parameter” expressing the average number of opportunities for taking a micro step for actors with $a_i = 0$. In the examples below, just the case of a constant rate ρ (without additional specification α) will be investigated. This corresponds to the situation where the probability to get the next opportunity for taking a micro step at any moment is uniformly distributed over the actors.

The objective function $f_i(\beta, x)$ is used to define the probability distribution of the change made by actor i , given that i was identified as having the opportunity to make one. It can be interpreted very loosely as a preference function for actor i with respect to the next state x of the network; β is a vector of statistical parameters. Assuming that the current network state is x_0 and actor i is the next to make a micro step, the possible outcomes of the micro step are $x_0^{\pm ij}$ for any $j \in \{1, \dots, n\} \setminus \{i\}$ plus the option not to change anything, formally denoted as $x_0^{\pm ii} = x_0$. Altogether, the decision is between n options, of which $n - 1$ concern the toggling of an outgoing tie variable X_{ij} . The probabilities for these options are defined as

$$P\{x_0 \text{ changes to } x_0^{\pm ij}\} = \frac{\exp(f_i(x_0^{\pm ij}))}{\sum_{h=1}^n \exp(f_i(x_0^{\pm ih}))}. \quad (2)$$

It was proved by McFadden (1974) that these are the probabilities obtained from *myopic stochastic maximization* of $f_i(\beta, x)$, where actor i chooses the option $j \in \{1, \dots, n\}$ yielding the highest value of

$$f_i(x_0^{\pm ij}) + V_j,$$

for the next obtained network $x_0^{\pm ij}$, where V_j are independent random variables all having the standard Gumbel distribution (of which the precise shape is not important here; it is merely a convenient choice leading to this nice explicit expression for the probabilities). This property implies that the

Figure 1. Flowchart of the simulation algorithm.

objective function may be regarded as representing the total result of the balance between preferences, opportunities, and restrictions, or gains and losses, where “total result” is understood in the sense of a short-term result because the maximization is myopic and ignores longer term strategic or other considerations. We may note that this way of modeling also rules out alternative rationality concepts like *satisficing* (Simon 1956).

The objective function $f_i(\beta, x)$ is modeled similarly. It is the linear predictor in a multinomial logit statistical model, defined as a linear combination of a set of components called *effects*,

$$f_i(\beta, x) = \sum_k \beta_k s_{ki}(x). \quad (3)$$

Here s_{ki} are actor-specific statistics expressing attributes of the actor and/or the actor's position in the network, weighted by the parameters β_k . The effects here represent the micro mechanisms, which are regarded as components of the preferences, opportunities, and restrictions, that jointly determine the probabilities of creating new ties and dropping existing ties. Examples will be given below; they include tendencies for actors to reciprocate ties, to show preferential

attachment, to cluster transitively, to select partners based on attribute homophily, and so on. The choice of the effects s_{ki} is a matter of model building; the determination of values of the parameters β_k is based on empirical data, as explained in the subsection ‘Data to Model: Estimation by Fitting Aggregated Micro Features’ subsection.

Model to Data: Simulation

At the core of the stochastic actor-based model is the simulation of a network evolution process, given a model specification S with a vector of parameter values $\theta = (\rho, \alpha, \beta)$.

The simulation algorithm is shown in Figure 1 in the shape of a structured flowchart according to Nassi and Shneiderman (1973). It shows the calculation of one “draw” from the conditional distribution $X(t_{\text{end}} | x_{\text{begin}}, t_{\text{begin}})$ or, in other words, one simulation of the evolution of a network change process starting in x_{begin} at time point t_{begin} , following the stochastic actor-based model expressed by rate function λ and objective function f , and ending at time point t_{end} . The first observation x_{begin} is taken as a given starting value of the evolution process. After this initialization, a loop is entered in which waiting times are drawn for all actors, according to their rate functions, leading to the identification of an actor who gets the opportunity to take a micro step. The outcome of this decision alters the network, and hence affects the conditions under which subsequent waiting times are drawn and decisions are made. This feedback loop ends as soon as model time reaches t_{end} . The network that accrued over time from the individual decisions is then reported as the outcome of the simulation process.

As described so far, the stochastic actor-based model is a probabilistic agent-based simulation model. What makes it a useful tool for data analysis is the possibility to fit it to data sets (or “empirically calibrate” it) by estimating the parameters (ρ, α, β) from empirical data about network dynamics.

Data to Model: Estimation by Fitting Aggregated Micro Features

Two frameworks are available for estimating parameters of stochastic actor-based models on a given data set: likelihood-based estimation (Koskinen and Snijders 2007; Snijders, Koskinen, et al. 2010) and equation-based estimation (Snijders 1996, 2001, 2005). From an inferential–statistical viewpoint, likelihood-based estimation and inference is preferable because it makes more efficient use of the information available in the data, and hence allows the detection of effects with higher statistical power and precision.

Equation-based estimation, however, is more directly connected to agent-based modeling and therefore we only explain this framework. It is also the default estimation algorithm used for fitting these models in the developed software.

For each element of the parameter vector $\theta = (\rho, \alpha, \beta)$, one equation is formulated, making use of corresponding statistics $u = (u^\rho, u^\alpha, u^\beta)$ defined as follows:

$$\begin{aligned} u^\rho(x) &= \sum_{i,j} |x_{ij} - (x_{\text{begin}})_{ij}| \\ u_k^\alpha(x) &= \sum_{i,j} a_{ki}(x_{\text{begin}}) |x_{ij} - (x_{\text{begin}})_{ij}| \\ u_k^\beta(x) &= \sum_i s_{ki}(x), \end{aligned} \quad (4)$$

the index k referring to the elements of the vectors α and β , and the corresponding elements of the vectors a_i and s_i . The intuition behind the choice of statistics in equation (4) is that for each parameter, the corresponding statistic, when evaluated on simulated networks $x(t_{\text{end}})$, will typically become larger as the parameter increases, thus ensuring model identifiability. The best-fitting parameter vector $\hat{\theta}$ is defined as the one for which the expected and observed values of u are the same, that is, which solves the system of *estimating equations*²

$$E_{\hat{\theta}}(u(X(t_{\text{end}}))) = u(x_{\text{end}}), \quad (5)$$

where $u(x_{\text{end}})$ is the vector of observed values of the statistics. By construction, θ and u are vectors with the same dimension. Under mild regularity conditions, the solution to equation (5) will be locally unique and often globally unique. For further details, the review of this estimation method by Bowman and Shenton (1985) is recommended.

The effects $s_{ki}(x)$ are summaries of the personal network, or network neighborhood, of actor i . They represent the mechanisms of the micro model because the preferences, opportunities, and constraints of the actors are assumed to be depend on this network neighborhood, and the probability of creating and dropping ties depends through equation (2) on the resulting changes in these effects, weighted by the parameter β_k . Since the effects are locally defined, estimation by means of equation (5) indeed is based on aggregated local information: the “aggregated micro” features discussed above. This also implies that for the estimation of these models, information about the entire observed network x_{end} is not required; it suffices to have the values of the “target statistics” vector $u(x_{\text{end}})$.

In practice, the expected values at the left-hand side of equation (5) can be determined only by computer simulation, and the model parameters are estimated following the stochastic approximation algorithm described in Snijders (2001, 2005). Thus, simulation enters our approach at different stages. At the most basic level, simulations as described in the subsection ‘Model to Data: Simulation’ subsection are used to generate the network dynamics. These simulations are repeatedly executed inside an iterative procedure of stochastic approximation to estimate the parameters α_k and β_k , with changing trial values for these parameters in each iteration. Once the estimates have been obtained, the simulations are used to obtain a random sample from the distribution of networks implied by this particular set of parameter estimates.

Micro-level Mechanisms

The specification of the actor-based model is given by the list of “effects” $s_{ki}(x)$ included in the objective function (equation 3). These represent the drives of the actors and can be used after aggregation to estimate the parameters and assess aspects of the fit of the model.

Models of stepwise increasing complexity are being considered. The following steps are roughly in line with the recommendations for model specifications of Snijders, van de Bunt, et al. (2010).

1. The starting model is an extremely simple actor-based model, accounting only for the *density* of the network and the tendency toward *reciprocation*. The objective function is

$$f_i(x) = \beta_1 \sum_j x_{ij} + \beta_2 \sum_j x_{ij}x_{ji}. \quad (6)$$

2. Next, it is assumed that actors have a tendency toward transitive closure (“friends of friends are my friends”) and possibly also toward local hierarchy (“when I am the advisor of someone’s advisor, I’ll not consider this someone to be my own advisor”). These can be represented (going back to Davis 1970; Holland and Leinhardt 1976) by triadic subgraph counts in the personal network of i :

$$f_i(x) = \beta_1 \sum_j x_{ij} + \beta_2 \sum_j x_{ij}x_{ji} + \beta_3 \sum_{j,h} x_{ih}x_{hj}x_{ij} + \beta_4 \sum_{j,h} x_{ih}x_{hj}x_{ji}. \quad (7)$$

Parameter β_3 , the weight of *transitive triplets*, represents the strength of transitive closure. Parameter β_3 , the weight of *three cycles*, inversely represents hierarchy.

3. Third, it is permitted that actors have differential tendencies to nominate few or many others and that actors are differentially attracted to others depending on their numbers of choice made (out-degrees) and/or received (in-degrees). Some of such effects are feedback effects at the individual level. There will be positive feedback if those already making many nominations may persist or further increase in doing so; and if those receiving many choices may be more popular directly because of this (the “Matthew effect”: de Solla Price 1976; Merton 1968). The latter effect was studied also by Barabási and Albert (1999), who coined the label “scale-free” for the networks resulting from this single mechanism. Translating these tendencies into effects in the actor-based model, we consider here, first, the *out-degree activity* effect: Higher out-degrees lead to activity, that is, sustained or even further increased out-degrees. Second comes the *in-degree popularity* effect: Higher in-degrees lead to popularity, that is, sustained or even further increased in-degrees (the Matthew effect). The third effect along these lines is the *out-degree popularity* effect, expressing that higher out-degrees lead to popularity, that is, sustained or even further increased in-degrees. In terms of parameter estimation by estimating equation (4), estimating these three effects will fit, respectively, the out-degree variance, the in-degree variance, and the covariance between in- and out-degrees. Including these effects leads to the objective function

$$f_i(x) = \beta_1 \sum_j x_{ij} + \beta_2 \sum_j x_{ij}x_{ji} + \beta_3 \sum_{j,h} x_{ih}x_{hj}x_{ij} + \beta_4 \sum_{j,h} x_{ih}x_{hj}x_{ji} \\ + \beta_5 \sum_j x_{ij} \sum_h x_{hj} + \beta_6 \sum_j x_{ji} \sum_h x_{jh} + \beta_7 \sum_j x_{ij} \sum_h x_{jh}. \quad (8)$$

4. Next to these tendencies depending only on network structure, it will be considered that the behavior of actors also depends on their own attributes and those of their potential network partners. The best known of these tendencies is homophily, the tendency to be linked to others who are similar to oneself (Lazarsfeld and Merton 1954; McPherson, Smith-Lovin, and Cook 2001). For a categorical actor variable V , this is represented by the *same V* effect, which is the term

$$\beta_8 \sum_j x_{ij} I\{v_i = v_j\}. \quad (9)$$

For a numerical actor variable V , homophily can be represented by the V similarity effect,

$$\beta_8 \sum_j x_{ij} (\text{sim}(v_i, v_j) - \overline{\text{sim}}(v)). \quad (10)$$

Here sim denotes the dyadic similarity transformation, defined by

$$\text{sim}(v_i, v_j) = 1 - \frac{|v_i - v_j|}{\text{range}(v)},$$

and the mean $\overline{\text{sim}}(v)$ of all dyadic similarity values is subtracted as a way of centering.

In addition, for numerical actor variables, it can be meaningful to include the V sender and V receiver effect, defined, respectively, as

$$v_i \sum_j x_{ij} \text{ and } \sum_j x_{ij} v_j,$$

and representing that higher values of v_i are associated with a higher attractiveness of making or receiving nominations.

5. Finally, depending on the results of the model obtained thus far, further model improvement may be attempted.

The Actor-based Model and Global Network Structure

The purpose of this article is to show how the actor-based model, explained in the previous section, may be used to study macro features of networks. In this section, we first discuss the selection of 10 macro features being considered here and then give the technical elaboration of how the link is made between the micro-level model and an observed network.

Indicators of Global Network Structure

Various aspects of macro-level network structure have been studied in the literature. Here, we consider the following aspects: reachability and transitivity/clustering, which provide the basis of small world structures (Watts 1999); degree distributions, defining the scale-free property of de Solla Price (1976) and Barabási and Albert (1999); and hierarchy (Krackhardt 1994).

An even more fundamental aspect for directed networks is reciprocity; but this can be totally represented by a single aggregated micro feature, the proportion of ties that are reciprocated, which is represented by parameter β_2 in our objective function, and therefore is of a trivial nature in micro–macro considerations.

Reachability is usually expressed by the distribution of geodesic distances. For this purpose, we here consider graph distances while disregarding directionality of ties. In the first place, note that two nodes are at infinite distance if there is no path between them. A weak component in a directed graph (we shall further call this simply a component) is a set of nodes such that between each pair of nodes in this set there is a path connecting these nodes (disregarding directionality of ties); and the set of nodes cannot be enlarged by adding nodes while retaining the validity of this property. Therefore, geodesic distances between nodes are finite if and only if the two nodes are in the same component. Large graphs with average degrees higher than 1 tend to have a giant component, that is, a component comprising almost all nodes in the graph (Erdős and Rényi 1960).

This leads to the first group of statistics. All of these are “proper macro” features in the sense of subsection ‘A Taxonomy of Network Macro Features’ subsection.

1. *Size of the largest component*, C_1 ; the number of actors in the largest component.
2. *Number of components*, N_C .
3. *Diameter of largest component*, D_1 ; longest path distance (disregarding directionality of ties) between any pair of actors in this largest component.

Next the values of the finite geodesic distances (path lengths) can be considered. One approach is to consider the entire distribution of geodesic distances; this distribution is used for goodness-of-fit assessment in Hunter et al. (2008). As low-dimensional summaries, one may use quantiles of this distribution, for example, the median (Robins et al. 2005).

4. *Median geodesic distance*, $G_{0.5}$.

The *small world property* for networks with many nodes was defined by Watts (1999) as having low density, high transitivity (also called *clustering*), and small path lengths. A better descriptive than density is average degree, which is a multiple of density but not sensitive to the number of nodes, as it is a mean of an actor-level statistic.

To measure transitivity, we use

5. the transitivity coefficient of Frank (1980),

$$T = \frac{\sum_{i,j,h} x_{ij}x_{jh}x_{ih}}{\sum_{i,j,h} x_{ij}x_{jh}}, \quad (11)$$

the number of transitively closed triplets ($i \rightarrow j \rightarrow h$ and $i \rightarrow h$) divided by the number of potentially closed triplets ($i \rightarrow j \rightarrow h$).

For the *degrees* in a directed graph, one must differentiate between in- and out-degrees. These are actor-level descriptives, so any summary of them is of the aggregated micro type. The finest detail is obtained by considering the degree distribution—a bivariate distribution in the case of directed graphs. Next to the average degree, a first-order summary is given by the following three, which all are standardized in some sense.

6. *Variance of the in-degrees* divided by the mean degree, $\tilde{V}_{\text{in}}$; the degree variance was proposed as a descriptive statistic by Snijders (1981), and here it is divided by the mean degree so that in case of a Poisson distribution the value is 1;
7. *variance of the out-degrees* divided by the mean degree, $\tilde{V}_{\text{out}}$;
8. *correlation between in- and out-degrees*, $r_{\text{in,out}}$.

The very long-tailed power distributions of degrees that were studied by de Solla Price (1976) and Barabási and Albert (1999) are not a good approximation for most social networks between humans; among the reasons are the costs involved in maintaining ties, limiting the occurrence of very high degrees. If more detail is desired than variances and correlations of degrees, the entire degree distributions may be considered.

Krackhardt (1994) studied ways to express the extent of *hierarchization* of a network. We shall use two measures proposed by him.

9. *Graph hierarchy*, measuring the extent to which paths in the network run in one direction only. The measure for graph hierarchy is defined using the transitive closure of the original graph, which is the directed graph with the same node set in which a link $i \rightarrow j$ exists whenever there exists a directed path $i \rightarrow h_1 \rightarrow h_2 \rightarrow \dots \rightarrow j$ in the original graph. The graph hierarchy measure H is defined as the number of asymmetric dyads in the transitive closure (unordered pairs (i,j) for which $i \rightarrow j$ or $j \rightarrow i$ but not both) divided by the number of

connected dyads (unordered pairs (i, j) for which $i \rightarrow j$ or $j \rightarrow i$ or both). The index is one if all paths run in one direction only, and zero if all connected pairs of nodes (i, j) are connected by paths $i \rightarrow j$ as well as $j \rightarrow i$.

10. *Least upper boundedness.* To explain this feature, we interpret the network as implying status attribution, so that if there is a direct tie $i \rightarrow j$ but also if there is a path $i \rightarrow j$ from i to j , then j is regarded as “higher” than i . Here, we use the definition used above where $i \rightarrow j$ indicates the existence of a path from i to j , but now enrich it with reflexivity, that is, we define $i \rightarrow i$ for all actors i . Least upper boundedness measures the extent to which for any pair of actors (i, j) there is a unique “lowest” third actor who is “higher” than both i and j . Formally this is defined as follows. For a pair (i, j) , a least upper bound is an actor h such that $i \rightarrow h$ and $j \rightarrow h$, and such that for every h' with the same property, it holds that $h \rightarrow h'$. In terms of hierarchies, h is interpreted as the lowest level individual in the hierarchy who is higher than or equal to i as well as j . In a pyramidal hierarchy with arrows pointing upward to the top, each pair of actors has a least upper bound. Krackhardt’s degree of least upper boundedness L is defined as one minus the number of pairs who do not have a least upper bound, divided by the maximum possible number of pairs without a least upper bound, given the component sizes of the network. Thus, it is 1 for strictly hierarchical and 0 for totally nonhierarchical networks.

Summarizing, we consider four features expressing connectedness: C_1 , D_1 , N_C , and $G_{0.5}$; the transitivity coefficient T ; three features related to the degree distribution, V_{in} , V_{out} , and $r_{in,out}$; and the hierarchy measures H and L . Of these, T , V_{in} , V_{out} , and $r_{in,out}$ are aggregated micro-level characteristics, which therefore should be easy to fit by an agent-based model just by including parameters reflecting these graph features. The others, C_1 , D_1 , N_C , $G_{0.5}$, H , and L , are proper macro features the sense of subsection ‘A Taxonomy of Network Macro Features’ subsection and are the main focus of interest of our micro–macro study.

These measures all are defined on the directed graph, disregarding any exogenous variables that also may be of interest; in particular, they do not refer to homophily. In studies specifically concerned with homophily it would be interesting to consider similar statistics taking account of actor characteristics, for example, median geodesic distance of same-gender pairs and median geodesic distance of different-gender pairs (Steglich 2007), or network autocorrelation indices (Steglich et al. 2010).

Relating the Actor-based Models to Single-network Observations

In this article, we wish to explore how well the actor-based model for network dynamics can be tuned as a micro model to represent macro features of a single (cross-sectional) observation of a network. It may be noted that this departs from the more common use of the actor-based model for analyzing longitudinal network data (Snijders, van de Bunt, et al. 2010). The correspondence between the model and a single-observed network is specified by considering the *model parameters for which the observed state is in a short-term dynamic equilibrium*, defined as follows: Starting with the observed network and letting the model run for some period such that every actor makes an average of λ changes in his or her outgoing ties, a distribution of networks is obtained that has, for all aggregated micro statistics $\sum_i s_{ki}(x)$ corresponding to parameters in the model, the same average as the empirically observed value. Thus, the observed network is used as the starting as well as the ending observation, as a reflection of the equilibrium concept. This is implemented by applying the estimation method explained in subsection ‘Data to Model: Estimation by Fitting Aggregated Micro Features’ subsection to the observed network as if this was observed at two repeated moments,³ while fixing the rate parameter at λ . The choice of the value λ has consequences for this definition of short-term equilibrium. On one hand, λ needs to be sufficiently high to give the simulation process enough time to “get away” from the starting network and reach an equilibrium state. On the other hand, it must not be too high because there might be a possibility of near degeneracy of the long-term stationary distribution (for $\lambda \rightarrow \infty$). This is a well-known problem for ERGMs (Snijders et al. 2006), which also occurs for stochastic actor-based models (Steglich 2006). To sidestep this issue, the notion of “short-term dynamic equilibrium” is used here (see also Quintane et al. 2011). The short term is operationalized as $\lambda = 20$ changes on average by each actor. This was checked for robustness by running a few models with $\lambda = 50$ and $\lambda = 100$, for which the same results were obtained.⁴

Examples

We now proceed to the illustration of the model fitting and goodness-of-fit assessment for two data sets. The fit criteria will be the global network features of Indicators of Global Network Structure subsection, and successive model specifications follow the sequence of models in the subsection ‘Micro-level Mechanisms’.

Table 1. Descriptives for Glasgow Friendship Network.

Number of actors, n	129
Average degree, $\bar{d}$	3.60
Proportion of ties being reciprocated, r	0.63
Largest component size, C_1	126
Number of components, N_C	4
Diameter, D_1	11
Median geodesic distance, $G_{0.5}$	5
Transitivity, T	0.44
Scaled in-degree variance, $\tilde{V}_{in}$	1.22
Scaled out-degree variance, $\tilde{V}_{out}$	0.85
Correlation in- and out-degrees, $r_{in,out}$	0.43
Graph hierarchy, H	0.37
Least upper boundedness, L	0.06

Data Set 1: Friendship in Secondary School

The data modeled in this section are the third observation collected for the older of two cohorts in the *Teenage Friends and Lifestyle Study*, observed in 1997 in a fourth grade of a secondary school in Glasgow. The study was executed by Lynn Michell and Patrick West of the Medical Research Council/Medical Sociology Unit, University of Glasgow. Earlier publications about this network data set include Michell and Amos (1997) and Pearson and West (2003). The network of 129 pupils analyzed here is the third wave of the network study in Steglich, Snijders, and West (2006). It is a friendship network, and each pupil was requested to nominate up to six friends in the same cohort. Of the available covariates we only use gender.

Table 1 gives a descriptive overview of this network. The average degree $\bar{d} = 3.6$ and proportion of reciprocation $r = 0.63$ are quite in line with other friendship networks. The values $C_1 = 126$, $N_C = 4$ imply that the largest component spans almost the whole network, and in addition there are three isolated nodes. This is also quite a usual situation. The diameter $D_1 = 11$ and median geodesic distance $G_{0.5} = 5$ seem relatively large for a network of $n = 129$ nodes. The transitivity coefficient $T = 0.44$ again is quite usual. The in-degree variance is slightly larger than for a Poisson distribution, and the out-degree variance slightly smaller. For the out-degrees, this is expected given the upper limit of six nominations. For the two hierarchy measures, there seems to be little experience about their values, and we just see how they are represented by the fitted models.

We fit a sequence of models of increasing complexity to this network, simulating and estimating the models with the RSiena package of the statistical

Table 2. Model 1 for Glasgow Friendship Network: Reciprocity Only.

Effect	$\hat{\beta}_k$	(SE)
Out-degree	−2.47	(0.05)
Reciprocity	2.78	(0.10)

Figure 2. Distribution of macro features for Glasgow network, model 1.

system R (Ripley et al. 2012). The maximum out-degree is limited in the model specification to six, because this was a requirement in the data collection, which hereby was also observed in the simulations. Each model defines a probability distribution over the set of directed graphs on 129 nodes. The question of interest is how well the observed characteristics of this network fit within the estimated distribution of graphs. In the following, each model is represented by a random sample of 1,000 graphs drawn from the distribution for the estimated parameters.

The implied distributions of the network characteristics are plotted as violin plots (Hintze and Nelson 1998), which are a combination of a boxplot and a kernel density plot. The observed values are superimposed as red dots, with printed numerical values, and linked by a line. Dotted lines give the upper and lower 2.5 percent values of the cumulative distribution. The figure contains the violin plots for all the 10 characteristics, centered so that the medians (black dots) are

Table 3. Model 2 for Glasgow Friendship Network: Reciprocity and Triadic Effects.

Effect	$\hat{\beta}_k$	(SE)
Out-degree	-2.72	(0.07)
Reciprocity	2.64	(0.14)
Transitive triplets	0.63	(0.07)
Three cycles	-0.63	(0.13)

Figure 3. Distribution of macro features for Glasgow network, model 2.

horizontally aligned, and scaled so that the plots fit in a common figure. The figure was obtained using the *sienaGOF* function, programmed by Josh Lospinoso, of the *RSienaTest* package.

For the first model, representing only the tendency to reciprocity, the parameter estimates are given in Table 2 and the distribution of macro-level descriptives in Figure 2.

The kernel density plots in the figure demonstrate that for the first four features (size of largest components, number of components, diameter, and median geodesic distance) the distribution has a small number of integer values; for example, for the diameter, values 6 and 7 have the highest occurrence in the distribution, there are some occurrences of 5, 8, and 9, and the observed value 11 does not occur among the 1,000 sampled values. The distribution of

Table 4. Model 3 for Glasgow Friendship Network: Reciprocity, Triadic Effects, and Degree-related Effects.

Effect	$\hat{\beta}_k$	(SE)
Out-degree	-0.60	(0.96)
Reciprocity	2.72	(0.51)
Transitive triplets	0.78	(0.25)
Three cycles	-0.59	(0.54)
In-degree—popularity	0.06	(0.41)
Out-degree—popularity	-0.26	(0.26)
Out-degree—activity	-0.21	(0.06)
Out-degree ≥ 1	-3.48	(0.91)

least upper boundedness (henceforth abbreviated as *lubness*) has a bimodal shape, while the other features have distributions that are nearly symmetric and continuous. The figure shows that the observed values for diameter, median geodesic distance, transitivity, and hierarchy are totally outside the values obtained for the 1,000 simulated networks; for most other features the observed values are in the tails of the simulated distributions, and only for the scaled out-degree variance the observed value is in the middle part of the distribution. Thus, the representation of most of the descriptives by the model is quite poor. But a poor representation by this oversimple model was expected, and Figure 2 is intended mainly as a baseline for comparison for the other models.

The next model (Table 3) represents local structure (Holland and Leinhardt 1976) by assuming that the actors have tendencies to transitive closure and to favoring, or avoiding, three cycles in their personal networks.

Figure 3 shows that the fit for largest component size and number of components now is good, but for scaled out-degree variance it has deteriorated; for several of the other features there is a moderate improvement, but still the overall fit on the macro level is quite poor. It is striking that the transitivity coefficient T_i of the aggregated micro kind, is not represented well by this model, although it does incorporate a parameter for transitivity. The reason is that this parameter corresponds to the numerator of equation (11) but not the denominator, and the latter is not fitted well at all. The denominator is closely related to the covariance of the in- and out-degrees, and will be fitted by the next model.

When adding the degree-related effects, the fit of the entire degree distribution also was studied (without presenting the plot here), and it appeared that the number of actors with out-degree 0 (whose observed number was 8) was underrepresented by this model. This may be because there were some pupils

Figure 4. Distribution of macro features for Glasgow network, model 3.

who did not take the network survey seriously and mentioned no nominations at all. Therefore, a separate effect was added defined by a dummy variable that is 0 if the out-degree is 0 and 1 if the out-degree is at least 1, representing the micro-level tendency to take the survey seriously and else give no response at all. The model is presented in Table 4 and the goodness-of-fit plot in Figure 4.

With this model the fit has improved quite a lot, starting to be close to acceptable. The only feature for which the fit is poor is the graph diameter, where the observed value 11 is higher than all simulated values; and the median geodesic distance, where almost all sampled graphs have a value 4 with a few values 3, but the observed value is 5. For all other features, the observed values are within the middle 95 percent parts of the simulated distributions. It may be noted as a sideline that for the scaled in-degree variance and the lubness the simulated distributions have a heavy right tail with a few outliers.

As a next step, the effect of gender homophily was added to the model. This is an overriding effect in all child and adolescent friendship networks. However, for the fit of our 10 macro-level features, this did not lead to important changes, and for space reasons we do not present the results for this model.

Thus, the planned steps in our model sequence lead to a model that fits well with respect to most macro features under consideration, but not on the two functions of the distribution of geodesic distances: diameter and median geodesic. The geodesic distances in the fitted distribution are shorter than in the

Figure 5. A directed k -triangle for $k = 4$, that is, a directed four triangle.

Figure 6. Geometrically weighted edgewise shared partners (GWESP)-related component of tie value for $\alpha = .69$, dependent on number of intermediaries.

observed network—in other words, the fitted actor-based model produces networks that are too closely connected. To achieve a better fit in this respect, other specifications were explored by considering degree assortativity (Newman 2002; Snijders, van de Bunt, et al. 2010) and different specifications of transitivity. Incorporating degree assortativity did not yield any improvement, but different ways of modeling transitivity did. Here we followed the developments of Snijders et al. (2006) obtained for fitting ERGMs, a tie-based approach to network modeling. To explain these developments, consider Figure 5, which shows a transitive four triangle, that is, a configuration where a tie $i \rightarrow j$ closes four two paths $i \rightarrow h \rightarrow j$. The mentioned publication found that, when modeling networks by ERGMs, the number of transitive triplets is usually not a good representation of transitive closure, because it implies that

Figure 7. Distribution of macro features for Glasgow network, model 5.

the conditional log odds of a tie depends linearly on the number of intermediates (two paths closed by this tie), whereas in reality this increase in log odds is less than linear. To express this mathematically we use the specification of Snijders et al. and the parameterization defined by the “geometrically weighted edgewise shared partners” (GWESPs) of Hunter (2007). However, this concept here is specified in an actor-based way, by counting configurations in the local neighborhood of a given actor, rather than in the tie-based way of the models in the ERGM family, for which the GWESP statistic was first developed. We define the actor-based GWESP effect, in direct analogy to the corresponding global statistic of Hunter, by

$$\text{GWESP}(i, \alpha) = \sum_{k=1}^{n-2} e^{\alpha} \{1 - (1 - e^{-\alpha})^k\} \text{EP}_{ik}, \quad (12a)$$

where EP_{ik} (for “edgewise partners”) is the number of nodes j such that $i \rightarrow j$ and there are exactly k other nodes h for which there is the two path $i \rightarrow h \rightarrow j$. An equivalent way of writing this is

$$\text{GWESP}(i, \alpha) = \sum_{j=1}^n x_{ij} e^{\alpha} \left\{ 1 - (1 - e^{-\alpha})^{\sum_{h=1}^n x_{ih} x_{hj}} \right\}, \quad (12b)$$

where the convention is used that $x_{jj} = 0$ for all j .

Table 5. Model 5 for Glasgow Friendship Network: With Geometrically Weighted Edgewise Shared Partner Effect.

Effect	$\hat{\beta}_k$	(SE)
Out-degree	-1.80	(0.31)
Reciprocity	2.77	(0.20)
GWESP ($\alpha = .69$)	4.10	(0.21)
Complete triads	-0.32	(0.06)
In-degree—popularity	0.03	(0.03)
Out-degree—popularity	-0.24	(0.05)
Out-degree—activity	-0.14	(0.03)
Out-degree ≥ 1	-1.62	(0.70)
Same gender	0.49	(0.10)

Note: GWESP = geometrically weighted edgewise shared partners.

The parameter α is a tuning parameter that may range from 0 to ∞ . For all α , it holds that $\text{GWESP}(0, \alpha) = 0$, $\text{GWESP}(1, \alpha) = 1$, and $\text{GWESP}(k, \alpha)$ increases with k to a maximum slightly less than e^α . For $\alpha = 0$, the coefficients $e^\alpha \{1 - (1 - e^{-\alpha})^k\}$ are equal to 1 for all $k \geq 1$, and for $\alpha \rightarrow \infty$ they tend to k . Since we can write

$$\sum_{j,h} x_{ih}x_{hj}x_{ij} = \sum_{k=1}^{n-2} k \text{EP}_{ik} ,$$

this implies that for $\alpha \rightarrow \infty$ the regular number of transitive triplets is approached, while for smaller α the extra contribution of a high number of intermediaries h is downweighted. We used the value $\alpha = \log(2) = 0.69$, which often is a good value (Snijders et al. 2006); some experimentation showed that here it performs quite well compared to other values of α . The coefficients $e^\alpha \{1 - (1 - e^{-\alpha})^k\}$ used in equation (12a) are plotted in Figure 6 for $\alpha = .69$. This shows how the values increase very little after $k = 4$, and even the largest number of intermediaries still does not have twice the worth for increasing the probability of a direct tie as the value of one intermediary.

It should be noted that although the GWESP statistic is not triadic but depends on higher-order configurations, it is still a locally defined micro configuration because only those ties are considered that are part of the personal network, that is, the set of actors immediately connected to the focal actor i .

To obtain a final model, we replaced the two triadic effects (transitive triplets and three cycles) by the term

Figure 8. Sensitivity of transitivity T and size of largest component (C_1) to coefficient of geometrically weighted edgewise shared partners (GWESP) effect ($\alpha = .69$). Observed values combined with parameter estimate in model 5 indicated by black squares.

$$\beta_3 \text{GWESP}(i, \alpha) + \beta_4 \text{CompleteTriads}_i,$$

where CompleteTriads_i is the number of complete triads in which actor i is involved; a complete triad is a triad (i, j, h) in which all six ties $i \rightleftharpoons j \rightleftharpoons h \rightleftharpoons i$ are present. These two effects combined appeared to give a clearly better fit to the local structure of this friendship network than the combination of the transitive triplets and three-cycles effects.

The results are shown in Table 5. The GWESP statistic, representing transitivity, has a positive and strongly significant effect; by contrast, the complete triads effect is negative, expressing that the other parameters by themselves would overpredict the number of complete triads, and a negative parameter is required as a counterbalance.

This leads to a clearly better fit of the macro-level features, where now for all the descriptives the observed value is even in the middle 90 percent region of the fitted distribution (see Figure 7). All of the effects in model 5 are needed to accomplish this: Dropping any of them moves the observed values for some of the statistics outside the middle 95 percent region.

Concluding, to obtain a good fit for the macro-level features of this data set, we need an actor model where the usual effects of reciprocation and dependence

on in- and out-degrees are included, in which transitive closure is represented in a somewhat complicated way, although still in line with other recent literature about modeling cross-sectionally observed networks, and which also includes gender homophily. Two proper macro features, the graph diameter and the median geodesic distance, proved to be most resistant to modeling, although with the final model they also are represented well. The other emergent “proper macro” features, the number of components and the extent of hierarchization, are well represented also by a simpler model, containing reciprocity, triadic, and degree-related effects. It should be noted here that friendship networks may show some tendency toward hierarchy but usually not a strong one, and also for this network hierarchy was not pronounced at all.

Sensitivity to the Transitivity-related Coefficients

As a further elaboration, we study for this friendship network how macro features of the generated networks depend on the coefficients expressing transitive closure in the network dynamics. This is interesting because there is one coefficient expressing transitive closure (the β_{GWESP} coefficient) that achieved good fit of the last model above, while another coefficient, also expressing transitive closure (the transitive triplets effect, denoted by β_{TT}) did not. A closer inspection therefore seems warranted.

So, first we study sensitivity to the GWESP effect, estimated in model 5 in Table 5 as 4.10; and after that, sensitivity to the transitive triplets effect. The network features whose sensitivity is investigated are the size of the largest component C_1 and the index of transitivity T , because those show most clearly the consequences of varying this parameter. Median geodesic distance would be a candidate for plotting too, but it is sensitive to component size as geodesic distances will be smaller in smaller components, and therefore we give primacy to C_1 .

It may be expected that the dependence of T will be an increasing function of β_{GWESP} , while the dependence of C_1 will be a decreasing function. The precise shape will depend on all the other parameters, and the parameters are related in ways that are hard to predict a priori. The question now is, how to control for other features of the network. Changing one parameter in these models while keeping the others fixed may lead to quite unrealistic networks, having implausible values for the average degrees, degree variances, and so on. Therefore, we control not by keeping other parameters fixed, but by keeping other aggregated micro features of the network fixed—on average in the distribution of networks (Steglich 2007). The statistical methodology, in particular the estimation by the method of estimating equation (5), is

helpful here. We shall use various different values of the GWESP coefficient β_{GWESP} within the context of model 5. All other parameters are estimated by the method of estimating equations, conditional on this prescribed value of β_{GWESP} . This ensures that the expected values of the aggregated micro-level statistics corresponding to the parameters in the model are equal to the observed values. These statistics are average degree, number of reciprocated ties, number of complete triads, variances and covariance of in- and out-degrees, number of actors with out-degree zero, and number of same-gender ties. In this way, the parameters, except for the coefficient of the GWESP effect, are automatically chosen in such a way that the average simulated values of these statistics remain constant, so that we are estimating the effect of coefficient β_{GWESP} while controlling for these eight features of the network. This is arguably a more meaningful way of studying sensitivity of macro features to micro-level parameters than straight simulation without prior empirical calibration of other model components.

The average values of the size of the largest component C_1 and the transitivity coefficient T are plotted in Figure 8 as a function of β_{GWESP} . The observed values and the parameter value $\hat{\beta}_{\text{GWESP}} = 4.1$ of Table 5 are indicated by black squares. It appears that T is a smoothly increasing function, as expected. The size of the largest component decreases very slowly until $\beta_{\text{GWESP}} = 6$, but then drops dramatically, implying that the networks generated have many, relatively small, connected components.

Recall that the distributions of networks considered are designed to be short-term equilibrium configurations, obtained after starting with the observed friendship network and giving each actor on average 20 opportunities of changing an outgoing tie. When the change process goes smoothly, this will result in a state of short-term dynamic equilibrium, and we made some checks that the results are the same as using 10 or 50 steps. For the higher values of β_{GWESP} , however, the drop in C_1 shows that there is quite an upheaval of the network structure, and it may be expected that for $\beta_{\text{GWESP}} > 6$ allowing an average of more than 20 steps would lead to a further breakup of the network, and thereby still lower average values of C_1 at the moment where some kind of equilibrium is reached. Space limitations keep us, however, from exploring this further in this article.

To get an impression of the difference between the results of the GWESP effect, with its reduction of the value of many intermediaries for the attractiveness of a tie, and the transitive triplets effect, which uses the number of intermediaries linearly, a similar sensitivity study was done but now for model 4, which is model 3 extended by the effect of gender homophily to make the model more closely comparable to model 5. The inclusion of

Figure 9. Sensitivity of size of largest component (C_1) and transitivity T to coefficient of transitive triplets effect. Observed values and parameter estimates indicated by black squares.

gender homophily reduces the parameter estimate for transitive triplets to $\beta_{TT} = 0.48$. Figure 9 gives the plot for the average values of the size of the largest component C_1 and the transitivity coefficient T , now as a function of the transitivity parameter in this model, where again all other parameters are estimated, which here means a control for the average degree, number of reciprocated ties, number of three cycles, variances and covariance of in- and out-degrees, number of actors with out-degree 0, and number of same-gender ties.

We see here a quite different picture compared to Figure 8. Note that in both pictures the parameter primarily affects the extent of transitivity in the network, and the estimate is in the middle of the studied range of parameters. The situation for $\beta_{TT} = 0$ in Figure 9 already differs from that for $\beta_{GWESP} = 0$ in Figure 8, yielding a higher average value for T , because the control for the number of complete triads in Figure 9 is replaced in Figure 8 by controlling for the number of three cycles. For values of β_{TT} from 0.0 to 0.40, the three-cycle effect is estimated as positive, thanks to the high reciprocation of ties leading to a value of T that is not very low, and thereby taking over the representation of transitivity. The horizontal (β) scales of both figures are comparable in the sense that in both cases the parameter ranges from 0 to a value where the average transitivity parameter is about 0.7. In Figure 9 also, the average largest component size drops for parameters somewhat higher

than the estimated value, but it drops only to 120 for $\beta_{TT} = 1$ whereas in Figure 8 it goes down to 38 for $\beta_{GWESP} = 10$. In the case of a high transitivity parameter, the largest component merely loses a few members, whereas for a high GWESP parameter the network breaks up in a number of much smaller components.

It can be concluded that, although Figures 4 and 7 already showed that models including the GWESP effect give a better representation of the global network features than analogous models with the transitive triplets effect, the sensitivity study with Figures 7 and 9 shows that for larger values of the corresponding parameters, the networks obtain completely different structures. A further analysis would be needed to obtain further insights into understanding why exactly this happens. For the case of our data set, however, the empirical analysis presented above implies that the GWESP effect with the sensitivity analysis of Figure 8 clearly is more in line with empirical reality.

Data Set 2: Advice Seeking in a Law Firm

As a quite different example, we now present results for an advice network. Due to the inequality in expertise and status that is associated with advice, advice networks tend to have quite different structures than friendship networks. The network considered here is an advice network in a law firm, studied by Lazega (2001). The actors are the 71 lawyers working in the firm. The question posed to them was as follows. "Here is the list of all the members of your Firm. Think back over the past year, consider all the lawyers in your Firm. To whom did you go for basic professional advice? For instance, you want to make sure that you are handling a case right, making a proper decision, and you want to consult someone whose professional opinions are in general of great value to you. By advice I do not mean simply technical advice." Thus, the tie $i \rightarrow j$ means that i states that he or she asks advice from j . As a covariate we use seniority, that is, rank order of entry into the firm.

Table 6 presents the descriptives. The average degree of 12.6 is rather high, which corresponds to other indicators of high connectedness: There is only one component, with a diameter of 3, and the median geodesic distance is 2. There is a large heterogeneity between individuals, reflected by scaled variances of the in-degrees and out-degrees being much larger than 1. The graph hierarchy of 0.16 seems a low number, but this should be related to the fact that the network is quite dense, implying that it is to be expected that most directed paths will be reciprocated by a path in the opposite direction. The lubness measure for hierarchy, 1, is the maximum possible value, in support of the hierarchical nature of advice asking.

Table 6. Descriptives for Advice Network Between Lazega’s Lawyers.

Number of actors, n	71
Average degree, $\bar{d}$	12.6
Proportion of ties being reciprocated, r	0.39
Largest component size, C_1	71
Number of components, N_C	1
Diameter, D_1	3
Median geodesic distance, $G_{0.5}$	2
Transitivity, T	0.44
Scaled in-degree variance, $\tilde{V}_{in}$	5.62
Scaled out-degree variance, $\tilde{V}_{out}$	4.10
Correlation in- and out-degrees, $r_{in,out}$	0.14
Graph hierarchy, H	0.16
Least upper boundedness, L	1.00

Table 7. Model I for Advice Relations: Only Reciprocity.

Effect	Parameters	(SE)
Out-degree	−0.983	(0.037)
Reciprocity	0.734	(0.075)

Figure 10. Distribution of macro features for advice network between Lazega’s lawyers, model I.

Table 8. Model 2 for Advice Relations: Reciprocity and Triadic Effects.

Effect	Parameters	(SE)
Out-degree	−1.738	(0.061)
Reciprocity	0.966	(0.104)
Transitive triplets	0.158	(0.008)
Three cycles	−0.149	(0.018)

Figure 11. Distribution of macro features for advice network between Lazega's lawyers, model 2.

A similar sequence of models is presented as for the first example. First a nearly trivial model, with only a reciprocity effect, is presented (Table 7). Figure 10 shows that most of the features have observed values very far from the simulated distribution. In all figures, features that have only one value, the same for all simulated graphs and for the observed network, are omitted.

Table 9. Model 3 for Advice Relations: Reciprocity, Triadic, and Degree-related Effects.

Effect	Parameters	(SE)
Out-degree	−0.926	(0.166)
Reciprocity	0.947	(0.109)
Transitive triplets	0.212	(0.016)
Three cycles	−0.095	(0.025)
In-degree—popularity	0.006	(0.006)
Out-degree—popularity	−0.060	(0.014)
Out-degree—activity	−0.025	(0.004)

Figure 12. Distribution of macro features for advice network between Lazega’s lawyers, model 3.

The second model (Table 8), which adds effects of transitive triplets and three cycles, already has a dramatically improved fit for the macro-level features, as is shown in Figure 11. However, for four of the features (hierarchy

Table 10. Model 4 for Advice Relations: With Seniority Effects.

Effect	Parameters	(SE)
Out-degree	-1.054	(0.146)
Reciprocity	0.968	(0.114)
Transitive triplets	0.192	(0.014)
Three cycles	-0.050	(0.029)
In-degree—popularity	0.007	(0.007)
Out-degree—popularity	-0.071	(0.013)
Out-degree—activity	-0.018	(0.004)
Seniority receiver	-0.008	(0.002)
Seniority sender	0.011	(0.003)
Seniority similarity	1.060	(0.172)
Seniority of indirect ties	-0.007	(0.003)

and characteristics of the degree distribution), the observed values still are very extreme in the fitted distribution.

The hierarchy in this network is not only local—which could possibly be represented by triadic effects—but also global: High-ranking individuals ask little advice and get many advice questions, and the converse holds for low-ranking individuals. This is reflected by the large values of the scaled variances of in- and out-degrees. The fact that nevertheless the correlation between in-degrees and out-degrees is weakly positive can be attributed to a center–periphery structure. This global hierarchy can be represented to some extent by the degree-related effects that are added in Table 9.

In Figure 12, we see that now all macro-level features are well fitted by the model, except for the hierarchy measure, for which the observed value still is much higher than the simulated distribution.

The variable that is a priori most adequate to represent hierarchy is seniority (rank order of entry in the firm; Lazega and van Duijn 1997). Of this variable, the usual effects of similarity, sender, and receiver are included, but the fit of the model was improved by also including the so-called *indirect ties effect*, defined as the sum of seniority values of advisors of advisors—those to whom, in the directed network, there is a geodesic distance of two:

$$\sum_j (1 - x_{ij}) (\max_h x_{ih} x_{hj}) v_j, \quad (13)$$

where v_j is the seniority of actor j (which in this formula is somebody at directed distance 2 from actor i). The results are in Table 10. For the

Figure 13. Distribution of macro features for advice network between Lazega's lawyers, model 4.

interpretation of all seniority effects, it should be noted that a low value of seniority means earlier entrance in the firm, and therefore higher status. The effect of seniority of advisors of advisors is significant ($t = -0.007/0.003 = -2.3, p < 0.05$) and negative; it is desirable to ask advice from individuals who themselves ask advice from high-ranking third persons, to whom one does not connect directly. The seniority effects contribute to the improvement of the fit of the hierarchy measure; however, the observed hierarchy still is quite extreme in the distribution, as can be seen in Figure 13.

As a final model component, the GWESP statistic, explained above, is added, to represent a looser transitivity structure than can be represented by the number of transitive triplets. In Table 11, it can be seen that this effect is significant ($t = 2.045/0.272 = 7.5, p < 0.001$); but—unlike the case of the friendship network treated above—this does not take away the significance of the triadic effects.

Table 11. Model 5 for Advice Relations: Including also the GWESP Representation of Transitive Closure.

Effect	Parameters	(SE)
Out-degree	-1.745	(0.168)
Reciprocity	1.054	(0.128)
Transitive triplets	0.121	(0.016)
Three cycles	-0.055	(0.028)
In-degree—popularity	-0.011	(0.008)
Out-degree—popularity	-0.062	(0.013)
Out-degree—activity	-0.021	(0.005)
GWESP ($\alpha = .69$)	2.045	(0.272)
Seniority receiver	-0.002	(0.003)
Seniority sender	0.009	(0.003)
Seniority similarity	1.110	(0.197)
Seniority of indirect ties	-0.004	(0.002)

Note: GWESP = geometrically weighted edgewise shared partners.

Figure 14 shows that here also, this effect gives the final push to make the fit adequate: The observed values for all features fall within the central 90 percent interval of the estimated distribution. While always adding more effects will yield a better representation of a given data set, it is interesting that in both examples the analysis led to a model having the GWESP term as the main representation of transitive closure, and this element was required in the model in order to represent the macro features of the network to a satisfactory extent. Whether this holds more generally will have to be seen in the analysis of further network data sets.

Discussion

This article has presented a way to synthesize micro–macro modeling with statistical goodness of fit considerations, in the context of modeling network structure by a statistical model defined by micro-level behavioral rules. Network models are especially suitable for such a synthesis because they have simultaneously a micro-level structure, in the form of network neighborhoods of individual nodes (“personal networks,” Degenne and Forsé 1999) or subgraph counts (“local structure,” Holland and Leinhardt 1976; “motifs,” Milo et al. 2002); and a macro-level structure (topology). Networks can be regarded as part of the social structures transforming micro-level behavior to macro-level properties. As descriptives of the macro structure, we distinguished between aggregated micro-

Figure 14. Distribution of macro features for advice network between Lazega's lawyers, model 5.

level characteristics, which can be defined on the basis of averaging or otherwise aggregating characteristics of local network neighborhoods of the actors, and proper macro characteristics, which cannot be in this way reduced to local characteristics. The first type is important in its own right, but for micro–macro studies especially the second type is relevant. From the point of view of micro–macro studies this synthesis adds the consideration of empirical data in all their realistic messiness, rather than only of “stylized facts,” as the link with empirical reality. From the point of view of statistical modeling of networks, this synthesis adds the consideration of emergent network-level properties as a major criterion for deciding about goodness of fit. Thus, the aim is to promote a more empirical orientation of micro–macro modeling, a better theoretical understanding of actor-based models for network dynamics, and in general a combination of theoretical and statistical approaches.

Summary of Empirical Illustrations

We studied the correspondence between micro-level model and macro-level features of the network by an intensive consideration of two example empirical networks. The first network considered was a network of friendship nominations between 129 actors; the second was an advice seeking network of 71 actors. We considered a set of 10 macro-level characteristics, of which 6 are proper macro features, expressing connectivity and hierarchy, while 4 are aggregations of node-level and triadic local properties.

For the friendship network, their observed values were represented rather well by a locally defined model containing reciprocity, triadic effects, and degree-related effects. The connectivity of the observed graph, as represented by diameter and median geodesic distance, was however still outside the range of the networks simulated according to this model, suggesting that these effects imply a too homogeneously distributed community structure, a “smaller world” than actually observed. To get a really close representation, it was needed to model transitive closure not by the traditional triadic effects—number of transitive triplets and number of three cycles in the personal network—but rather by the recently proposed effect of so-called *GWESP* (see Snijders et al. 2006; Hunter 2007), combined with the tendency not to create complete triads (in which all six possible links are present). Also the *GWESP* effect leads to transitive closure because indirect connections imply a higher attractiveness of creating and maintaining a direct link; but whereas in the transitive triplets effect the attractiveness of the tie depends linearly on the number of indirect connections it would close, for the *GWESP* effect this attractiveness still is increasing less than linearly in the number of indirect connections: The marginal effect of additional indirect connections is decreasing. This more involved representation of transitive closure, together with the effect of sex homophily, led to an excellent fit: For all 10 macro-level features, the observed values were within the middle 90 percent part of their distribution in the fitted distribution of networks.

For the advice network, a more extensive micro-level model was required to get a reasonable approximation for the observed macro-level features. As structural effects, this model again included reciprocity, triadic effects, and degree-related effects. In addition, it required effects of seniority, an individual attribute expressing hierarchy and therefore intimately connected to the advice process (Blau 1955; Lazega 2001). The structural measure of hierarchy introduced by Krackhardt (1994), however, was still not well represented by this model. In this case also the situation improved considerably by the introduction of the *GWESP* effect into the micro-level model, and brought the observed values of all macro-

level features within the middle 90 percent part of the distribution. The fact that a transitive closure effect (GWESP) improved the global fit of the model on the hierarchy measures illustrates the double meaning of transitive closure: It models group formation (as could be seen in the friendship data set) as well as hierarchy within these groups (in the advice network).

Secondary Methodological Innovation

For the elaboration of the above examples, two side issues were addressed, both of which are potentially useful instruments independent of this article.

First, it was studied how to construct an actor-based model for a single (cross-sectional) observation of these networks. This deviates from the usual purpose of actor-based network modeling, which is longitudinal. This deviation was followed because it leads to the consideration of the structure of a single network, and the population of networks from which it can be regarded as a representative element, without dependence on an earlier observation of the same network. This “short-term equilibrium” specification of the model is achieved by requiring that the observed network is both the center and the starting value of a longitudinal network evolution process in which the number of change opportunities per actor is fixed to some high (but not too high) value. Given the near-degeneracy problems that “long-term equilibrium” network models are facing (Snijders et al. 2006; Steglich 2006), and considering that in social science research, we hardly ever face long time frames in the sense of these models, the short-term equilibrium concept may turn out to be quite useful in empirical research.

Second, a method was elaborated for studying the sensitivity to a parameter in the agent-based model while controlling for other features of the generated networks. In order to keep model-derived networks close to empirical data, this was done by equating the observed values of aggregated micro descriptives to the corresponding averages in the network simulated from the model (see also Steglich 2007). The method was illustrated when studying the details of transitive closure in the friendship network. It turned out that size and number of components were much more “tunable” through the GWESP parameter (a specification of transitivity where the number of shared partners has a marginally decreasing impact) than through the transitive triplets parameter (a specification where the impact is linear). This kind of sensitivity study, which involves the estimation of “control” parameters to ensure good fit on potentially correlated outcome dimensions while studying variations in a (set of) parameter/parameters of interest, may be useful for avoiding model artifacts in agent-based modeling (Steglich 2007).

Further Outlook

The main purpose of this article was to demonstrate how micro–macro modeling and statistical modeling can be brought together in network analysis. Of course other, perhaps extended, sets of macro-level features can be used. The principle can be applied equally well to longitudinal analyses using the actor-based model; see Lospinoso (2012).

For the examples, the standard approach to model specification (Snijders, van de Bunt, et al. 2010), involving a micro-model based on structural characteristics of the personal network and a few attributes, yielded a reasonable fit to the macro-level features, which however was not excellent for the emergent macro features. The need for including the GWESP effect to achieve a good fit of the macro-level features is, in our view, quite interesting. This shows that for transitive closure, the additional value to the focal actor of more and more indirect connections (friends of friends, advisors of advisors) is decreasing. The same assumption was proposed for ERGMs (Snijders et al. 2006) to obtain models for which the estimation algorithm converges, and now appears to be useful also for actor-based modeling, to achieve a good fit of macro-level features. This is a new finding that should be explored also in longitudinal models and for other networks.

The study of sensitivity to parameters led to unexpected differences between models with the GWESP and triadic specifications of transitive closure, when the transitivity parameter is strong. This may be interesting to pursue further and may help in understanding the differences between these specifications of transitivity.

For agent-based modelers, the simulation-based partial fitting of a model to a selection of important macro features of the “aggregate micro” kind may be a tool to explore further in their own research. It will make sense whenever there is reason to assume that a multi-agent simulation model leads to data that are flagrantly off the mark on some such features, compared to empirical data sets. In such situations, the equation-based estimation of auxiliary parameters in the model, for which stochastic approximation may be used, can improve empirical validity on such dimensions, and will help avoid the generation of theory that is irrelevant to the empirical world.

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Notes

1. *R* is a general statistical software system (R Development Core Team 2011), and the incorporation of these methods in an *R* package allows combining them with many other statistical methods.
2. The name “method of moments” is also used; the name of “estimating equations” is more general, allowing also that function $u(x)$ in equation (5) depends on θ .
3. The computer implementation of the estimation algorithm in the RSiena package (Ripley et al. 2012) does not run when two subsequently observed networks are identical; therefore, one of them is slightly disturbed by changing one tie, which is such a little change that it does not meaningfully change the results.
4. If there is near degeneracy, this would affect what would be expected to happen for values of, say, $\lambda = 10,000$ or $\lambda = 1,000,000$ changes by all actors; this is left out of consideration.

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